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# Neutrino Production of Delta Resonance

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## Outline

- Motivation
- Single pion (resonance) production
  - Neutrinoproduction
  - Comparison with additional data:
    - Ratio of RES to QE
    - $\frac{d\sigma}{dQ^2}$(RES) and $\sigma_{tot}$(RES)
    - Electroproduction data
- Conclusions and Outlook

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Paschos, Sakuda, Yu, PRD69(2004)014013/hep-ph 0308130

Paschos, Sakuda, Schienbein, Yu, hep-ph/0408185

# Motivation

- Neutrino Hadron Interactions @ $E_\nu \simeq \mathcal{O}(1\ \text{GeV})$
  ['atmospheric neutrino energies']
- Discussion of form factors of $\Delta$ resonance

→ current/future LBL experiments: K2K, MiniBoone, MINOS, CERN-GS, JHF . . .

⇝ determine neutrino properties: main underlying motivation

## Neutrinoproduction of $\Delta$–resonances

The double-differential cross section

$$\frac{d\sigma}{dQ^2 dW^2} = \frac{G_F^2}{16\pi M_N^2} \sum_{i=1}^{3} [K_i \widetilde{W}_i]$$

- $G_F$: Fermi constant, $M_N (N = n, p)$: Nucleon mass
- Kinematic factors:
  $K_1(Q^2, E_\nu)$, $K_2(Q^2, E_\nu, W)$, $K_3(Q^2, E_\nu, W)$
- Structure functions:
  - $\widetilde{W}_1$, $\widetilde{W}_2$, $\widetilde{W}_3$ can be expressed in terms of helicity amplitudes
- Helicity amplitudes:
  - $T_{3/2,1/2}$, $T_C$, $U_{3/2,1/2}$, $U_C$, $U_D$
  - depend on $f(W)$ and form factors (*)
    $(C_i^V, C_i^A, i = 1, ..., 5)$
- Breit – Wigner factor $f(W)$ :

$$f(W) = \frac{\sqrt{\frac{\Gamma_\Delta(W)}{2\pi}}}{(W - M_\Delta) - 1/2 i \Gamma_\Delta(W)}$$

Schreiner von Hippel, NPB58(1973)333

(*) Paschos, Sakuda, Yu, PRD69(2004)014013

- Form Factors from Paschos et al. [1]
  - Vector Form Factors:

$$
\begin{aligned}
C_3^V(Q^2) &= \frac{C_3^V(0)}{\left[1+\frac{Q^2}{M_V{}^2}\right]^2}\left(\frac{1}{1+\frac{Q^2}{4\,M_V{}^2}}\right) \\
C_4^V(Q^2) &= -\frac{M_N}{W} C_3^V(Q^2) \\
C_5^V(Q^2) &= 0 \\
C_6^V(Q^2) &= 0 \quad \text{(CVC)}
\end{aligned}
$$

  - Axial Form Factors:

$$
\begin{aligned}
C_5^A(Q^2) &= \frac{C_5^A(0)}{\left[1+\frac{Q^2}{M_A{}^2}\right]^2}\left(\frac{1}{1+\frac{Q^2}{3\,M_A{}^2}}\right) \\
C_4^A(Q^2) &= -\frac{1}{4} C_5^A(Q^2) \qquad [2] \\
C_3^A(Q^2) &= 0
\end{aligned}
$$

Fit parameters: $C_3^V(0)$, $C_5^A(0)$, $M_V$, $M_A$

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Note:

- $d\sigma|_{C_6^A} \propto m_l^2 \rightarrow$ neglected
- $C_3^V(Q^2)$ and $C_5^A(Q^2)$ give dominant contribution to cross section
- All the form factors need to be multiplied by $\sqrt{3}$ due to $< \Delta^{++}|V_\alpha|p >= \sqrt{3} < \Delta^{+}|V_\alpha^{em}|p >$.

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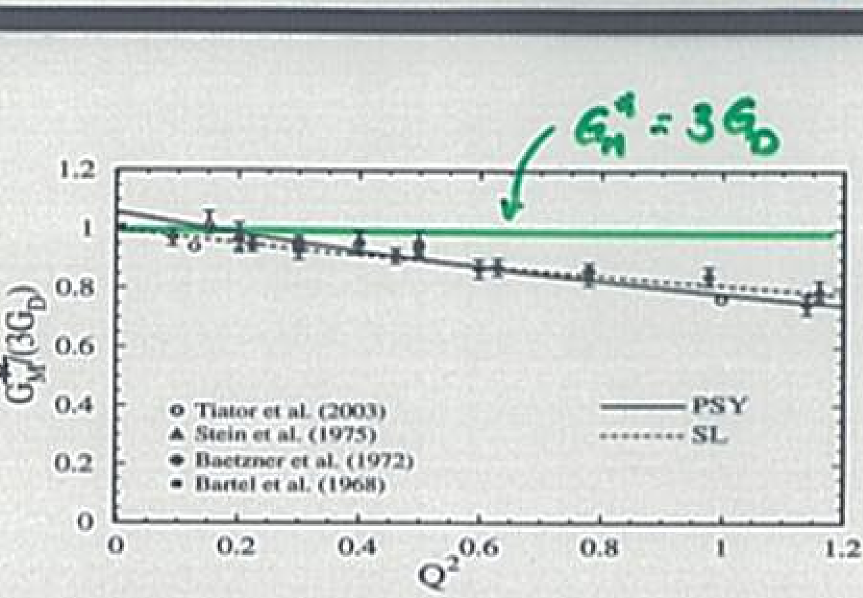
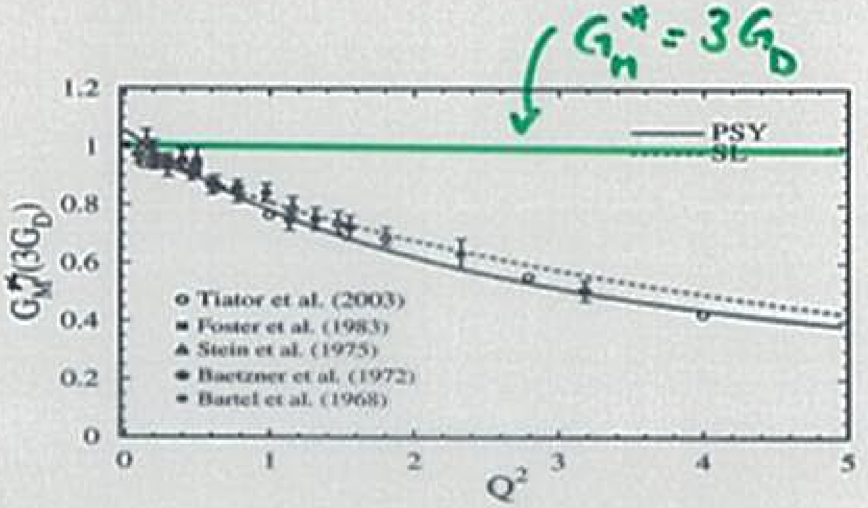
[1] Paschos, Sakuda, Yu, PRD69(2004)014013

