Issues on Neutrino-Nucleus Reactions in the Quasi-Free Delta Production Region

Ryoichi SEKI

Department of Physics and Astronomy, California State University, Northridge, Northridge, CA 91330, USA
and *W. K. Kellogg Radiation Laboratory, California Institute of Technology, Pasadena, CA 91125, USA*
Hiroki NAKAMURA
Department of Physics, Waseda University, Tokyo 169-8555, Japan

Abstract

A brief overview is presented of various issues involved in phenomenological and theoretical works on charge-current neutrino-nucleus reactions associated with the quasi-free $\Delta(1232)$ production. An assessment of the present status of the works is made with respect to the objective of this conference, Sub-Dominant Oscillation Effects in Atmospheric Neutrino Experiments.

1. Introduction

Above the quasi-elastic neutrino-reaction energy, the prominent process in the charge-current neutrino-nucleus reactions is the quasi-free $\Delta(1232)$ production. We present a brief overview of the present phenomenological and theoretical studies on the process and discuss what one could focus on for further progress. Note that higher-mass resonance and multi-pion productions, as well as a diffractive non-resonance background do contribute to the neutrino reaction cross sections in this region. They are needed to be included for a realistic description of the neutrino reactions in this region, and their treatment combined with the $\Delta(1232)$ production requires more refined consideration.

This presentation is not meant to be a comprehensive review, but a short overview based on the formalism described below, in which we believe various physics involved in the process emerges most clearly. We do not discuss, for example, work of Valencia/Granada School, which follows a different formalism by systematically applying a nuclear many-body theory starting from Fermi gas. The reader can find their work on electron scattering in [1], and on neutrino scattering in [2] and J. Nieves's talk in this conference.

The inclusive neutrino-nucleus (ν, ℓ) cross section is written in linear re-

sponse formalism as

$$\frac{d\sigma}{dE_\ell d\Omega_\ell} = \frac{k_\ell}{8(2\pi)^4 M_A^2 E_\nu} \int d^3\mathbf{p} F(\mathbf{p}, \mathbf{q}, \omega) |\mathcal{M}_{\nu N}|^2 , \tag{1}$$

where E_ν is the incident neutrino energy, E_ℓ and Ω_ℓ are the energy and solid angle of the scattered lepton, respectively, and M_A is the target nuclear mass. $\mathcal{M}_{\nu\mathrm{N}}$ is the invariant on-shell neutrino-nucleon scattering amplitude, depending on the Mandelstam variables (expressed in terms of the four momenta of the leptons and the nucleon). $F(\mathbf{p}, \mathbf{q}, \omega)$ includes all relevant information of the initial nuclear state and also the final-state interactions between the lepton and the final-state nucleus. F depends on $\mathbf{p}$, the momentum of the initial nucleon, and $\mathbf{q}$ and ω, the momentum and energy transfer to the nucleus, respectively.

$F(\mathbf{p}, \mathbf{q}, \omega)$ describes the response of the nucleus to the disturbance generated by the neutrino probe, $|\mathcal{M}_{\nu N}|^2$. Here, we are applying the widely used approximation that neutrino interacts only directly with the nucleon. (See also Subsection 3.1.) $F(\mathbf{p}, \mathbf{q}, \omega)$ is expressed in the form of two-particle Green's function as

$$\begin{aligned} F(\mathbf{p}, \mathbf{q}, \omega) &= \langle A | a_\mathbf{p}^\dagger a_{\mathbf{p}+\mathbf{q}} \delta(\hat{H} - E_A - \omega) a_{\mathbf{p}+\mathbf{q}}^\dagger a_\mathbf{p} | A \rangle \\ &\approx \frac{1}{2M_A} \int d\omega' P_h(\mathbf{p}, \omega') P_p(\mathbf{p} + \mathbf{q}, \omega - \omega') , \end{aligned} \tag{2}$$

