How to test QE neutrino-nucleus interaction model using the data QE lepton-nucler interaction [1]

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[1] "Sub-dominant oscillation effects in atmospheric neutrino experiments", ICRR, 10^{th} December 2004

In GeV energy region the CC QE neutrino interaction with nuclei gives main contribution to detected muons.

- Precise description of neutrino-nucleus interaction is important in analysis data used to determine neutrino properties
- The inclusive QE process $^{16}O(\nu, \mu)$ is the main source of the '1 RING ' events in water Cherenkov detector
- The semi-inclusive $A(\nu, \mu p)B$ reactions may be observed in fine grained detector. The kinematics of an outgoig proton and lepton can be used to accurately reconstruction the energy of incomig neutrino.
- The $A(\nu, \mu p)B$ process can be used for evaluation of non-QE interactions contribution in the '1 RING' events

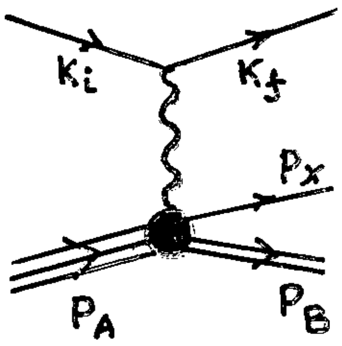
Monte-Carlo codes to simulate the response of neutrino detector are based on Fermi gas model.

- They take into account Fermi motion of the nucleons inside the nucleus and Pauli blocking effect
- The uncertainties of the existing data of QE neutrino-nucleus interaction don't allow estimate the accuracy of this model

Electron-nucleus scattering high-precision data and relativistic models.

- Fermi gas model neglect some important nuclear effects
- Relativistic Distorted Wave Impulse Approximation (RDWIA), Relativistic Optical Model Eikonal Approximation, and Relativistic Multiple Scattering Glauber Approximation (RMSGA) explain well experimental electron QE cross-section in range of nuclei from carbon to lead

Semi-inclusive Cross-Section



$$k_i = (\varepsilon_i, \bar{k}_i)$$

$$k_f = (\varepsilon_f, \bar{k}_f)$$

$$p_x = (\varepsilon_x, \bar{p}_x)$$

$$p_B = (\varepsilon_B, \bar{p}_B)$$

$$p_A = (m_A, 0)$$

$$q = k_i - k_f = (\omega, \bar{q}), \quad Q^2 = -q^2$$

Electron QE scattering

$$\frac{d^6\sigma}{d\varepsilon_f d\Omega_f d\varepsilon_x d\Omega_x} = \frac{p_x \varepsilon_x}{(2\pi)^3} \frac{\varepsilon_f}{\varepsilon_i} \frac{\alpha^2}{Q^4} L^{el}_{\mu\nu} (W^{\mu\nu})^{el}$$

Neutrino QE scattering

$$\frac{d^6\sigma}{d\varepsilon_f d\Omega_f d\varepsilon_x d\Omega_x} = \cos^2(\theta_C) G^2 \frac{p_x \varepsilon_x}{(2\pi)^5} \frac{|\bar{k}_f'|}{\varepsilon_i} L^{cc}_{\mu\nu} (W^{\mu\nu})^{cc}$$

G is Fermi constant and θ_C is Cabibbo angle

Hadron tensor

$$W^{el(cc)} = \frac{1}{8 m_A \varepsilon_x \varepsilon_B} \overline{\sum} \sum \int d^3\bar{p}_B |\langle B_f \bar{p}_x \mid J^{el(cc)}_\mu \mid A_i \rangle|^2$$
$$\delta(p_x + p_B - q - p_A)$$

Lepton tensor

$$L^{el}_{\mu\nu} = l^{\mu\nu}_s + h l^{\mu\nu}_A \qquad L^{cc}_{\mu\nu} = l^{\mu\nu}_s - l^{\mu\nu}_A$$

h is helicity of high energy electron and $l_S(l_A)$ is symmetrical (antisymmetrical) component

Five-folded cross-section

For exclusive reactions in which only a single discrete state or narrow resonance of target is exited ($^{16}O(e,e'p)^{15}N$ or $^{16}O(\nu,\mu p)^{15}O$) we use recoil factor

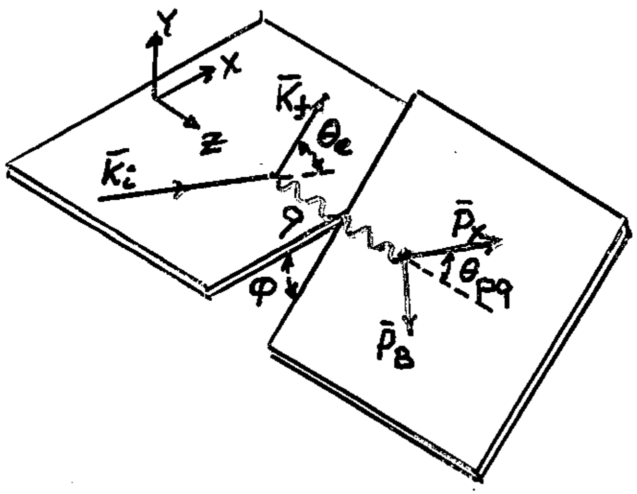
$$R = \int \delta(\varepsilon_x + \varepsilon_B - \omega - m_A) d\varepsilon_x = \left| 1 - \frac{\varepsilon_x(\bar{p}_x \bar{p}_B)}{\varepsilon_B p_x^2} \right|^{-1}$$

