Idea of Resolving Neutrino Parameter Degeneracy

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Abstract

To determine neutrino parameters unknown to date from oscillation experiments, it is essential to resolve neutrino parameter degeneracies. We first of all review on how the neutrino parameter degeneracies occur and discuss a recent proposal to lift the neutrino parameter degeneracies by using two identical detectors, one in Kamioka and the other in Korea. We also demonstrate the possible impact of astrophysical neutrino sources to be measured at neutrino telescope on the determination of neutrino mass hierarchy and leptonic CP phase.

1. Introduction

The present neutrino experimental data [1, 2, 3] exhibit that the atmospheric neutrino deficit points toward a maximal mixing between the tau and muon neutrinos, whereas the solar neutrino deficit favors a not-so-maximal mixing between the electron and muon neutrinos. Those precision results provide robust evidemce for the neutrino oscillation. Nevertheless, there exist the oscillation parameters which are still unknown to date. They are the third mixing angle θ_{13} for which we have an experimental bound from CHOOZ reactor experiments [4], the mass hierarchy, normal ($\Delta^2_{31} > 0$) versus inverted ($\Delta m^2_{31} < 0$), which is closely related with the sign of the mass squared difference Δm^2_{31} , and the leptonic CP phase δ.

Since neutrino beams such as T2K [5], NOνA [6] and MINOS [7] are sensitive to the unknown neutrino parameters, $\sin^2 2\theta_{13}, \delta_{CP}$ and neutrino mass hierarchy, we can expect that long baseline experiments will determine those quantities in the future. However, it has been addressed that the determination of the unknown neutrino parameters mentioned above leads to correlations and degeneracies when it comes to the extraction of the individual parameters. This fact makes it difficult to determine uniquely the values of the oscillation parameters.

The purpose of this talk is, first of all, to review on the neutrino parameter degeneracies encountered when we try to determine the individual unknown parameters from neutrino oscillation experiments and then to introduce recent proposal to resolve the parameter degeneracies by using two identical detectors. In particular, in this talk, we introduce recent study of the possible impact of astrophysical neutrino sources to be measured at neutrino telescope on the determination of neutrino mass hierarchy and leptonic CP phase, and discuss future prospect for resolving the neutrino parameter degeneracies by using the astrophysical neutrino sources.

2. Review of Neutrino Parameter Degeneracies

It has been known that there exist three kinds of parameter degeneracies : the intrinsic (θ_{13}, δ) degeneracy [8], the degeneracy of flipping the sign of Δm^2_{31} [9], and the degeneracy of θ_{23} octant [10, 11]. The latter two parameter degeneracies can be simply understood by considering the probability in two-neutrino oscillation with a single mixing angle and a single mass-squared difference Δm^2 given as

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follows,

$$P_{\alpha\beta} = \delta_{\alpha\beta} - (2\delta_{\alpha\beta} - 1)\sin^2 2\theta \sin^2(\frac{\Delta m^2 L}{4E}). \tag{1}$$

It is easy to see from Eq.(1) that the probability is invariant under flipping the sign of Δm^2 or switching the angle θ with its complementarity angle $\pi/2 - \theta$. Therefore, in the light of the neutrino probability given above, the two sets, $(\theta, \Delta m^2)$ and $(\pi/2 - \theta, -\Delta m^2)$, are not physically different. The above probability is therefore two-fold degenerate simply due to $(\theta, \Delta m^2)$ and $(\pi/2 - \theta, \Delta m^2)$ or due to $(\theta, \Delta m^2)$ and $(\theta, -\Delta m^2)$. On the other hand, the ν_e-appearance probability in the framework of three neutrino flavors with matter effect A,

$$\begin{aligned} &\left\{ \begin{array}{l} P(\nu_\mu \to \nu_e) \\ P(\bar{\nu}_\mu \to \bar{\nu}_e) \end{array} \right\} = \sin^2\theta_{23}\sin^2 2\theta_{13}\frac{\sin^2((\Delta m^2_{31} \mp A)L/4E_\nu)}{(1 \mp A/\Delta m^2_{31})^2} \\ &+ \frac{\Delta m^2_{21}}{\Delta m^2_{31}}\sin 2\theta_{23}\sin 2\theta_{13}\sin 2\theta_{12}\cos(\delta \pm \Delta m^2_{31}L/4E_\nu)\cdot \\ &\qquad \cdot\frac{\sin((\Delta m^2_{31} \mp A)L/4E)\sin(AL/4E_\nu)}{(1 \mp A/\Delta m^2_{31})(A/\Delta m^2_{31})} \\ &+ (\frac{\Delta m^2_{21}}{\Delta m^2_{31}})^2\cos^2\theta_{23}\sin^2 2\theta_{12}\frac{\sin^2(A/4E_\nu)}{(A/\Delta m^2_{31})^2}, \end{aligned} \tag{2}$$