where $a_\mathbf{p}$ and $a_\mathbf{p}^\dagger$ are the annihilation and creation operators of a nucleon, $|A\rangle$ is the target nucleus in the ground state, and E_A is its energy. The delta function (operator) ensures the final state of the process to be physical (on-shell). In the second step in Eq. (2), $F(\mathbf{p}, \mathbf{q}, \omega)$ is approximately factorized as a product of single-hole and single-particle Green's functions, $P_h(\mathbf{p}, \omega)$ and $P_p(\mathbf{p}, \omega)$, respectively. $P_h(\mathbf{p}, \omega)$ is (apart from a simple kinematic factor) the probability of finding a nucleon of the momentum $\mathbf{p}$ and removal energy ω in the nucleus, and is referred as the spectral function. $P_p(\mathbf{p}, \omega)$ is a similar quantity for adding a nucleon in the nucleus and contains information of the final-state interactions.

$|\mathcal{M}_{\nu N}|^2$ is the νN cross section apart from a kinematic factor, and

$$|\mathcal{M}_{\nu N}|^2 \propto \eta_{\mu\nu} T^{\mu\nu} , \tag{3}$$

where $\eta_{\mu\nu}$ is the leptonic tensor and $T^{\mu\nu}$ is the nucleonic tensor. For the quasi-elastic reaction, $T^{\mu\nu}$ is expressed as a product of the nucleon current. For the quasi-free Δ production, it is expressed as a product of the $N - \Delta$ transition currents J's:

$$T_{\mu\nu} = \sum_{spin} J_\mu^- J_\nu^+ \times (\mathrm{Breit - Wigner}) , \tag{4}$$

where (Breit-Wigner) describes the decaying state of $\Delta(1232)$. J's expressed as a linear combinations of the transition form factors C's

$$J_\mu^+ = \bar{U}^\rho(p')\Gamma_{\rho\mu}(C_3^V(-t), C_4^V(-t), C_5^V(-t), C_5^A(-t); p', p)u(p) , \qquad (5)$$

where U^ρ and u are the spin $3/2$ (Rarita-Schwinger) spinor and the nucleon spinor, respectively. The invariant principles dictates that Γ depends also on other form factors, but $C_3^V(-t)$ and $C_5^A(-t)$ are most dominant and closely studied.

We will first examine in Section 2 lepton-nucleon reactions, because they serve as an input for nuclear reaction calculations; and second, nuclear reactions in Section 3. The concluding remarks are presented in Section 4.

2. Input: Lepton-Nucleon Reactions

2.1. Electron scattering

Around 1980, Bodek et al.[3, 4] analyzed inclusive (e, e') SLAC data from p and D, and extracted p and n responses, W_1 and W_2, phenomenologically. W_1 and W_2 are related to the inclusive cross section through the well-known expression,

$$\frac{d^2\sigma}{d\Omega dE_\ell} = \sigma_{Mott}[W_2 + 2\tan^2\theta \cdot W_1] , \qquad (6)$$

where σ_{Mott} is the Mott scattering cross section. As this expression shows, W_1 and W_2 are inclusive quantities, including all other processes in addition to the $\Delta(1232)$ production.

Subsequently, exclusive, detailed data have been obtained at Jefferson Lab, and their analyses have been carried out by the use of phenomenological/theoretical models. The data and analyses have been recently reviewed by Burkert and Lee [5], which serves as a useful reference.

As described in the review, several theoretical models (approaches) have been considered:

- Unitary isobar model, MAID [6] and Jlab/Yeveran [7].

- Multi-channel K-matrix model, SAID [8].

- Dynamical model, Sato-Lee [9] and Dubna-Mainz-Taiwan [10].

All these models seem to have an excellent agreement with proton data of explicit single pion production observables in the region relevant to the $\Delta(1232)$ production. In addition to these theoretical models, we add a phenomenological model,

- H2/D2 model [11].

The H2 model was constructed using a large body of inclusive electron-proton data from SLAC [12], but was found to agree with new data from Jefferson Lab [13, 14] to better than 5 % as well. The D2 model is a fit to Jefferson Lab data [15] only.

