Measurement of $\boldsymbol{\theta}_{23}$ with Atmospheric Neutrinos

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Abstract

The experimental study of the atmospheric, solar and reactor neutrino beams has resulted in the discovery of flavor neutrino oscillations and the measurement of several fundamental parameters of the neutrino mass matrix. Several new experimental programs for the study of neutrino flavor transitions with long baseline accelerator neutrino beams are in different stages of development, and in the next several years will should obtain more complete and precise measurements of the neutrino mass matrix parameters. The scientific potential of atmospheric neutrino studies is however not exhausted. One interesting direction for future studies is the search for additional terms in the flavor evolution Hamiltonian that could be present as subleading contributions. However there is also the potential for precision measurements, in particular for the mixing angle θ_{23}. The present determination of this angle is compatible with 45°, possibly indicating the existence of a symmetry in the ν sector. It is of great theoretical interest to measure the deviation of the θ_{23} value from the "maximal" value, and the sign of this deviation. Oscillations driven by the well measured "solar parameters" Δm^2_{12} and θ_{12} have effects of the sub–GeV atmospheric neutrinos that are realistically measurable, and that allow to measure the angle θ_{23}. In these effects the mixing angle enters in a form ($\propto \sin^2\theta_{23}$) that allows to distinguish between θ_{23} and $(90° - \theta_{23})$ resolving the "octant" ambiguity that is present in measurements that are only sensitive to $\sin^2 2\theta_{23}$. Because of the small size of these effects, their detection requires higher statistics, and an improved control of the systematic errors are needed.

1. Introduction

Measurements of atmospheric neutrinos have lead to the discovery of the phenomenon of neutrino oscillations, and to the measurements of two parameters of the neutrino mass matrix, the mixing angle θ_{23} and the squared mass difference Δm^2_{23} [1]. At the same times the study of solar neutrinos and reactor neutrinos has

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lead to the measurements of two more parameters of the neutrino mass matrix, θ_{12} and Δm^2_{12} [2, 3]. For the third mixing angle (θ_{13}) one has ony some an upper limit [4], while the CP violating phase is not measured. These measurements of the neutrino mixing have revealed a structure strikingly different from the quark mixing matrix, with two large mixing angles. The most puzzling, and potentially interesting results is that the measured value of the angle θ_{23} is compatible, and close to the value $\pi/4$. The determination of a value of the mixing angle at (or very close) to $\pi/4$ would be a compelling indication of the existence of a symmetry (for the exchange of ν_μ and ν_τ neutrinos) with potentially deep implications for the construction of solutions to the flavor problem and the understanding of physics beyond the Standard Model; it is therefore of central importance to measure precisely the mixing angle θ_{23} and determine (or obtain a more stringent upper limit on) the deviation of θ_{23} from the "symmetry" value of $\pi/4$.

2. The "standard method" for the measurement of $\sin^2 2\theta_{23}$

The most sensitive measurement of θ_{23} has been obtained with the observation of the zenith angle distribution of multi-GeV μ–like events. Because of the isotropy of the primary cosmic rays, and the sphericity of the Earth, in the absence of oscillations (or other forms of non–standard propagation) the atmospheric neutrino flux is up–down symmetric. In the presence of oscillations the rate of events generated by up–going neutrinos is reduced because a fraction of the ν_μ and $\overline{\nu} + \mu$ is transformed into tau (anti)–neutrinos. In the approximation of considering only the shorter wavelength oscillations and neglecting θ_{13} one has:

$$\frac{N_\mu}{N_\mu^0} \simeq 1 - \sin^2 2\theta_{23} \; \left\langle \sin^2 \left(\frac{\Delta m^2_{23} \, L}{4 \, E_\nu} \right) \right\rangle \tag{1}$$

where N_μ (N_μ^0) is the measured (predicted in the absence of oscillations) rate of μ–like events, and the average is performed over the energy and pathlength of the neutrinos that contribute to the detected signal. Given the order of magnitude of the squared mass difference Δm^2_{23}, and because of the very large difference in pathlength for up–going and down–going neutrino, it is a reasonable approximation to assume that the oscillating term in (1) is unity for down–going neutrinos, and (after averaging over rapid oscillations) is approximately 1/2 for up–going (long pathlength) neutrinos. Using multi–GeV events, where there is a good correlation between the directions of the detected muon and the parent neutrinos, one can therefore extract the mixing parameter as:

$$\sin^2 2\theta_{23} \simeq 2 \; \left[1 - \frac{N_\mu(\text{Up})}{N_\mu^0(\text{Up})} \right] \simeq \sin^2 2\theta_{23} \; \left[1 - \frac{N_\mu(\text{Up})}{N_\mu(\text{Down})} \right] \tag{2}$$

where we have indicated as N_μ(Up) and N_μ(Down) the measured rated of up and down–going neutrinos, and have used the up–down symmetry for the no–oscillation rate.

To perform this measurement one has to select a neutrino (and detected muon) sufficiently large, so that the angular correlation between neutrino is sufficiently good, that is multi–GeV neutrinos. The "useful' event rate is of order: $N_\mu^0(\text{Up}) \simeq N_\mu^0(\text{Down}) \simeq N_\mu(\text{Down}) \simeq 5.7$ (Kton year)$^{-1}$ and the corresponding statistical error of the mixing parameter determination is:

$$\delta(\sin^2 2\theta_{23})_{\text{stat}} \simeq \sqrt{\frac{3}{\text{N}_{\text{Down}}}} \simeq \frac{0.07}{\sqrt{\text{Exposure(SKI)}}} \tag{3}$$

where the exposure of SKI (corresponding to 4 years of live–time) is approximately 90 Kton years. A very large exposure corresponding to 50 times SKI would reduce the statistical error to the level of $\delta(\sin^2 2\theta_{23})_{\text{stat}} \simeq 0.01$. The systematic error of the measurement is dominated by uncertainties in the up–down symmety of the event rates, and it should be possible to keep it below the statistical error

