RCCN Workshop Dec. 9, 2004

Three-flavor subleading effects and systematic uncertainties in Super-Kamiokande

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Includes work in progress with
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Outline:

- Notation
- Archeo-phenomenology (10-20 years ago)
- Current phenomenology (2004)
- Features of 3ν effects including LMA
- Numerical expectations
- Is SK limited by systematics?
- Conclusions

Notation: Mass spectrum

normal

$$(m_1^2,\, m_2^2,\, m_3^2) = \left(-\frac{\delta m^2}{2},\, +\frac{\delta m^2}{2},\, \pm\Delta m^2\right) + m_\nu^2$$

inverted

$\Delta m^2 = \frac{1}{2}(\Delta m_{13}^2 + \Delta m_{23}^2)$

Notation: Mixing matrix (CP conserved for simplicity)

$\delta_{\mathrm{CP}} = 0$

$\delta_{\mathrm{CP}} = \pi$

$$U_{\alpha i} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & \pm s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}(\pm s_{13}) & +c_{12}c_{23} - s_{12}s_{23}(\pm s_{13}) & s_{23}c_{13} \\ +s_{12}s_{23} - c_{12}c_{23}(\pm s_{13}) & -c_{12}s_{23} - s_{12}c_{23}(\pm s_{13}) & c_{23}c_{13} \end{pmatrix}$$

$$[\ \delta_{\mathrm{CP}} = 0 \ \rightarrow\ \delta_{\mathrm{CP}} = \pi\ ] \qquad \Longleftrightarrow \qquad [\ +s_{13} \ \rightarrow\ -s_{13}\ ]$$

(it doesn't mean that s_{13} can be negative; it's just $\cos\delta_{\mathrm{CP}}$ which changes sign)

Notation: Interaction MSW term in matter

$$A(x) = 2\sqrt{2}\,G_F\,N_e(x)\,E$$

$N_e \sim 2\ \mathrm{mol/cm^3}$ (mantle)
$N_e \sim 5\ \mathrm{mol/cm^3}$ (core)

Matter effects typically (but not necessarily) relevant when: $A/(m_i^2 - m_j^2) \sim O(1)$

$$\frac{A}{\Delta m^2} \simeq 1.33 \left(\frac{2.3 \times 10^{-3}\ \mathrm{eV}^2}{\Delta m^2}\right)\left(\frac{E}{10\ \mathrm{GeV}}\right)\left(\frac{N_e}{2\ \mathrm{mol/cm^3}}\right)$$

O(1) in: MultiGeV data;
Stopping muons;
Tau-appearance sample

$$\frac{A}{\delta m^2} \simeq 3.82 \left(\frac{8.0 \times 10^{-5}\ \mathrm{eV}^2}{\delta m^2}\right)\left(\frac{E}{1\ \mathrm{GeV}}\right)\left(\frac{N_e}{2\ \mathrm{mol/cm^3}}\right)$$

O(1) in: SubGeV data;
Atm. bkgd to SN relic ν;
Low-energy K2K ν

Note: Relevant signs (leading to different physics)

$+\Delta m^2 \to -\Delta m^2$ Flips hierarchy

$+A \to -A$ Flips (anti)neutrinos

$+s_{13} \to -s_{13}$ Flips CP parity

Archeo-phenomenology: about 20 years ago

Interest in "solar corrections" to atmospheric neutrino oscillations, as well as in "atmospheric corrections" to solar neutrino oscillations, is rather old (80s).

E.g., "corrected" mass eigenvalues and mixing angles *(in constant matter)* can be found (with earlier refs.) in the classic review by Kuo and Pantaleone (1989):

$$\frac{\sin 2\theta_{13}}{\sin 2\widetilde{\theta}_{13}} = \sqrt{\left(\frac{A}{\Delta m^2 + \frac{\delta m^2}{2}\cos 2\theta_{12}} - \cos 2\theta_{13}\right)^2 + \sin^2 2\theta_{13}}$$

(1,2) effect on (1,3) mixing in matter

$$\frac{\sin 2\theta_{12}}{\sin 2\widetilde{\theta}_{12}} \simeq \sqrt{\left(\frac{A\cos^2\theta_{13}}{\delta m^2} - \cos 2\theta_{12}\right)^2 + \sin^2 2\theta_{12}}$$

(1,3) effect on (1,2) mixing in matter

$$\widetilde{m}_i^2 \simeq \widetilde{m}_i^2(m_j^2, \theta_{12}, \theta_{13}, A)$$

(1,2)-(1,3) effects on squared masses

Archeo-phenomenology: about 10 years ago (pre-SK, pre-CHOOZ)

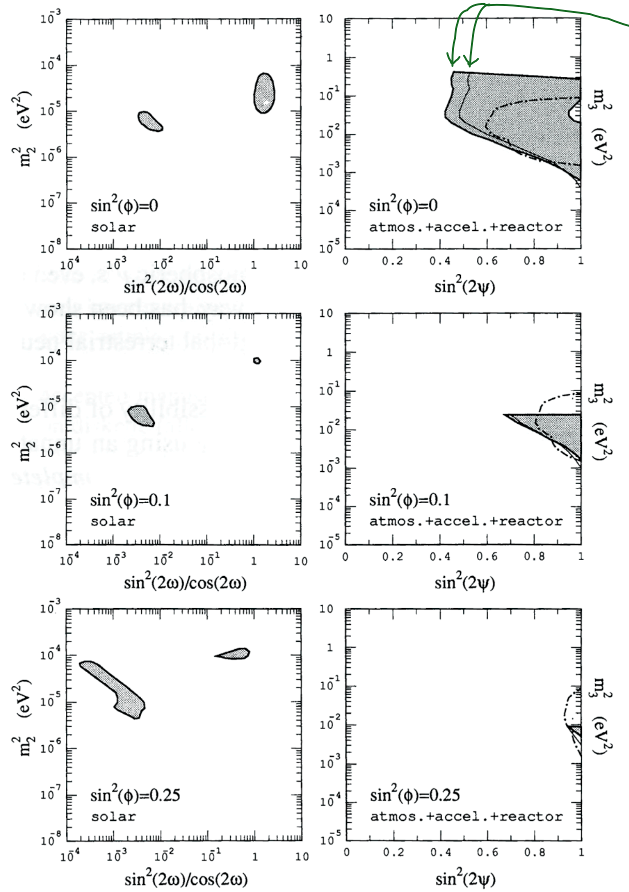


Fig. 8. Comparison of the global solutions obtained in our analysis at the first step (thin solid lines) and at the second step (thick solid lines, grey areas). The difference is modest for atmospheric neutrinos, and graphically unobservable for solar neutrinos. This proves the stability and consistency of our hierarchical scenario. Also shown are the boundaries of the regions preferred by the multi-GeV Kamiokande data (dash-dotted lines). See the text for details.

