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## Test of Unitarity by using $\nu_\mu \rightarrow \nu_\mu$ Oscillations

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### Abstract

In the case of large 1-3 mixing angle as  $\sin^2 2\theta_{13} \geq 0.03$ , we investigate the possibility for measuring the leptonic CP phase by using only  $\nu_\mu \rightarrow \nu_\mu$  oscillations independently of  $\nu_\mu \rightarrow \nu_e$  oscillations. As the result, we find that the CP phase can be measured best at the energy around  $E = 0.43\text{GeV}$  and the baseline length  $L = 5000\text{km}$  without strongly depending on the uncertainties of other parameters. In this region, the CP phase effect remains even after averaging over neutrino energy. We also find that there is a CP sensitivity even in the short baseline length  $L \leq 1000\text{km}$  if  $\Delta m_{31}^2$  is determined with the uncertainty of about 0.1%. In the T2KK experiment, we explore the possibility for measuring the CP phase assuming 0.1% uncertainty of  $\Delta m_{31}^2$ . As the result, we find that some information of the CP phase can be obtained.

### 1. Introduction

The finite mass of neutrinos and the mixings among different flavors have been confirmed in various neutrino experiments. The aim of next generation experiments is to explore the 1-3 mixing angle  $\theta_{13}$ , the sign of  $\Delta m_{31}^2$  and the CP phase  $\delta$ . One of the turning points is whether  $\theta_{13}$  can be determined by the next generation reactor experiments and superbeam experiments. In the case of  $\sin^2 2\theta_{13} \geq 0.03$ ,  $\theta_{13}$  will be found in the next generation experiments without being affected by the  $\theta_{13}$ - $\delta$  ambiguity. In such cases, it is suggested that the remaining four-fold degeneracies can also be solved within a few decades [1]. In particular, an illuminating plan with two detectors, so called Tokai-to-Kamioka-Korea (T2KK) experiment, was proposed in refs. [2]. They investigated the capability of this experiment. As a result, in the case of large  $\theta_{13}$ , the mass hierarchy is determined and CP phase can be also measured. In addition, the precision measurement of other parameters is possible.

In this talk, we concentrate on the case of  $\sin^2 2\theta_{13} \geq 0.03$  and we explore a new possibility for measuring the CP phase in  $\nu_\mu \rightarrow \nu_\mu$  oscillations independently of  $\nu_\mu \rightarrow \nu_e$  oscillations based on the works [3, 4]. If we assume the unitarity in three generations, the channel of  $\nu_\mu \rightarrow \nu_e$  oscillations should be related to that of  $\nu_\mu \rightarrow \nu_\mu$  oscillations. Therefore, the inconsistency between these channels gives an evidence of new physics and we will have some constraints to the unified theory in the high energy physics.

The CP dependence of the probabilities related to the superbeam experiments are given by

$$P_{\mu e} = A_{\mu e} \cos \delta + B_{\mu e} \sin \delta + C_{\mu e}, \quad (1)$$

$$P_{\mu \mu} \simeq A_{\mu \mu} \cos \delta + C_{\mu \mu}, \quad (2)$$

$$P_{\mu \tau} \simeq B_{\mu \tau} \sin \delta + C_{\mu \tau}, \quad (3)$$

where  $A$ ,  $B$  and  $C$  are the function of parameters except for  $\delta$  [5, 6]. If we assume the unitarity in three generations, the sum of three probabilities has to be one for any value of  $\delta$ . So, we obtain the relations among these coefficients as

$$A_{\mu e} + A_{\mu\mu} = 0, \quad (4)$$

$$B_{\mu e} + B_{\mu\tau} = 0, \quad (5)$$

$$C_{\mu e} + C_{\mu\mu} + C_{\mu\tau} = 1. \quad (6)$$

It is well known that the order of magnitude for each coefficient is given by

$$A_{\mu\mu} = -A_{\mu e} = O(\alpha s_{13}), \quad (7)$$

$$B_{\mu\tau} = -B_{\mu e} = O(\alpha s_{13}), \quad (8)$$

$$C_{\mu e} = O(\alpha^2) + O(s_{13}^2), \quad (9)$$

$$C_{\mu\mu} = C_{\mu\tau} = O(1), \quad (10)$$

where  $\alpha = \Delta m_{21}^2 / \Delta m_{31}^2$ . Here, we should note that  $A_{\mu\mu}$  is the same order as  $A_{\mu e}$ . So, in the case of large  $\theta_{13}$ , the CP phase effect in  $\nu_\mu \rightarrow \nu_\mu$  oscillations is not so small. The observation in this channel may be difficult compared with that in  $\nu_\mu \rightarrow \nu_e$  oscillations because of the large CP independent terms, but there may be a possibility to detect CP phase effect in  $\nu_\mu \rightarrow \nu_\mu$  oscillations independently from  $\nu_\mu \rightarrow \nu_e$  oscillations. Accordingly, we can test the unitarity and may be obtain the clue of new physics.