This simple and robust method for extracting the mixing angle θ_{23} has the limitation that is sensitive to the angle only in the functional form $\sin^2 2\theta_{23}$. The mixing angle θ_{23} is defined in the interval $0 \leq \theta_{23} \leq \pi/2$, this leaves an "octant ambiguity" in the mixing angle. A mixing angle $\theta_{23} \leq 45°$ indicates that the overap of ν_3 with ν_μ is larger than the overlap with ν_τ $(|\langle\nu_\mu|\nu_3\rangle| > |\langle\nu_\tau|\nu_3\rangle|)$ while $\theta_{23} \geq 45°$ indicates the opposite. As an illustration the measurements of the mixing angle $\sin^2 2\theta_{23} = 0.96 \pm 0.01$ would corresponds to two disjoint intervals in θ_{23} corresponding to: $\sin^2\theta_{23} \in [0.4, 0.43]$ and $\sin^2\theta_{23} \in \oplus[0.57, 0.6]$. Note also that because the function $\sin^2\theta_{23}$ has a maximum at $\theta_{23} = \pi/4$, that is close to the measured value of the the mixing angle, the sensitivity of the measurement is reduced by the jacobian factor connecting θ_{23} to the mixing parameter $\sin^2\theta_{23}$. As an example a measurement compatible with maximal mixing: with a small error, such as: $\sin^2 2\theta_{23} = 1 \pm 0.01$ corresponds to a larger allowed interval in the mixing angle: $\sin^2\theta_{23} = 0.5 \pm 0.05$.

3. "Solar" parameters and Atmospheric Neutrinos

The solar neutrino and KamLand[2, 3] experiments have obtained a remarkably precise determination of two parameters of the neutrino mass matrix:

$$\Delta m^2_{12} = 7.9^{+0.6}_{-0.5} \times 10^{-5}\ \ \text{eV}^2, \qquad \theta_{12} \simeq (32.3 \pm 2.5)°. \tag{4}$$

The "solar" squared mass difference Δm^2_{12} is approximately 30 times smaller than $|\Delta m^2_{23}|$, and generates oscillations with a 30 times longer period, however

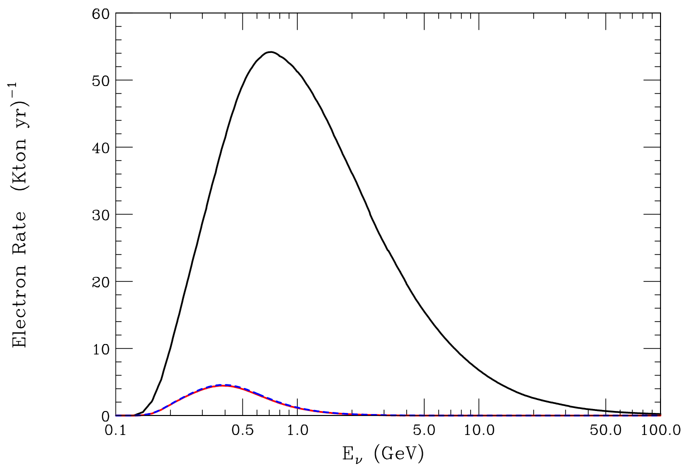

Fig 1. The thick solid curve shows the rate of e–like events (in the form $E_\nu\, dN_e/dE_\nu$) as a function of the neutrino energy. The rate is calculated in the absence of oscillations summing over the ν_e and $\overline{\nu}_e$ contributions and integrating over all zenith angles. The thin dashed line shows the disappearance of e–like effects due to $\nu_e \to \nu_{\mu,\tau}$ oscillations. The oscillation probability is calculated using the best fit point for $(\Delta m^2_{12},\ \theta_{12})$ obtained from KamLand and the solar neutrino experiments, and is independent from the $\{23\}$ parameters of the ν mass matrix. The thin solid line shows the appearance of e–like effects due to $\nu_\mu \to \nu_e$ oscillations, calculated with the same $\{12\}$ parameters and $\sin^2\theta_{23} = 0.5$. In this case the appearance and disappearance effects cancel each other nearly exactly.

the effects of these oscillations are significant for sub–GeV neutrinos. A general discussion of three flavor oscillations is fairly complicated, but it simplifies significantly with the assumption that θ_{13} vanishes. In this case it is simple to demonstrate that the probabilities for transitions that involve ν_e's (or $\overline{\nu}_e$'s) can be simply expressed in terms of the functions $P^{12}_{\rm 2flav}$ and $\overline{P}^{12}_{\rm 2flav}$ that depends only on three variables: $P^{12}_{\rm 2flav}(\varepsilon_{12}, \theta_{12}, \Theta_{\rm z})$, where $\varepsilon_{12} = E/\Delta m^2_{12}$ and $\Theta_{\rm z}$ is the zenith angle):

$$P(\nu_e \to \nu_e) \;=\; 1 - P^{12}_{\rm 2flav} \tag{5}$$

$$P(\nu_e \to \nu_\mu) \;=\; P(\nu_\mu \to \nu_e) = P^{12}_{\rm 2flav}\ \cos^2\theta_{23} \tag{6}$$

$$P(\nu_e \to \nu_\tau) \;=\; P(\nu_\tau \to \nu_e) = P^{12}_{\rm 2flav}\ \sin^2\theta_{23} \tag{7}$$

(and similarly for antineutrinos). These oscillation probabilities are rather large for sub–GeV neutrinos, however the net effects on the electron neutrino flux is reduced because of a cancellation effect between appearance and disappearance effects.

