
Constraint on θ_{13} from the Super-Kamiokande Atmospheric Neutrino Data

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Abstract

We report on the results of three-flavor oscillation analysis using Super-Kamiokande I atmospheric neutrino data, with the assumption of one mass scale dominance ($\Delta m_{12}^2=0$). No significant enhancement due to matter effect, which occurs when neutrinos propagate inside the Earth on the condition of $\theta_{13}\neq 0$, cannot be seen in multi-GeV ν_e -rich sample and a constraint on θ_{13} is given from Super-Kamiokande data only. Both normal and inverted mass hierarchy hypotheses are tested and consistent each other.

1. Introduction

Neutrino oscillation in muon neutrinos have been established by atmospheric neutrino and long-baseline neutrino experiments [1, 3, 4]. These results can be well explained by two-flavor oscillation model between ν_μ and ν_τ . According to the standard neutrino oscillation theory, oscillations among three-flavor neutrinos can be described using two parameters of mass square difference (Δm_{12}^2 , Δm_{23}^2) and neutrino mixing matrix (PMNS matrix) [5], which is expressed by four parameters (θ_{12} , θ_{23} , θ_{13} and CP-violation phase δ). Δm_{12}^2 and θ_{12} are considered to be responsible for the oscillations measured with solar and reactor neutrinos, and recent result gave a constraint of $\Delta m^2=7.9_{-0.5}^{+0.6}\times 10^{-5}\text{eV}^2$ and $\tan^2\theta=0.40_{-0.07}^{+0.10}$ [6], assuming $\nu_e \leftrightarrow \nu_x$ two-flavor oscillation. Muon neutrino oscillation observed by atmospheric neutrino measurements is considered to be driven by Δm_{23}^2 and θ_{23} . A constraint of $\sin^2 2\theta>0.92$ and $1.5\times 10^{-3}<\Delta m^2<3.4\times 10^{-3}\text{eV}^2$ is given in two-flavor oscillation scheme [3]. No evidence was reported in $\bar{\nu}_e$ oscillation from short-baseline reactor neutrino experiment, such as CHOOZ [7], and θ_{13} is constrained to be considerably smaller ($\sin^2\theta_{13}\lesssim 0.05$).

If we assume $|\Delta m_{23}^2|\gg\Delta m_{12}^2$ and the contribution of Δm_{12}^2 term is neglected in atmospheric neutrinos, oscillation probabilities can be much simply expressed without any approximation. We have only to consider the oscillation driven by Δm_{23}^2 (one mass scale dominance) [8, 9]. The following equations shows the oscillation probabilities in the case of vacuum propagation:

$$\begin{aligned}
\text{Prob}(\nu_e \rightarrow \nu_e) &= 1 - \sin^2 2\theta_{13} \sin^2\left(\frac{1.27\Delta m_{23}^2 L}{E}\right) \\
\text{Prob}(\nu_\mu \rightarrow \nu_e) &= \sin^2 \theta_{23} \sin^2 2\theta_{13} \sin^2\left(\frac{1.27\Delta m_{23}^2 L}{E}\right) \\
\text{Prob}(\nu_\mu \rightarrow \nu_\mu) &= 1 - 4 \cos^2 \theta_{13} \sin^2 \theta_{23} (1 - \cos^2 \theta_{13} \sin^2 \theta_{23}) \sin^2\left(\frac{1.27\Delta m_{23}^2 L}{E}\right)
\end{aligned}$$

In this scheme, oscillation probabilities can be expressed with only three parameters, Δm_{23}^2 , θ_{23} and θ_{13} , even in the case of matter oscillation. These becomes consistent with two-flavor $\nu_\mu \leftrightarrow \nu_\tau$ oscillation at the limit of $\theta_{13}=0$. Hereafter Δm^2 is used instead of Δm_{23}^2 .

The three-flavor oscillations in the multi-GeV energy range around 1–10 GeV can be drastically changed by matter effect [10, 11, 12] even if θ_{13} is small. The Earth matter effect can resonantly enhance oscillations concerned with electron neutrino, while oscillations for anti-neutrino is suppressed. Fig. 1 shows the oscillation probability of $\nu_e \rightarrow \nu_\mu$ as a function of neutrino energy and zenith angle. The oscillation amplitudes are locally enhanced for neutrinos which propagate in the core or mantle region of the Earth. This effect is expected to be appeared as the marginal excess of upward-going ν_e -rich sample.