## 2. Search for $E$ - $L$ Regions with Large $\cos \delta$

In order to investigate the CP phase effect in  $\nu_\mu \rightarrow \nu_\mu$  oscillations, at first, we numerically explore the energy  $E$  and the baseline length  $L$  where  $A_{\mu\mu}$  is large, without limiting the setup of T2KK. We use the parameters

$$\sin^2 \theta_{12} = 0.3, \quad \sin^2 \theta_{23} = 0.5, \quad \sin^2 2\theta_{13} = 0.15, \quad (11)$$

and

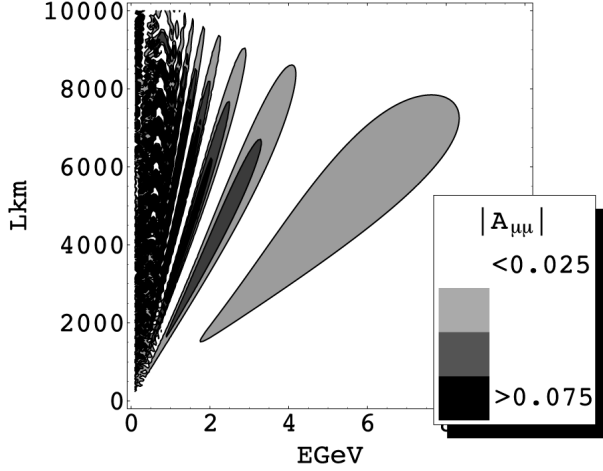
$$\Delta m_{31}^2 = 2.5 \times 10^{-3} \text{eV}^2, \quad \Delta m_{21}^2 = 8.1 \times 10^{-5} \text{eV}^2. \quad (12)$$

In fig. 1,  $|A_{\mu\mu}|$  takes large value in black region. So, we found that  $|A_{\mu\mu}|$  becomes large in the low energy and long baseline length, namely in the region with large  $L/E$ . This region may be explored by the very long baseline experiments or atmospheric neutrino experiments with large detector. Then, where is the best point in this region? Are there any chance to measure the CP phase in short baseline experiments by using  $\nu_\mu \rightarrow \nu_\mu$  oscillations?

In order to investigate the behavior of  $A_{\mu\mu}$  more accurately, we use the approximate formula of  $A_{\mu\mu}$  derived in ref [4] as

$$A_{\mu\mu} \simeq \underbrace{\frac{4J_r \Delta_{21} (a - \Delta_{21} \cos 2\theta_{12})}{\tilde{\Delta}_{21}^2} \sin^2 \tilde{\Delta}'_{21}}_{A_1} - \underbrace{\frac{4J_r \Delta_{21}}{\tilde{\Delta}_{21}} \sin \tilde{\Delta}'_{21} \sin(2\tilde{\Delta}'_{31} - \tilde{\Delta}'_{21})}_{A_2}. \quad (13)$$

Here,  $J_r = s_{12}c_{12}s_{23}c_{23}s_{13}c_{13}^2$ ,  $\Delta_{ij} = \Delta m_{ij}^2 / (2E)$ ,  $a \simeq 7.56 \times 10^{-5} \cdot \rho Y_e$ ,  $\rho$  is the matter density,  $Y_e$  is the fraction of electrons,  $\tilde{\Delta}_{ij} = \lambda_i - \lambda_j$ ,  $\tilde{\Delta}'_{ij} = \tilde{\Delta}_{ij} L / 2$  and



**Fig. 1** Region with large  $|A_{\mu\mu}|$ . In the black region, the magnitude of  $|A_{\mu\mu}|$  becomes large.

$\lambda_i$  is the effective mass of  $i$ -th neutrino divided by  $(2E)$  given as

$$\lambda_1 \simeq \frac{\Delta_{21} + a - \sqrt{(a - \Delta_{21} \cos 2\theta_{12})^2 + \Delta_{21}^2 \sin^2 2\theta_{12}}}{2} \quad (14)$$

$$\lambda_2 \simeq \frac{\Delta_{21} + a + \sqrt{(a - \Delta_{21} \cos 2\theta_{12})^2 + \Delta_{21}^2 \sin^2 2\theta_{12}}}{2} \quad (15)$$