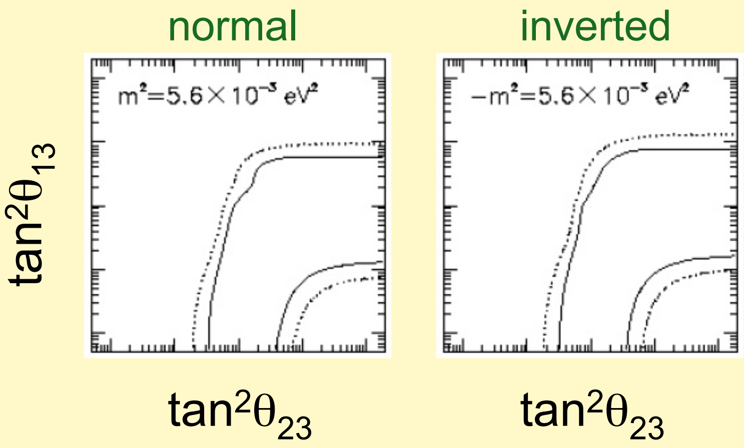
Fogli, Lisi, Montanino, Astrop. Phys. (1995): 3ν analysis of solar, atm., reac., and accel. data, at and beyond 0th order in $\delta m^2/\Delta m^2$

Included effect of Δm^2 as low as 10^{-3} eV^2 on solar neutrinos. Results: no observable change on solar ν solutions. Observable effects only from nonzero θ_{13} mixing. (Still true today).

Included effect of δm^2 as high as 10^{-4} eV^2 (LMA) on atm. neutrinos, through full 3ν numerical evolution in five Earth shells. **Results: small but observable changes on atmospheric ν solution, even at θ_{13}=0.** (Still true today).

In particular, note atm. solution shifted to smaller mixing by δm^2>0 at θ_{13}=0.

Archeo-phenomenology: about 10 years ago (pre-SK, pre-CHOOZ)



Fogli, Lisi, Montanino, Scioscia,hep-ph/9607251
(analysis at negligible δm^2, e.g., SMA):

- Full mixing space: two octants and log $\tan^2\theta$
- Effects of $\theta_{13}\neq0$ and of **hierarchy** on atm. ν

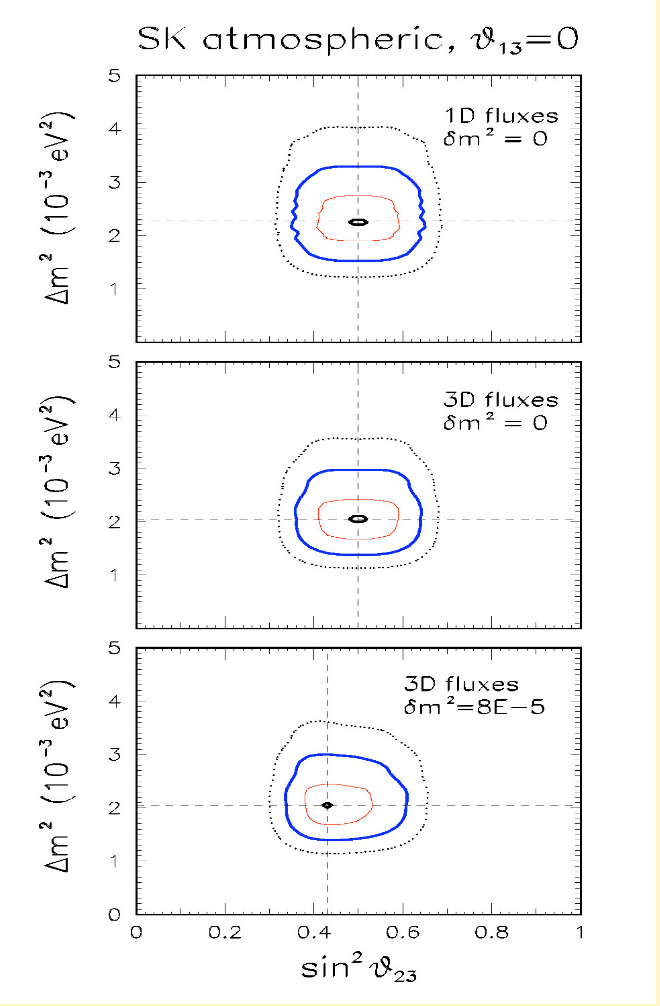
Many numerical and/or analytical studies of subleading three-neutrino effects by different research groups in the last decade, and especially after release of first SK atmospheric data and after confirmation of LMA solution.

Effects well understood analitically for constant matter (in the general case), and for mantle-core step-like matter (at least in the limit of $\delta m^2=0$).

Numerical calculations unavoidable for accurate estimates and data analyses.

Current phenomenology (2004)

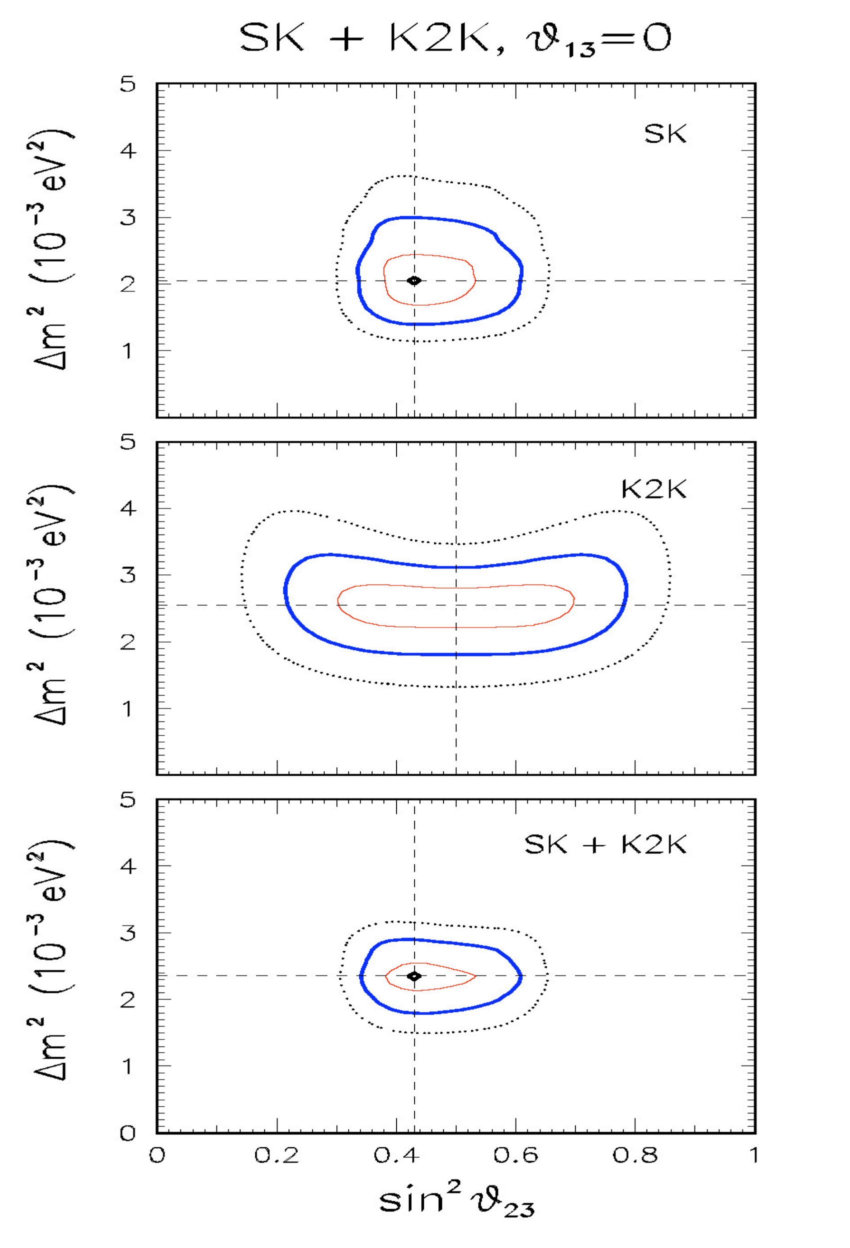
- Combination of all data (CHOOZ-dominated) prefers $\theta_{13} \cong 0$ (many analyses)
- For $\theta_{13} \cong 0$, SK data slightly prefer $\theta_{23} < \pi/4$ *(Gonzalez-Garcia, Maltoni, Smirnov)*
- Effect at $\theta_{13} = 0$ statistically small, but not smaller than others we take care of…