Neutron (e, e') data are scarce and with larger uncertainties, as they have to be extracted from $D(e, e'p)$. New Jefferson Lab data are found in [16]. A new precision experiment BONUS (E03-012) is being planned at Jefferson Lab by measuring the recoil proton [Private communication with C. E. Keppel.] We note that new precision experiments are also being carried out at Jefferson Lab on deuterium [E02-109; Spokespersons, M. E. Christy and C. E. Keppel] and also on nuclei [E04-001 (Jupiter); Spokespersons, A. Bodek and C. E. Keppel.]

Overall, precise electron reaction data and their reliable analyses are (and are becoming) available, so as to determine the vector transition form factors rather well.

2.2. Neutrino Reactions

Following the detailed theoretical work of Adler [17] and the comprehensive review of Llewellyn Smith [18] around 1970, Rein-Sehgal analysis [19] in 1980's has been serving as the standard description of the resonance production processes. The single pion production process has been re-examined in the last several years [20, 21, 22]. The $N - \Delta$ transition form factors used in these works are similar to each other, but are different in detail. Single-pion production cross sections by the use of the form factors are usually compared to ANL [23] and BNL [24] data from the proton and deuterium targets. As the data are weighted with the neutrino flux energy distribution, they serve only as weak constraints on the form factors.

The different form factors that have been used do affect nuclear calculations and yield different cross sections, sometimes appreciably. The data were reported two decades ago. Clearly new measurements are critically needed by the use of improved technology, so as to determine the axial transition form factors reliably.

3.   Neutrino-Nucleus Reactions

Neutrino-nucleus reactions based on the present formalism are discussed by H. Nakamura in this conference, focusing on quasi-elastic scattering by incorporating the inclusive electron-nucleus scattering data. As most discussions are

also applicable to quasi-free Δ production, we will not repeat them here. Note that H. Nakamura's talk also includes some results in quasi-free Δ production.

Instead, here we discuss what we believe to be prominent issues in the treatments of nuclear structure and reactions in neutrino process. The issues are:

- Spectral functions.

- Final-state interactions.

- Exchange current.

- Other approaches.

3.1. Spectral Functions

As noted in Section 1, the initial nuclear state needed is the one-hole state of the target nucleus, as the hole state is generated by the neutrino knocking out a nucleon. The spectral function describes this state as a function of energy to remove a nucleon and of its momentum. The energy is not discrete because a hole state has a finite life time. Deeply bound states such as those of the s-shell have a quite broad distribution. The momentum distribution is also spread out, extending beyond the Fermi momentum and beyond the momenta involved in shell-model and mean-field calculations. This is because of short-range nucleon correlations of high-momentum components, generated by the short-distance nuclear interactions. These features are much different from those of simple Fermi gas model, which continues to be widely used in the Monte Carlo analysis of experimental data. Note that high-momentum nucleons tend to alter, for example, the angular distributions of neutrino-nucleus reactions, though they affect less the total cross sections.

When the removal energy is summed, the spectral function yields the momentum distribution of a nucleon in the nucleus. The high momentum components play an important role in neutrino-nucleus reactions in the GeV region. A useful general review of nuclear momentum distributions is found in [25].

The most detailed calculation of the spectral function has been carried out by O. Benhar and his collaborators for ^{12}C, ^{16}O, and other nuclei [26], based on a nuclear many-body calculation with correlated nuclear-state basis, combined with shell model and local density approximation. Cross sections calculated by the use of these spectral functions have been reported at the NuInt04 conference on neutrino scattering in comparison to the case of electron scattering [27]. Note that different recent calculations of the spectral functions are available, for example, by Ciofi degli Atti and his collaborators based on a simpler but more physical approach [28].

3.2. Final State Interactions

For the outgoing nucleon, eikonal approximation has been applied within an optical potential description [29] to inclusive neutrino-nucleus reactions [30]. The present application remains to be of a simple estimate, and should be improved.

Monte Carlo simulation codes used in data analysis are basically a classical description, and they, as well as the eikonal calculations, need to include nuclear medium corrections in nucleon-nucleon cross sections. The corrections are quite substantial in low-energy nucleon-nucleon scattering [31], which is involved in the final-state interactions of low-momentum transfer (to the nucleon).