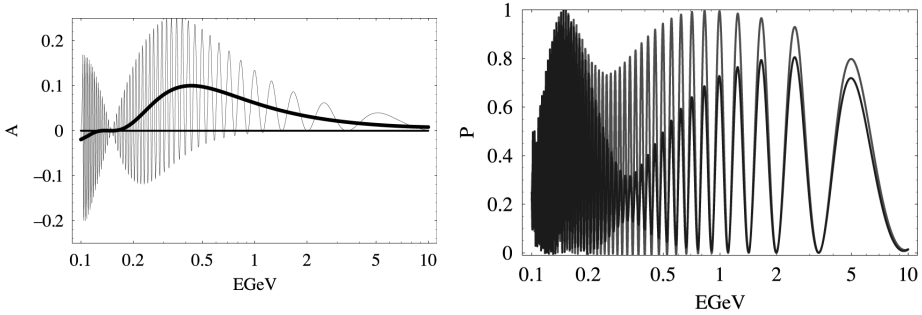
$$\lambda_3 \simeq \Delta_{31}. \quad (16)$$

In the derivation of (13), we took the approximation  $\lambda_1 < \lambda_2 \ll \lambda_3$ ,  $a \ll \lambda_3$  and  $s_{13}^2 \ll 1$ . Then, we have left only leading order terms of small quantities,  $\Delta_{21}$ ,  $\lambda_1$ ,  $\lambda_2$  and  $s_{13}$ . In order to be a good approximation for the region with large  $L/E$ , we did not neglect the term with the order of  $O(\Delta'_{21})$  included in the oscillating part. In eq.(13),  $A_{\mu\mu}$  is represented as the sum of two terms  $A_1$  and  $A_2$ .  $A_1$  is slowly oscillating term according to the change of energy as controlled by  $\tilde{\Delta}'_{21}$ .  $A_2$  is rapidly oscillating term as controlled by  $\tilde{\Delta}'_{31}$ . In the small  $L/E$  region,  $A_1$  can be neglected and the main contribution comes from  $A_2$ . As the value of  $L/E$  increases,  $A_1$  also gives the contribution and  $A_2$  oscillates faster. Therefore, only  $A_1$  remains in the region with sufficiently large  $L/E$  and after averaging over the energy. The total behavior of  $A_{\mu\mu}$  can be described as the oscillation around the average value determined by  $A_1$ . The coefficient of sine function in  $A_1$  is given by

$$\frac{4J_r \Delta_{21} (a - \Delta_{21} \cos 2\theta_{12})}{\tilde{\Delta}_{21}^2} = \frac{4J_r \Delta m_{21}^2 (2aE - \Delta m_{21}^2 \cos 2\theta_{12})}{(2aE - \Delta m_{21}^2 \cos 2\theta_{12})^2 + \Delta m_{21}^4 \sin^2 2\theta_{12}}. \quad (17)$$

If we use the parameters  $\sin^2 2\theta_{23} = 1$  and  $\sin^2 \theta_{12} = 0.31$ , it is found that the value of local maximum is given by

$$A_1^{\max} = \frac{\sin 2\theta_{13}}{4} \sin^2 \left( \frac{\sqrt{2} \Delta m_{21}^2 \sin 2\theta_{12} L}{4E_\ell} \right) \quad (18)$$



**Fig. 2** Energy dependence of  $A_{\mu\mu}$  and  $P_{\mu\mu}$ . Left and right figures show the magnitude of  $A_{\mu\mu}$  and  $P_{\mu\mu}$ . In the left figure, grey and black lines represent the magnitudes of  $A_{\mu\mu}$  and  $A_1$  respectively. In the right figure, red (grey) and blue (black) lines correspond to  $\delta_{true} = 0^\circ$  and  $180^\circ$ .

at the energy

$$E_\ell = \frac{\Delta m_{21}^2 (\cos 2\theta_{12} + \sin 2\theta_{12})}{2a} = 0.43 \text{ GeV} \cdot \frac{\Delta m_{21}^2}{8.1 \times 10^{-5} \text{ eV}^2} \cdot \frac{3.3 \text{ g/cm}^3}{\rho}. \quad (19)$$

If we fix the energy at this value and from the maximal condition

$\sqrt{2}\Delta m_{21}^2 \sin 2\theta_{12} L / 4E_\ell = (2n+1)\pi/2$  ( $n = 0, 1, 2, \dots$ ) for (18), we can determine the baseline length  $L_\ell$  as