and obtain cross-section in form

$$\frac{d^5\sigma^{el}}{d\varepsilon_f d\Omega_f d\Omega_x} = R \frac{p_x \varepsilon_x}{(2\pi)^3} \frac{\varepsilon_f}{\varepsilon_i} \frac{\alpha^2}{Q^4} L^{el}_{\mu\nu} (W^{\mu\nu})^{el}$$

$$\frac{d^5\sigma^{cc}}{d\varepsilon_f d\Omega_f d\Omega_x} = R \cos^2(\theta_C) G^2 \frac{p_x \varepsilon_x}{(2\pi)^5} \frac{k_f}{\varepsilon_i} L^{cc}_{\mu\nu} (W^{\mu\nu})^{cc}$$

In electron scattering experiments the five-fold cross-section is measured holding variables (ω, q) and p_x fixed.



Y
X
Z
$\bar{K}_i$
$\bar{K}_f$
θ_e
q
φ
$\bar{p}_x$
θ_{pq}
$\bar{p}_B$

Inclusive cross-section

In reference frame

$$\hat{z} = \bar{q}/\mid \bar{q} \mid \quad \hat{y} = \bar{k}_i \times \bar{k}_f/\mid \bar{k}_i \times \bar{k}_f \mid$$

the inclusive cross-sections have form

$$\frac{d^2\sigma^{el}}{d\varepsilon_f d\Omega_f} = \sigma_M\{V_L R_L + V_T R_T\}$$

where the Mott cross-section

$$\sigma_M = \alpha^2 \cos^2(\theta/2)/4\varepsilon_i^2 \sin^4(\theta/2)$$

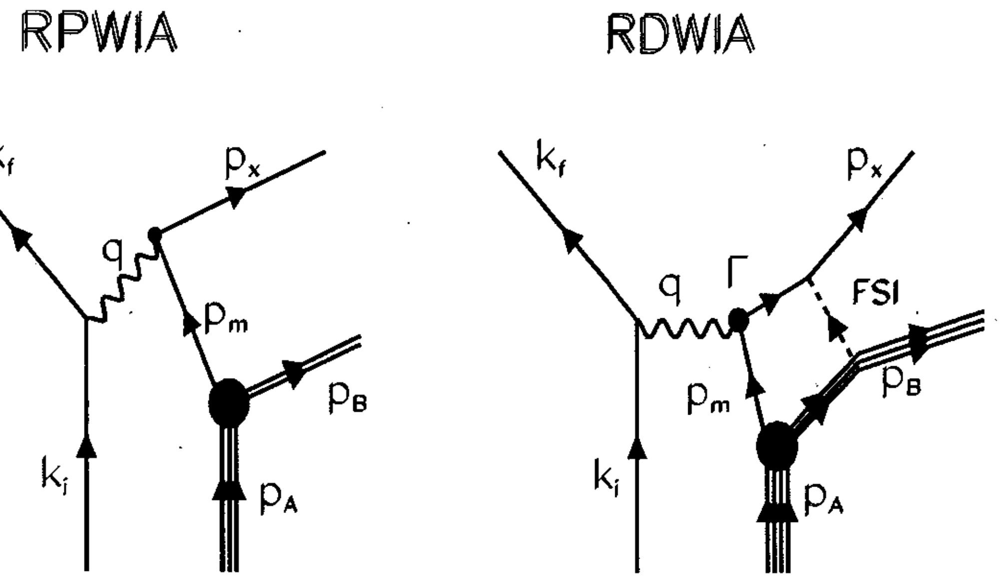
$$\frac{d^2\sigma^{cc}}{d\varepsilon_f d\Omega_f} = \cos^2(\theta_C)\frac{G^2}{4\pi^2} \mid \bar{k}_f \mid \varepsilon_f\{V_L R_L + V_T R_T + V_{zz} R_{zz} - V_{0z} R_{0z} \pm V_{xy} R_{xy}\}$$

Nuclear response functions

$$R_L = W^{00} \sim \langle J^0(J^0)^+\rangle$$
$$R_T = W^{xx} + W^{yy} \sim \langle J^x(J^x)^+\rangle + \langle J^y(J^y)^+\rangle$$
$$R_{0z} = W^{0z} + W^{z0} \sim \langle J^0(J^z)^+\rangle + \langle J^z(J^0)^+\rangle$$
$$R_{xy} = i(W^{yx} - W^{xy}) \sim 2Im(\langle J^y(J^x)^+\rangle)$$

The coefficients obtained from lepton tensor

$$V_L = 1 + \tilde{k}\cos(\theta)$$
$$V_T = 1 - \tilde{k}\cos(\theta) + \frac{\varepsilon_i \mid \bar{k}_f \mid \tilde{k}}{\mid \bar{q} \mid^2}\sin^2(\theta)$$



RPWIA
RDWIA
k_f
p_x
q
p_m
p_B
k_i
p_A
k_f
p_x
q
Γ
FSI
p_m
p_B
k_i
p_A

RDWIA

Bound nucleon wave functions

$$\Psi_i(r) = \begin{pmatrix} u_i \\ v_i \end{pmatrix}$$

are the solution of a Dirac equation, derived within relativistic mean field approach from a relativistic Lagrangian with scalar and vector potentials.