In figure 1 we show the no–oscillation prediction for the e–like event rate,

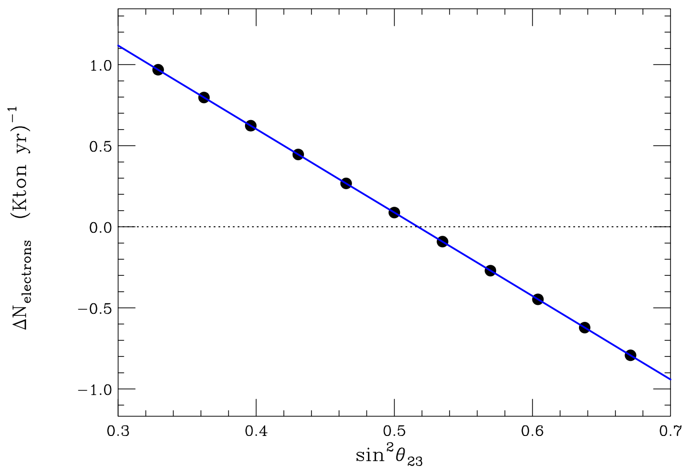

Fig 2. Deviation $\Delta N_e = N_e - N_e^0$ of the e–like event rate from the no–oscillation expectation plotted as a function of $\sin^2\theta_{23}$. The calculation assumes $\theta_{13} = 0$, and the best fit point of the combined solar and KamLand data [3] for Δm^2_{12} and θ_{12}.

based on the Bartol [15] neutrino fluxes, the same figure shows separately the disappearance (due $\nu_e \to \nu_{\mu(\tau)}$ flavor transitions) and appearance (due to $\nu_\mu \to \nu_e$ transitions) effects predicted from the measurements of solar neutrino oscillations (and assuming $\theta_{13} = 0$. The disappearance effect can be calculated specifying only the "solar" parameters (Δm^2_{12} and θ_{12}), while the appearance effect depends also the θ_{23} angle (set to 45° in the figure). Note that "solar oscillations", are important only for sub–GeV up–going neutrinos, when the quantum phase difference $\varphi_{12} = \Delta m^2_{12}\, L/(2E_\nu)$ is sufficienty large. Note also that for the choice $\theta_{23} = 45°$ the appearance and disappearance effects are nearly exactly canceling. This can be easily understood observing that the deviation of the charged current event rate for ν_e interactions is given by:

$$\begin{aligned}\Delta N_{e^-} = N_{e^-} - N^0_{e^-} &= N^0_{e^-}\ \langle P(\nu_e \to \nu_e)\rangle + N^0_{\mu^-}\ \langle P(\nu_\mu \to \nu_e)\rangle - N^0_{e^-} \\ &= -N^0_{e^-}\ \left\langle P^{12}_{\rm 2flav}\right\rangle + N^0_{\mu^-}\ \left\langle P^{12}_{\rm 2flav}\right\rangle \cos^2\theta_{23} \\ &= N^0_{e^-}\ \left\langle P^{12}_{\rm 2flav}\right\rangle\ \left[-1 + r_{\mu e}\cos^2\theta_{23}\right] \qquad (8)\end{aligned}$$

(and similarly for anti–neutrinos). In equation (8) N_e (N_e^0) is the measured (no–oscillation prediction) for the ν_e rate, and $r_{\mu e} = N^0_\mu/N^0_e$ is the ratio between the μ–like and e–like rate; the average sign indicates averaging over the energy and zenith angle of the neutrinos that contribute to the signal. Note that the deviation from the no–oscillation expectation depend linearly on the mixing parameter $\cos^2\theta_{23}$, as is illustrated in fig. 2 that shows the deviation from the expected rate $\Delta N_e = N_e - N_e^0$ as a function of $\sin^2\theta_{23}$. The comparison between the no–oscillation expectation and the data, allows to extract the combination $P_{2\text{flav}}^{12}$ $[-1 + r_{\mu e} \cos^2\theta_{23}]$. This quantity is the product of two terms, the first depends only on the "solar" [1–2] parameters, while the second one: $[-1 + r_{\mu e} \cos^2\theta_{23}]$ depends only on θ_{23}. The measurements of the total rate of e–like events allows therefore in principle to extract the mixing angle θ_{23} as: The mixing parameter $\sin^2\theta_{23}$ can be extracted

$$\sin^2\theta_{23} \simeq \left(1 - \frac{1}{r_{\mu e}}\right) - \frac{\Delta N_e}{N_e^0}\,\frac{1}{r_{\mu e}\,\langle P_{2\text{flav}}^{12}\rangle} \simeq \frac{1}{2} - \frac{\Delta N_e}{N_e^0}\,\frac{1}{2\,\langle P_{2\text{flav}}^{12}\rangle} \qquad (9)$$

However, at low energy the no–oscillation μ/e ratio is very close to the value $r_{\mu e} = 2$, reflecting the fact that in the chain decay of a charged pions results in the production of one ν_e and two ν_μ's of approximately the same energy, therefore the most probable value for the factor $-1 + r_{\mu e}\cos^2\theta_{23}$ is approximately zero for the best fit value $\theta_{23} \simeq 45^\circ$. For the (SK) allowed interval $\sin^2 2\theta_{23} \geq 0.92$, that corresponds to $\theta_{23} = 45 \pm 8.2$ degrees, or to $\sin^2\theta_{23} = 0.5 \pm 0.141$ the factor that controls the solar oscillation effects can vary in the interval:

$$-0.283 \leq -1 + r_{\mu e}\cos^2\theta_{23} \leq 0.283 \qquad (10)$$

The e–like rate can therefore be enhanced or suppressed with respect to the no–oscillation prediction. An enhancement (suppression) implies $\theta_{23} < 45^\circ$ ($\theta_{23} > 45^\circ$). The effects we are discussing here are small, and can only be detected with high statistics and a good control of systematic errors. As an order of magnitude estimate, the sub–GeV e like event rate has the numerical value:

$$N_e^{\text{SG}} \simeq 29 + 2.3\,(-1 + 2\,\cos^2\theta_{23})\;(\text{Kton yr})^{-1} \qquad (11)$$

