Uncertainties in HKKM Calculation

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Abstract

The uncertainties in the calculation of atmospheric neutrino flux are discussed based on HKKM04 [1]. The uncertainties are classified in 2 categories; one is those inherited from the inputs, and the other introduced by the calculation processes. We summarize the second kind uncertainties first, then discuss the first kind ones, using the accurately measured muon flux data. In 1 – 10 GeV region, the uncertainty for the absolute value is estimated as $\sim 10\%$, and less than 1 – 2 % for the $(\nu_\mu + \bar{\nu}_\mu)/(\nu_e + \bar{\nu}_e)$ ratio.

1. Introduction

The uncertainties in the calculation of atmospheric neutrino flux are classified in two categories; the first kind uncertainty is the ones inherited from the inputs, such as the uncertainties from the the primary flux and the interaction model. The second kind uncertainty is the ones introduced in the calculation processes, such as the approximations in the calculations, and the statistical errors.

We consider the atmosphere model as the source of second kind uncertainty, since the air density structure is measured by the meteorological office and huge world wide data are available now. Essentially the second kind uncertainties could be reduced with a sophisticated simulation code and a fast computer system.

The second kind uncertainties are partly discussed in HKKM04. For the statistical errors and the size of simulation sphere, we estimated they are smaller than 1–2 % for $E_\nu \lesssim 10\%$. Here, we study the second kind uncertainties from the large size virtual detector and the atmosphere model. Then, the first kind uncertainties are discussed using the accurately measured muon flux data [5, 4, 6]. Thanks to the precise measurements by AMS and BESS, the uncertainty of the primary cosmic ray fluxes below 100 GeV is considerably reduced ($\sim 5\%$). Therefore, the precise muon flux data would be a good calibrator for the hadronic

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interactions below 100 GeV.

2. Uncertainties Introduced by the Large Virtual Detector

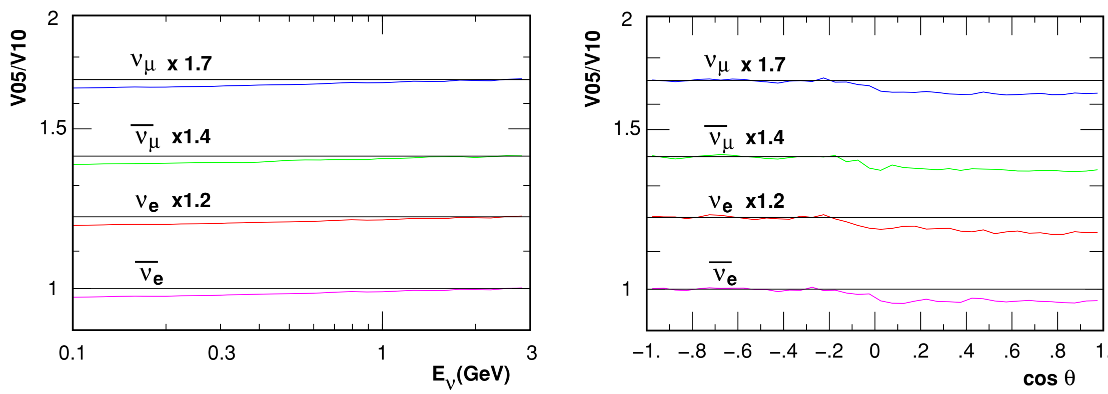

Fig 1. Left panel: Ratio of all direction averaged flux with smaller virtual detector (V05) to that with the regular virtual detector (V10). Right panel: Zenith angle dependence of the ratio at $E_\nu = 0.3$ GeV.

To examine the effect of the large size virtual detector, we take the statistics with a smaller size concentric virtual detector (V05, $R = 558$ km, 5° from the center of the Earth) and compared the results with that of regular virtual detector in HKKM04 (V10, $R = 1117$ km, 10° from the center of the Earth). The comparison of the neutrino fluxes determined by the virtual detectors V10 and V5 is shown in Fig. 1.