[2] Schreiner, von Hippel, NPB58(1973)23

## Magnetic $N\Delta$ transition form factor $G_M^\star$ [1]




$$G_D = \frac{1}{\left(1+\frac{Q^2}{M_V^2}\right)^2}$$




- $N\Delta$ FF $G_M^\star$ drops faster than nucleon FF $G_D$ with increasing $Q^2 \Rightarrow \Delta$ resonance larger than nucleon
- PSY FF [2] and SL FF [3] agree well with data

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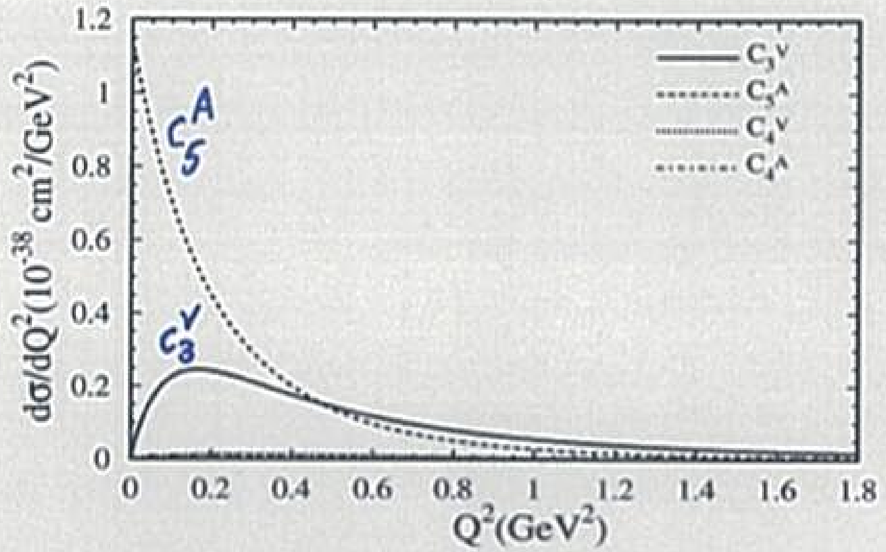
[1] L. Tiator, et al., Eur. Phys. J. A17 (2003) 357

[2] Paschos, Sakuda, Yu, PRD69(2004)014013

[3] T. Sato, D. Uno and T.-S. H. Lee, PRC67 (2003) 065201

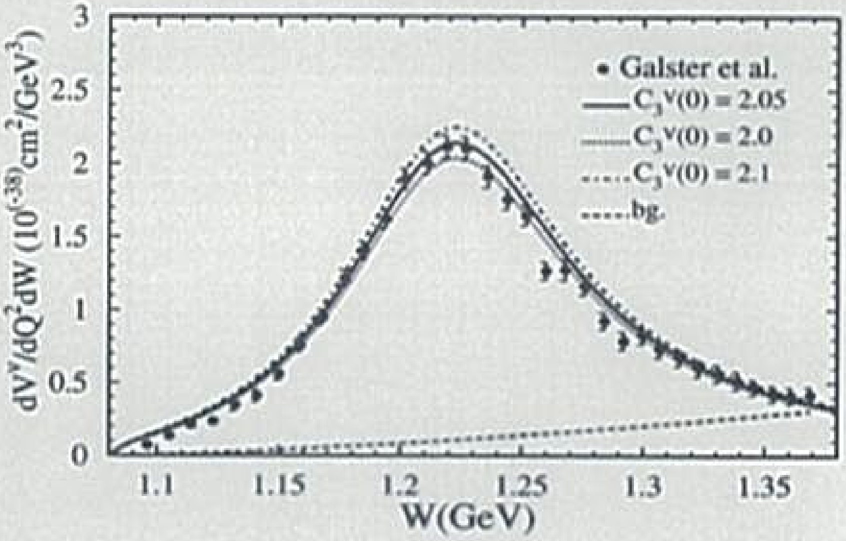
# Contribution of Form Factors to the Cross Section




- $C_3^V$ and $C_5^A$ dominate the cross section
- The contributions from $C_4^V$ and $C_4^A$ are very small

⇒ The excitation of the Δ-resonance is well described by <u>two form factors</u>

## Comparison of the Vector Contribution to $\nu + p \to \mu^- + p + \pi^+$ with renormalized Electroproduction Data




$$\frac{dV^\nu}{dQ^2dW} = \frac{G^2}{\pi}\ \frac{3}{8}\ \frac{Q^4}{\pi\alpha^2}\ \frac{d\sigma^{\mathrm{em},\,I=1}}{dQ^2dW}$$

- Electroproduction data for the reactions $e + p \to e + \left\{ \begin{array}{l} p\,\pi^0 \\ n\,\pi^+ \end{array} \right.$ [1]
- Subtraction of the $I = 1/2$ background (dashed line)
- Renormalize to the reaction $\nu + p \to \mu^- + p + \pi^+$