Let us note here on the occasionally raised question of how to treat the final-state interactions by differentiating the inclusive process (no outgoing nucleon is measured) and the semi-inclusive process (the outgoing nucleon is measured): The difference in two theoretical treatments is clearly understood in the optical potential description [32].

For the outgoing pion, a Monte Carlo algorithm, originally developed for pion physics [33], is currently in use in the Monte Carlo simulation codes and is under a good control. Note, however, that a low-energy pion in nuclear medium receives a strong dispersive effect (described by the real part of the pion-nucleus optical potential) in addition to an absorptive (collision) effect (described by the imaginary part), as also noted in [33]. The former is significant for low-energy pions, for which the cross sections are strongly energy-dependent, and should be included in Monte Carlo simulation.

3.3. Exchange Current

Almost all high-energy neutrino-nucleus calculations and Monte Carlo simulation codes do not account for effects associated with exchange currents in nuclei. Thus, for example, off-mass/energy shell contributions are not included in them, and theoretically most importantly, the current conservation is violated. Proper inclusion of the effects is difficult, and is not done satisfactorily even in electron scattering works.

Physics of this issue may play, however, a more important role in high energy neutrino reactions than in corresponding electron reactions, because of the axial currents. Quenching of 20 - 30 % in $g_A(GT)$ is well known in beta decays, and such effect has been incorporated in some of solar neutrino calculations. In high-energy neutrino reactions, we may have a strong modification of the form factors themselves beyond that of the coupling constants. This issue has been studied previously [34], but would deserve a closer examination. It is a complicated

matter, as pion dynamics in nuclei has to be carefully sorted out, together with the parallel view of a possible modification of nucleon structure in nuclei. Note that a series of the many-body theory investigation of the axial form factors in nuclei has been made by D. Riska and his collaborators [35].

In this connection, we emphasize that the rigorous determination is vital of the axial form factors of the nucleon in free space. There is an urgent need for more detailed, reliable neutrino scattering experiments from the proton and deuterium. Furthermore, the exchange effects in neutrino-deuterium reactions perhaps should be re-examined for the extraction of the axial form factors, as the existing work is nearly two decade old [36].

3.4. Other Approaches

- We have already noted work of Valencia/Granada School, which uses a different many-body theory approach, incorporating many of comments made here. Please see J. Nieves's talk in this conference.

- T. W. Donnelly , I. Sick, and their collaborators proposed a (super)scaling approach [37] based on the observation that the simple Fermi gas model works fairly well in electron scattering. An extension to neutrino scattering has been recently reported [38]. In this approach, it has not been clarified what physics is included in the modifying factor of the Fermi gas model, and thus how reliable the extension to neutrino scattering is from electron scattering, because the currents involved in the two processes are different.

4. Summary

We conclude this presentation by briefly assessing the present status of works in the Delta region by listing what we consider to be its most important aspects:

- Quality: Presently available calculations and codes include most of important physics at various degrees, with the exception of the physics associated with the current conservation.

- Predictability: More tuning of the calculations is desirable to precise electron-scattering data currently available or becoming available.

- Experiment: More precise neutrino-nucleon data are critically needed.

- Codes: It is perhaps the best time for upgrading the existing codes to create codes of the next generation by including all available nuclear-physics

information and knowledge that have been accumulated, for detailed examination of atmospheric neutrino experiments and for the upcoming high quality experiments such as those at J-Parc and Fermi Lab.

References

[1] A. Gil, J. Nieves, and E. Oset, Nucl. Phys. A627 (1997) 543; and references therein.

[2] J. Nieves, J. E. Amaro, and M. Valverde, nucl-th/0408008.

[3] A. Bodek et al., Phys. Rev. D20 (1979) 1471.

[4] A. Bodek and J. L. Richie, Phys. Rev. D23 (1981), 1070.

[5] V. D. Burkert and T.-S. H. Lee, nucl-ex/0407020.

[6] D. Drechsel, S. S. Kamalov, and L. Tiator, Nucl. Phys. A645 (1999) 145.

[7] I. G. Aznauryan, Phys. Rev. C67 (2003) 015209; ibid., C68 (2003) 065204.