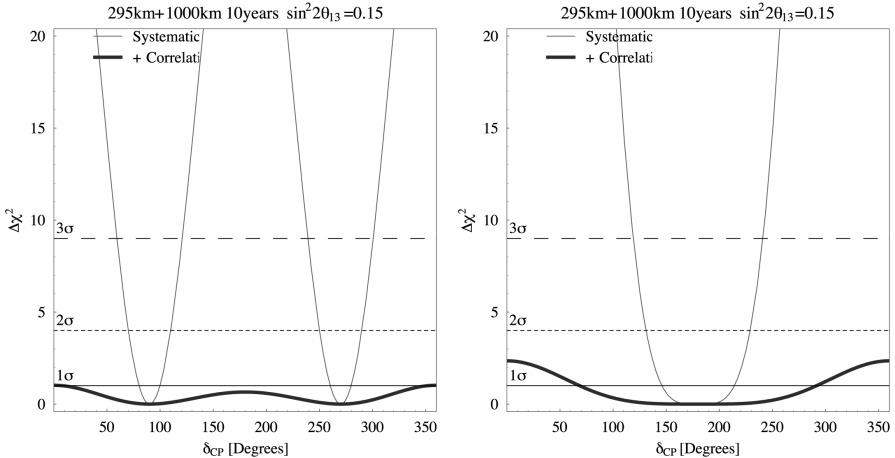
$$L_\ell = \frac{\sqrt{2}E_\ell (2n+1)\pi}{\Delta m_{21}^2 \sin 2\theta_{12}} = 5000 \text{ km} \cdot (2n+1) \cdot \frac{8.1 \times 10^{-5} \text{ eV}^2}{\Delta m_{21}^2} \cdot \frac{\rho}{3.3 \text{ g/cm}^3}. \quad (20)$$

If we use the average density calculated in the PREM [7] corresponding to each baseline,  $A_1$  becomes maximal at  $L_\ell = 5000 \text{ km}$  and  $10000 \text{ km}$  in the earth mantle. We also find from (20) that  $A_1^{\max}$  attains to about 0.1 in the case of  $\sin^2 2\theta_{13} = 0.15$ .

In fig. 2, we show the energy dependence of  $A_{\mu\mu}$  and  $P_{\mu\mu}$  at  $L = 5000 \text{ km}$ . In the left figure, grey and black lines represent the magnitudes of  $A_{\mu\mu}$  and  $A_1$  respectively. We can see that  $A_1$  takes maximal value 0.1 around  $E = 0.43 \text{ GeV}$  as expected from (13). Even after averaging over the low energy region, the CP phase effect due to the first term remains. Namely, we need not always the good energy resolution for the measurement of the CP phase effect in this region. This means that the CP phase effect appears in the measurement of total rates of  $\nu_\mu$  events. In addition, good energy resolution of a detector will provide further information from also rapidly oscillating term. In the right figure, red (grey) and blue (black) lines correspond to the value of  $P_{\mu\mu}$  in the case of  $\delta_{true} = 0^\circ$  and  $180^\circ$ . We can see that the CP phase effect changes the value largely even in survival probability.

### 3. Simulation of CP Sensitivity

Next, let us describe an experimental setup in order to see the CP phase effect in  $\nu_\mu \rightarrow \nu_\mu$  oscillations. We assume the JPARC beam and the water cherenkov detector with total fiducial mass of 500kt. We consider the two patterns of the location of the detectors. One is  $L = 295 \text{ km}$  (250kt) + 1000km (250kt) as the T2KK setup and the other is  $L = 5000 \text{ km}$  (500kt). We assume ten years run of only neutrinos. We calculate  $\Delta\chi^2$  by using only  $\nu_\mu \rightarrow \nu_\mu$  oscillations because we would like to learn the possibility for measuring the CP phase independently



**Fig. 3**  $\Delta\chi^2$  at 295km+1000km. The pink (thin) line represents  $\Delta\chi^2$  calculations including only systematics and the blue (thick) calculations including also parameter uncertainties. Left and right figures correspond to  $\delta_{true} = 90^\circ$  and  $180^\circ$ . These figures are calculated under the assumption of normal hierarchy.

from  $\nu_\mu \rightarrow \nu_e$  oscillations. As signal, we use CC events and QE events and as background we consider NC events. Here, in order to avoid the double counting, we take the normalization free for QE events. We also include systematics, signal and background normalization and tilt. About the parameter uncertainties, we assume 5% for three mixing angles, 1% for  $\Delta m_{31}^2$ , 4% for  $\Delta m_{21}^2$  and 5% for  $\rho$  as 1- $\sigma$  error. Later, we change the uncertainty of  $\Delta m_{31}^2$  to 0.1%. We use the GLOBES software for calculating the  $\Delta\chi^2$  [7].