It should be noted that the sign of Δm^2 is not completely known from our current knowledge of neutrinos. The situations of matter oscillation are exchanged between neutrinos and anti-neutrinos for inverted mass hierarchy case ($\Delta m^2 < 0$), i.e., oscillation probabilities for anti-neutrinos will be resonantly enhanced and suppressed for neutrinos in multi-GeV energy range.

2. Analysis

We used Super-Kamiokande I 1489 days fully-contained (FC) and partially-contained (PC) data and 1646 days upward-going muon data [3]. One hundred year corresponding Monte-Carlo events are generated and analyzed as well as real data for comparison. FC events deposit all of their Cherenkov light in the inner detector, while PC events have exiting tracks and some energy deposit in the outer detector. Upward-going muons are the high energy muons, which are generated in the surrounding rocks and intersect the detector. The sensitive neutrino energy differs from sample to sample and the order of their neutrino energy is 1, 10, 10–100 GeV, for FC, PC and upward-going muon events, respectively. FC data are separated according to their number of Cherenkov rings and particle identification into “single-ring μ -like”, “single-ring e -like” and “multi-ring μ -like” events. In addition FC events are separated into “sub-GeV” and “multi-GeV”

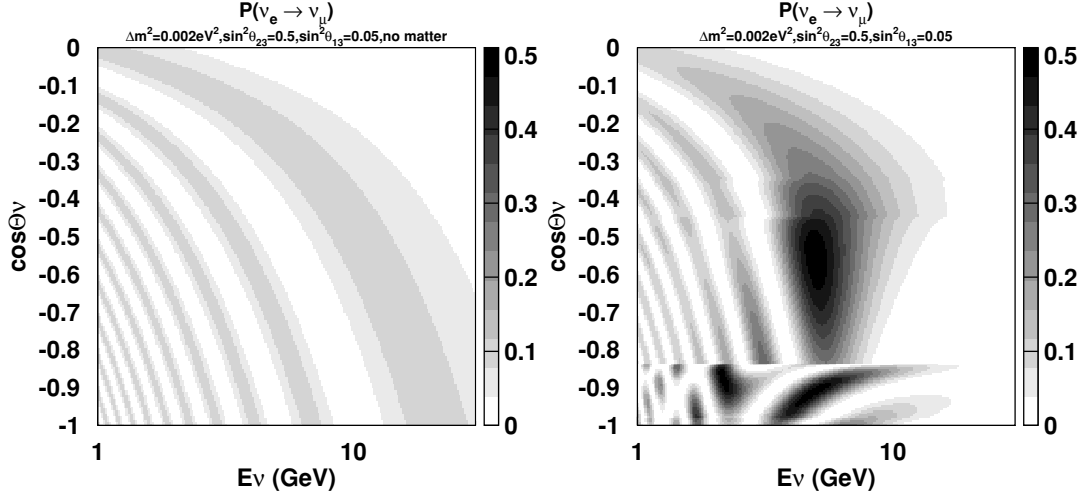


Fig 1. $\nu_e \rightarrow \nu_\mu$ oscillation probabilities without matter effect (left) and with matter effect (right), as a function of zenith angle ($\cos \Theta_\nu$) and energy (E_ν). Oscillation parameters of $\Delta m^2 = 2.0 \times 10^{-3} \text{eV}^2$, $\sin^2 \theta_{23} = 0.5$, $\sin^2 \theta_{13} = 0.05$ are assumed.

according to their visible energy by 1.33 GeV. PC events are divided into “PC stop” and “PC through” sample according to the amount of charge deposit in outer detector, which separation method is described in [2]. In addition to these samples, we introduce new data sample, named as “multi-ring e -like”, in order to increase statistics and sensitivity of ν_e -induced events in multi-GeV energy range. Multi-ring e -like events are required:

- Fully-contained, multi-ring and visible energy is greater than 1.33 GeV
- e -like pattern is identified for the brightest Cherenkov ring
- Event topology is likely to multi-ring e -like according to likelihood function

Many backgrounds are expected in the selection of ν_e -induced events in this energy range because energetic hadronization shower events due to π^0 production make event pattern similar to electromagnetic shower in water Cherenkov detector. We adopted likelihood method to increase the purity of ν_e -induced events and reduce backgrounds. The likelihood function for multi-ring e -like sample is defined using the following parameters considering the energy dependence:

- Particle identification likelihood parameter
- Energy fraction of the brightest Cherenkov ring
- Number of accompanied decay-electrons

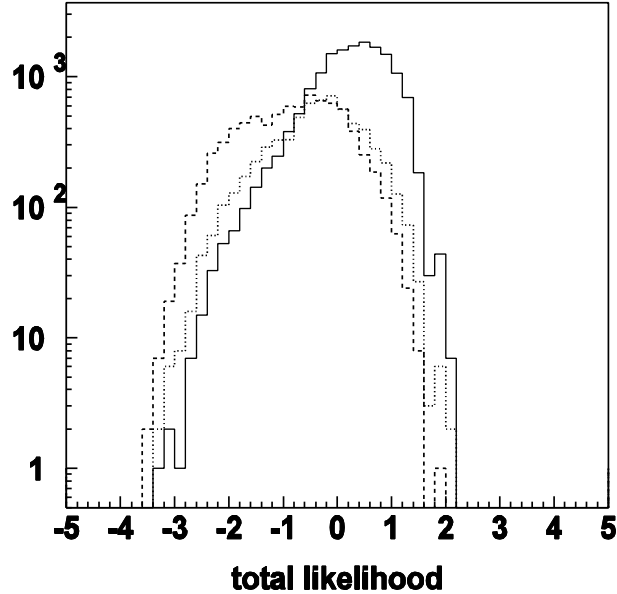


Fig 2. Distributions of multi-ring e -like likelihood for ν_e charged-current (CC) (solid), ν_μ CC (dashed) and neutral-current (NC) events (dotted). Events which likelihood value is greater than zero are selected as multi-ring e -like sample.

- Distance between decay-electron and primary particle vertices

Fig. 2 shows multi-ring e -like likelihood distributions for events induced by ν_e charged-current (CC), ν_μ CC and neutral current (NC) interactions, respectively. Events caused by CC ν_e are discriminated and tend to have larger likelihood value. We selected the events which likelihood value is greater than zero. Table 1 shows the fraction of the neutrino interaction mode in multi-ring e -like sample. 74% of multi-ring e -like events are estimated to be caused by ν_e CC interactions according to Monte Carlo simulation study.

| | w/ L cut | w/o L cut |
|--------------|----------|-----------|
| ν_e CC | 73.5 | 52.0 |
| ν_μ CC | 11.4 | 26.7 |
| NC | 15.1 | 21.3 |

Table 1. Fraction of neutrino interaction modes in multi-ring e -like sample with and without likelihood (L) cut (unit is percentage).

We introduce χ^2 test to perform three-flavor oscillation analysis. All events are divided into 37 momentum bins (10+5 bins for FC single- and multi-ring e -like,

8+4 bins for FC single- and multi-ring μ -like, 4+4 bins for PC stop and through, and 1+1 bin for upward-going stop and through muons) and each momentum bin is also divided into 10 bins equally spaced between $\cos \Theta = -1$ and $\cos \Theta = +1$, where $\cos \Theta$ is cosine of zenith angle of particle direction. In total, number of bins amounts to 370 bins. Number of events in each bin is compared with expectation and χ^2 value is calculated according to Poisson probability distribution defined by the following expression:

$$\chi^2 = \sum_{n=1}^{370} \left[2 \left\{ N_{exp}^n \left(1 + \sum_{i=1}^{44} f_i^n \cdot \epsilon_i \right) - N_{obs}^n \right\} + 2N_{obs}^n \ln \left(\frac{N_{obs}^n}{N_{exp}^n \left(1 + \sum_{i=1}^{44} f_i^n \cdot \epsilon_i \right)} \right) \right] + \sum_{i=1}^{44} \left(\frac{\epsilon_i}{\sigma_i} \right)^2$$

where

| | |
|--------------|--|
| N_{obs}^n | Number of observed events in n -th bin |
| N_{exp}^n | Number of expected events in n -th bin |
| ϵ_i | i -th systematic error term |
| f_i^n | Systematic error coefficient |
| σ_i | 1 sigma value of systematic error |