1, 2, 3σ contours ($\Delta\chi^2$=1, 4, 9)
from our analysis
(note linear scale on both axis)

~ -0.5σ shift of Δm^2 from 1D to 3D

~ -0.5σ shift of $\sin^2\theta_{23}$ due to LMA



Best fit SK+K2K (our analysis):
$\Delta m^2 = 2.3 \times 10^{-3}$ and $s^2_{23} = 0.43$

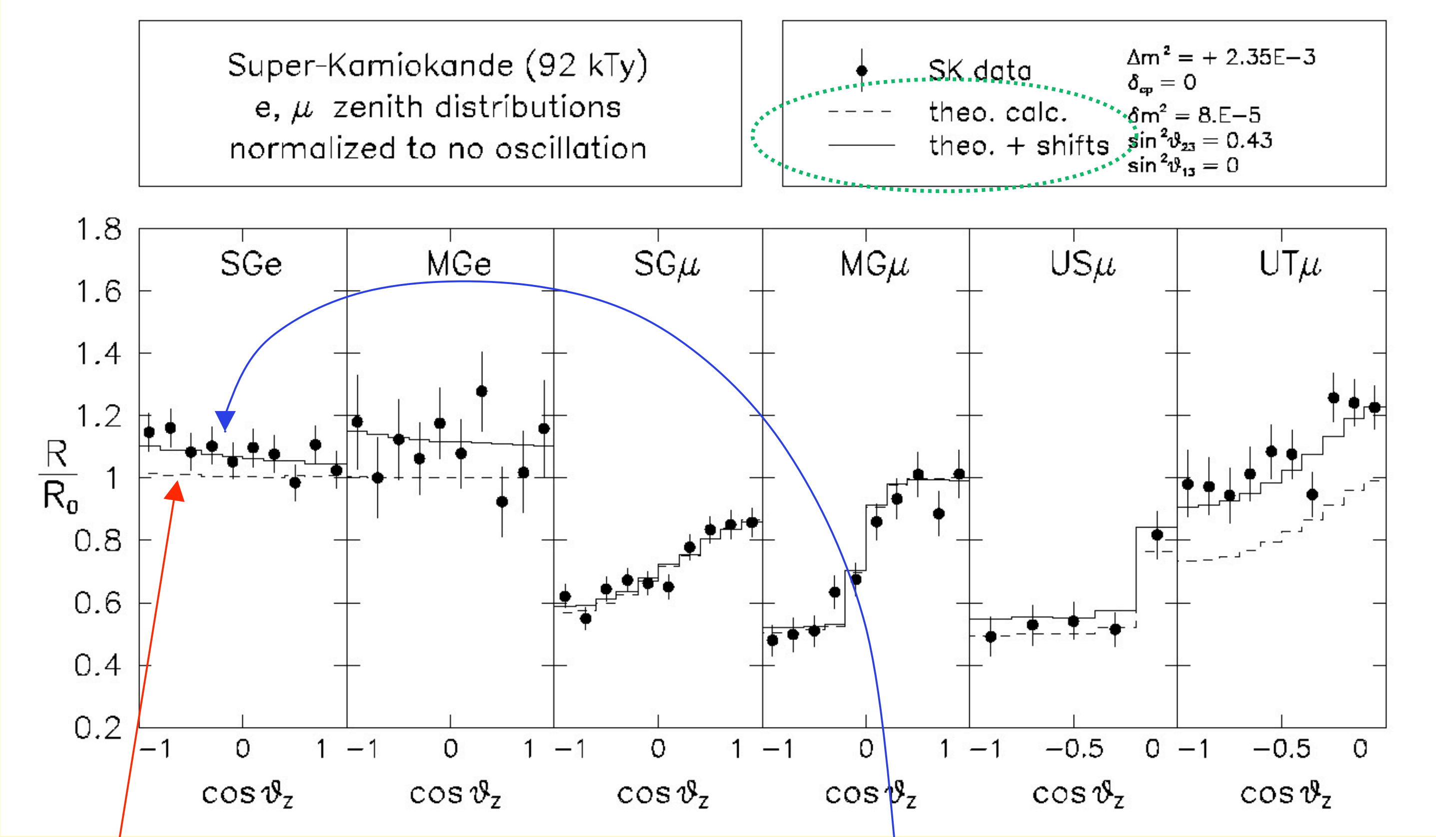
Combination of SK with K2K increases Δm^2 slightly and reduces its +error (+ and - errors become ~symmetrical)

Errors on $\sin^2\theta_{23}$ remain asymmetrical as a consequence of LMA effect.

Message:

If we take care of 1D →3D fluxes and of K2K data impact, we have no reason to neglect LMA-induced effects on parameter estimation, even if they are rather small.