We find the differences are $\lesssim 2$ % in all directional average for $E_\nu \gtrsim 0.1$ GeV, and $\sim$ 0.8 % for $E_\nu \gtrsim 1$ GeV. The "true flux" would be obtained in the limit $R_d \to 0$. However, if the center is not a singular point of the neutrino flux, the difference to the true value would decrease as $O(R_d^2)$ with R_d. We estimate the error calculated with the regular virtual detector in HKKM04 is $\lesssim$ 2.5 % for $E_\nu \gtrsim 0.1$ GeV, and ~ 1 % for $E_\nu \gtrsim 1$ GeV in all direction average.

The difference between the atmospheric neutrino fluxes calculated with V10 and V05 are almost constant for down going directions ($\cos\theta > 0$) and are negligibly small for upward going directions ($\cos\theta < 0$). Therefore, the zenith angle dependence of the error would be simple, when the flux is averaged over the azimuth angles. It is ~ 5 % at $E_\nu = 0.1$ GeV and ~ 2 % at $E_\nu = 1$ GeV for downward going directions, and is negligibly small for the upward going directions. It is still difficult to discuss the the azimuth angle dependence of the error with the statistics we have. It could be larger than above estimation.

3. Uncertainties Due to the Atmosphere Model

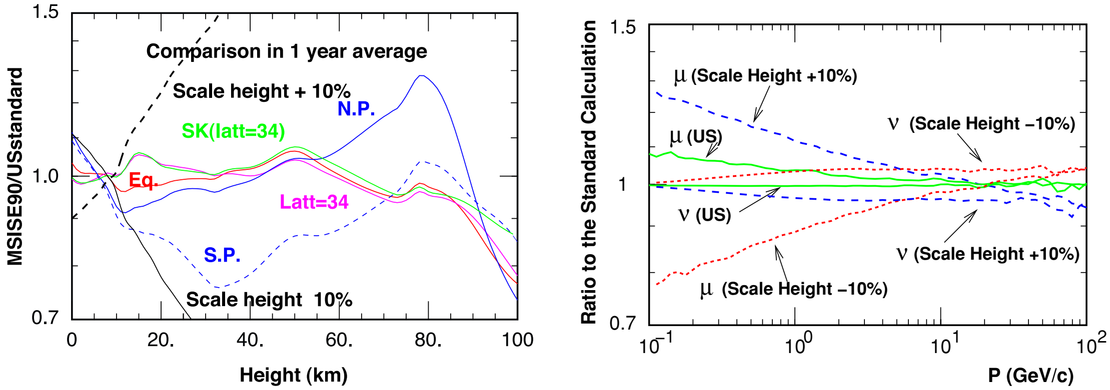

Fig 2. Left panel: Ratio of air-densities to the US-standard '76[7]. Right panel: Ratio of atmospheric muon and neutrino fluxes calculated with different atmosphere models to those of the calculation based on the MSIS atmosphere model.

In left panel of Fig. 2, we show the comparison of US-standard '76 atmosphere model [7], and a little more realistic model MSIS [8], which included the latitude and seasonal variations of atmosphere. In the right panel, we show the corresponding the variations of the fluxes of atmospheric muons and neutrinos at the ground. We find the US-standard '76 atmosphere is not so bad as the one year average. Also the variation of atmospheric neutrino flux is small for the change of the atmosphere model even with the change of the scale height by $\pm$ 10 %. We may conclude that the US-standard '76 atmosphere model does not introduce a sizable uncertainty to the one-year-average atmospheric neutrino fluxes.

The flux of atmospheric muons at the ground level varies largely with the change of atmosphere model. This is a crucial feature for the atmospheric muons, since we are going to study the hadronic interactions with them. We use the air density structure measured by the meteorological office, in the followings analysis.