→ Comparison with the vector contribution to $\nu + p \to \mu^- + p + \pi^+$ denoted by $V^\nu$ ↔ $C_3^V(Q^2)$ part

- find good agreement using $C_3^V(0) = 2.05 \pm 0.05$

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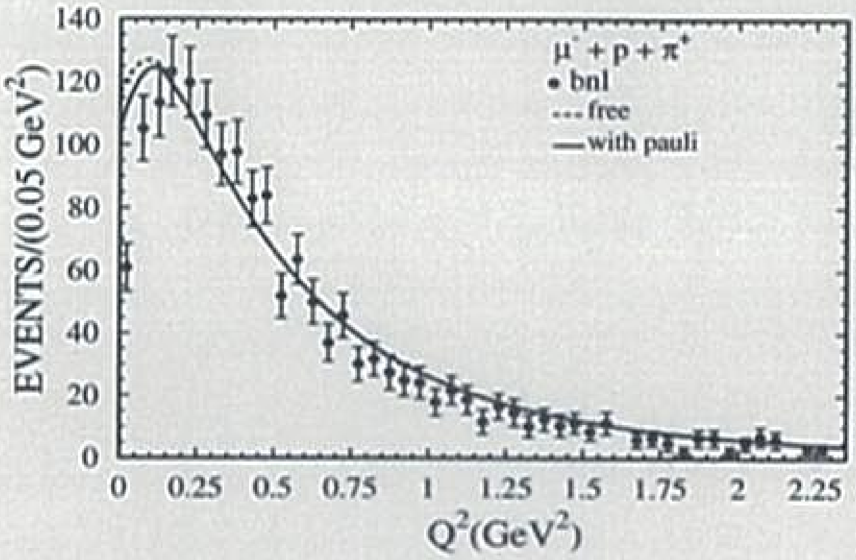
[1] Galster et al., PRD5(1972)519

## $Q^2$-spectrum of the process $\nu p \to \mu^- p \pi^+$ in comparison with BNL data [1]




- $0.5 < E_\nu < 6\,\mathrm{GeV}$, $W \leq 1.4\,\mathrm{GeV}$
- BNL data as histograms, averaged over neutrino flux, and with unspecified normalization

→ fix normalization such that area under theoretical curve and histogram is the same for $Q^2 \geq 0.2\,\mathrm{GeV}^2$

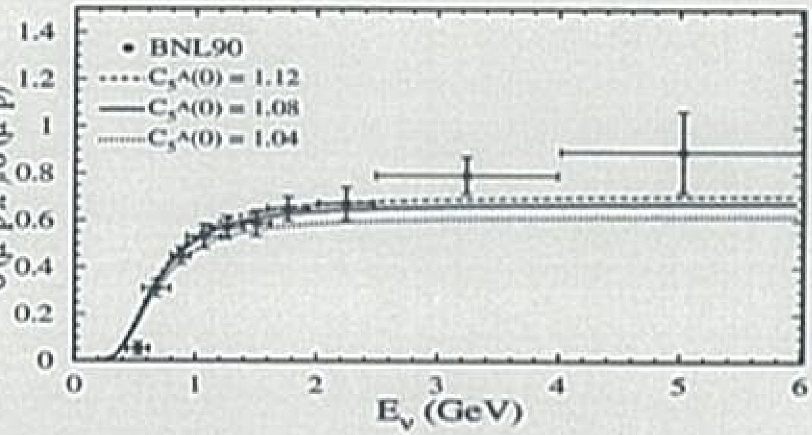
- Reduce nuclear effects: fit to all data with $Q^2 \geq 0.2\,\mathrm{GeV}^2$
- best fit result: $\chi^2/\mathrm{d.o.f.} = 1.04$ $[Q^2 \geq 0.2\,\mathrm{GeV}^2]$

paramters: $C_3^V(0) = 1.95\,, \quad C_5^A(0) = 1.2$
$M_V = 0.84\,\mathrm{GeV}\,, \quad M_A = 1.05\,\mathrm{GeV}$


[1] Kitagaki et al., PRD34(1986)2554

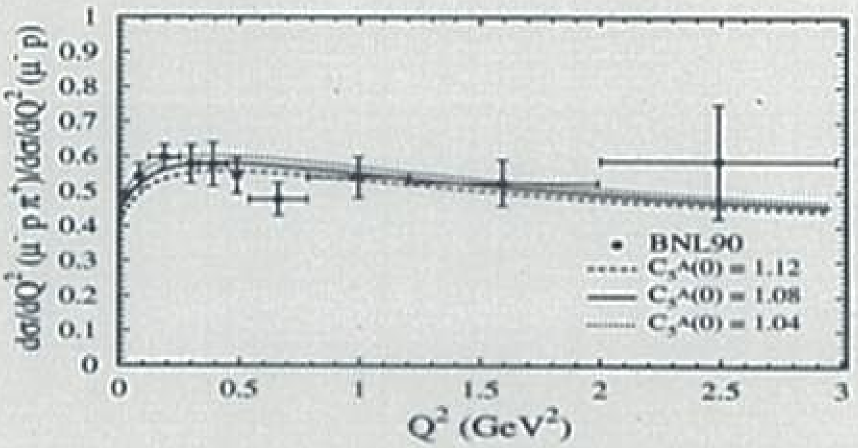
# Ratio of $\mathbf{RES}(\mu^- p\pi^+)$ to $\mathbf{QE}\ (\mu^- p)$ [1]