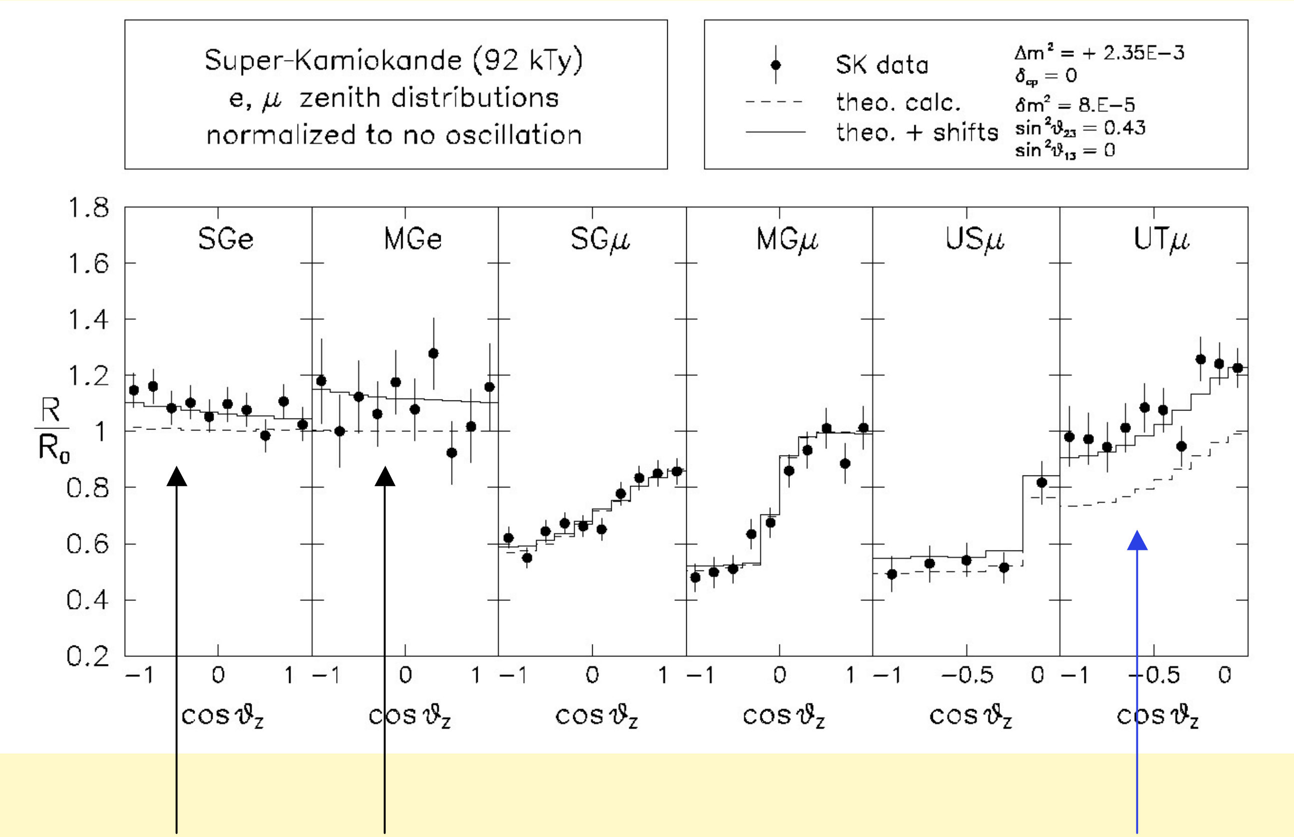
How small is small (in the zenith distributions) ?



This small at best fit! … and smaller than systematic shifts!

The electron excess would become a deficit in 2nd octant (s^2_{23} =0.57).
Despite being very small, the effect gives $\Delta\chi^2 \sim 2$ from s^2_{23} = 0.43 to 0.57

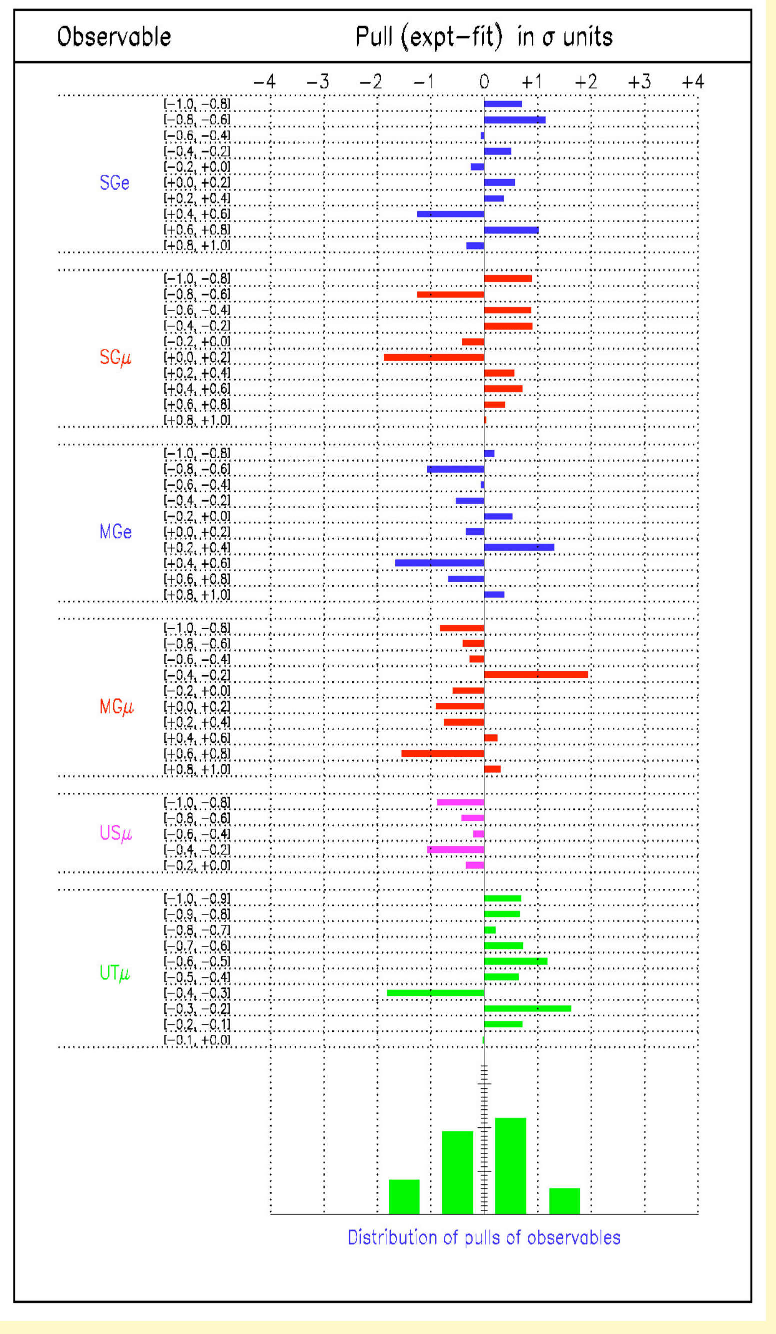
More on systematics



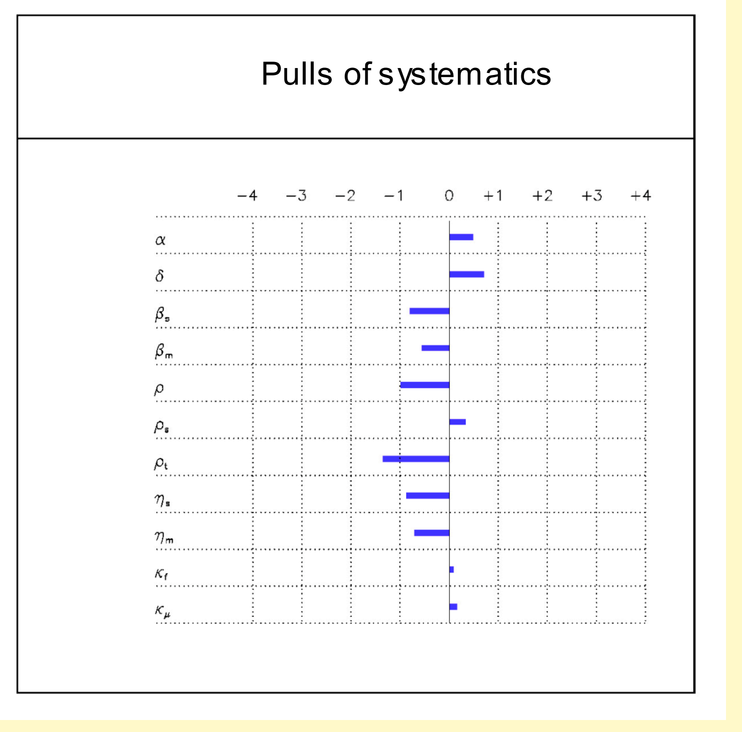
We cannot be sure that there is a real SG or MG excess …

… as far as we believe (a posteriori) that there is no real UT-muon excess !