The change of air density structure works differently for muons and neutrinos. At low energies, the air density structure mainly related to the muon flux through the muon decay. Therefore, the muon flux decreases with the increase of the scale height. For the neutrinos, the dependence of the pion interaction – decay competition on the air density works effectively even at the low energies. Therefore, the neutrino flux increases with the increase of the scale height.

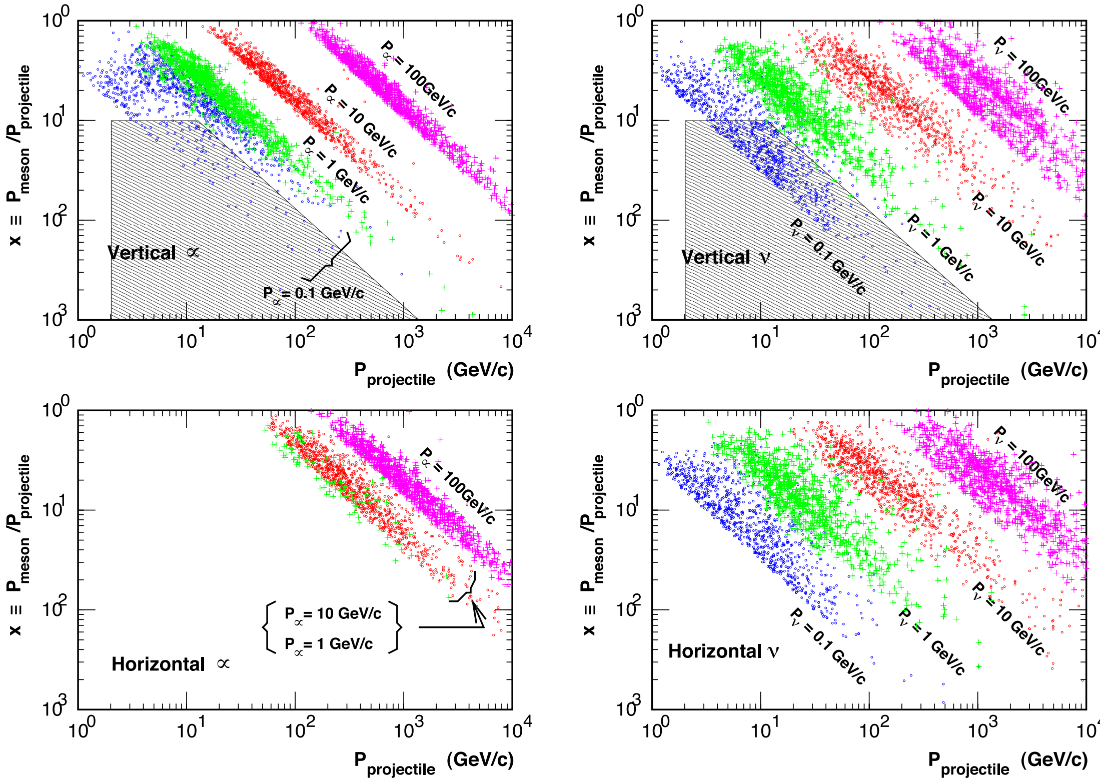

Fig 3. The meson distributions in the $(P_{projectile}, x)$ plane of the cosmic ray – air nuclei hadronic interactions relevant to the atmospheric muons and neutrinos with fixed momentums. The left upper panel is for vertical muons, right upper panel for vertical neutrinos, left lower panel for horizontal muons, and right lower panel for horizontal neutrinos.