Fig. 3 shows the CP sensitivity at  $L = 295\text{km}+1000\text{km}$ . In the left and right figures, we set  $\delta_{true} = 90^\circ$  and  $180^\circ$ , respectively. The horizontal line represents the test value of  $\delta$  and the vertical line represents the value of  $\Delta\chi^2$ . In this calculation, we assume that hierarchy is already determined as normal. Pink (thin) lines represent  $\Delta\chi^2$  for including systematics only. On the other hand, blue (thick) lines represent  $\Delta\chi^2$  including also the effect of parameter uncertainties. In both cases, it is found that we cannot measure the CP phase effect well in  $\nu_\mu \rightarrow \nu_\mu$  oscillations because of the parameter uncertainties.

In fig. 4, we show  $\Delta\chi^2$  at the baseline length  $L = 5000\text{km}$ . Other conditions are the same as those in fig. 3. Compared to the previous case,  $\Delta\chi^2$  is not affected by the parameter uncertainties very much. In fig. 4, we can see that the allowed range of  $\delta$  at 2- $\sigma$  C.L. is about  $120^\circ$ . Although the precision is not so good due to the small statistics in  $L = 5000\text{km}$ , there is a possibility for measuring the CP phase by only  $\nu_\mu \rightarrow \nu_\mu$  oscillations.

#### 4. Correlation of CP Phase with $\delta(\Delta m_{31}^2)$

Next, we explain that the uncertainty of  $\Delta m_{31}^2$  is related to the precise determination of  $\delta$  in more detail by using the analytical expression and explore the possibility for the improvement. For the case of  $O(\Delta'_{21}) \ll 1$ , the probability  $P_{\mu\mu}$  is reduced to the well known expression

$$P_{\mu\mu} \simeq 1 - \sin^2 \Delta'_{31} - \frac{8J_r \Delta'_{21}}{\tilde{\Delta}'_{21}} \sin \tilde{\Delta}'_{21} \sin \Delta'_{31} \cos \Delta'_{31} \cos \delta, \quad (21)$$

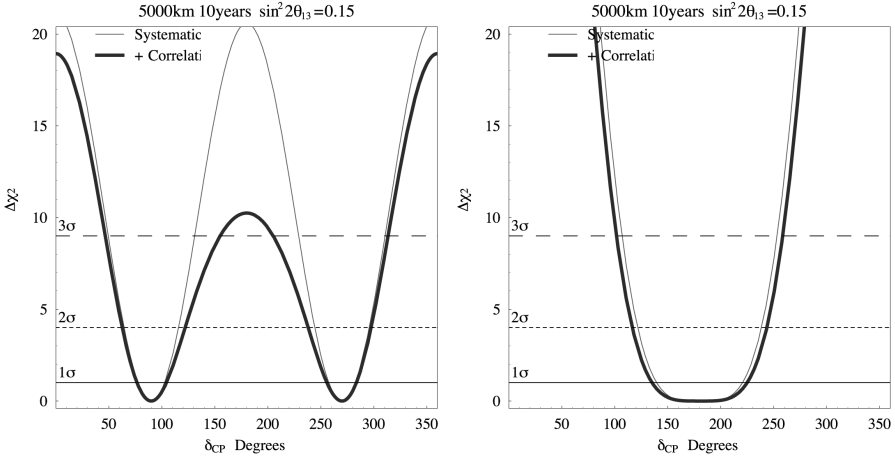


Fig. 4  $\Delta\chi^2$  at  $L = 5000\text{km}$ . Conditions except for the baseline length are the same as in fig.3.

as shown in ref. [4]. If we perform the replacements  $\Delta_{31} \rightarrow \Delta_{31}(1 + \epsilon)$  and  $\delta \rightarrow \delta'$ , the probability changes as

$$P'_{\mu\mu} \simeq 1 - \sin^2 \Delta'_{31} - \frac{4J_r \Delta_{21}}{\tilde{\Delta}_{21}} \sin \tilde{\Delta}'_{21} \sin 2\Delta'_{31} \left( \cos \delta' + \frac{\Delta_{31} \epsilon \tilde{\Delta}_{21} L}{8J_r \Delta_{21} \sin \tilde{\Delta}'_{21}} \right) \quad (22)$$

up to the leading order of  $s_{13}$  and  $\epsilon$ . Here, we should note that the energy dependence of the correction term from the uncertainty of  $\Delta m_{31}^2$  is approximately equal to that of  $\cos \delta'$  term. Namely, we cannot distinguish two probabilities  $P_{\mu\mu}(\Delta_{31}(1 + \epsilon), \delta')$  and  $P_{\mu\mu}(\Delta_{31}, \delta)$  when the relation

$$\cos \delta' \simeq \cos \delta - \frac{\Delta_{31} \epsilon \tilde{\Delta}_{21} L}{8J_r \Delta_{21} \sin \tilde{\Delta}'_{21}} \simeq \cos \delta - \frac{\Delta m_{31}^2 \epsilon a L}{8J_r \Delta m_{21}^2 \sin \frac{aL}{2}} \quad (23)$$