Why is normalization systematically increased at low and high energy, but not in between? Symptom of two different effects?



Unfortunately, no evident candidate(s) selected from pull analysis of observables and systematics (yet).



Difficult to test if an excess is physical or fake, despite its effects on $\Delta\chi^2$. Also: Flux, detector, and cross-section errors induce *partially degenerate* shifts.

Features of 3ν effects including LMA

Discussed in the general case by Peres and Smirnov (1999,2004).
(Also: Gonzalez-Garcia and Maltoni, 2003)

Do-it-yourself derivation (for constant density and CP symmetry):

1) Take the oscillation probability in vacuum:

$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4\sum_{i<j} U_{\alpha i}U_{\alpha j}U_{\beta i}U_{\beta j}\sin^2\left(\frac{m_i^2 - m_j^2}{4E}L\right)$$

2) Replace vacuum → matter values (e.g., use Kuo & Pantaleone 1989):

$$m_i^2 \longrightarrow \widetilde{m}_i^2, \quad \theta_{ij} \longrightarrow \widetilde{\theta}_{ij}$$

3) Estimate electron excess as:

$$\frac{N_e}{N_e^0} - 1 = (P_{ee} - 1) + rP_{e\mu}, \quad \big(r = \phi(\nu_\mu)/\phi(\nu_e)\big)$$

4) After suitable (sometimes tricky) approximations, you get …

$$\frac{N_e}{N_e^0} - 1 \simeq \sin^2 2\widetilde{\theta}_{13} \sin^2\left(\Delta m^2 \frac{\sin 2\theta_{13}}{\sin 2\widetilde{\theta}_{13}} \frac{L}{4E}\right) \times (r s_{23}^2 - 1)$$

1st

$$+ \sin^2 2\widetilde{\theta}_{12} \sin^2\left(\delta m^2 \frac{\sin 2\theta_{12}}{\sin 2\widetilde{\theta}_{12}} \frac{L}{4E}\right) \times \left\{ \begin{array}{l} (r c_{23}^2 - 1) \\ \\ + r s_{13} c_{13}^2 \sin 2\theta_{23} (\tan 2\widetilde{\theta}_{12})^{-1} \end{array} \right.$$

2nd

3rd term

for neutrinos in normal hierarchy and $\delta_{CP}=0$; otherwise:

$+\Delta m^2 \longrightarrow -\Delta m^2$ Flips hierarchy

$+A \longrightarrow -A$ Flips (anti)neutrinos

$+s_{13} \longrightarrow -s_{13}$ Flips CP parity

1st term generated by θ_{13} only; sensitive to hierarchy, not to CP

2nd term generated by **LMA** only; not sensitive to CP or hierarchy

3rd term generated by **LMA** and θ_{13}; sensitive to CP, not hierarchy

SubGeV energies:

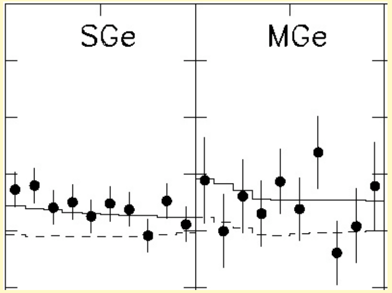
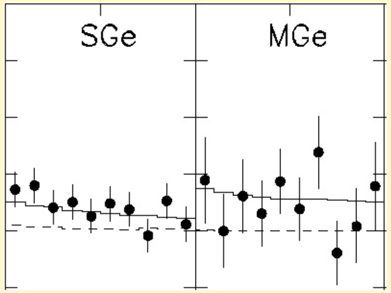
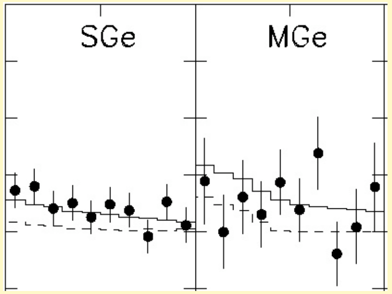
$\widetilde{\theta}_{13} \simeq \theta_{13}$	▶ 1st term ~ $\frac{1}{2}\sin^2 2\theta_{13}(r s_{23}^2 - 1)$ at large L
$r \simeq 2$	▶ 1st (2nd) term negative (positive) for $s_{23}^2 < 1/2$
$\tan 2\widetilde{\theta}_{12}(\nu) < 0$	▶ 3rd term typically negative for $\delta_{\mathrm{CP}} = 0$

MultiGeV energies:

$\widetilde{\theta}_{12} \simeq \pi/2$	▶ 2nd and 3rd (LMA) terms suppressed
$r \simeq 3.5$	▶ 1st term positive in allowed SK region

Note: The surviving (1st) MultiGeV term must include mantle-core interference effects in realistic estimates (Petcov, Akhmedov, Smirnov, ….). These and other effects are always accounted for, in numerical evolution of (anti)neutrino amplitudes along Earth density profile.

Numerical examples for $\Delta m^2 = +2.3 \times 10^{-3}\ \mathrm{eV}^2$ (N.H.)

$\delta m^2 = 0$ $s^2_{23} = 0.4$ $s^2_{13} = 0.05$		SG: 1st term < 0 (1st octant) 2nd term = 0 (no LMA) 3rd term = 0 (no LMA) MG: 1st term > 0 (nonzero 13 mixing)
$\delta m^2 = \mathrm{LMA}$ $s^2_{23} = 0.4$ $s^2_{13} = 0$		SG: 1st term = 0 (zero 13 mixing) 2nd term > 0 (1st octant) 3rd term = 0 (zero 13 mixing) MG: 1st term ~ 0 (zero 13 mixing)
$\delta m^2 = \mathrm{LMA}$ $s^2_{23} = 0.5$ $s^2_{13} = 0.05$ $\delta_{\mathrm{CP}} = \pi$		SG: 1st term ~ 0 (maximal 23 mixing) 2nd term ~ 0 (maximal 23 mixing) 3rd term > 0 (interfer. at $\delta_{CP}=\pi$) MG: 1st term > 0 (nonzero 13 mixing)

In all cases, systematic-shifted predictions (solid lines) enhance excess or “undo” deficit

Let us quantify the (unshifted) theoretical electron distributions in zenith angle through the following quantities:

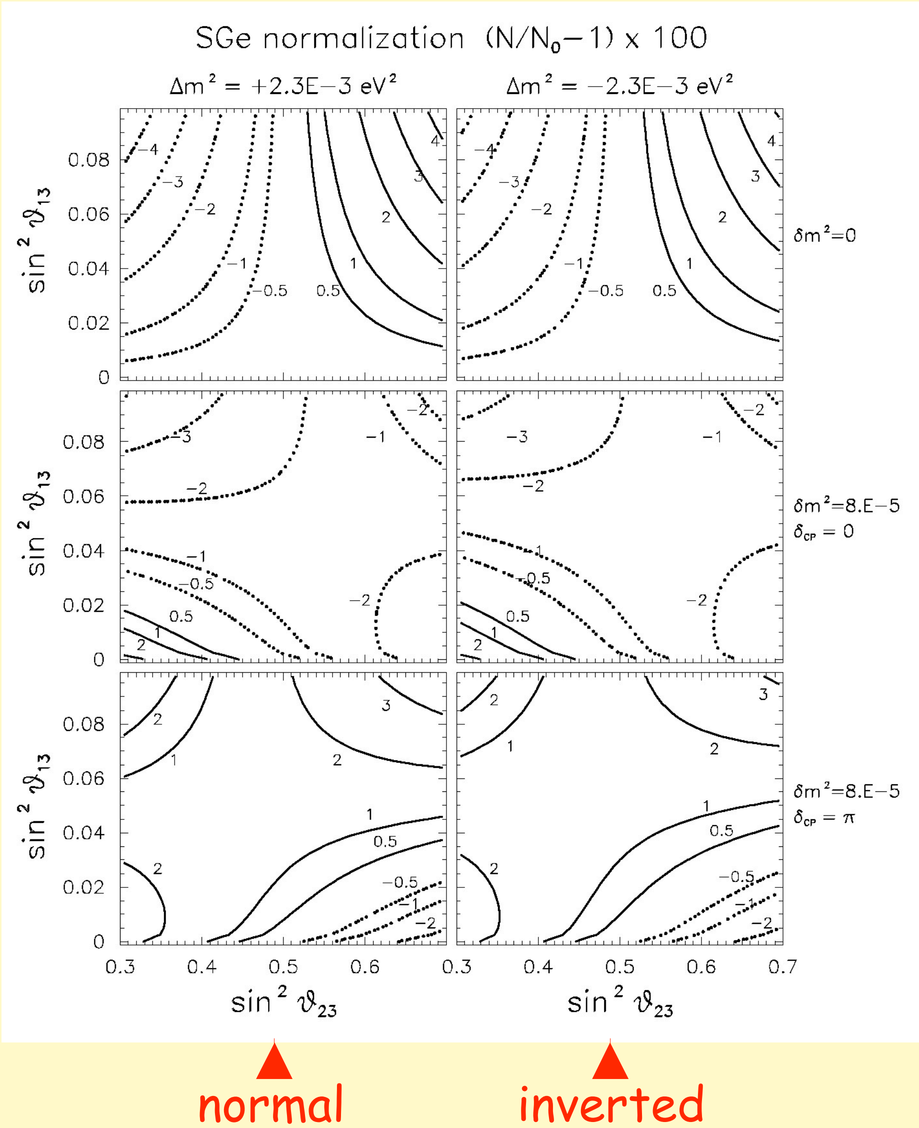
a) SGe fractional excess (total on all bins) w.r.t. to no oscillation (*depends on absolute normalization*)

b) SGe fractional deviation of up/down asymmetry* w.r.t. no oscill. (*independent of absolute normalization*)

c) MGe fractional deviation of up/down asymmetry w.r.t. no oscill. (*independent of absolute normalization*)

* UP=first three bins; DOWN=last three bins

The following calculations refer to $\Delta m^2 = 2.3 \times 10^{-3}\ \mathrm{eV}^2$



$$\delta m^2 = 0$$

Only 1st term present;
Zero at s^2_{13}=0 and s^2_{23}~1/2

$$\delta m^2 = \mathrm{LMA}, \quad \delta_{\mathrm{CP}} = 0$$

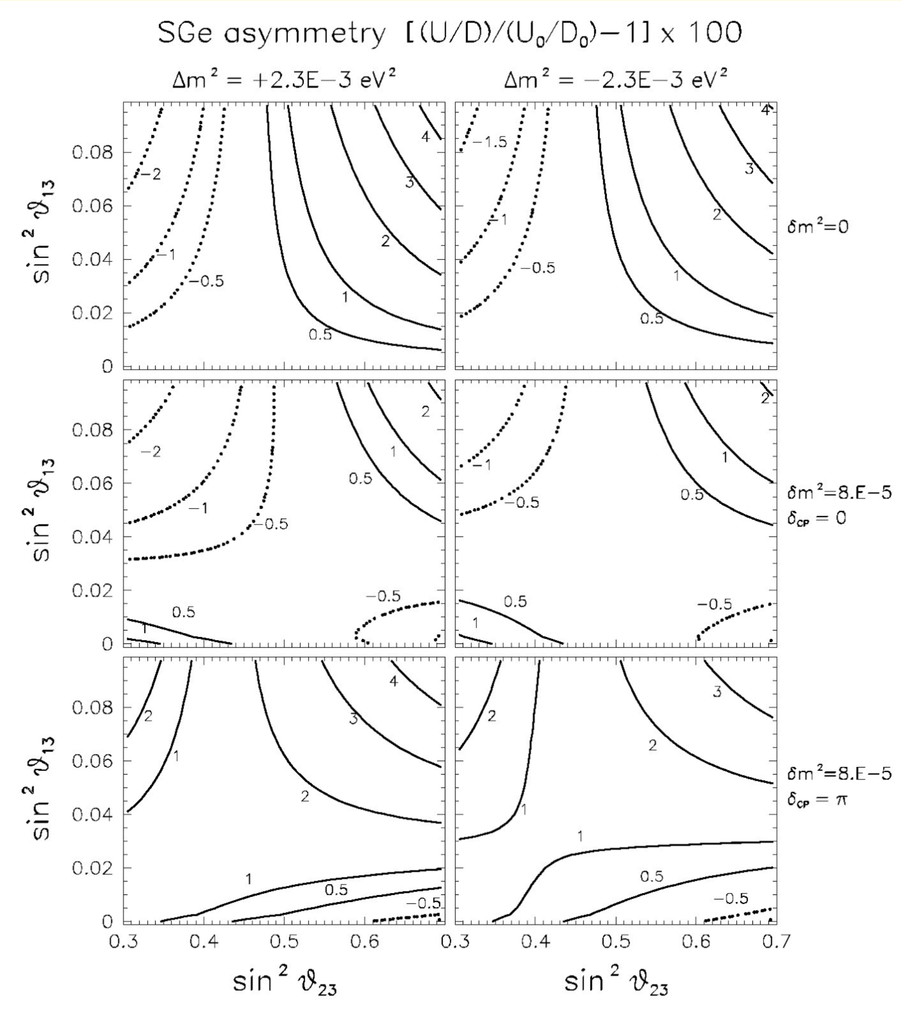
At s^2_{13}=0, nonzero values from 2nd term (>0 for s^2_{23}<1/2);
At s^2_{23}~1/2, negative contributions from 3rd term

$$\delta m^2 = \mathrm{LMA}, \quad \delta_{\mathrm{CP}} = \pi$$

2nd term as above, but 3rd term flips sign

(Note: if s^2_{13}~0.04 in the future, SGe excess would prefer $\delta_{CP}=\pi$ over δ_{CP}=0!