N_{exp}^n is calculated using Monte Carlo events with the correction of oscillation probability and systematic uncertainties. We considered 44 systematic error sources, which come from sample normalization, neutrino flux, neutrino interactions and event selection. Most of them are common with those listed in [3], and additional systematic uncertainties with respect to backgrounds of e -like sample and upward-going muons, sample normalization of e -like events are estimated. f_i^n is calculated and tabulated in advance for every bin and systematic error source. A global scan is carried out on a $(\log_{10}(\Delta m^2), \sin^2 \theta_{23}, \sin^2 \theta_{13})$ grid minimizing χ^2 with respect to 44 systematic error parameters. ϵ_i is to be selected so that distributions between data and expectation are fitted, i.e., χ^2 value becomes minimum. We used ϵ_i so that the first derivative of χ^2 with respect to ϵ_i becomes zero ($\frac{\delta \chi^2}{\delta \epsilon_i} = 0$), which can be obtained by solving linear equations [13]. Since this equation have non-linear terms in case of our χ^2 definition, we used the approximate solution obtained by iteration method.

The oscillation probability is calculated considering matter effect for neutrinos which propagates in the Earth. Matter density is considered according to the distance from the Earth center. We adopted a model that the Earth consists of four layer of a constant matter density (core 1: $R \leq 1221$ km, $\rho = 13.0$ g/cm³, core 2:

1221 < R ≤ 3480 km, $\rho = 11.3$ g/cm³, mantle: 3480 < R ≤ 5701 km, $\rho = 5.0$ g/cm³, surface: 5701 < R ≤ 6371 km, $\rho = 3.3$ g/cm³). The calculation method of matter oscillation probabilities in constant density is based on [12]. In order to compensate insufficient statistics of Monte Carlo events, probabilities are averaged out with regard to oscillation phase when neutrino propagates more than two times of oscillation wavelength in the mantle region of the Earth.

3. Results

As a result of the global scan on the oscillation parameter grid assuming normal mass hierarchy, the minimum χ^2 value of $\chi^2_{min} = 375.82/368$ DOF is obtained at the grid point of $(\Delta m^2, \sin^2 \theta_{23}, \sin^2 \theta_{13}) = (2.5 \times 10^{-3} \text{ eV}^2, 0.5, 0.0)$, which is consistent with $\nu_\mu \leftrightarrow \nu_\tau$ two-flavor oscillation. Fig. 3 shows the zenith angle distribution of each data sample overlapped with non-oscillated and best-fitted expectations. The fitted distributions well agree with data. Fig. 4 shows the UP/DOWN asymmetry as a function of momentum, where UP (DOWN) means number of events in $-1.0 < \cos \Theta < -0.2$ ($0.2 < \cos \Theta < 1.0$). UP/DOWN asymmetry $(\text{UP} - \text{DOWN}) / (\text{UP} + \text{DOWN})$ distributions are consistent with flat and also fitted expectation. No significant excess due to matter effect is seen in upward-going multi-GeV e -like sample. The 90 % (99 %) confidence level allowed region is defined to be $\chi^2 = \chi^2_{min} + 4.6$ (9.2) and obtained as shown in Fig. 5. The region of $\sin^2 \theta_{13} < 0.14$ and $0.36 < \sin^2 \theta_{23} < 0.65$ is allowed at 90% confidence level.

At last, we tested inverted mass hierarchy hypothesis. Water Cherenkov detector, such as Super-Kamiokande, cannot discriminate neutrino or anti-neutrino event-by-event basis. However mass hierarchy affects number of events and energy spectrum of ν_e -rich sample, since neutrino cross sections and fluxes differ. Because of lower cross section of $\bar{\nu}_e$, the enhancement in multi-GeV ν_e -rich sample is expected to be reduced and therefore constraint on θ_{13} will be weakened for inverted mass hierarchy case. The allowed region assuming inverted mass hierarchy is also obtained and shown in Fig. 6. $\chi^2_{min} = 376.76/368$ DOF is obtained at the grid point of $(\Delta m^2, \sin^2 \theta_{23}, \sin^2 \theta_{13}) = (-2.5 \times 10^{-3} \text{ eV}^2, 0.525, 0.00625)$. This χ^2_{min} value is very close to that for normal hierarchy case, therefore both hypothesis are allowed by Super-Kamiokande data.