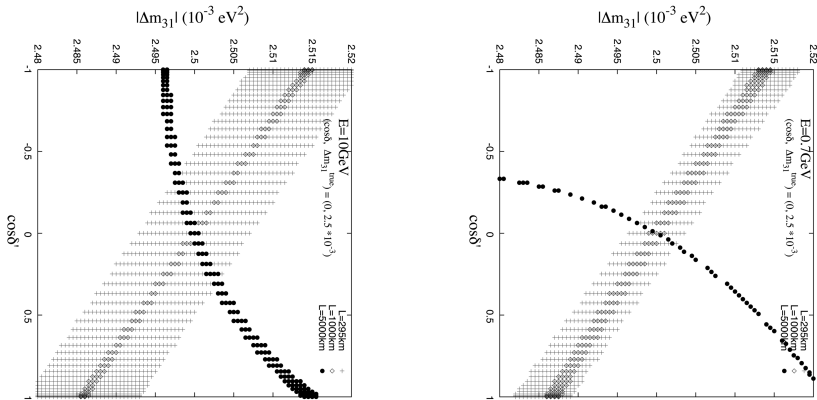
is satisfied, where we use the approximation  $\tilde{\Delta}_{21} \simeq a$ . This means that the value of the CP phase cannot be determined if  $\epsilon$  is larger than a certain value. This is the correlation between  $\delta$  and the uncertainty of  $\Delta m_{31}^2$ , and is a serious obstacle for measuring the CP phase in  $\nu_\mu \rightarrow \nu_\mu$  oscillations. The small value of the CP sensitivity in the baseline  $L = 295\text{km} + 1000\text{km}$  is due to this correlation.

Next, let us estimate the magnitude of  $\epsilon$  giving the small CP sensitivity. If we substitute  $\rho = 3.3\text{g/cm}^3$ ,  $Y_e = 0.494$ ,  $J_r = s_{12}c_{12}s_{23}c_{23}s_{13}c_{13}^2 \simeq 0.43$ ,  $\Delta m_{31}^2 = 2.5 \cdot 10^{-3}\text{eV}^2$  and  $\Delta m_{21}^2 = 8.1 \cdot 10^{-5}\text{eV}^2$  into (23), we obtain

$$\cos \delta' \simeq \cos \delta - \frac{5 \cdot 10^{-2} \epsilon L}{\sin(2.67 \cdot 10^{-4} L)}. \quad (24)$$

In the case of relatively short baseline, the above relation is further reduced to

$$\cos \delta' \simeq \cos \delta - 178\epsilon \quad (25)$$



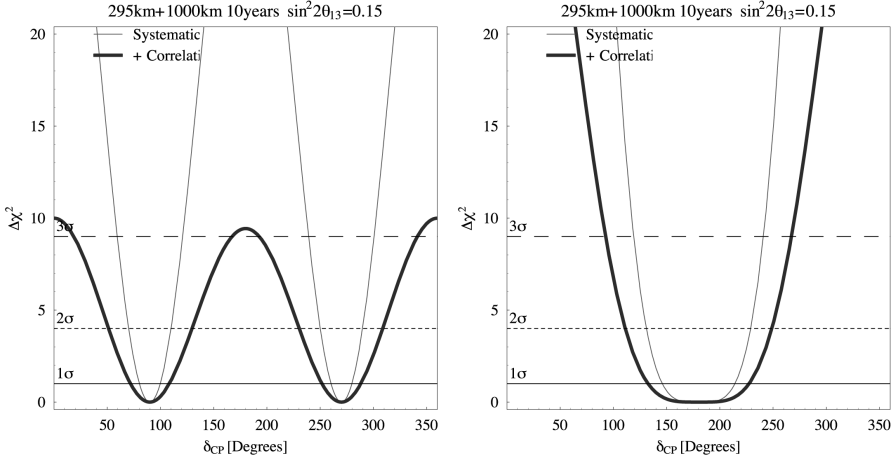
**Fig. 5** Correlation of  $\delta$  with the uncertainty of  $\Delta m_{31}^2$ . In left and right figures, we take  $E = 10$  GeV and  $0.7$  GeV. Regions with same color have almost the same probability given by  $|P_{\mu\mu}^{true} - P_{\mu\mu}| < 0.001$  ( $0.00005$ ) in the left (right) figure. Different colors correspond to the different baseline lengths.

and does not depend on the baseline length. Let us illustrate the meaning of this relation in fig. 5.