4. Phase Space in Hadronic Interaction Relevant to Muons and Neutrinos

In Fig. 3, we plotted the phase space of mesons in the cosmic ray – air nuclei hadronic interactions, relevant to the muons and neutrinos with fixed momentums, $P_{\mu,\nu} = 0.1, 1.0, 10, 100$ GeV/c at ground. Note, the distributions are narrow strips for muons with $P_\mu \gtrsim 1$ GeV/c. The meson distributions for neutrino with $P_\nu \gtrsim 1$ GeV/c are wider than those for muons, but could be well reconstructed by the superposition of the meson distribution for muons with neighboring momentums. The meson distributions for the horizontal neutrinos are similar to those for vertical neutrinos, then they could be also reconstructed by the superposition of the meson distribution for muons.

For the muons with $P_\mu \lesssim 1$ GeV/c, The meson distribution is largely deformed from that of higher energies. The peak of the distribution moves slowly to the lower projectile momentum, but the distribution has a large overlap with that for muons with $P_\mu = 1$ GeV/c. This is understood by the fact that most of the muons observed with this momentum at the ground are actually produced with higher momentums ($\gtrsim 1$ GeV/c) at the higher altitude, but they lose the energy in air before reaching the ground. The muons created by the mesons in the hatched area decay quickly, and mostly do not reach the ground.

The meson distribution for neutrinos with $P_\nu \lesssim 1$ GeV/c movies to lower projectile momentum without a large modification of the shape. Therefore, the meson distributions for the neutrino in these momentums are not reproduced by the superposition of those for muons. The interactions relevant to the neutrinos with $P_\nu \lesssim 1$ GeV/c are not calibrated by the muon flux at the ground level.

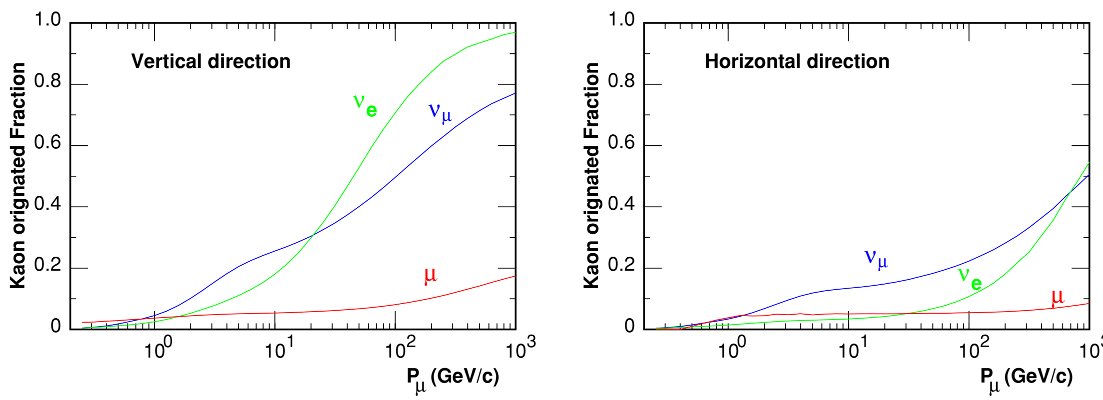

Fig 4. The K-meson contribution to the vertical muons and neutrinos(left panel), and to the horizontal muons and neutrinos (right panel).

There is another limitation at higher energies, since the contribution of K-meson is largely different for muons and neutrinos. In Fig. 4, we depicted the contribution of K-mesons to the atmospheric muons and neutrinos as the function of particle energies. At 100 GeV/c, the K-meson contribution is more than 50 % to the neutrinos, while it is still less than 10 % to the muons, for vertical directions. In Fig. 3, we find 2 clusters in the meson distribution for 100 GeV/c neutrinos. Each cluster stands for pion and K-meson contributions.

We may conclude that the hadronic interactions relevant to the atmospheric neutrinos in $1-10$ GeV/c are well calibrated by the muons in $1-30$ GeV/c at the ground level. Note, the momentum range for muons is wider than that for neutrinos, since the meson distribution for neutrinos is wider than that for muons. We need a new data or a theoretical idea, for the study of hadronic interaction relevant to the neutrinos outside of 1 – 10 GeV/c.

