Flux Calculation Results and Systematic Errors in BGLRS

Giles BARR
University of Oxford, Department of Physics, Denys Wilkinson Building, Keble Road, Oxford, OX1 3RH, United Kingdom.

Abstract

Neutrinos from cosmic rays have already been a powerful tool for measuring the properties of neutrinos and the prospects of a larger detector in the future promises even more sensitive measurements. To fully extract the particle physics from this powerful natural source will require a careful understanding of the expected unoscillated fluxes. This paper takes a detailed look at some of the components of one of the calculations of the atmospheric neutrino fluxes.

1. Introduction

There are now about eight different groups who have made a Monte-Carlo calculation of the expected fluxes of neutrinos from cosmic ray induced showers in the atmosphere. These calculations have grown quite sophisticated, since the Earth's magnetic field breaks the (almost) spherical symmetry of the planet. Consequently, detailed calculation must consider cosmic ray showers at all points and in all orientations over the surface of the globe. Neutrinos which do not pass near the detector region are discarded. This makes the practical challenges of such a calculation quite large and several careful considerations or 'tricks' are needed to make the calculation complete in a reasonable time.

The challenges posed by the exciting new possibilities with a mega-ton scale water Cerenkov detector as described elsewhere in these workshop proceedings present us with the opportunity to develop really sophisticated techniques for removing the uncertainties in the atmospheric neutrino flux.

This paper presents some of the internal details and checks which have been made with one of the calculations, the 'Bartol Flux Calculation'. Although the underlying particle production generator, TARGET has always been three-dimensional in nature, the atmospheric transport part of the calculation has until recently used a one-dimensional approximation in which all particles in the shower are rotated to travel along the same direction as the primary particle. Since only particles traveling on a direct line towards the detector need be considered in

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this approximation, the calculation was substantially less demanding in terms of computer time. The transportation code has recently been upgraded to remove this approximation and results are now available from the three-dimensional calculation [1].

This paper first describes some of the details of the techniques used in the 3-dimensional version of the calculation. Other details are given in ref. [1]. As this is a specialized workshop more technical issues than usual are aired here. This is followed by a description of some of the features of the fluxes. The final section discusses the sensitivity of the calculation to some of the most important external inputs, the atmospheric density model, the primary cosmic ray flux and details of the hadronic interactions.

2. Calculation Details

The calculation begins with particles injected at a height of 80 km above the surface of the Earth. These are then tracked forwards and backwards from this point using differently adapted code. The forward tracing involves deciding at what depth (in g/cm^2) this particle will interact and then stepping through the atmosphere applying energy loss, bending in the geomagnetic field and accumulating the depth traversed until the interaction point is reached. Secondary particles are tracked using the same code and for mesons and muons, the possibility of them decaying is included in a similar manner to the interactions but with a counter representing the internal time-to-decay in the particle centre of mass frame. Relatively large steps are permitted because, at the final particle decay or interaction, correct accounting of the fractional remaining step is included. To avoid rounding errors involved with a coordinate system with origin at the centre of the Earth, a local coordinate system starting at the injection point is used during the forward tracking. The variable which is stepped is the particle height and at the end of each step, the local zenith angle, grammage and displacement vector are computed. The step size is usually 300 m: for particles below 10 GeV the step size is reduced to 30 m when they are within 10 km of the Earth's surface (where the density is higher) and all particles are tracked with 30 m steps once their energy goes below 200 MeV in the region where ionisation loss is high.

The backward tracking is performed to determine if the primary trajectory is above or below cutoff [1]. Here, the centre of the Earth is used as origin and the algorithm for choosing the step size decides at each step depending on the local track curvature in the geomagnetic field (so that the overall deviation within a step is around 5 mrad).