Dependence on hierarchy small since Δm^2-driven oscillation mostly averaged out



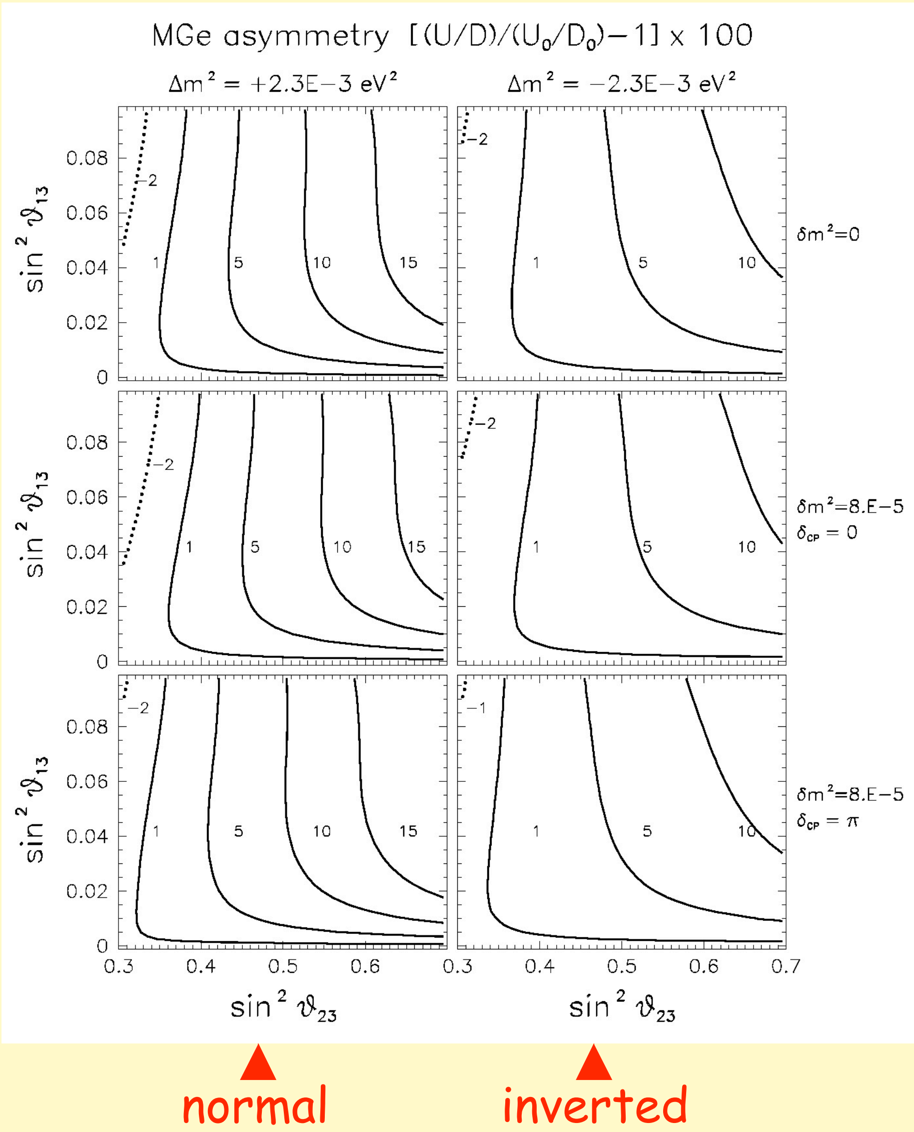
normal

inverted

Behavior of asymmetry iso-lines qualitatively similar to total excess

Dependence on hierarchy a bit larger since Δm^2 oscillation not fully averaged in "down" bins

In both cases, typical SGe effects are at O(1%) level. Need to reach this level of accuracy in stat+syst errors to claim evidence.



More reasonable prospects for MGe asymmetry, although mainly in the 2nd octant. May hope to see ~10% effect with some luck.

Dependence on hierarchy significant.

Dependence on LMA and CP largely (although not completely) lost.
1st term (1-3 mixing) dominant.

Note that, if s^2_{13}~few% fixed by future experiments, MGe asymmetry could provide a measurement of s^2_{23} for given hierarchy
(large literature on this topic)

Is SK limited by systematics?

It seems that, to see subleading LMA effects in SGe sample, stat and syst errors must reach (sub-)percent level.
Less stringent requirements for 1-3 mixing effects in MGe sample.

Since systematics are hard to reduce, it is legitimate to ask what happens by reducing only statistical errors significantly (say, up to 1/10, equivalent to ten years of Hyper-K operation).

Unexpected trend occurs: Parameter estimation improves as $\sim\sqrt{\text{(time)}}$ by increasing statistics, and never reaches a "plateau". This seems to happen in some prospective high-statistics SK MC simulations (e.g., Moriyama at NOW 2004); we also find a similar trend (not shown).

Looks like SK is not limited by systematics !
But this might be too good to be true…

$$\chi^2_{\rm SK} = \min_{\{\xi_k\}} \left[\sum_{n=1}^{55} \left(\frac{\tilde{R}_n^{\rm theo} - R_n^{\rm expt}}{\sigma_n^{\rm stat}} \right)^2 + \sum_{k,h=1}^{11} \xi_k \, [\rho^{-1}]_{hk} \, \xi_h \right]$$

In terms of the pull method, vanishing statistical errors imply that N bin observables must be ~exactly matched by theoretical predictions, up to shifts induced by K systematic sources

$$R_n^{\rm theo} \to \tilde{R}_n^{\rm theo} = R_n^{\rm theo} + \sum_{k=1}^{11} \xi_k c_n^k$$

But N equations with K<<N unknowns have no solutions, and will make the χ^2 explode outside the starting MC point, providing "perfect" parameter estimation for ∞ statistics, even with large systematics (20%, 30% !)

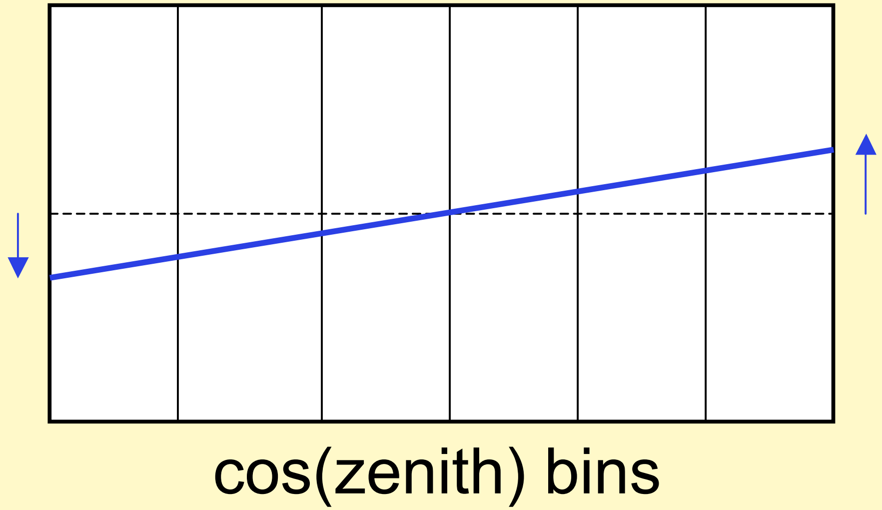
Q.: Is this a reasonable limit? A.: Probably NO.