5. Uncertainties Due to The Interaction Model

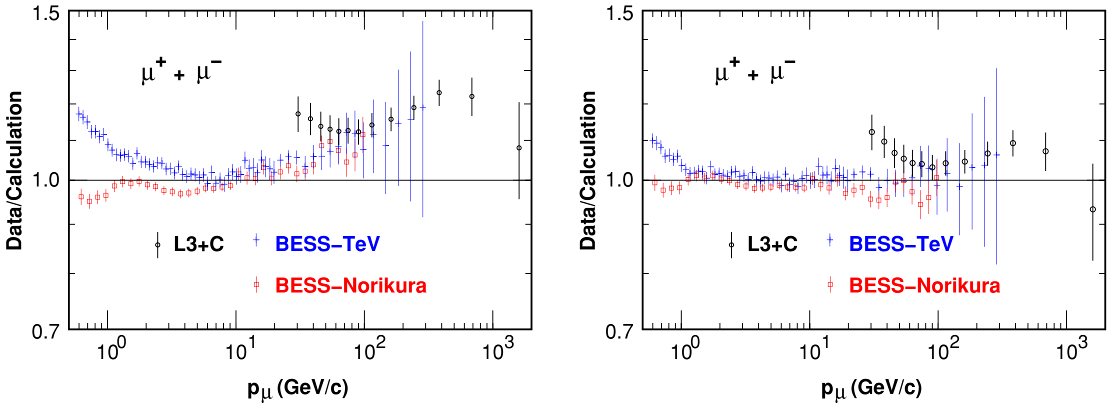

Fig 5. The ratio of observed muon fluxes to the calculated ones, for HKKM04 (left panel) and with modified interaction model (right panel).

In the left panel of Fig. 5, the calculated muon fluxes by HKKM04 are compared with the accurate measurements at Tsukuba (BESS-TeV) [4], Mt. Norikura(BESS) [5], and CERN (L3+C) [6]. Note, the muon fluxes are calculated for different sites separately, but only the ratios are plotted in Fig. 5. The observation at Mt. Norikura was carried out at 2770m asl, but the problem of muon flux for $\lesssim$1 GeV/c still remains at this altitude.

We find the agreement of the calculated muon fluxes by HKKM04 with the observed ones are $\lesssim$ 5% in 1 – 30 GeV/c region. Therefore, we may consider the uncertainty of HKKM04 in 1 – 10 GeV is less than 10 % in the absolute value, assuming the experimental errors of $\sim$ 5 % in the muon observations. However, the difference is larger for $P_\mu \lesssim$ 1 GeV and $P_\mu \gtrsim$ 30 GeV. Note, the large differences seen for Tsukuba and Norikura observations at lower momentums are due to the air density structure.

We have modified the DPMJET-III, to get a better agreement between calculations and observations. As the modification we have changed the average energy of the secondary mesons which have the same valence quark as the projectile. The change rate is determined for each kind of quark as a function of the projectile energy. We also assume the iso-symmetry that the change rates for u-quark and d-quark in $p + air$ interactions are the same as those for d-quark and u-quark in $n + air$ interactions respectively. In $p, n + air$ interactions, the change rates for π^+ and k^+ are the same, that for k^0 is the 1/2 of π^-, and no change is applied to K^-. The nucleon average energies are also changed to balance the total energy. The variation rates are tuned to minimize the difference between calculations and observations.

In the right panel of Fig. 5, the muon fluxes calculated with the modified interaction model are compared with the observed ones, showing a better agreements in 1 – 300 GeV/c. In this calculation, we used the air density structure measured by the meteorological office, then the difference between Tsukuba and Norikura becomes smaller.