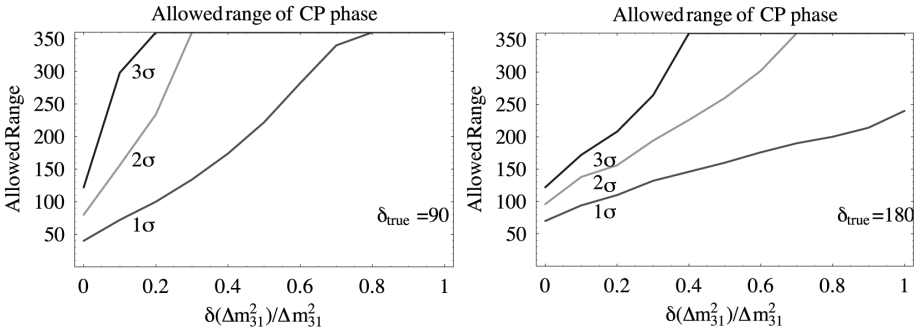
In fig. 5, regions with same color have almost the same probability as that for  $\delta_{true} = 90^\circ$ . More concretely, the region with  $|P_{\mu\mu} - P_{\mu\mu}^{true}| \leq 0.001$  ( $0.00005$ ) is plotted in the left (right) figure. The horizontal axis and vertical axis are taken as  $\cos \delta'$  and  $\Delta m_{31}^2$  ( $\epsilon$ ) respectively. Left and right figures represent the case for  $E = 10$  GeV and  $0.7$  GeV. Red (light grey), blue (dark grey) and black colors correspond to the cases for  $L = 295$  km,  $1000$  km and  $5000$  km. It is obvious that the value of  $\delta$  can be determined by measuring the probabilities for two different energies for the case of  $L = 5000$  km from fig. 5, because a superposition of the black curves for the two different energies (left and right figures) would lead to a clear intersection point representing the allowed region. In contrary, we cannot determine the value of  $\delta$  for a relatively short baseline like  $L = 295$  km and  $1000$  km even if the probabilities are measured for two different energies, because of an almost identical overlap of regions with the same color for the two different energies (left and right figures). The overlapping is over the whole range of angles  $0^\circ \leq \delta \leq 360^\circ$ . We can determine the slope of the coefficient of  $\epsilon$ , namely  $-1/178$  in (25). From this observation, we conclude that the uncertainty of  $\Delta m_{31}^2$  of more than  $0.6\%$  prevents us from determining  $\delta$  from  $\nu_\mu \rightarrow \nu_\mu$  oscillations only in relatively short baselines. Or in other words, for the case of short baseline length the determination of the CP phase  $\delta$  will become possible, if the uncertainty can be decreased below  $0.6\%$ , as demonstrated in the following section.

## 5. Possibility for the Test of Unitarity in T2KK

The uncertainty of  $\Delta m_{31}^2$  can be reduced up to  $1\%$  at the T2K experiment and the NO $\nu$ A experiment [8, 10, 11]. However, it is required that the uncertainty has to be reduced one more order of magnitude in order to receive the sensitivity for the CP phase as discussed in the previous section. Here, we simply assume that this is realized before the T2KK experiment starts. In ref. [4], we have discussed the possibility for diminishing the uncertainty of  $\Delta m_{31}^2$  and furthermore measuring the CP phase in  $\nu_\mu \rightarrow \nu_\mu$  oscillations by using the result of the T2KK experiment



**Fig. 6**  $\Delta\chi^2$  at  $L = 295\text{km}+1000\text{km}$  with 0.1% uncertainty of  $\Delta m_{31}^2$ . Conditions except for the uncertainty of  $\Delta m_{31}^2$  are the same as in fig.3.



**Fig. 7**  $\delta(\Delta m_{31}^2)$  dependence of CP sensitivity at  $L = 295\text{km}+1000\text{km}$ . In left and right figures, we take  $\delta_{true} = 90^\circ$  and  $180^\circ$ . Three lines show 1,2 and 3- $\sigma$  C.L. lines respectively. We fix the 1-3 mixing as  $\sin^2 2\theta_{13} = 0.15$ .

only.

In fig. 6, we show the CP sensitivity at  $L = 295\text{km}+1000\text{km}$  as in fig. 3 but with 0.1% error of  $\Delta m_{31}^2$ . We found that the allowed range becomes about  $160^\circ$  ( $140^\circ$ ) at 2- $\sigma$  when  $\delta_{true} = 90^\circ$  ( $180^\circ$ ). So, we can measure the CP phase even at these baselines by using only  $\nu_\mu \rightarrow \nu_\mu$  oscillations.

In fig. 7, we show how the CP sensitivity changes according to the uncertainty of  $\Delta m_{31}^2$ , where we fix  $\theta_{13}$  as  $\sin^2 2\theta_{13} = 0.15$ . As a result, we obtain the information of the CP phase from  $\nu_\mu \rightarrow \nu_\mu$  oscillations when the uncertainty of  $\Delta m_{31}^2$  is below 0.6% as expected from the consideration in the previous section.