4. Summary

In summary, three-flavor oscillation analysis assuming one mass scale dominance ($\Delta m^2_{12} = 0$) is performed with Super-Kamiokande I FC+PC+Up μ combined dataset. Multi-ring e -like sample, which is discriminated using likelihood method, are newly introduced to increase the statistics of electron neutrinos and improve

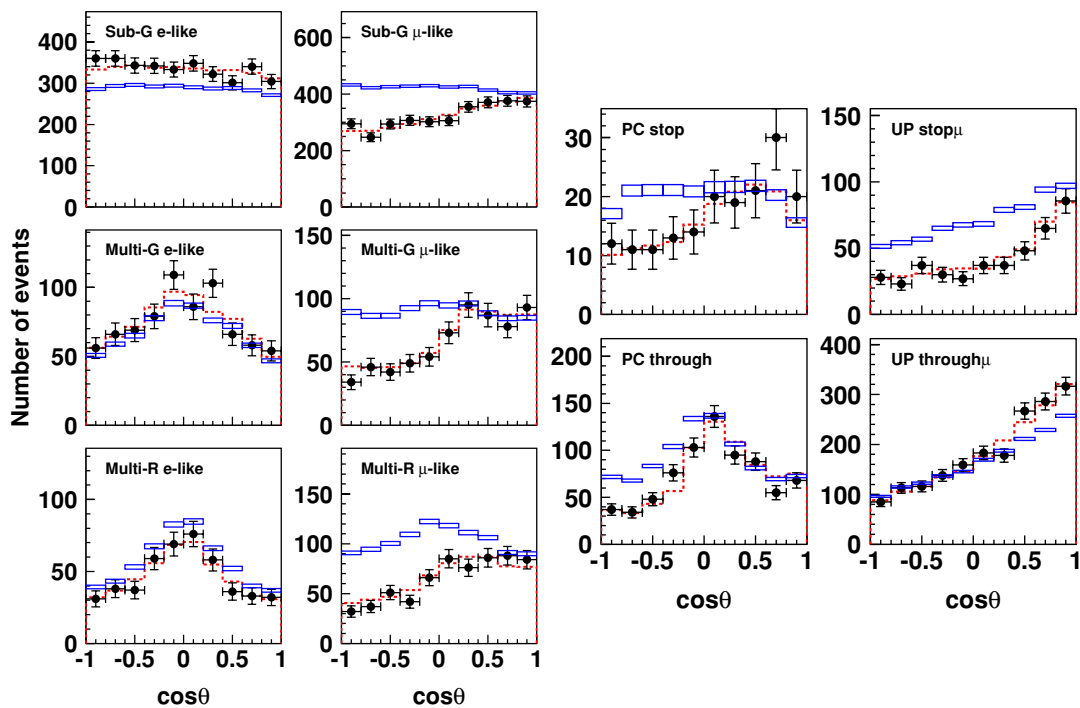


Fig 3. Zenith angle distributions of FC e -like, μ -like, PC, and upward-going muons are shown for data (filled circles with error bars), Monte-Carlo distributions without oscillation (box) and best-fitted distributions (dashed).

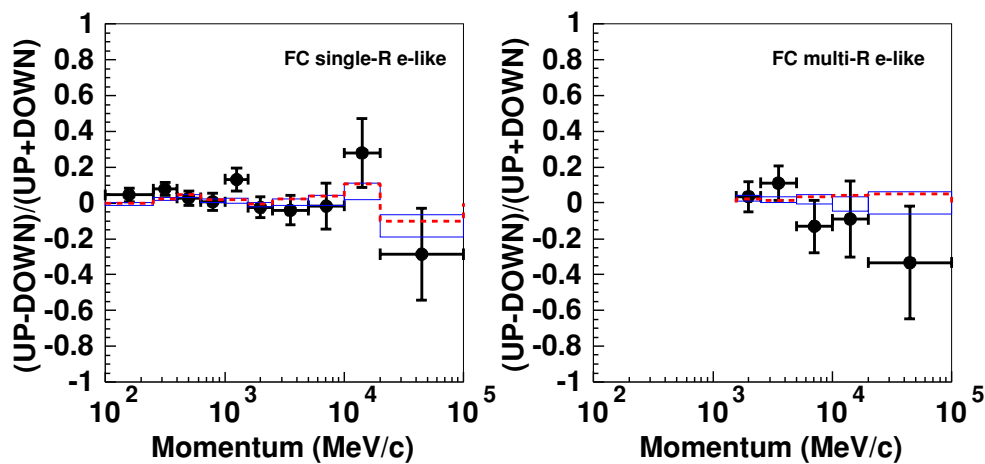


Fig 4. UP/DOWN asymmetry $(UP-DOWN)/(UP+DOWN)$ as a function of momentum for FC single- (left) and multi-ring (right) e -like events. Filled circles represents data, box represents non-oscillated Monte-Carlo and dashed line represents best-fitted distributions.