## Ratio of total cross sections




$\frac{\sigma(RES)}{\sigma(QE)}$

## Ratio of $Q^2$ distributions

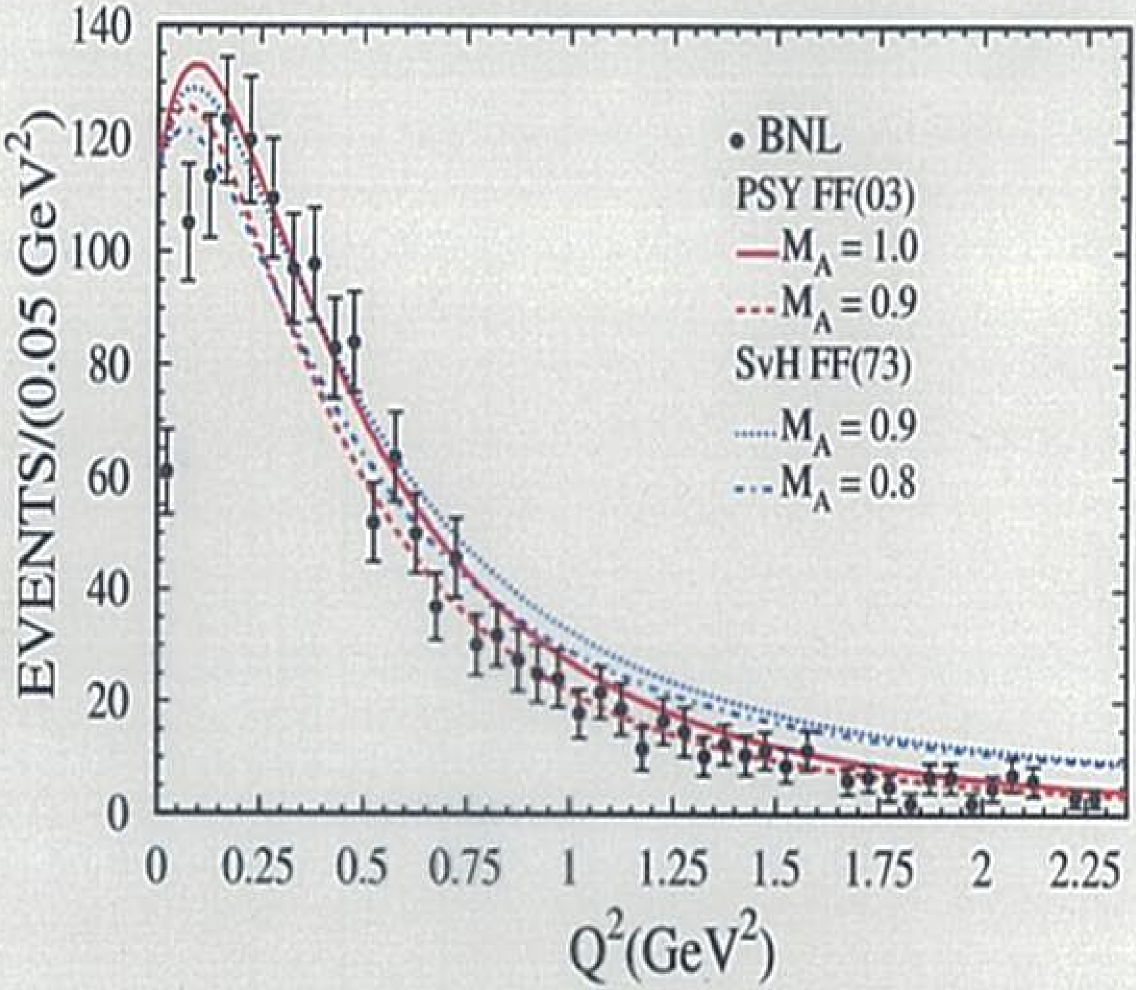



$\frac{\frac{d\sigma}{dQ^2}(RES)}{\frac{d\sigma}{dQ^2}(QE)}$

- Ratio $\equiv \frac{\mathbf{RES}(\mu^- p\pi^+)}{\mathbf{QE}(\mu^- p)}$ $(M_A = 1.05\ \mathrm{GeV})$
- Reduced flux- and experimental uncertainty
- find very good agreement
- Also good agreement at low $Q^2 < 0.2\ \mathrm{GeV}^2$


[1] T. Kitagaki et al., PRD42(1990)1331

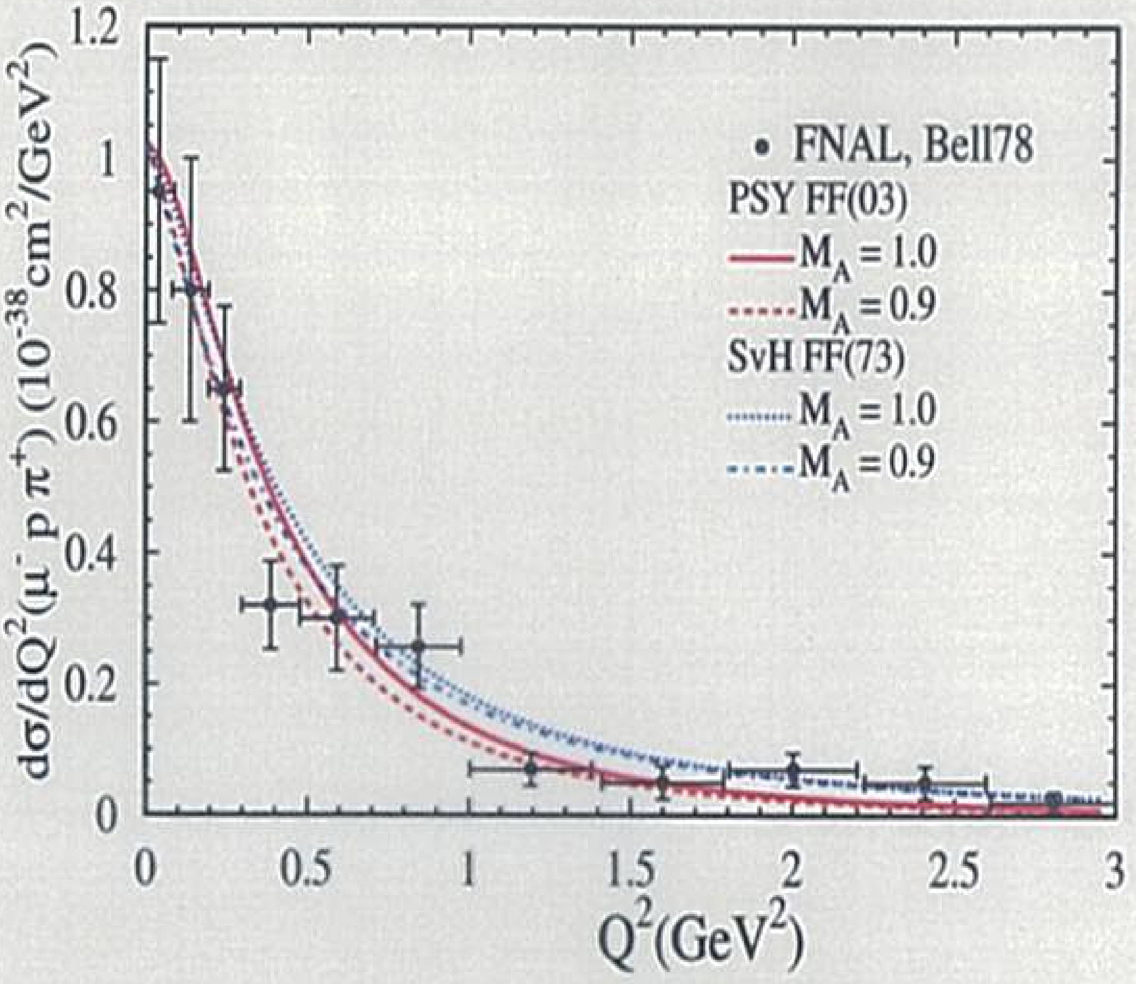
## $Q^2$-spectrum of the process $\nu p \to \mu^- p \pi^+$ in comparison with BNL data [1]