Ejectile nucleon wave functions

$$\Psi_f(r) = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$$

can be written as

$$\Psi_f(r) = \begin{pmatrix} \psi_+ \\ \frac{\bar{\sigma}\bar{p}}{E+M+S-V}\psi_+ \end{pmatrix}$$

where S and V are scalar and vector components of phenomenological optical potential of nuclei with energy E. The range applicability A=12÷208 and energies E=20÷1040 MeV.

ψ_+ can be related with a two-component wave function Φ by Darwin factor $D(r)$

$$\Psi_+ = D^{1/2}\Phi$$

$$D(r) = [E + M + S(r) - V(r)]/(E + M)$$

and Φ is solution of Schrodinger equation.

Vertex operators for nucleon current

The electromagnetic vertex function Γ^{μ} for a free nucleon can be represent by three different operator which a related by Gordon identity. Usually cc2 definition is used

$$\Gamma^{el}_{\mu} = F_{1}(Q^{2})\gamma_{\mu} + i\frac{k}{2M}F_{2}(Q^{2})\sigma_{\mu\nu}q^{\nu}$$

where F_1 and F_2 are Dirac and Pauli nucleon form-factors, k is anomalous magnetic moment, and $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_{\mu}\gamma_{\nu}]$.
Current conservation is restricted by replacing

$$J^{z} = \frac{\omega}{|\,\bar{q}\,|}J^{0}$$

The charge current vertex function

$$\Gamma^{cc}_{\mu} = F^{V}_{1}(Q^{2})\gamma_{\mu} + \frac{i}{2}F^{V}_{2}(Q^{2})\sigma_{\mu\nu}q^{\nu} - G_{A}(Q^{2})\gamma_{\mu}\gamma_{5} + F_{p}(Q^{2})q_{\mu}\gamma_{5}$$

where F^{V}_{1} and F^{V}_{2} are isovector Dirac and Pauli nucleon form-factors, G_A and P_p are axial and psevdoscalar form-factors and

$$G_{A} = \frac{g_{A}}{(1 + Q^{2}/M_{A}^{2})^{2}} \qquad F_{p} = \frac{2MG_{A}}{m_{\pi}^{2} + Q^{2}}$$

m_{π} and M_A are the pion and axial masses respectively, g_A=1.267

$$V_{zz} = 1 + \tilde{k}\cos(\theta) - 2\frac{\varepsilon_i \mid \bar{k}_f \mid \tilde{k}}{\mid \bar{q} \mid^2}\sin^2(\theta)$$

$$V_{0z} = \frac{\omega}{\mid \bar{q} \mid}(1 + \tilde{k}\cos(\theta)) + \frac{m_l^2}{\mid \bar{q} \mid \varepsilon_f}$$

$$V_{xy} = \frac{\varepsilon_i + \varepsilon_f}{\mid \bar{q} \mid}(1 - \tilde{k}\cos(\theta)) - \frac{m_l^2}{\mid \bar{q} \mid \varepsilon_f}$$

where $\tilde{k} = \mid \bar{k} \mid / \varepsilon_i$

Current operator in Impulse Approximation

The current matrix element J^μ calculate in framework of the Impulse Approximation. In this model it is assumed

- incident lepton interacts with only one nucleon
- nuclear current is the sum of single nucleon currents

$$J^\mu(\bar{q}) = \sum_{i=1}^{A} j_i^\mu(\bar{q})$$

- the states of the target and residual nuclei describe by independ particle model wave functions

Matrix elements of the nuclear current operator in relativistic model

$$J^\mu(\bar{q}) = \int d\bar{r}\bar{\Psi}_f(\bar{r})\Gamma^\mu(\bar{q})exp(\imath\bar{q}\bar{r})\Psi_i(\bar{r})$$

is calculated with relativistic wave functions for initial bound Ψ_i and final scattering $\bar{\Psi}_f$ states

PWIA

In Plane Wave Impulse Approximation (PWIA) the knockout A(e,e' p)B cross-section factorizes in form

$$\frac{d\sigma}{d\varepsilon_f d\Omega_f d\varepsilon_x d\Omega x} = \frac{\varepsilon_x p_x}{(2\pi)^3}\sigma_{eN} S(E_m, P_m)$$

where σ_{eN} is the cross-section for scattering of an electron by the off-shall nucleon.
$S(p_m, E_m)$ is probability that removal of a nucleon with momentum p_m will result in final state of residual nuclei with missing energy $E_m = M_B + M - M_A$

In PWIA the momentum distribution $\rho(p_m)$ for a fully occupied orbital with total angular momentum j is normalized to its occupancy

$$4\pi \int dp_m \rho_j(p_m) p_m^2 = 2j + 1$$

From measured cross-section the distorted momentum distribution

$$\rho^D(\bar{p}_m, \bar{p}_x) = \frac{(2\pi)^3}{\varepsilon_x p_x} \frac{1}{\sigma_{eN}} \frac{d\sigma}{d\varepsilon_f d\Omega_f d\varepsilon_x d\Omega_x}$$

can be obtained.