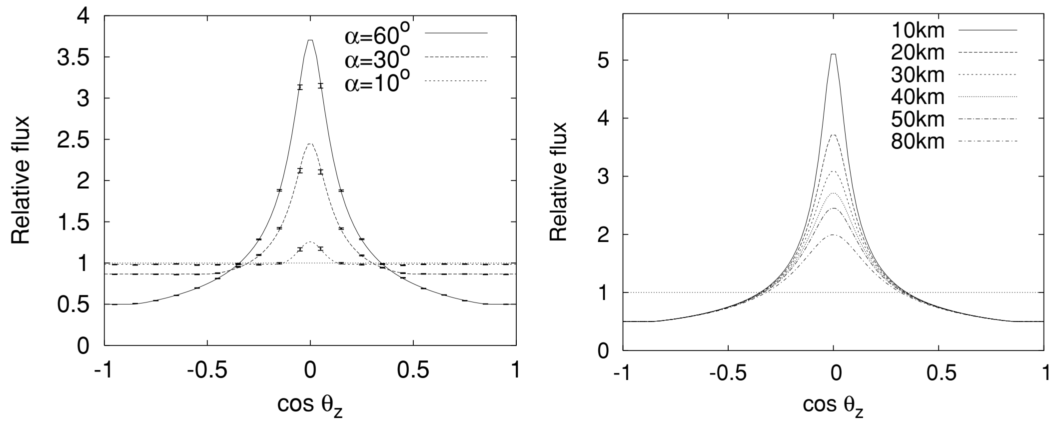

Fig 1. Geometrical effects associated with the 3-D atmospheric neutrino calculation (from [3]). The left plot shows the effect when a 'bend' of 10°, 30° and 60° is introduced in the shower at an altitude of 20 km. The points indicate that the Monte-Carlo calculation can reproduce these effects. The right plot shows how the effect depends on the altitude where the bend occurs (for a fixed 60° bend).

3-D effects

The removal of the 1-dimensional approximation causes a strong change in the fluxes at low energy near the horizon. These effects were first seen by the FLUKA group [2] and explained in terms of geometric effects by Lipari [3]. Figure 1 shows the effect when a single fixed angle bend is introduced into the path of the cosmic ray and it's descendents at a fixed altitude using an analytic expression from [3]. The effect increases the closer the bending occurs to the Earth's surface. The effect is important near the horizon where there is a large angle between the primary cosmic ray and the neutrino, i.e. at low energy.

The above discussion shows the necessity for removing the 1-dimensional approximation from the calculation. This requires that particles be generated all over the globe in all directions and then the ones which hit the detector accumulated. The next sections of this paper describe the considerations for how the detector may be artificially enlarged in the calculation to save computer time, how large the detector may practically be and considerations for the 'detector angle' weight.

One of the new components of the calculation when working in three dimensions is the details of how to collect particles at the detector site. Two techniques were developed and shown to give consistent results. The main technique uses an artificially enlarged spot following the surface of a sphere to represent the detector. This sphere is centered at the centre of the Earth with a radius of