- $0.5 < E_\nu < 6\,\mathrm{GeV}$, $W \leq 1.4\,\mathrm{GeV}$
- $Q^2 \geq 1\,\mathrm{GeV}^2$:
  - PSY FF are better than SvH FF
  - Note: Form factors become insensitive to $M_A$ for large $Q^2$


[1] Kitagaki et al., PRD34(1986)2554

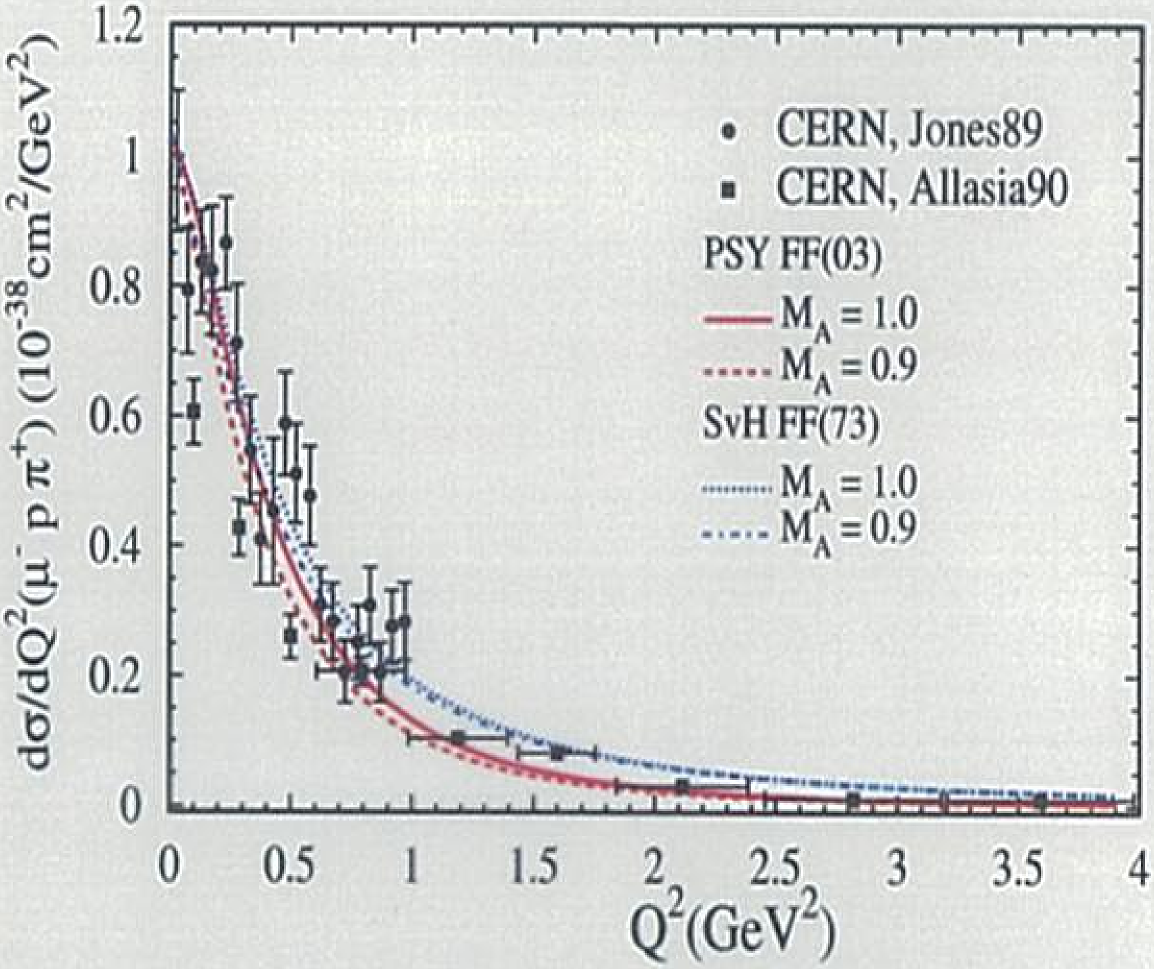
## $Q^2$-spectrum of the process $\nu p \to \mu^- p \pi^+$ in comparison with FNAL data [1]




- $< E_\nu > = 15\,\mathrm{GeV}$, $W \leq 1.4\,\mathrm{GeV}$
- $Q^2 \geq 1\,\mathrm{GeV}^2$:
  - PSY FF are lower than SvH FF
  - Note: Form factors become insensitive to $M_A$ for large $Q^2$