Use of this equation implies a statistical error for the estimate of $\sin^2\theta_{23}$:

$$\delta(\sin^2\theta_{23})_{\text{stat}} \simeq \frac{0.12}{\sqrt{\text{Exposure(SKI)}}}. \qquad (12)$$

Also the survival probabilities $P_{\nu_\mu\to\nu_\mu}$ and $P_{\overline{\nu}_\mu\to\overline{\nu}_\mu}$ are modified because of the presence of the "solar" parameters. It can be shown that they take the form:

$$P(\nu_\mu \to \nu_\mu) = 1 - \frac{\sin^2 2\,\theta_{23}}{2}\;\left\{1 - \Re\left[e^{i\varphi(L/E_\nu)}\right]\right\} - \cos^4\theta_{23}\;P_{2\text{flav}}^{12} \qquad (13)$$

where $\varphi(L/E_\nu)$ is a real phase, that depends also on the squared mass differences Δm^2_{12} and Δm^2_{23}. Comparing the expression (13) to the well known formula for 2–flavor oscillations:

$$P(\nu_\mu \to \nu_\mu) = 1 - \frac{\sin^2 2\,\theta_{23}}{2}\;\left\{1 - \Re\left[\exp\left(i\,\frac{\Delta m^2_{23}\,L}{2E_\nu}\right)\right]\right\} \qquad (14)$$

we can see that they differ because of the presence of the extra term $\cos^4\theta_{23}\ P^{12}_{\rm 2flav}$ and for the more complex form of the phase that controls the "standard" oscillations (with amplitude $\sin^2 2\theta_{23}$). This difference in phase arises because in the 2–flavor case there is a single quantum phase difference to consider $\varphi_{23} \equiv \varphi_{13} = \Delta m^2 L/(2E)$, while in the more general case the mass splitting between $|\nu_1\rangle$ and $|\nu_2\rangle$ results in the existence of two independent quantum phase differences, that combine to generate a more complex oscillation structure. However, averaging over the fast oscillations results in approximately the same results for both the simple 2–flavor approximation and the general case with non–vanishing 1–2 mass splitting. What remains after the averaging is the additional term $(-\cos^4\theta_{23}\ P^{12}_{\rm 2flav})$. The deviation of the ν_μ charged current interaction rate from the no–oscillation case has therefore a disappearance contribution:

$$[\Delta N_\mu]_{\rm disapp} \simeq -N_\mu^0\ \left\{\sin^2 2\theta_{23}\ \left\langle \sin^2\left(\frac{\Delta m^2_{23}\, L}{4\, E_\nu}\right)\right\rangle + \cos^4\theta_{23}\ \langle P^{12}_{\rm 2flav}\rangle\right\} \qquad (15)$$

The disappearance has two contribution, a "standard" one that can be attributed to $\nu_\mu \to \nu_\tau$ transitions, and a smaller additional one that can be attributed to $\nu_\mu \to \nu_e$ transitions. This has to be combined with the appearance effect due to $\nu_e \to \nu_\mu$ transitions:

$$[\Delta N_\mu]_{\rm app} \simeq N_e^0\ \cos^2\theta_{23}\ \langle P^{12}_{\rm 2flav}\rangle \qquad (16)$$

Combining the appearance and disappearance effects one obtains:

$$\frac{N_\mu}{N_\mu^0} \simeq 1 - \sin^2\theta_{23}\ \left\langle \frac{\Delta m^2_{23}\, L}{4\, E_\nu}\right\rangle - \cos^2\theta_{23}\ \langle P^{12}_{\rm 2flav}\rangle\ \left[\cos^2\theta_{23} - \frac{1}{r_{\mu e}}\right] \qquad (17)$$

Note that the second term in the right–hand side of equation (17), that is the correction to the two flavor formula has (quite obviously) the *opposite* sign with respect to the deviation of the e–like rate from the no–oscillation expectation, that is the appearance effect for e–like events is associated with an extra contribution to the μ–like disappearance that produces a larger effect. This extra contribution to the μ–like event rate has approximately the same absolute size as the e–like appearance. Note again that for θ_{23} close to 45° the effect vanishes, because $r_{\mu e} \simeq 2$. This effect on the μ–like event rate is also measurable, however since this effects is combined with the "standard" 2–flavor oscillations, one needs to have a good determination of Δm^2_{23} that control (together with θ_{23}) the size of the [23] oscillations.

An illustration of these effects is shown in fig. 3 that shows the deviation of the μ–like rate (integrated over all zenith angles) from the no–oscillation expectation plotted as a function of $\sin^2\theta_{23}$. Note that the dominant oscillation effect on