6. Uncertainties in Ratio of Different Kind Neutrinos

It is well known that the ratio $(\nu_\mu + \bar{\nu}_\mu)/(\nu_e + \bar{\nu}_e)$ is very stable at $\simeq 2$. This is due to the fact that the most of the neutrinos below 10 GeV are created by the $\pi - \mu$ decay, and is independent of the hadronic interaction model.

$$\begin{array}{ll} \pi^\pm \rightarrow & \mu^\pm + \nu_\mu(\bar{\nu}_\mu) \\ & \downarrow \\ & e^\pm + \nu_e(\bar{\nu}_e) + \bar{\nu}_\mu(\nu_\mu) \end{array} \tag{1}$$

In the μ-decay, the ν_e's carries a little larger energy than ν_μ's due to the muon spin effect, but the average energy ratios are fixed among 3 particles. However, the ratios of the μ-decay products' average energies to that of ν_μ's from π-decay vary a little with the air density, due to the energy loss of μ's in air. This is a possible source of the uncertainty for the $(\nu_\mu + \bar{\nu}_\mu)/(\nu_e + \bar{\nu}_e)$ ratio in this energy region.

$\nu_e/\bar{\nu}_e$ is determined only by μ^+/μ^- at decay, then it is sensitive to the π^+/π^- production ratio in the hadronic interactions. $\nu_\mu/\bar{\nu}_\mu$ varies with the interaction model and the air-density. However, the magnitude of each dependence is smaller than that of $\nu_e/\bar{\nu}_e$ or $(\nu_\mu + \bar{\nu}_\mu)/(\nu_e + \bar{\nu}_e)$, since the difference is diluted by the $\nu_\mu(\bar{\nu}_\mu)$ created in the π-decay.

In Fig. 6, we plotted the variation of neutrino ratios, by the change of atmosphere model, and by the modification of the interaction model. Even with the variation of the scale height of atmosphere by ± 10 %, the variation of $(\nu_\mu + \bar{\nu}_\mu)/(\nu_e + \bar{\nu}_e)$ is small. Also the difference between original and modified interaction models is very small as expected. We may conclude that the uncertainty of $(\nu_\mu + \bar{\nu}_\mu)/(\nu_e + \bar{\nu}_e)$ in HKKM04 is smaller than 1 – 2 % for $P_\nu \lesssim$ 10 GeV/c.

The difference of muon charge ratio between HKKM04 and the observation is $\sim$ 5 %, then we conclude the uncertainties are also $\sim$ 5 % for $\nu_e/\bar{\nu}_e$, and $\sim$ 3 % for $\nu_\mu/\bar{\nu}_\mu$ in 1 – 10 GeV/c. Note, the muon charge ratio at Norikura (2770m asl) is a little lower than that at Tsukuba ($\sim$ sea level). The energy loss of muons in air between two altitudes ($\sim$ 0.5 GeV, 250 g/cm^2) shift the same charge ratio point to the lower momentum at Tsukuba. At high energies, there may be lager uncertainties in the neutrino ratios, due to the K-meson contributions.

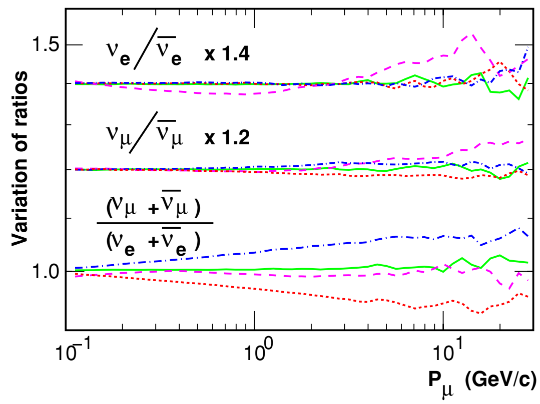

Fig 6. The variation of ratios between different kind neutrinos. Is shown the ratio to the HKKM04 calculation scheme with the measured air density structure. The solid lines shows that of HKKM04 calculation with US-standard '76, and dashed lines the calculation with modified interaction model. The dotted and dash dot lines show the variation with the air scale height changes by ± 10 %.