[1] J. Bell et al., PRL41(1978)1008

## $Q^2$-spectrum of the process $\nu p \to \mu^- p \pi^+$ in comparison with CERN data [1], [2]




- $10 < E_\nu < 200\,\mathrm{GeV}$, $W \leq 1.4\,\mathrm{GeV}$
- $Q^2 \geq 1\,\mathrm{GeV}^2$: PSY FF are better than SvH FF

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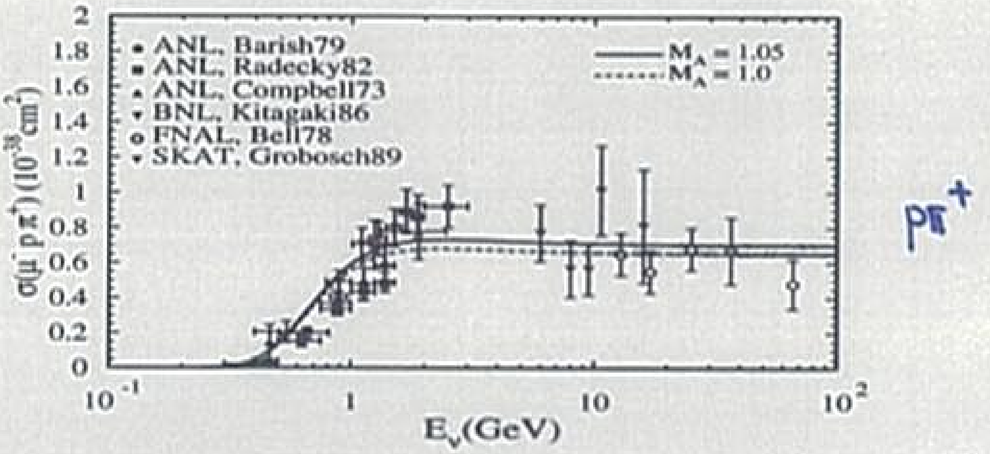
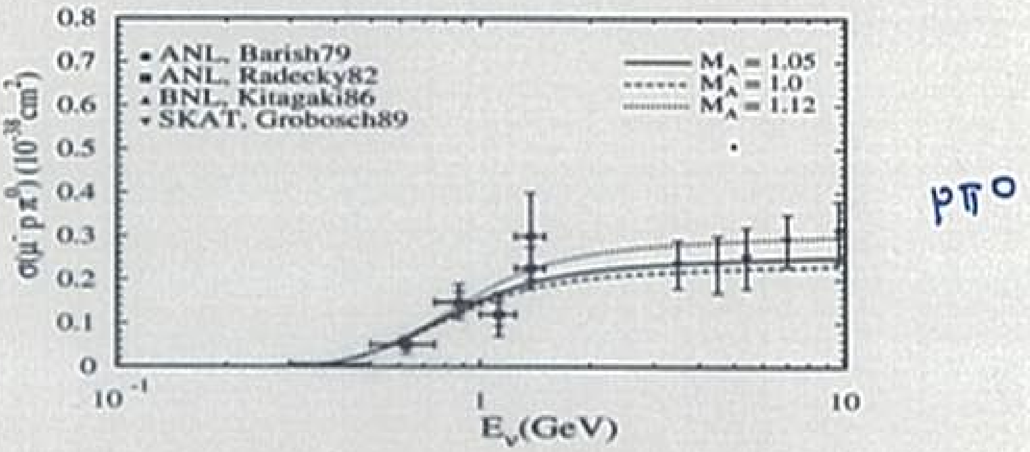
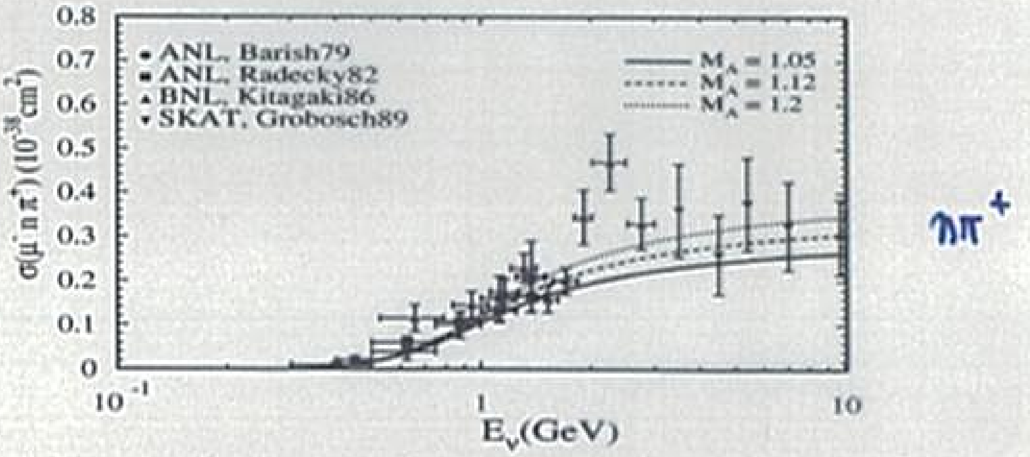
[1] G. T. Jones ZPC43(1989)527

[2] D. Allasia et al. NPB343(1990)285

# Charged current total cross sections of Delta Resoance in comparison with data




- good agreement

# Δ–resonance electroproduction

The fully-differential cross section [1]

$$\frac{d^4\sigma}{dQ^2 dW^2 d\Omega_\pi^*} = \frac{1}{\sqrt{4\pi}} \frac{d^2\sigma}{dQ^2 dW^2} \Big[ Y_0^0 - \frac{2}{\sqrt{5}} (\widetilde{\rho}_{(33)} - \frac{1}{2}) Y_2^0 + \frac{4}{\sqrt{10}} \widetilde{\rho}_{(31)} \mathrm{Re} Y_2^1 - \frac{4}{\sqrt{10}} \widetilde{\rho}_{(3\,-1)} \mathrm{Re} Y_2^2 \Big]$$