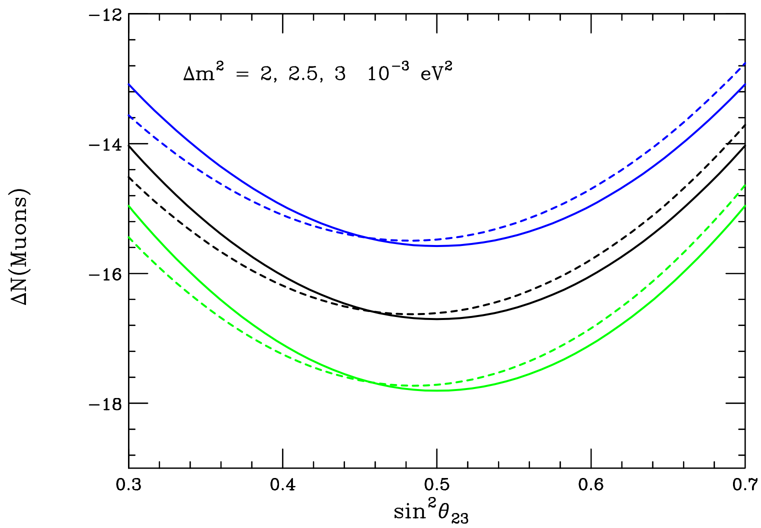

Fig 3. Deviation $\Delta N_\mu = N_\mu - N_\mu^0$ of the μ–like rate from the no–oscillation expectation plotted as a function of $\sin^2\theta_{23}$. The calculations assumes $\theta_{13} = 0$, the best fit point of the combined solar and KamLand data [3] for Δm^2_{12} and θ_{12}. The three solid curves are calculated in a two–flavor framework, and correspond to three values of Δm^2_{23}, while the three dashed curves are calculated for the same Δm^2_{23} values in a three neutrinos framework.

the μ–like rate is caused by $\nu_\mu \leftrightarrow \nu_\tau$ oscillations, that depends on Δm^2_{23}, and depend on the mixing angle θ_{23} with the form: $\propto \sin^2 2\theta_{23} = 4\,\sin^2\theta_{23}\,(1-\sin^2\theta_{23})$ that is symmetric around $\theta_{23} = 45°$, however the "solar oscillations" generate a correction that breaks this symmetry.

The study of the ratio N_e/N_μ allows to "sum" the two effects, and at the same time to cancel sources of systematic effects (like the absolute normalization of the fluxes).

4. Effects of a non–vanishing $\boldsymbol{\theta}_{13}$ and of the phase $\boldsymbol{\delta}$

The discussion of the previous paragraph assumed that the third mixing angle (θ_{13}) vanishes, however in the most general case this is clearly not the case, and the expressions for the flavor transitions become more complex. It is instructive to consider first the situation where θ_{13} is different from zero, but the effects of the "solar parameters" are negligible. This last condition implies considering the limit $\Delta m^2_{12} \to 0$ or $\theta_{12} \to 0$, this is of course incorrect, and therefore this case has only a pedagogical interest. In this case the oscillation probabilities involving ν_e can be written in the form

$$P(\nu_e \to \nu_\mu) \;=\; P(\nu_\mu \to \nu_e) = P^{13}_{\rm 2flav}\,\sin^2\theta_{23} \qquad (18)$$

$$P(\nu_e \to \nu_\tau) \;\; = \;\; P(\nu_\tau \to \nu_e) = P^{13}_{\rm 2flav}\; \cos^2\theta_{23} \tag{19}$$

(and similarly for $\overline{\nu}$). These expressions have the same structure as equations (6) and (7) with the exchange $\sin^2\theta_{23} \leftrightarrow \cos^2\theta_{23}$. The function $P^{13}_{\rm 2flav}$ has the identical mathematical form as form as $P^{13}_{\rm 2flav}$ but needs the repacements $\theta_{12} \to \theta_{13}$ and $\Delta m^2_{12} \to \Delta m^2_{13}$. It clearly follows that the modification of the neutrino flux can be written in analogy to equation (8) as:

$$N_e \simeq N_e^0 \times P^{13}_{\rm 2flav}\;\left[-1 + r_{\mu e}\sin^2\theta_{23}\right] \tag{20}$$

Note that in this case an excess of electrons implies an angle $\theta_{23} > \pi/4$. The survival probability for ν_μ (and similarly for $\overline{\nu}_\mu$) is given by a similar expression to (13):

$$P(\nu_\mu \to \nu_\mu) = 1 - \frac{\sin^2 2\,\theta_{23}}{2}\;\{1 - \Re\left[\exp\left(i\phi_{13}\right)\right]\} - \sin^4\theta_{13}\;P^{13}_{\rm 2flav} \tag{21}$$

that corresponds to equation (13) with the same substitutions as discussed before.

In general the effects of the solar parameters θ_{12} and Δm^2_{12} and θ_{13} will be both present, and their interference will also depend on the phase δ. In fig. 4 we show the deviation of the e–like ΔN_e plotted as a function of energy, assuming a large value of θ_{13} of 7.5°, and different values of θ_{23} and δ. One can see that in principle there are interesting effects that could be studied with very large events samples. In principle the effects of θ_{13} and of the "solar oscillations" are present in different energy regions, very low energy for the solar effects, and multi–GeV ($\Delta m^2_{23} \simeq V\,E_\nu$) for θ_{13}, however the interference between the two effects (that depends on δ) [8] complicates the picture. Note also that the study of the θ_{13} induced oscillations has the potential to determine the sign of Δm^2_{23} (and the hierarchy of the ν masses) since the MSW resonance is present for ν ($\overline{\nu}$) when Δm^2_{23} positive (negative).

5. Super–Kamiokande data

The detector that has the potential to detect the effects discussed above, is clearly Super–Kamiokande, however one will need to colkect a significantly larger statistics than what has been obtained until now.