The main states and measured occupancy of ^{16}O
1p1/2 discrete state - E_m=12.13 MeV and $n_\alpha \approx$1.4
1p3/2 discrete state - E_m=18.45 MeV and $n_\alpha \approx$2.6
1s1/2 continue state - $E_m \approx$40.2 MeV and $n_\alpha \approx$2

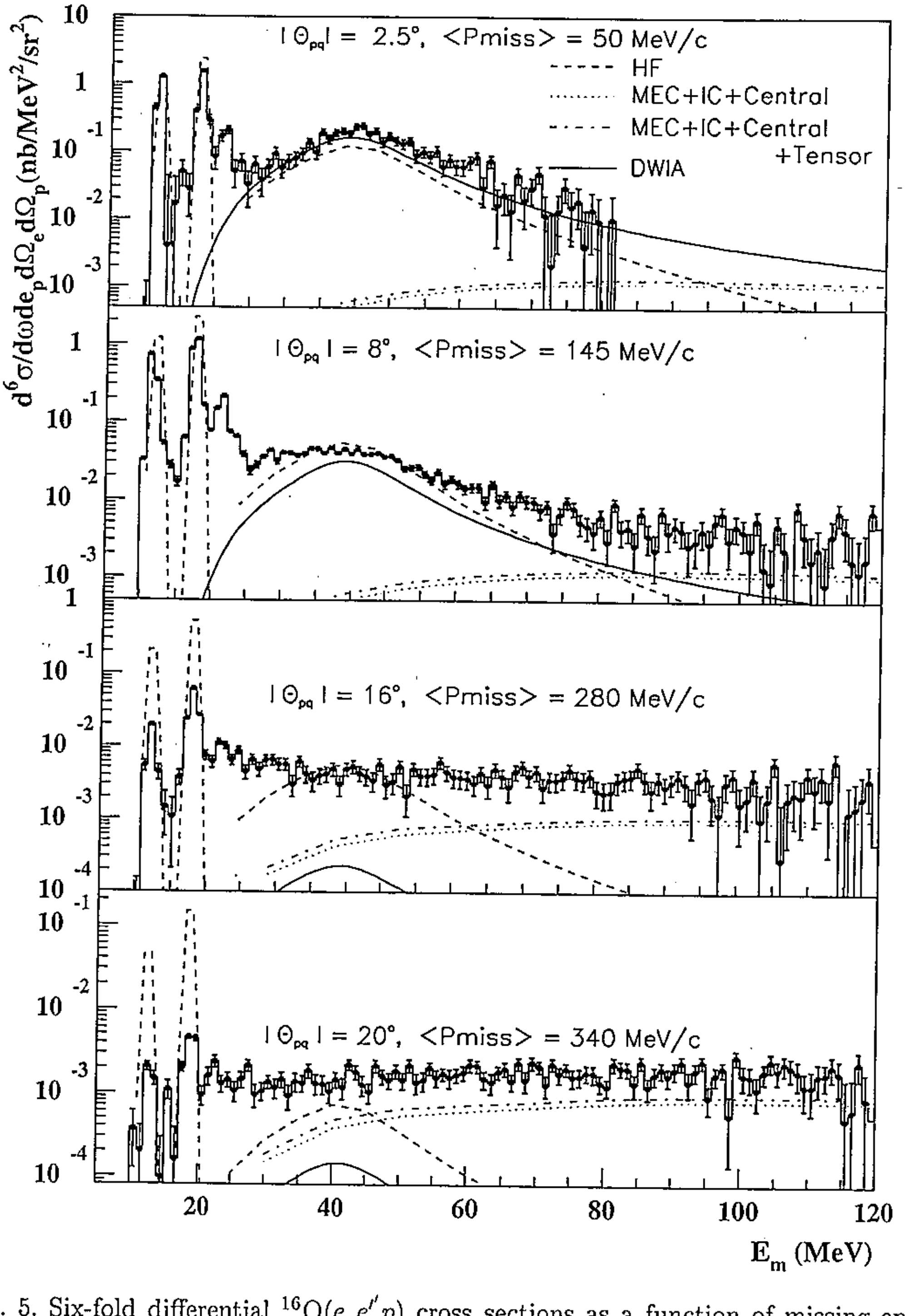


Fig. 5. Six-fold differential $^{16}O(e, e'p)$ cross sections as a function of missing energy for four different average values of missing momentum. The solid (dashed) lines represent the Kelly [9] (Ryckebusch *et al.* [16,24–27]) single nucleon knockout calculations folded with the Lorentzian paramterization of Mahaux [28]. The dotted Ryckebusch *et al.* calculation shows the $(e, e'pp)$ and $(e, e'pn)$ contributions due to pion-exchange currents, intermediate $\Delta(1232)$ creation, and central short-range correlations, while the dot-dashed calculation also includes tensor correlations. The prominence of the broad peak centered at $E_{\text{miss}} \approx 40$ MeV, which is primarily due to knockout from the $1s_{1/2}$-state, decreases with increasing p_{miss}. Figure courtesy N. Liyanage.

RESULT

Several codes have been developed based on RDWIA model and applied to QE electron scattering data.