Finally, we show the  $\theta_{13}$  dependence of CP sensitivity in fig. 8. We fix the uncertainty of  $\Delta m_{31}^2$  as 0.1%. As we expected from the analytical expression of  $A_{\mu\mu}$ , which is proportional to  $s_{13}$ , the sensitivity for  $\delta$  becomes worse gradually according to the decrease of  $\theta_{13}$ . As a result, we obtain the information of the CP phase in the case of  $\sin^2 2\theta_{13} > 0.03$  at the 1- $\sigma$  level.



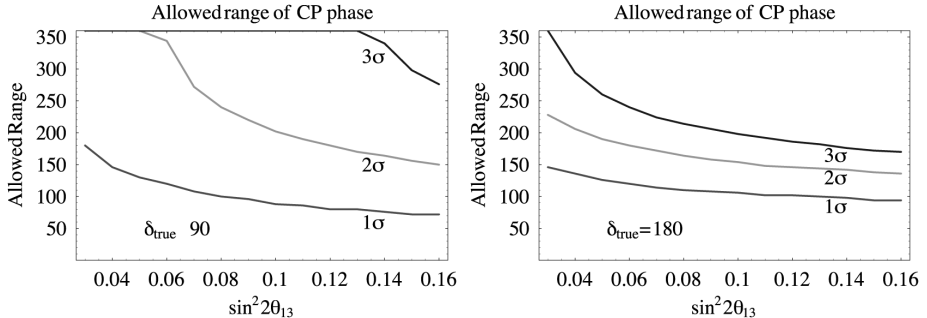


Fig. 8  $\theta_{13}$  dependence of CP sensitivity at  $L = 295\text{km}+1000\text{km}$ . We fix the uncertainty of  $\Delta m_{31}^2$  as 0.1. Other conditions are the same as in fig.7.

## 6. Summary

In summary, we have explored a new possibility for measuring the CP phase by only  $\nu_\mu \rightarrow \nu_\mu$  oscillations independently of  $\nu_\mu \rightarrow \nu_e$  oscillations in the case that  $\sin^2 2\theta_{13}$  is larger than 0.03 and has been determined in the next generation reactor experiments. If we can measure the CP phase in two different channels independently and there is a difference for these values, this would be considered as the sign of new physics. Below, the results are listed.

- At first, we have investigated the energy and the baseline where the  $\cos\delta$  term included in  $P_{\mu\mu}$  becomes large by using both numerical and analytical methods. As the result, we found from (19) and (20) that the coefficient  $A_{\mu\mu}$  has its largest value around  $E = 0.43\text{GeV}$  and  $L = 5000$  and  $10000\text{km}$  in the earth mantle. The difference of the probabilities attains the maximal value about 0.2 due to the CP phase effect even after averaging over the energy.
- Next, we have considered the same beam and the detector as in the T2HK experiment but the baseline (fiducial mass)  $L = 295\text{km}$  (250kt)+1000km (250kt) and  $L = 5000\text{km}$  (500kt) and have calculated the CP sensitivity by using  $\chi^2$  method. As the result, the allowed range becomes  $120^\circ$  ( $120^\circ$ ) at  $2\text{-}\sigma$  in  $L = 5000\text{km}$  when  $\delta_{true} = 90^\circ$  ( $180^\circ$ ). On the other hand, we have almost no sensitivity for the CP phase in the case of  $L = 295\text{km}+1000\text{km}$  because of the parameter uncertainties.
- We have shown that the precise measurement of  $\Delta m_{31}^2$  is particularly important in determining the value of  $\delta$ . If the uncertainty of  $\Delta m_{31}^2$  will be less than 0.6%, a certain sensitivity for  $\delta$  will be obtained even in the relatively short baseline like  $L \leq 1000\text{km}$ .
- We have explored the possibility for measuring the CP phase in the T2KK setup assuming the uncertainty of  $\Delta m_{31}^2$  as 0.1%. As the result, the allowed range is improved up to about  $160^\circ$  ( $140^\circ$ ) at  $2\text{-}\sigma$  when  $\delta_{true} = 90^\circ$  ( $180^\circ$ ).

In future, the mixing angles and the mass squared differences are precisely measured in various kinds of experiments. If  $\theta_{13}$  is found in the next generation reactor experiments in addition to this improvement, it will be very important to consider the strategy for exploring the new physics. As one of the strategies, the possibility for measuring the CP phase in  $\nu_\mu \rightarrow \nu_\mu$  oscillation is interesting.

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