A reanalysis of the SK atmospheric neutrino data has been presented at this workshop by Shoei Nakayama [17]. In this analysis the prediction for the atmospheric neutrino event rates was calculated fixing the values of Δm^2_{12} and $\sin^2\theta_{12}$ at the best fit point for the solar and Kamland data, and the value of Δm^2_{23} at the best fit point of the standard SK analysis ($\Delta m^2_{23} = 2.5 \times 10^{-3}$ eV^2) and the angle θ_{23} is estimated in a three–neutrino framework. Because of the

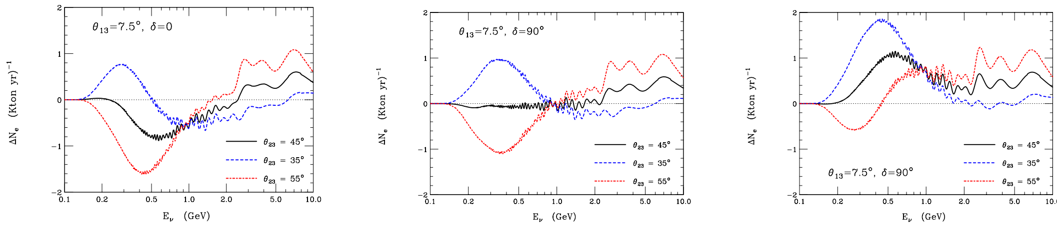

Fig 4. Left panel: deviations of the e–like event rate from the no–oscillation expectation calculated for three values of θ_{23} that correspond to $\sin^2 2\theta_{23} = 1$ and $\sin^2 2\theta_{23} = 0.9$. Center panel: as before for $\delta = 90°$. Right panel: as before for $\delta = 180°$.

sub–dominant “solar oscillations” the e–like rate is modified with an effect that depends on θ_{23}. Most of the effect is present for electron momentum is in the range $100 \leq p_e \leq 400$ MeV. In this energy range the effect appears as nearly isotropic because of the poor correlation between θ_e and θ_ν, even if only up–going neutrinos have a pathlength sufficiently long to be affected by the long–wavelength solar oscillations. For $\sin^2 2\theta_{23}$ the effects of appearance and disappearance cancel each other nearly exactly, and the e–like rate calculated including oscillation differs from the no–oscillation calculation by less than 0.5%, while values of $\sin^2\theta_{23}$ at the limit of the allowed interval ($\sin^2\theta_{23} = 0.4$ or 0.6) correspond to deviations of +2% (-2.5%) with respect to the no oscillation prediction. The effect of the solar oscillations is reduced at higher momentum ($p_e > 400$ MeV), however the detectability of the effects is improved because of the smaller $\theta_{\nu e}$ angle, and the effect is of order ±1% for $\sin^2\theta_{23} = 0.4$ (0.6). Similarly for $\sin^2\theta_{23} < 0.5$ (> 0.5) the rate of μ–like events has a slightly larger (smaller) reduction than what is expected in a simple 2–flavor $\nu_\mu \leftrightarrow \nu_\tau$ framework. The fit of the $\sin^2\theta_{23}$ parameter results in best fit point $\sin^2\theta_{23} = 0.5$ that corresponds to the published SK analysis [1], however the allowed interval is not symmetric around this value, and values of $\sin^2\theta_{23} \leq 45°$ are more favored. Qualitatively this result had been found also indicating before [5, 6, 9]. The contribution of Nakayama also address the question if a larger statistics would allow to distinguish between $\sin^2\theta_{23} = 0.4$ and $\sin^2\theta_{23} = 0.6$ that both corresponds to $\sin^2 2\theta_{23} = 0.96$. The interesting result is that 20 years of SK data taking, would result in $\Delta\chi^2 \simeq 2$ between the true value of the parameter and its “mirror” value, making the same assumptions about systematic errors. Assuming a reduction of systematic errors to 1/4 of the present estimate would improve the significance of the discrimination between the two solutions at the level of $\Delta\chi^2 \simeq 5$.

A reanalysis of the sensitivity of the SK experiment to the measurement of θ_{13} was presented by K. Okumura [16].

A general analysis of the potential of a large exposure for a SK–like water detector has been discussed by M. Shiozawa [18], that demonstrates the potential of a

6. Systematics errors

For the detection of the small effects discussed at this workshop, the control of systematic errors is crucial, in fact already now, as stressed by Gonzalez–Garcia [6] in her contribution [6] the description of systematic errors control the size and shape of the allowed region for the neutrino mass matrix. With larger exposures, the reduction of the systematic errors will become of essential importance (see also the contributions of K. Okumura and M. Shiozawa [17, 18]).

Moreover it will be very important to parametrize and describe the systematic uncertainties in complete and detailed way, in order to determine realistic allowed intervals for the physical parameters that are studied.

As discussed in the SK papers and contributions, there are three classes of systematic uncertainties: (i) the estimate of the neutrino fluxes, (ii) the modeling of the neutrino cross section and (iii) the modeling of the detector performance. The first two classes of uncertainties (and possible methods to reduce these uncertainties) have been the object of several contributions at the workshop.

General discussions of the calculation of the atmospheric neutrino fluxes have been presented by M. Honda [22] and G. Barr [21] at this meeting. This calculation requires two fundamental ingredients: a knowledge of the primary cosmic ray fluxes, and a description of the hadronic interactions, that we will discuss in the following.

6.1. Primary Cosmic Ray Fluxes

The main uncertainty of the description of the low energy cosmic rays (that are the sourc of sub–GeV events) is the existence of a large ($\sim 20\%$) discrepancy between the observations of the BESS [10] and AMS [11] detectors (that are in good agreement with each other) and the lower normalization data of Caprice [12]. Solved this problem, to reduce the normalization uncertainty to the level of few percent, it is necessary to include an accurate description of solar modulation, correlating the cosmic rays flux to the continuous observations of neutron monitors. The BESS detector [19] has performed 5 different measurements of the cosmic ray fluxes from 1997 to 2002 when the solar increased from minimum to maximum, measuring as expected that at low energy the cosmic ray flux decreases for a more intense solar activity. The data in first approximation can be well rep-