- **Jeferson Lab. Experiment E89-003 $^{16}O(e, e'p)^{15}N$, E_e=2.442GeV, θ_e=23.36^o, Q^2=0.8GeV^2**
  (K.G.Fissum et al. nucl-ex/0401021)
  Calculations are given by J.Kelly (Phys.Rev.C60,044609, (1999)) **and J.M.Udias et al.** (Phys.Rev.Lett. 83,5451, (1999)).
  Normalization factors are 0.73 and 0.72 for 1p1/2 and 1p3/2 states

- **Kelly's result in comparison with different data sets**

- **Inclusive cross-sections calculated by A.Meucci et al.** (Phys.Rev. C67,054601,2003) **in comparison with $^{16}O(e, e')$ data of ADONE-Frascati experiment** (M.Anghinolfi et al. (Nucl.Phys. A602,405 (1996)) **and predictions of RFGM (P_F=225 MeV,ϵ_B=27 MeV)**

- **Meucci et al.** (Nucl.Phys. A739,277,2044)
  The flux-averaged (Los-Alamos neutrino spectrum) QE cross-section integrated over the muon energy gives 11.15 × 10^{-40} **cm^2 and experimental value of** (10.6±0.3±1.8)×10^{-40} **cm^2** (L.B.Auerbach et al. Phys.Rev. C66,015501 (2002)) ; $^{12}C(\nu_\mu, \mu)$

- **Meucci et al.** $R = \sigma_{DWIA}/\sigma_{PWIA} \approx 0.66$ **at** $E_\nu = 0.3$ **GeV and** $R \approx 0.81$ **at** $E_\nu = 1$ **GeV**
  Maieron et al. $R = \sigma_{DWIA}/\sigma_{RFGM} \approx 1$ **at** $E_\nu = 0.3$ **GeV and** $R \approx 0.85$ **at** $E_\nu = 1$ **GeV** (nucl-th/0407094)

- $\sigma_{Meucci}/\sigma_{Maieron} \approx 0.7$ **at** $E_\nu \geq 0.3$ **GeV**

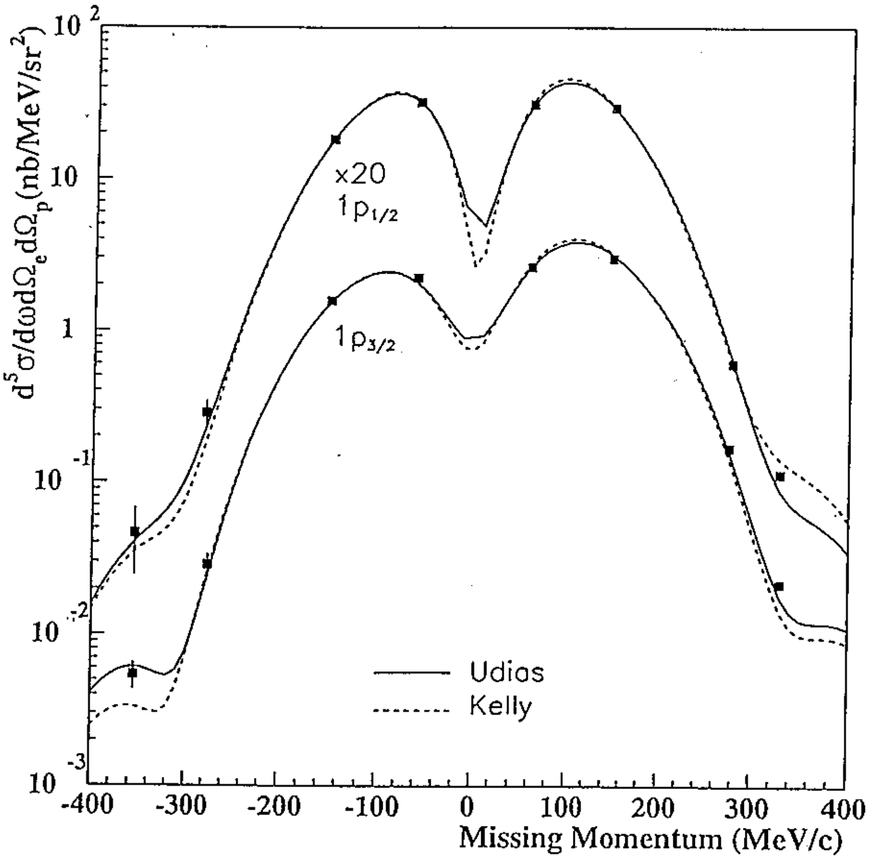
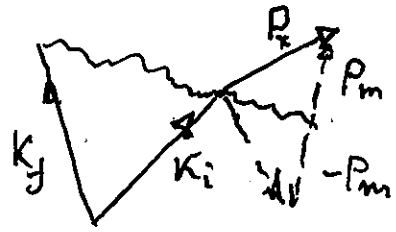


Fig. 2. Five-fold differential cross sections obtained in perpendicular kinematics for the knockout of $1p$-shell protons from ^{16}O as a function of missing momentum. Details pertaining to the calculation represented by the solid (dashed) line may be found in [20] ([9]). Figure courtesy J. Gao.