- $\frac{d^2\sigma}{dQ^2 dW^2} = N\, \Sigma_{i=1}^3 K_i \widetilde{W}_i$
- Spin density matrix elements: $\widetilde{\rho}_{(33)}$, $\widetilde{\rho}_{(31)}$, $\widetilde{\rho}_{(3\,-1)}$
- Spherical harmonic functions: $Y_l^m(\theta_\pi, \phi_\pi)$

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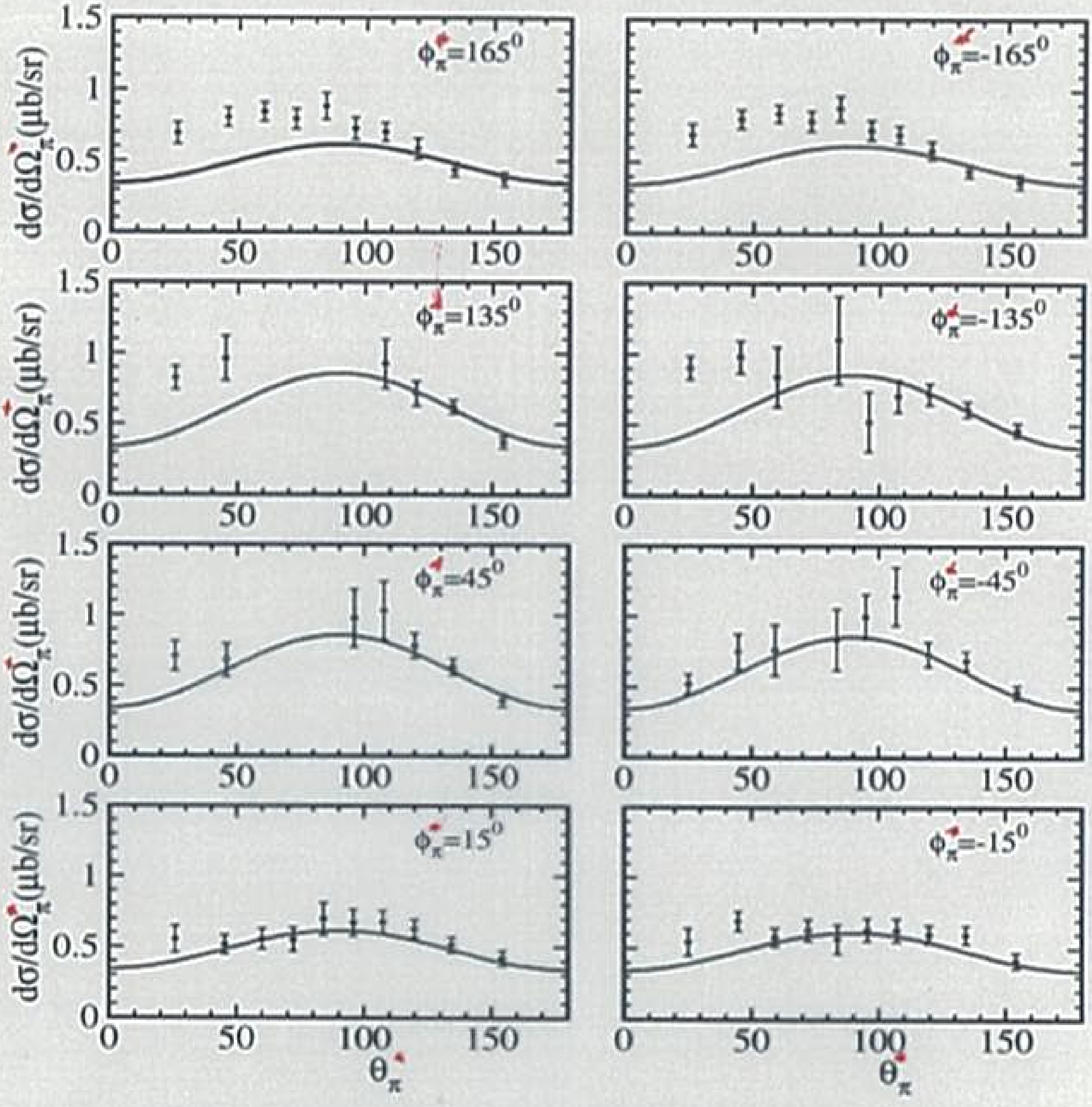
$$\Rightarrow \frac{d^4\sigma}{dQ^2 dW^2 d\Omega_\pi^*} = \frac{N}{4\pi} \Big[ \Sigma_{i=1}^3 K_i (\widetilde{W}_i - D_i \frac{(3\cos^2\theta_\pi^* - 1)}{2}) - 2\sqrt{3} \sin\theta_\pi^* \cos\theta_\pi^* \cos\phi_\pi^* (K_4 D_4 + K_5 D_5) - \sqrt{3} \sin^2\theta_\pi^* \cos 2\phi_\pi^* (K_6 D_6) \Big]$$

- Norm: $N = \frac{\pi\alpha^2}{2M_N^2 Q^2}$
- Pion polar and azimuthal angle: $\theta_\pi^*$, $\phi_\pi^*$
- Kinematic factors: $K_i$, $i = 1...6$
- Structure functions: $\widetilde{W}_i$, $D_i$

[1] Schreiner von Hippel, Nucl. Phys. **B58**, 333 (1973)

## Electroproduction data for the reaction $e + p \rightarrow e + p + \pi^0$ [1]




- $W = 1235\,\mathrm{GeV}$, $Q^2 = 2.8\,\mathrm{GeV}^2$, $E_e = 3.2\,\mathrm{GeV}$
- find good agreement for $\phi_\pi^* \leq 135°$