[8] R. A. Arndt et al., Phys. Rev. C52 (1995) 2120; Int. J. Mod. Phys. A18 (2003) 449.

[9] T. Sato and T.-S. H. Lee, Phys. Rev. C54 (1996) 2660; ibid., C63 (2001) 055201.

[10] S. S. Kamalov and S. N. Young, Phys. Rev. Lett. 83 (1999) 4494; and D. Drechsel, O. Hanstein, and L. Tiator, Phys. Rev. C64 (2002) 032201.

[11] C. E. Keppel, http://hallcweb.jlab.org/resdata/ and private communication with M. E. Christy.

[12] C. Keppel, PhD thesis (American Univ.), SLAC-R-694 (1994).

[13] I. Niculescu et al., Phys. Rev. Lett. 85 (2000) 1186.

[14] Jefferson Lab Hall C E94-110 Collaboration, nucl-ex/0410027.

[15] I. Niculescu et al., Phys. Rev. Lett. 85 (2000) 1182.

[16] A. V. Klimenko, PhD thesis (Old Dominion Univ., 2004).

[17] S. L. Adler, Ann. Phys. (N. Y.) 50 (1968) 189.

[18] C. H. Llewellyn-Smith, Phys. Rept. 3 (1972) 261.

[19] D. Rein and L. Sehgal, Ann. Phys. 133 (1981) 79; and D. Rein, Z. Phys. C35 (1987) 43.

[20] L. Alverez-Ruso, S. K. Singh, and M. J. Vicente Vacas, Phys. Rev. D69 (2004) 014013.

[21] T. Saito, D. Uno, and T.-S. H. Lee, Phys. Rev. C67 (2003) 065201.

[22] E. A. Paschos, L. Pasquali, and J.-Y. Yu, Nucl. Phys. B588 (2000) 263; and E. A. Paschos, J.-Y. Yu, and M. Sakuda, Phys. Rev. D69 (2004) 014013.

[23] S. J. Barish et al., Phys. Rev. D19 (1979) 2521.

[24] T. Kitagaki et al., Phys. Rev. D34 (1986) 2554.

[25] A. N. Antonov, P. E. Hodgson, and I. Zh. Petkov, *Nucleon Momentum and Density Distributions in Nuclei* (Clarendon Press, Oxford, 1988).

[26] Private communication with O. Benhar; O. Benhar, A. Fabrocini, S. Fantoni, and I. Sick, Nucl. Phys. A579 (1994) 493; and references therein.

[27] O. Benhar (in *Proc. Third Int. Workshop on Neutrino-Nucleus Int. in the few GeV region*), Nucl. Phys. B.–Proc. Suppl. 139 (2005) 15; H. Nakamura, R. Seki and M. Sakuda, ibid. 139 (2005) 201; and O. Benhar and N. Farina, ibid. 139 (2005) 230.

[28] C. Ciofi degli Atti, D. B. Day, and S. Liuti, Phys. Rev. C46 (1992) 1045.

[29] O. Benhar et al., Phys. Rev. C44 (1991) 2328.

[30] H. Nakamura, this proceedings; and O. Benhar, private communication.

[31] V. R. Pandharipande and S. C. Pieper, Phys. Rev. C45 (1992) 791.

[32] Y. Horikawa, F. Lenz, and N. C. Mukhopadhyay, Phys. Rev. C22 (1980) 1680.

[33] L. L. Saalcedo, E. Oset, M. J. Vicente Vacas, C. Garcia Recio, Nucl. Phys. A484 (1988) 557.

[34] S. K. Singh and E. Oset, Nucl. Phys. A5422 (1992) 587.

[35] M. Kirchbach and D. O. Riska, Nucl. Phys. A578 (1994) 511; M. Hjorth-Jensen et al., Nucl. Phys. A563 (1993) 525; and D. O. Riska, Phys. Rep. 181 (1989) 208.

[36] S. K. Singh and H. Arenhövel, Z. Phys. A324 (1986) 347.

[37] C. Marieron, T. W. Donnelly, and I. Sick, Phys. Rev. C65 (2002) 025502.

[38] J. E. Amaro et al., nucl-th/0409078.