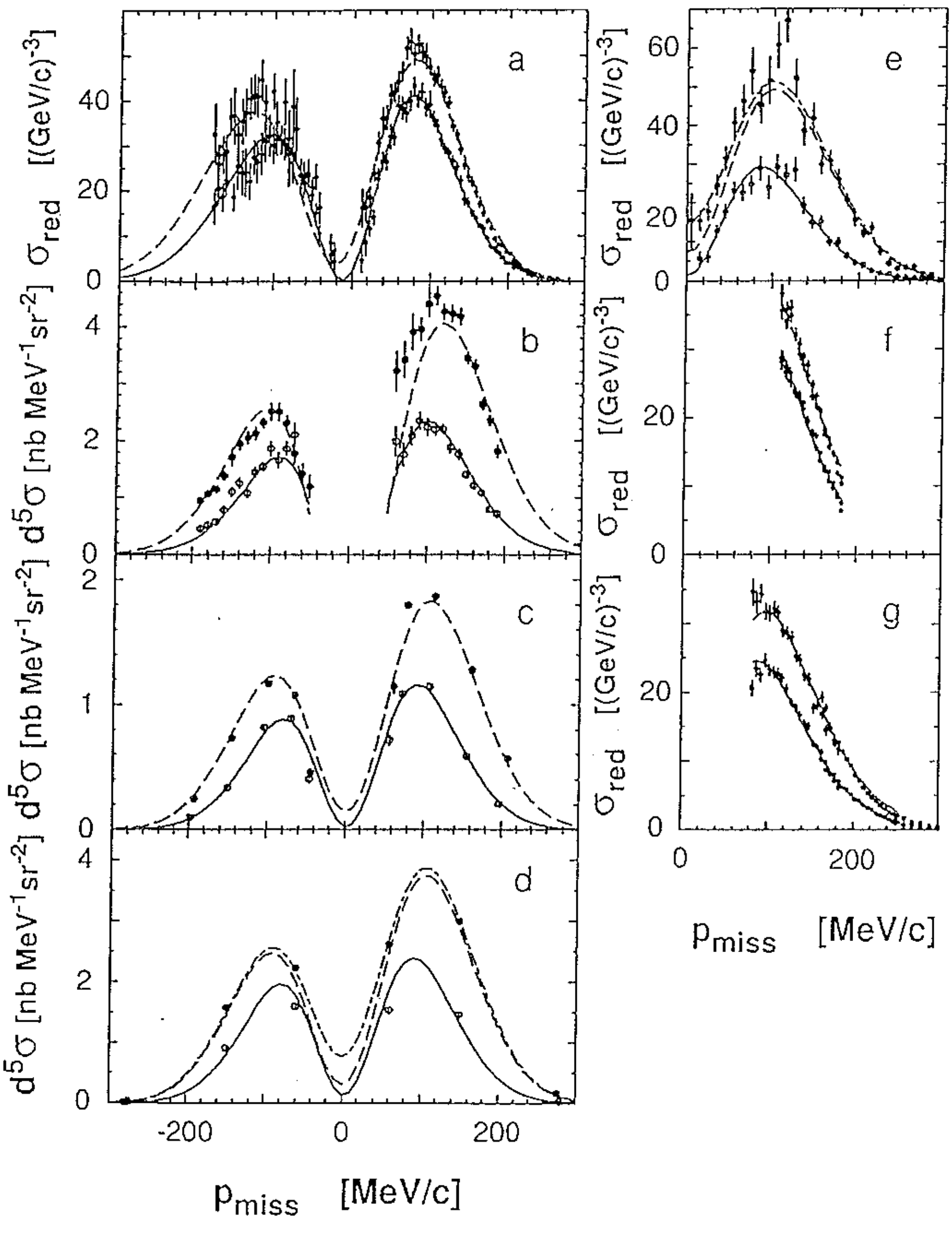
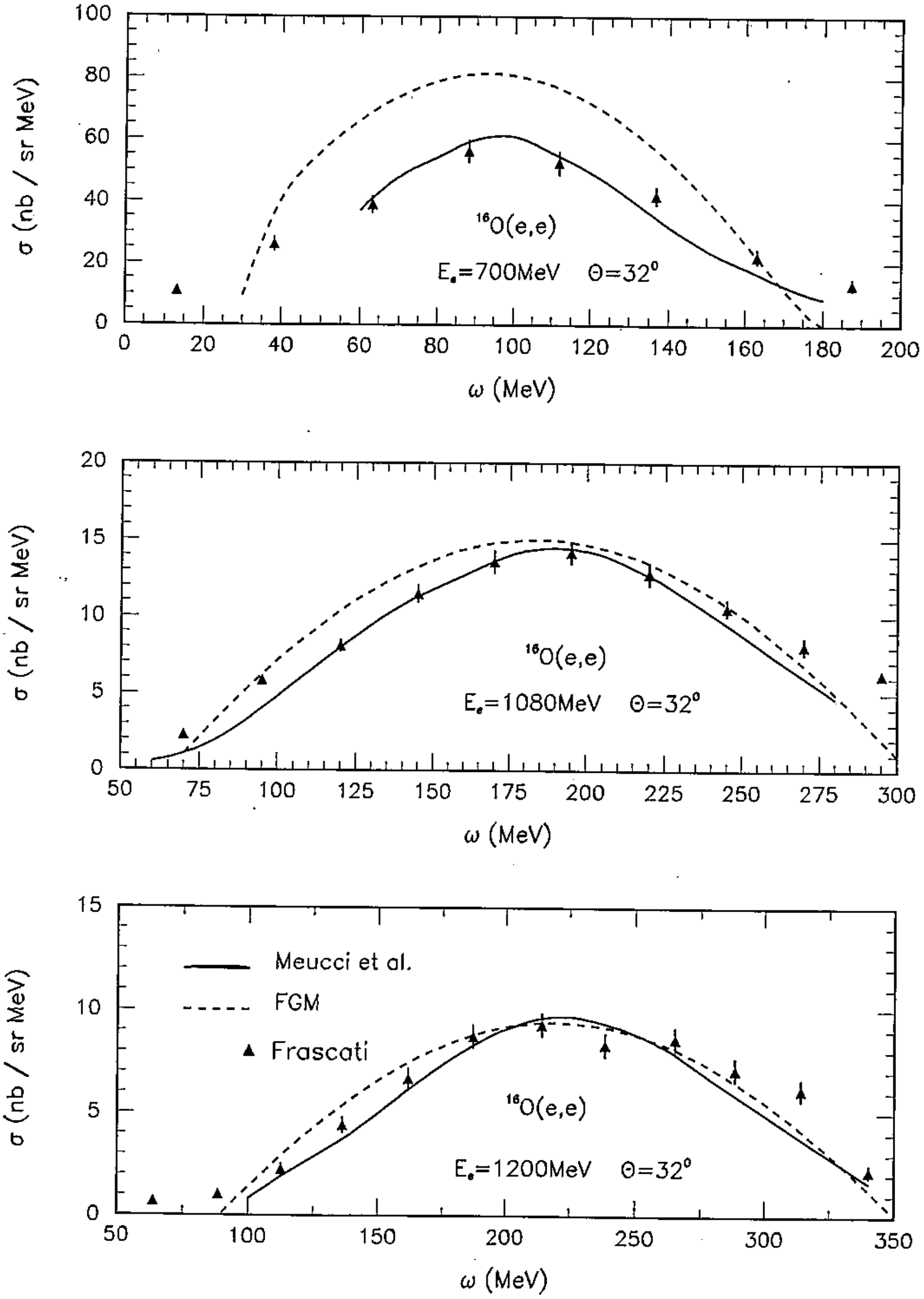
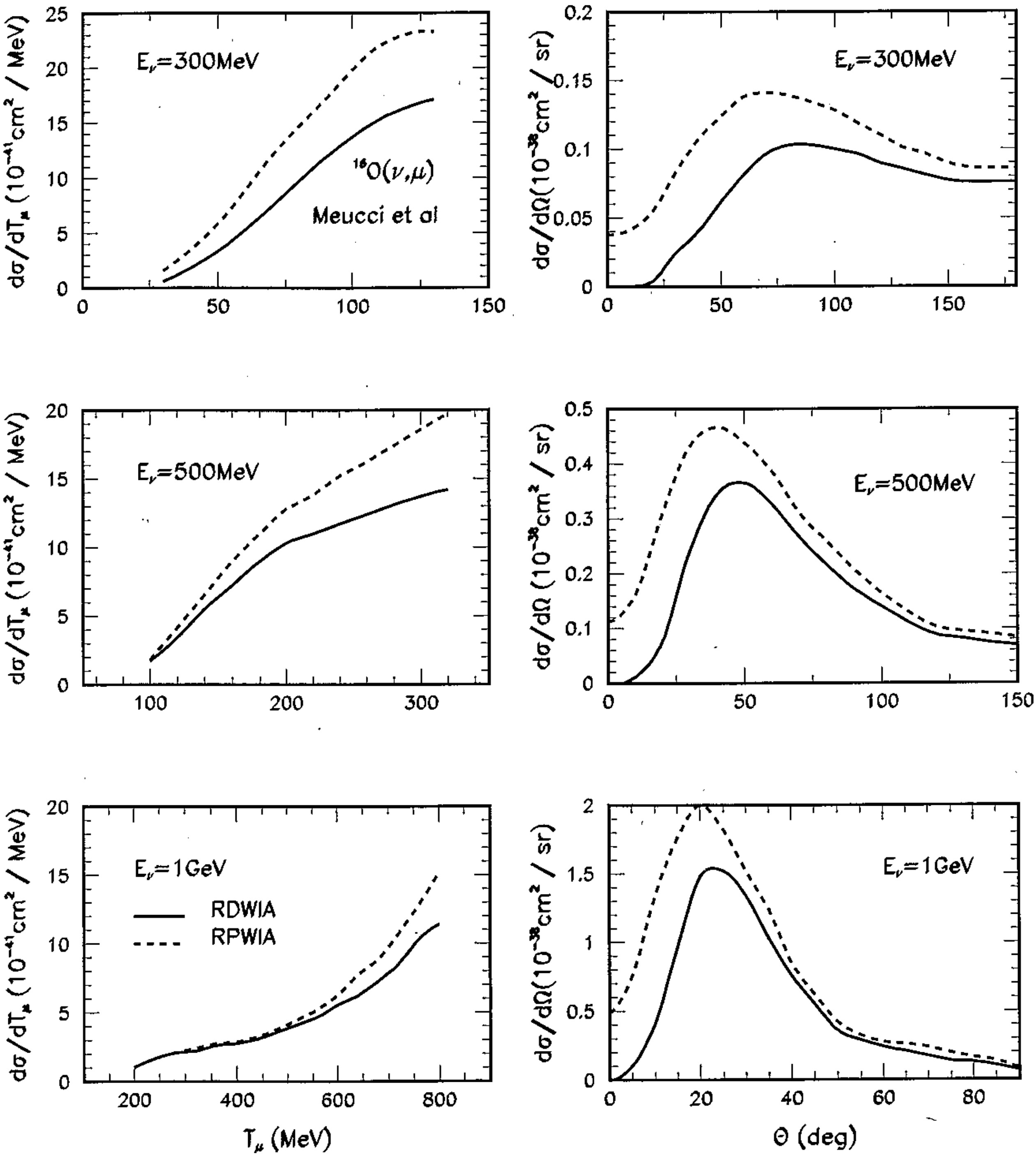