In fact, it implies two not-so-innocent assumptions:

1) We know all systematic error sources

2) We know exactly the effect of each source on each bin

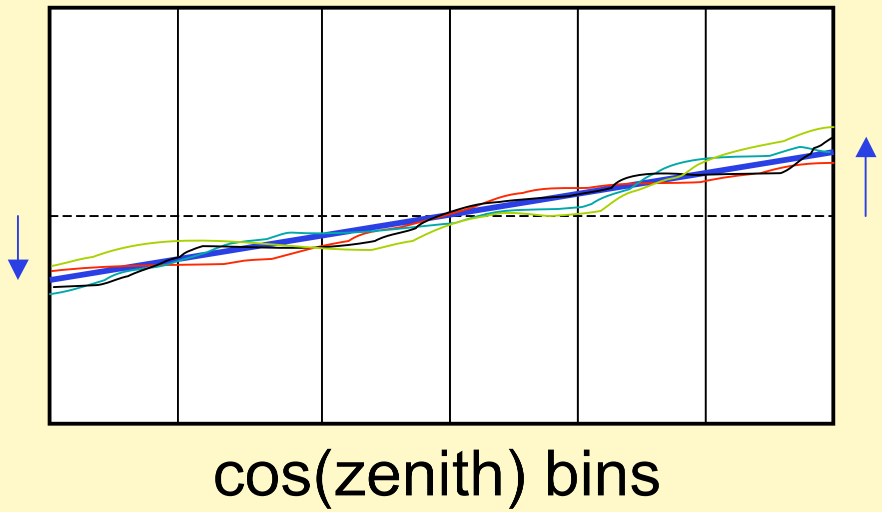
A simple example:



Parametrization of the up-down asymmetry error trhough a single source: a "tilt", linear in cos(θ)

(as currently done in the fit)

◀ This is an optimistic modelization!

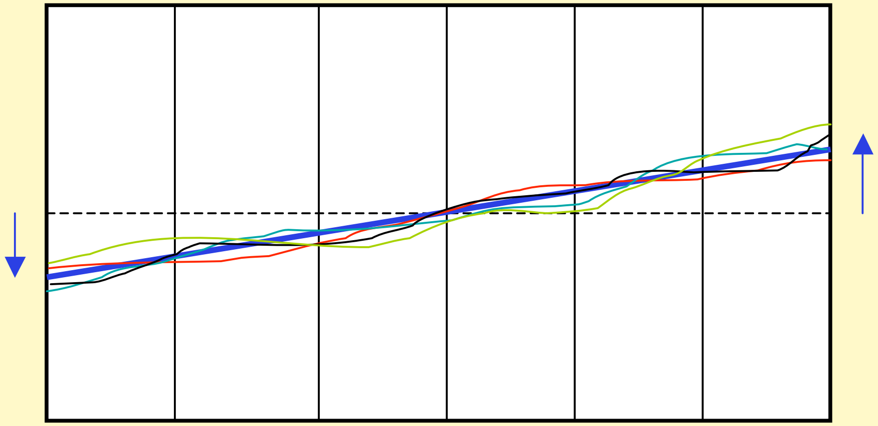


In general, we should expect either one error source with some tolerance on its shape, or more error sources with (slightly ?) different shapes

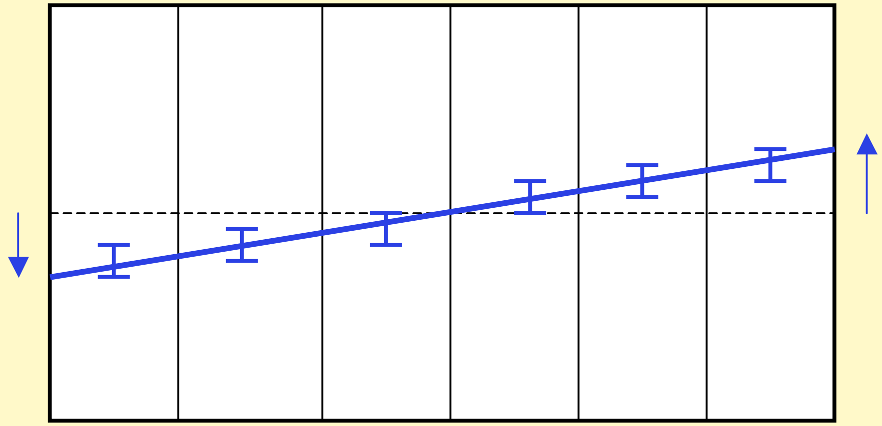
◀ This may be more realistic

Incomplete knowledge of error sources and shapes may be severe in some cases, e.g., in atmospheric neutrino flux spectra

This means that, on top of any dominant (correlated) systematic trend, we should allow small uncorrelated systematics, e.g.:



cos(zenith) bins



cos(zenith) bins

$$\chi^2_{\rm SK} = \min_{\{\xi_k\}} \left[\sum_{n=1}^{55} \left(\frac{\tilde{R}_n^{\rm theo} - R_n^{\rm expt}}{\sigma_n^{\rm stat}} \right)^2 + \sum_{k,h=1}^{11} \xi_k \, [\rho^{-1}]_{hk} \, \xi_h \right]$$

$$\frac{1}{\sigma_n^2(stat)} \to \frac{1}{\sigma_n^2(stat) + \sigma_n^2(syst)}$$

Uncorrelated systematics will prevent χ^2 "explosion" and determine the **ultimate sensitivity** for vanishing stat. errors. (Might also **alter current fit results.**)

Conclusions

Subleading three-flavor effects in atmospheric neutrinos have been studied for a long time and in different phenomenological aspects.

We finally know that LMA is true and its induced effects must be there. Currently, they help to fit the electron excess slightly better, for zero 1-3 mixing and for nonmaximal 2-3 mixing. But stat significance is small.

In SG sample, LMA effects may be entangled with 1-3 mixing effects, with some sensitivity to CP phase, and basically no sensitivity to hierarchy. Detection of typical effects requires error reduction to (sub)percent level.

In MG sample, LMA and CP effects are suppressed, but there is sensitivity to 1-3 and 2-3 mixing and to mass spectrum hierarchy. Error requirements may be less stringent than in SG sample (with some luck).

In any case, investigation of subleading effects at % level requires not only reduction of statistical errors but, at the same time, the evaluation of (so far neglected) subleading uncorrelated systematics. A challenging task!