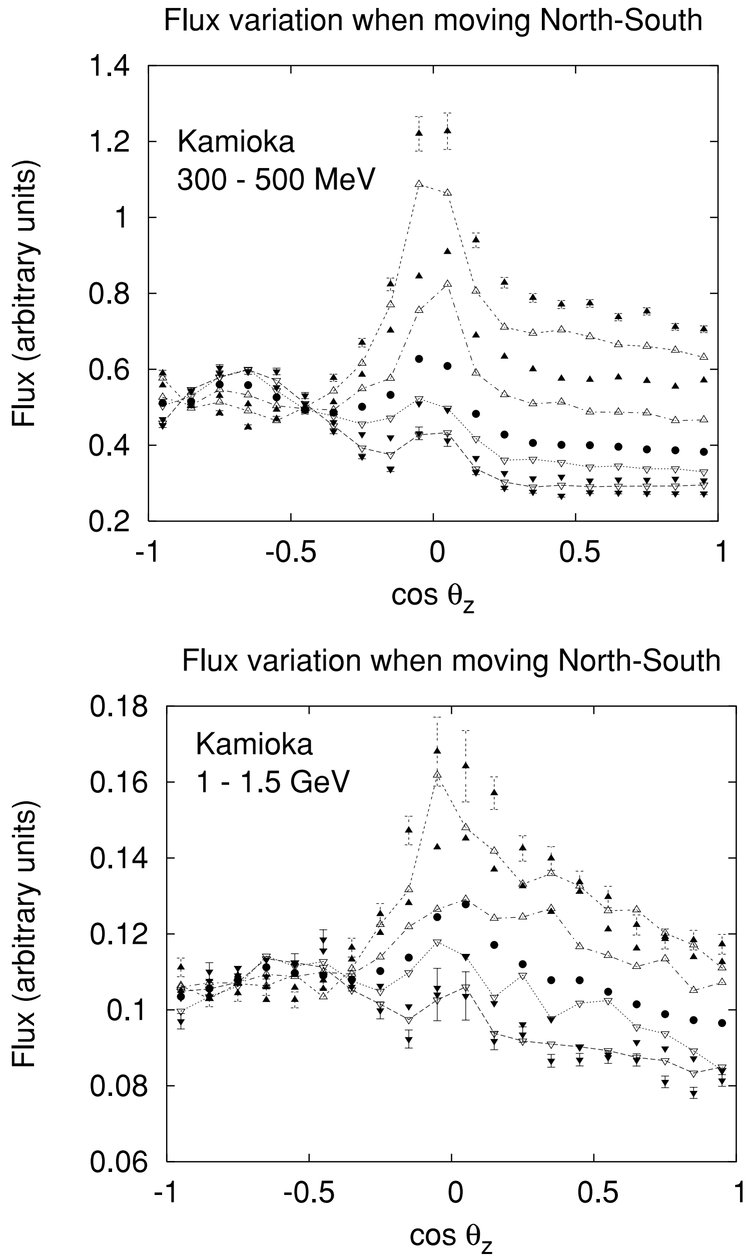

Fig 2. Effect of moving the detector centre in the North-South direction by up to 2000 km from Kamioka in steps of 500 km. Top: 300–500 MeV neutrino energy. Bottom: 1–1.5 GeV. The points at ±500 km and ±1500 km are joined by a line to guide the eye. The intermediate points are at 0 km and ±1000 km and ±2000 km. Error bars on the ±2000 km points represent the errors on the other displacements as well; except for the 0 km points (solid circles) where four times more statistics are used. Upward pointing triangles represent a displacement to the North and downward pointing triangles a displacement to the South.

$R_\oplus + a$ where a is the detector altitude with the spot centered at the detector location. Any neutrinos which extrapolate to intersect this spot are counted as hitting the detector.

The second technique was used as a cross check. It considers the detector as a point and computes the angle between the neutrino direction and the direction of the line joining the neutrino production point with the detector. Provided this angle is below 5°, the neutrino is accepted and weighted according to the distance between the production point and the detector. The technique is very useful for upward going neutrinos, but the downward going ones suffer from large statistical fluctuations.

Detector size

For the main technique, a study has been made to determine how large the spot can be without introducing a large error in the calculation. The maximum spot size is determined by how much the downward fluxes vary due to the changing local geomagnetic latitude. Since the real detector location is centered on the spot, a linear change across the spot does not affect the total flux collected in the spot; a deficit on one side is compensated by an increase on the opposite side of the spot. It is therefore possible to increase the spot size until non-linear effects become intolerable.

To evaluate the effect, the calculation was performed using a circular spot of radius 500 km for the detector with the location displaced along the (north, south) and (east, west) directions in steps of 500 km. The results of the scan in (north, south) directions around the Kamioka site are shown in figure 2 as a function of neutrino zenith angle $\cos\theta_\nu$. The changes in flux are quite dramatic for low energy neutrinos but are also present at higher energy.

The same points are plotted as a function of the (north, south) displacement for each $\cos\theta_\nu$ in figure 3. The upward going fluxes (left hand plot) change quite a lot, but are linear over the range studied. The downward going and horizontal fluxes however (right hand plot) show non linearities. This limits the size of the detector which can be used to around 500 km.