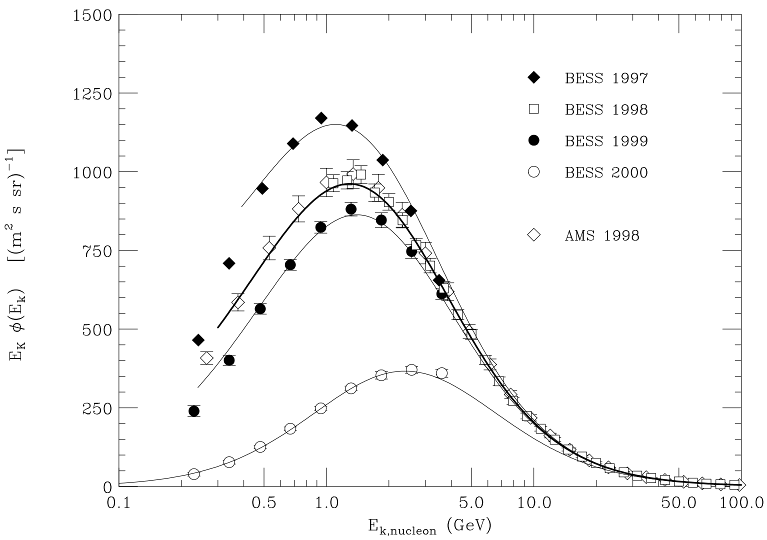

Fig 5. Proton measurements by BESS in 1997, 1998, 1999 and 2000. The curves are calculated assuming the same interstellar flux and describing the solar modulation with the Force Field Approximation.

resented by the "Force Field Approximation", where the effects of the solar wind are described as a potential, a cosmic ray particle with energy $E_{\rm i.s.}$ in interstellar space arrives at the Earth with energy $E_\oplus$:

$$E_\oplus \simeq E_{\rm (i.s.)} - |Z|\ V_{\rm wind}(t) \tag{22}$$

and the c.r. flux $\Phi_\oplus(E,t)$ in the vicinity of the Earth is related to the insterstellar flux $\Phi_{\rm i.s.}$ as:

$$\Phi_\oplus(E;\ t) \simeq \frac{p_\oplus^2}{p_{\rm i.s.}^2}\ \Phi_{\rm i.s.}[E + |Z|\, V_{\rm wind}(t)] \tag{23}$$

even if more sophisticated analysis could be required.

An important uncertainty is present at energies higher than $E_0 \simeq 100$ GeV, when the measurements of magnetic spectrometers are absent or have still small statistics [13]. This uncertainty becomes very important for the prediction of tne ν–induced upward–going muons.

6.2. Hadronic Interactions

The modeling of Hadronic interactions is the second crucial problem in the calculation of the atmospheric neutrino fluxes. Given the fact that a first principle calculation is not possible, this requires the fitting, interpolation adnd

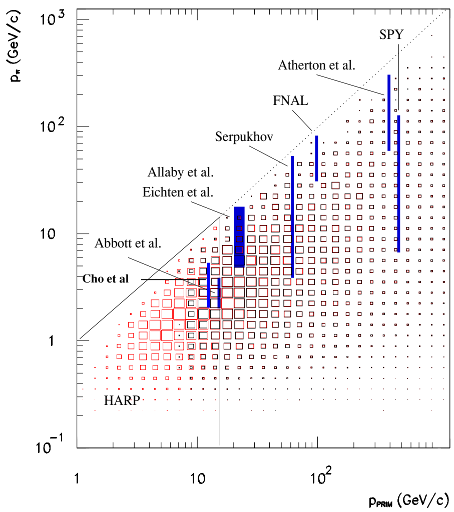

Fig 6. Bidimensional distribution in primary energy E_0 and meson energy $E_{\rm mes}$ for the neutrino interactions observable in an udnerground detector. The regions covered by existying measurements are also shown. See [23] for the references and new measurement.

extrapolation of sparse accelerator data. Real progress in this domain can only be achieved from new measurements a acceleratoe. The situation has been reviewed by Giles Barr [23] that has discussed what is the relevant kinematic region (in the plane $\{E_0, E_f\}$, that is projectile laboratory energy and final state particle energy), and which data sets are currently available.

New accelerator data should soon be available, with the potential to significantly reduce the existing uncertainties. The HARP experiment at CERN has completed the datat takin with the primary energy in the interval $E_0 = 3$–15 GeV and full coverage of the final state phase space. Hopefully the analysis of the data will soon be completed, since this would be important for the prediction of the sub–GeV neutrino fluxes.

The NA49 experiment is especially designed for the measurement of heavy interactions but, motivated by cosmic ray and atmospheric neutrino studies has performed measurements of p–Carbon interactions at $E_p = 100$ and 158 GeV, that should provide important constraints for the proprties of hadronic interactions in an important range.

Ther MIPP (Main Injector Particle Production Experiment) at Fermilab has the ambitious goal to obtain large statistics of unbiased hadron interactions with a variety of targets. Of particular interest for atmospheric neutrino studies is the program to obtain data on proton–Nitrogen interactions for a set of energies

(5, 15, 25, 50, 70 and 90 GeV) that together with NA49 and HARP will cover well the range of primary energy that is the source of contained neutrino interactions.

6.3. Atmospheric Muons

It is well known that atmospheric muons are produced in the same meson decays that are the source of atmospheric neutrinos, and their measurement can in principle provide important constraints of the energy spectrum and absolute normalization of the neutrino fluxes. However most muons below a few GeV decay in the atmosphere before reaching ground level (becoming the source of additional neutrinos), and are difficult to observe. The muon measurement is possible from high altitude balloons, and in principle the miost sensitive results are obtainable at altitude of 10–20 Km when the muon flux is at maximum. T. Sanuki [20] has reviewed the situation of the existing measurement both at ground level, and on ballons, with particular attention to the BESS data. The measurements obtained during balloon ascent still suffer from poor statistics, however precise measurement obtained at ground level, at different locations (different altitudes and different geomagnetic cutoff) provide important constraints. In fact it appears that the present models cannot reproduce in all details the observations. A full explanation of the origin of these discrepancies and of their significance is still incomplete (see [22, 20]).