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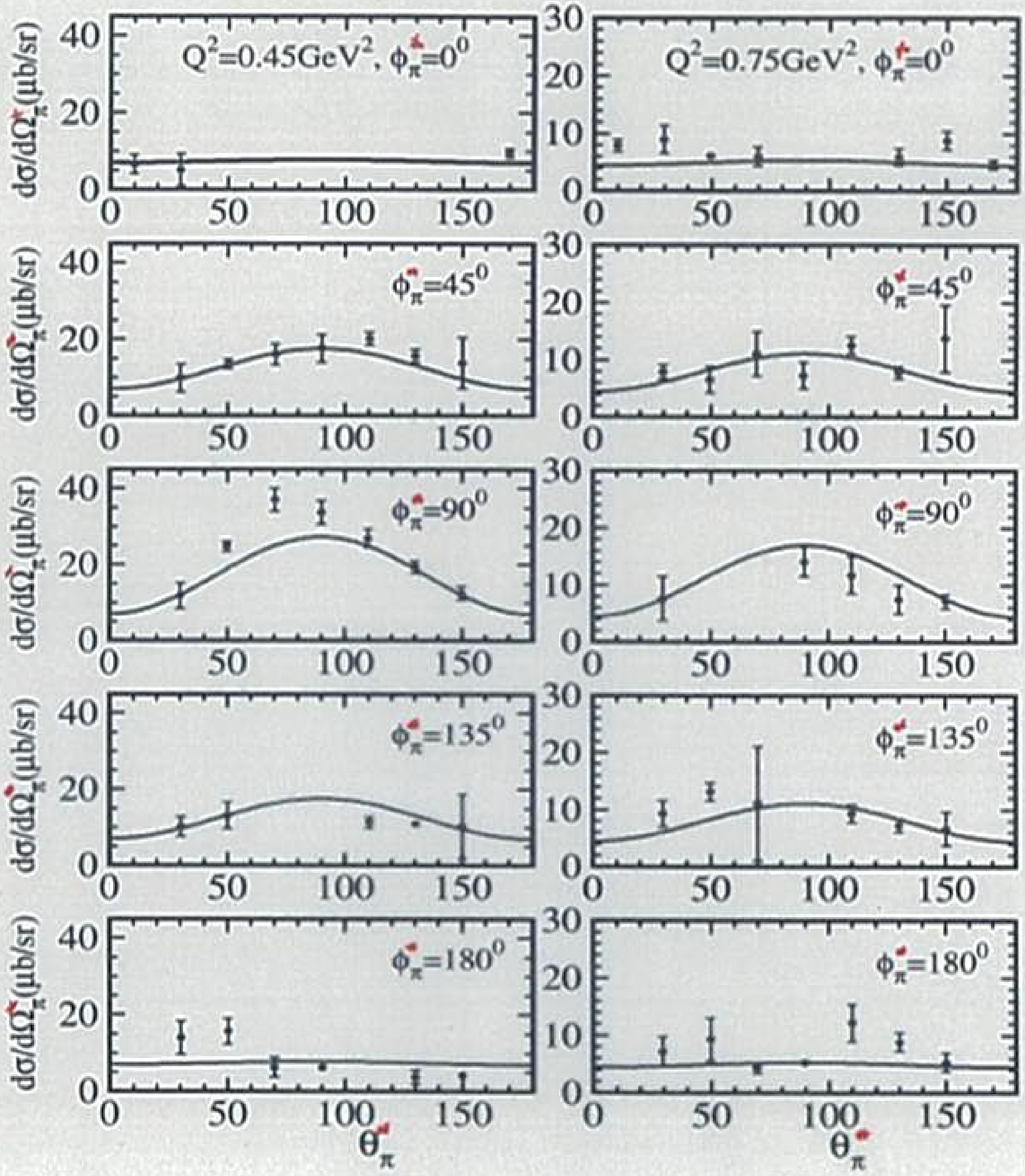
[1] V. V. Frolov et al, PRL82(1995) 45

## Electroproduction data for the reaction $e + p \to e + p + \pi^0$ [1]




- fixed: $W = 1232$ GeV
- find good agreement

[1] R. Siddle et al., NPB35(1971)93; J. C. Alder et al., NPB46(1972)573

# Comparison of PSY [1] and SL [2,3] models

| | PSY | SL |
|---|---|---|
| Form Factors | O | O |
| $\nu$- and electroprod. | O | O |
| $\pi$-angular distribution of electroprod. | O | O |
| ($\star$) Spin effect | X | O |
| Nuclear effects | O | X |

($\star$) Spin effect is small → PSY model works also well.

[1] Paschos, Sakuda, Yu, PRD69(2004)014013

[2] T. Sato and T.-S. H. Lee, PRC63 (2001) 055201

[3] T. Sato, D. Uno and T.-S. H. Lee, PRC67 (2003) 065201

# Conclusions

- Single pion neutrino production at low energies important to measure properties of neutrino oscillations
  - exist several more complicated calculations (more parameters, more complicated functional form of form factors)
    $O(20\%)$ theoretical uncertainty
  - our approach particularly simple (self-contained) $\rightarrow$ helpful for physical interpretation
- We have analysed the differential cross section $\frac{d\sigma}{dQ^2}$ in terms of two form factors
  - Two free parameters $M_V$, $M_A$, $(C_3^V(0), C_5^A(0))$
  - Good fit to BNL data for $Q^2 \geq 0.2\,\mathrm{GeV}^2$ $(\chi^2/d.o.f = 1.04)$
  - $Q^2 \leq 0.2\,\mathrm{GeV}^2$: theory is too high
    - Pauli correction is small
    - possible other medium effect?
    - Is resolved by taking ratio $\frac{\sigma(RES)}{\sigma(QE)}$

$\Rightarrow$ The neutrino production of $\Delta$-resonance can be described by two form factors: $C_3^V$, $C_5^A$
  - $C_3^V$ consistency check from electroproduction data
  - $C_5^A$ only from neutrino data

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- Comparison with existing neutrinoproduction data:
  - consistent
  - PSY FFs are better than SvH FFs.
- Comparison with existing electroproduction data:
  - reasonable agreement
  - need background from Isospin I =1/2 resonances and non resonant background

# Outlook

- include higher resonances and non resonant background
- more detailed comparison with electroproduction data and connection neutrinoproduction $\leftrightarrow$ electroproduction