Moving in the (east, west) direction about the Kamioka site produces smaller variations as shown in figure 4, since the Earth's magnetic field is constant as a function of geomagnetic longitude over short distances. The (north, south) variation is also reduced when considering a high geomagnetic latitude site such as Sudbury or Soudan as shown in figure 5. This can be understood because at Kamioka, the geomagnetic cutoff rigidity is around 17 GV for vertical downward cosmic rays, which is right in the middle of the energy region which is important for generating neutrinos. Any change in cutoffs by moving the location produces

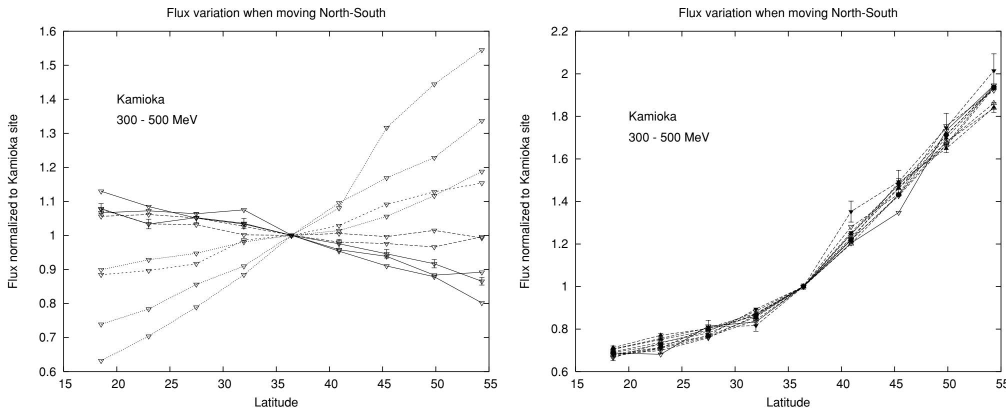

Fig 3. Effect of moving the detector centre in the North-South direction from Kamioka shown as a function of distance from the actual detector position for 300–500 MeV neutrino energy. The left plot shows the 9 upward $\cos\theta_\nu$ bins and the right plot shows the 11 downward and horizontal $\cos\theta_\nu$ bins.

a large effect in neutrino fluxes. At Sudbury however, the vertical cutoff is much lower, around 1–2 GV, too low for neutrino production, and so the effect of changing the location is nowhere near so severe. The effect is larger moving to the south (cutoff begins to play a role).

The above results have been processed to quantify the size of the non-linear variations and the results are given in [1]. The largest effect is in the (north, south) direction at Kamioka. For a 500 km circular spot detector, the error is 0.5% and varies quadratically with detector size. The variation in the (east, west) direction is about five times smaller as is the (north, south) variation at Sudbury.

Weights near the horizon

An awkward consequence of using the flat spot detector shape is that the area of the detector changes as a function of the zenith angle. The extreme case is a neutrino approaching exactly along the horizontal which sees a detector of zero area. There is a weight of $1/\cos\theta_\nu$, where θ_ν is the zenith angle of the neutrino, applied to correct for this. This causes a problem as $\cos\theta_\nu$ approaches zero because (a) the number of events tends to zero and (b) the weights diverge. This problem can be reduced by selecting primary zenith angles θ_p from a distribution which is flat in $\cos\theta_p$ and applying a weight of $\cos\theta_p$ (the correct distribution rises linearly with $\cos\theta_p$). For high energy showers in which $\theta_p \sim \theta_\nu$, the combined weight $\cos\theta_p/\cos\theta_\nu$ is nearly unity.