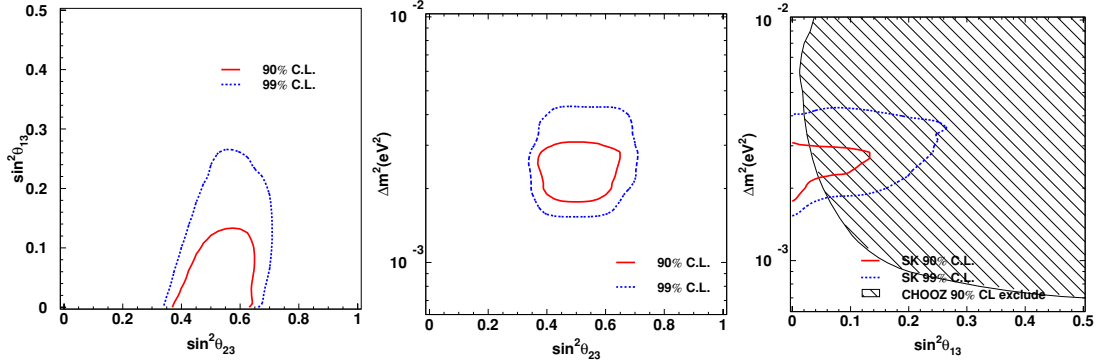


Fig 5. 90 % (solid line) and 99 % (dashed line) confidence level allowed regions are shown in $\sin^2 \theta_{13}$ vs $\sin^2 \theta_{23}$ (left), Δm^2 vs $\sin^2 \theta_{23}$ (middle), and Δm^2 vs $\sin^2 \theta_{13}$ (right). Normal mass hierarchy ($\Delta m^2 > 0$) is assumed. Shaded area in the right figure shows the excluded region by CHOOZ reactor neutrino experiment.

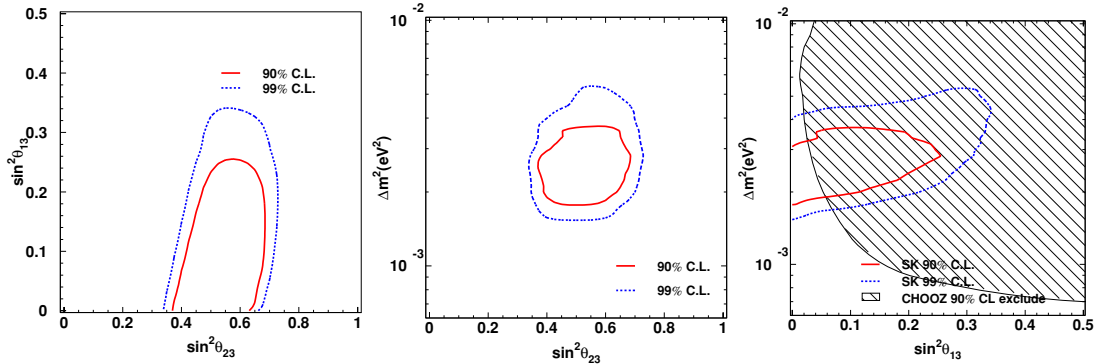


Fig 6. 90 % (solid line) and 99 % (dashed line) confidence level allowed regions assuming inverted mass hierarchy ($\Delta m^2 < 0$), shown in same orientation as in Fig. 5.

the sensitivity of θ_{13} with help of matter effect. The best-fitted parameters for three-flavor oscillation becomes $(\Delta m^2, \sin^2 \theta_{23}, \sin^2 \theta_{13}) = (2.5 \times 10^{-3} \text{ eV}^2, 0.5, 0.0)$ and the region of $\sin^2 \theta_{13} < 0.14$ and $0.36 < \sin^2 \theta_{23} < 0.65$ is allowed at 90 % confidence level, assuming normal mass hierarchy. No significant UP/DOWN asymmetry in e -like sample cannot be seen and the indication of non-zero θ_{13} signal has not been discovered from oscillation analysis results. We also test the inverted mass hierarchy case. There is no remarkable difference between these two hypotheses and both of them agree with our data.

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