6.4. The neutrino Cross section

A detailed knowledge of the neutrino cross section is also an important ingredient in the calculation of prediction of the rates of atmospheric neutrino events. For the range of E_ν that is most important for atmospheric neutrinos, the charged current cross section is dominated by low multiplicity final states, such as quasi–elastic scattering $\nu_\ell + n \to \ell^- + p$ (or $\overline{\nu}_\ell + p \to \ell^+ + n$) or single pion production. In this energy range is also very important to consider in detail the nuclear effects. The problem has been the subject of a series of specialized workshop [31]. Because of uncertainties in the description of hadronic structure at low Q^2, one has to rely on experimental results.

For sub–GeV neutrinos, the nuclear effects represent an important correction. In this field, very likely not all the information that has been obtained from recent electron scattering experiments on nuclei has been completely analysed and inserted in the ν–interaction modeling. This (rather complex) situation is reviewd in the contributions of [26, 27, 28, 29, 30].

The study of e–like experiments (both at the nucleon and nucleius level) cannot provide all the desired information about the ν–interaction properties, and it would be very desirable to use the intense beams of the long–baseline accelerator beams in preparation to perform experimental studies at the near detector sites.

Events collected at the near detectors of the K2K project, have provided already several important results. The data of the Kton water detector a have been discussed by Kameda [24], and the data of the new high resolution Scibar detector by Hasegawa [25]. The main motivation of the Scibar detector is the determination of the shape of the energy spectrum of the neutrino beam at the near site studying Quasi–elastic reactions. The absolute normalization of the K2K beam is not well known, since the beam is constructed for oscillation studies, and not for the measurements of cross section, however the measurements have shown to provide very valuable information.

7. Conclusions

The experimental study of atmospheric neutrinos has still the possibility to contribute important results to the study of the fundamental properties of the neutrinos. An important limitation of these fluxes is that they can only provide small event rates, however they offer the possibility to study the properties of neutrino propagation over a wide range on energy and pathlengths. This situation is very interesting for the study of additional "non–standard' terms in the flavor evolution Hamiltonian that can be present at the sub–leading level. On the other hand atmospheric neutrinos, if larger number of events will be available could also contribute to the precision measurements of the neutrino mass matrix, that are the goal of several projects that use long–baseline accelerator beams.

In this framework what appears to be the most interesting measurement is the determination of the mixing angle θ_{23}. The precise measurement of this parameter is of central importance because it could be the signature of symmetry, of potential deep significance for the understanding of the physics beyond the Standard Model. The comparison of the up and down–going rates of ν_μ and $\overline{\nu}_\mu$ in the multi–GeV range (when the directions of the detected muons and the parent neutrinos are well correlated) has provided the best determination of the angle (or better of the parameter $\sin^2 2\theta_{23}$). The error on this measurement is dominated by statistical uncertainties, and additional exposure of SK (or of a megaton extension of SK) should reduce the error on $\sin^2 2\theta_{23}$ as $\propto$ (Exposure)$^{-\frac{1}{2}}$.

A second method for determining the angle θ_{23} has been proposed in the last few years [7, 8, 9, 5], and relies on the study of flavor transitions driven by the [1–2] "solar" parameters. These oscillations generate $\nu_e \to \nu_{\mu,\tau}$ and $\nu_\mu \to \nu_e$ transitions that can be obseved as an enhancement of the e–like rate and an additional suppression of the μ–like rate if θ_{23} is in the first octant ($\theta_{23} < 45°$), and viceversa a suppression of the e–like rate and a reduced suppression of the μ–like rate if θ_{23} is in the seconf octant ($\theta_{23} > 45°$). The measurement of these effects allow not only to measure θ_{23} but also to solve the "octant ambiguity' that is present in all measurement that arte only sensitive to the mixing parameter $\sin^2 2\theta_{23}$.

The effects generated by the "solar oscillations" on atmospheric neutrinos are small, therefore they can be observed only using event samples that are several times larger that the the present (SKI) data set. However it has been shown [17, 18] that a 20 years exposure of SK can solve the octant ambiguity is θ_{23} is close to the edges of its allowed interval ($\theta_{23} \simeq 45 \pm 8$ degrees). The exposure of a megaton water detector would offer more interesting possibilities.

In case the third mixing angle θ_{13} is also close to its present upper limit, atmospheric neutrinos can also measure this parameter, and address the question of the hierarchy of the neutrino masses (or the sign of Δm^2_{23}). In this general case, in principle atmospheric neutrinos have also an interesting sensitivity to the CP violating phase δ.

To make the most of the additional data on atmospheric neutrinos it is essential to reduce the systematic errors on the calculated prediction. This represents a three–fold effort: (i) obtain an improved description of the primary cosmic rays, including the effects of solar modulations; (ii) collect and analyse additional data on hadronic interactions in accelerator experiments to improve the modeling of hadronic shower development, (iii) improve the description of the properties of neutrino interaction with nuclei. This last point can be realized with additional data on neutrino interactions in the near detectors of the planned LBL ν beams projects (and the K2K program has already collected valuable results). A better theoretical description of the nuclear effects is also important, and can be tested in electron scattering experiments. The significant (factor of 2 or better) reduction of all most significant systematic erors seems a realistic possibility.

With this reduction in systematic errors, a large detector exposure to the atmospheric neutrino fluxes would offer an important contribution to future studies of the fundamental properties of neutrinos, and to the measurements of their masses and mixing.

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