At low energy however, $\theta_p \neq \theta_\nu$ and the problem of the divergent weights

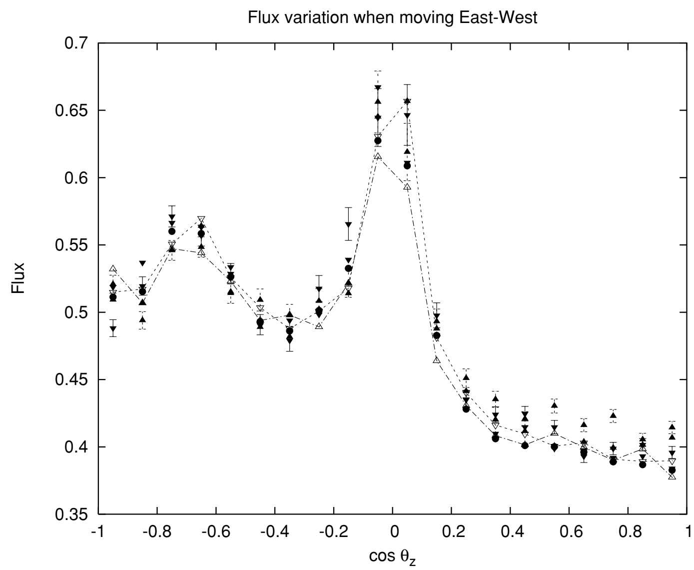

Fig 4. Effect of moving the detector centre in the East-West direction from Kamioka. 300-500 MeV neutrino energy.

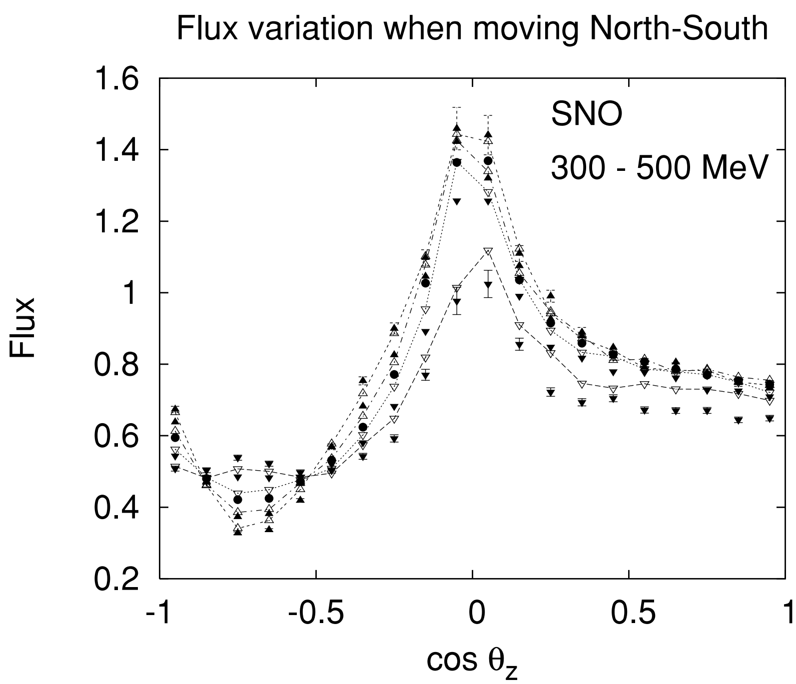

Fig 5. Effect of moving the detector centre in the North-South direction from Sudbury/Soudan. 300-500 MeV neutrino energy.