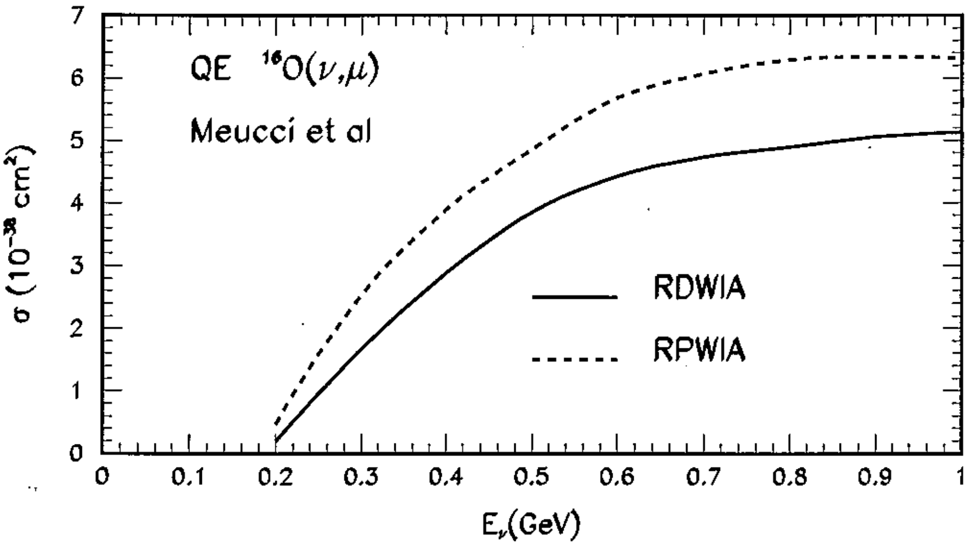
FIG. 24: Fits to various $^{16}O(e, e'p)$ data sets based on the HS bound-nucleon wave function and the EDAD1 optical potential. See Table IX for the key to the data-set labels. Open points and solid lines pertain to the $1p_{1/2}$-state, while solid points and dashed lines pertain to the $1p_{3/2}$-state. The dashed-dotted lines include the contributions of the positive parity $2s_{1/2}1d_{5/2}$-doublet to the $1p_{3/2}$-state. Panel (d) shows the data from this work.