7. Summary

We have studied the systematic uncertainties in the calculation of atmospheric neutrino fluxes based on HKKM04. The second kind uncertainties introduced by the calculation are reasonably small, including the uncertainties come from atmosphere model, and the large size virtual detectors.

As the primary cosmic ray fluxes are measured accurately below 100 GeV, the main source of the first kind uncertainty is the hadronic interaction. From the comparison of the distributions of the mesons in $(P_{projectile}, x)$ plane, we find the hadronic interaction relevant to atmospheric neutrinos in 1 – 10 GeV/c, can be calibrated using the muons in 1 – 30 GeV/c at the ground level. From the difference of calculated and observed muon fluxes, we may conclude that the uncertainties of the atmospheric neutrino fluxes are around 10 % for the absolute values in 1 – 10 GeV/c in HKKM04. Note, the second kind uncertainties are far smaller than the ones introduced by the interaction model.

The ratio $(\nu_\mu + \bar{\nu}_\mu)/(\nu_e + \bar{\nu}_e)$ is almost free from the first kind uncertainties, and the uncertainty is estimated less than 1 – 2 % in HKKM04. The ratio $\nu_\mu/\bar{\nu}_\mu$ and $\nu_e/\bar{\nu}_e$ are affected by the first kind uncertainties. However, from the comparison of muon charge ratio, the uncertainties are estimated $\lesssim$ 5 % for $\nu_e/\bar{\nu}_e$ and $\lesssim$ 3 % for $\nu_\mu/\bar{\nu}_\mu$ in 1 – 10 GeV/c.

For the muon fluxes $\gtrsim$ 30 GeV/c, HKKM04 fails to reproduce the observed ones within the observation errors. Then a modification of the interaction model is introduced with an assumption based on the quark model. With this modification,

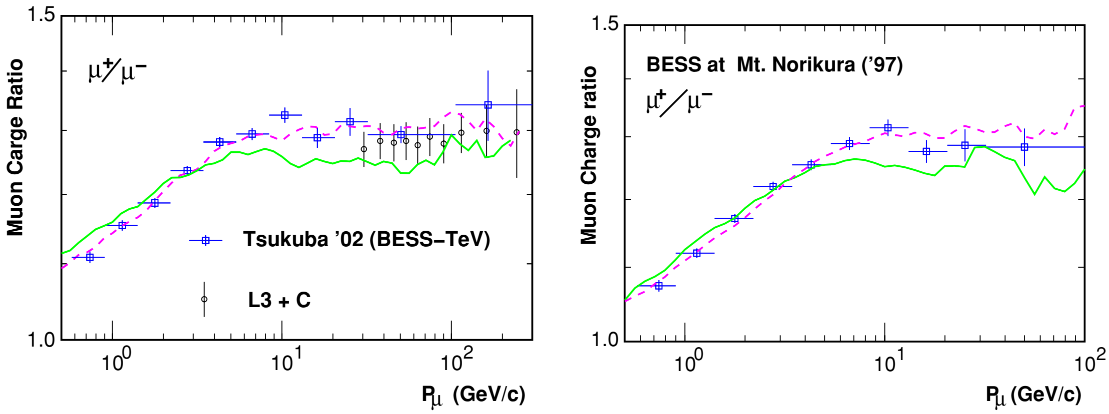

Fig 7. The comparison of calculated muon charge ratio with the observed ones for Tsukuba (left panel) and Norikura (right panel). The sold line show muon ratio with HKKM04, and dashed line show that with the modified interaction model.

the agreement of calculation and data for the muon flux becomes better in a wider momentum range, 1 – 300 GeV/c. The atmospheric neutrino fluxes are increased around 10 % at 100 GeV, and 15 % at 1 TeV with the extension of the modification to higher energies. However, the changes remain $\lesssim$ 5 %, below 10 GeV.

8. Acknowledgments

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