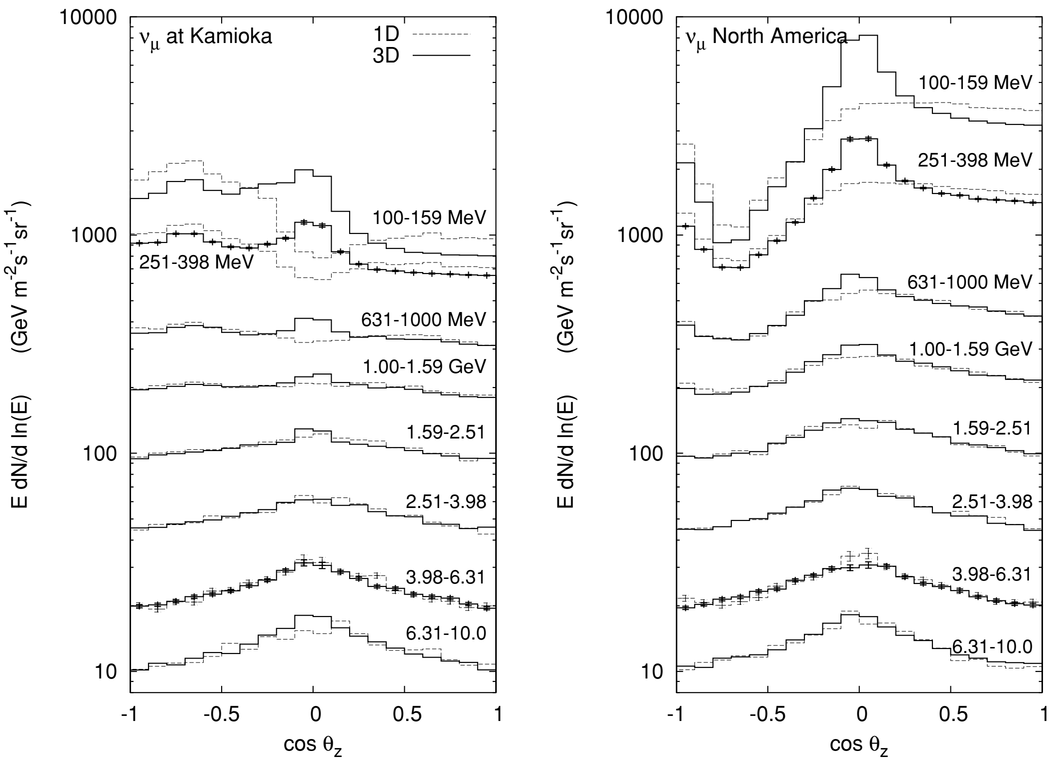

Fig 6. Results from the 3-D Monte-Carlo compared with the 1-D approximation at Kamioka (left) and Soudan (right) for different energy ranges.

is still present. The effect is removed by binning the weights in small bins in $\cos\theta_\nu$ (80 bins in the full range of $\cos\theta_\nu$) and applying the weights appropriate to the centre of each bin. This technique is described further in [4]. The level of bias is below 1% with the use of 80 bins.

3. Results

The main results from this calculation are described in [1]. Here, we briefly describe the features of the fluxes and refer the reader to [1] for a more comprehensive discussion including particle ratios. Figure 6 shows the fluxes of muon type neutrinos as a function $\cos\theta_\nu$ for different energy ranges. The left hand pane shows fluxes at Kamioka and the right hand one from North America. At low energy, there are substantially more downward neutrinos at the North American sites due to their higher geomagnetic latitude. The difference in upward fluxes is less marked because the particles come from a much larger area on the Earth and so some averaging takes place; nevertheless there are small variations which can be understood in terms of the varying magnetic fields. The 3-D and 1-D calculation results are shown together and have effects at low energy as described earlier. At higher energies, both the 3-D effects and the differences in geomagnetic location disappear. The high energy variation with zenith angle is caused by muons hitting the Earth's surface which occurs starting at lower energy for the vertical.

4. Systematic Checks

The calculation described above is sufficiently fast to be able to run many configurations with the computing time which is available. A study of how the fluxes depend when some of the input assumptions vary is in progress:

- The overall cross sections which are used to determine the interaction probability have been varied by 15%. This variation has been made independently of any hadron production probabilities which are considered in the program only once it is decided that an interaction should take place at a particular position. It is found that the fluxes increase by about 2–3% with this 15% change with ν_e changing a bit more than ν_μ. This is independent of neutrino energy up to 10 GeV with a slight decrease in the change at higher neutrino energies [5]. This small change is to be expected since the increase in cross section simply causes the shower to occur a little higher in the atmosphere - the neutrinos still (in general) get produced but at a slightly different altitude.