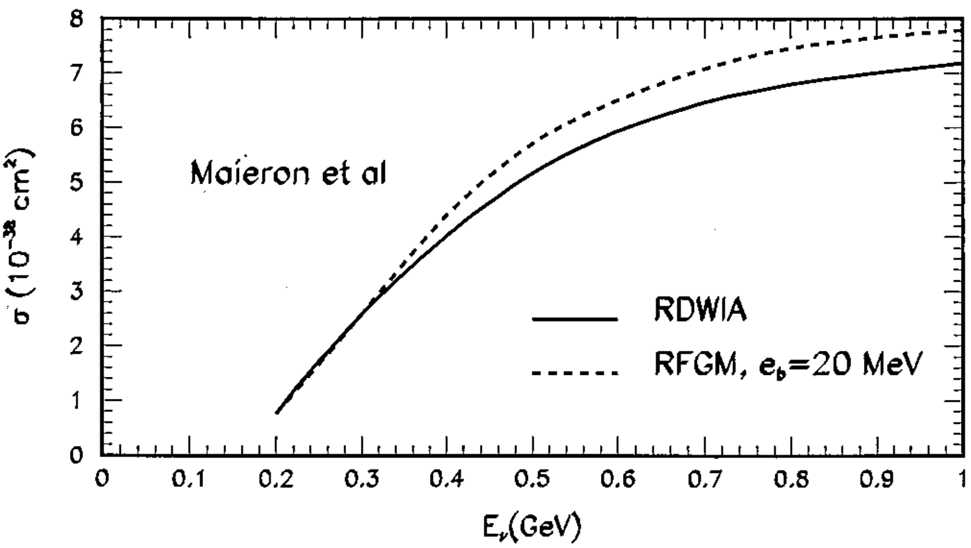


σ (nb / sr MeV)
^{16}O(e,e)
E_e=700MeV Θ=32°
ω (MeV)
σ (nb / sr MeV)
^{16}O(e,e)
E_e=1080MeV Θ=32°
ω (MeV)
σ (nb / sr MeV)
Meucci et al.
FGM
Frascati
^{16}O(e,e)
E_e=1200MeV Θ=32°
ω (MeV)



E_ν=300MeV
$^{16}O(\nu,\mu)$
Meucci et al
E_ν=500MeV
E_ν=1GeV
RDWIA
RPWIA
dσ/dT_μ (10^-41 cm² / MeV)
dσ/dΩ(10^-38 cm² / sr)
T_μ (MeV)
Θ (deg)


QE $^{16}O(\nu,\mu)$
Meucci et al
RDWIA
RPWIA
σ (10^{-38} cm^2)
E_ν(GeV)


Maieron et al
RDWIA
RFGM, e_b=20 MeV
σ (10^{-38} cm^2)
E_ν(GeV)

Conclusion

- RDWIA model explains well the the experimental electron-nuclei QE cross-sections.

- Comparison of RDWIA, RPWIA, and RFGM predictions with ^{16}O data shows that FSI effects are significant at electron energies less then 1 GeV.

- Two calculations of QE neutrino-nuclei cross-sections in the framework of RDWIA modelpredict different (%30) results. But RPWIA and RFGM results are always large.

- New careful calculations are required.