- A change in the atmospheric densities has been studied. As pointed put by Wentz [6], the atmospheric density as a function of altitude varies seasonally and by latitude. Models obtained from [7] were used. To emphasize the effect, the two extremes in atmospheric change - a polar atmosphere (70° latitude) in March and in September were used over the entire surface of the Earth [5]. Changes were small, perceivable at the 1% level, but at the limit of the statistical accuracy of the calculation.

- A preliminary attempt to characterize the effect of different hadron production has been made. A summary of the current situation is presented in [8]. A series of simple parameters were defined (a) for particular parent energies, independent of secondary particle energies to represent experimental normalization errors and errors in extrapolation of p_T into unmeasured regions, (b) in particular regions of (parent, secondary) phase space to represent regions where no measurements exist — particularly in regions where resonances make modeling difficult and (c) to represent kaon production uncertainties. These were then varied within the model and combined to produce a systematic error. An example of the error estimates is shown in figure 7 where the fractional error on both the electron neutrino and electron antineutrino fluxes is shown as a function of neutrino energy. The error on the ratio can also be obtained by this technique by evaluating the change in the ratio for each of the changes in the parameters and then determining the range of different ratios obtained. This is shown on figure 7. The effect of cancellation in flux uncertainties is clear, the error on the ratio is around a factor of 10 lower than the error on the individual fluxes.

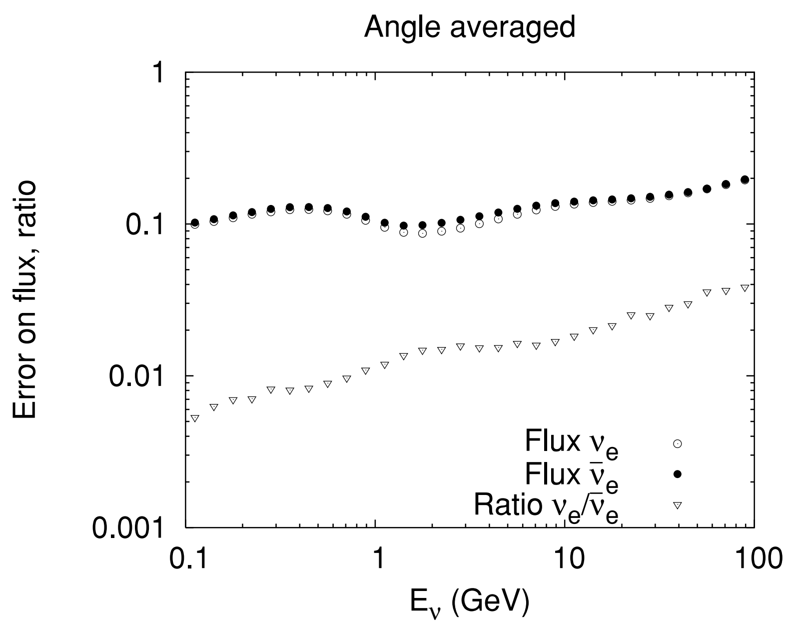

Fig 7. Example of the error estimate from hadron production (preliminary) using the hadron production variabilities described in the text for the ν_e and $\overline{\nu}_e$ fluxes and the ratio of $\nu_e/\overline{\nu}_e$ as a function of neutrino energy

5. Summary

The quality of atmospheric neutrino calculations continues to improve. Controlling the effects of some of the uncertainties, such as the 3-dimensional geometrical effects has developed considerably, by many groups performing the calculations, not just the one described in this paper. Dealing with the 3-dimensional effect has been challenging from the technical point of view. A series of checks is being performed to determine the size of the various input models which are needed in the calculation such as the the primary fluxes, atmospheric model and cross sections. The latter two of these are seen to have a small effect on the fluxes, even though they are known to have a larger effect on muon fluxes at a particular altitude. The difficult part of assigning errors to the fluxes and in particular the flux ratios is the assessment of the hadron production uncertainties. These are discussed in more detail in [8]. It is clearly difficult at the moment, but much improved hadron production data is just round the corner. Overall, the fascinating new measurements which are being planned to be made with atmospheric neutrinos as described at this workshop present an exciting challenge to those predicting atmospheric fluxes.

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