provides multi solutions to (θ_{13}, δ) pair and $(\Delta m^2_{31}, \delta)$ pair. It is clear that the first and the third terms in Eq.(2) are blind to the sign of Δm^2_{31}. The second term is also blind to the sign due to $\cos\delta$. So in that way, the sign of Δm^2_{31} causes another double degeneracy to the probability. It is well known as a matter of neutrino mass hierarchy, normal versus inverse hierarchy. The multiple possibilities in the pair of (θ_{13}, δ) for a given value of $\sin 2\theta_{13}\cos\delta$ lead to the eight-fold degeneracy with other two double degeneracies. The matter effect A gives little contribution to the degeneracies.

In fact, the intrinsic degeneracy is exact when $\Delta m^2_{21}/\Delta m^2_{31}$ is exactly zero. The degeneracy of flipping the sign of Δm^2_{31} is exact when the matter effect is absent. The degeneracy of octant for θ_{23} is exact when the oscillation parameter $\cos 2\theta_{23}$ is exactly zero. Therefore, even if we measure the values of the oscillation probabilities $P(\nu_{mu} \to \nu_e)$ and $P(\bar{\nu}_\mu \to \bar{\nu}_e)$, it is still difficult to determine uniquely the values of the oscillation parameters due to the eight-fold degeneracy.

To see how the eight-fold degeneracy is lifted, it is useful to consider the plots which give eight different points for the eight different solutions. An effort was made by Minakata *et al.* [12] to visualize the eight different points by plotting the trajectories of constant probabilities in the $(\sin^2\theta_{13}, s^2_{23})$ plane. Also, Yasuda [13] proposed a plot in the in the $(\sin^2\theta_{13}, 1/s^2_{23})$ plane, which offers the simplest way to visualize how the eight-fold degeneracy is lifted. Fig. 1 [13] shows the trajectories of solutions given by $P(\nu_\mu \to \nu_e)$=const. and $P(\bar{\nu}_\mu \to \bar{\nu}_e)$=const., which indicates the eight-fold degeneracy. The plot is very useful to see how the eight-fold degeneracy is resolved.

Very recently, Kajita *et al.* [14] showed that two identical detectors with each fiducial mass of 0.27 megaton water, one in Kamioka and the other in Korea, which receive the (anti-) muon neutrino beam of 4 MW power from J-PARC facility have potential of determining the neutrino mass hierarchy and discovering leptonic CP violation by resolving the parameter degeneracies. They also discussed a possibility that the same setting has capability of resolving the θ_{23} octant degeneracy in region where $\sin^2 2\theta_{23} \le 0.97$ at 2 σ C.L. even for very small value of θ_{13}. Here,

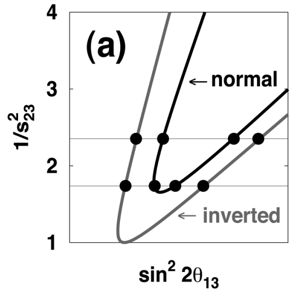

Fig. 1 Trajectories of solutions given by $P(\nu_\mu \to \nu_e)$ =const. and $P(\bar{\nu}_\mu \to \bar{\nu}_e)$ =const., and the eightfold degeneracy. [13]

we note that there are two merits of measuring neutrino beam generated at J-PARC in Korea [14]. One is that the contribution from $\Delta^2 m_{12} L/E$ becomes large, which is useful to determine the sign of Δm^2_{31}, and the other is that the correlation between CP phase and θ_{13} being measured in Korea is different from that in Superkamiokande. An interesting property phrased as decoupling between degeneracies was found in [14], which is valid to first-order in perturbation theory of the earth matter effect and may serve as a possible solution of a particular degeneracy without worrying about the presence of other degeneracies.

3. Astrophysical Neutrino Sources at a Neutrino Telescope

It has been proposed that a detection of astrophysical neutrinos at neutrino telescopes [16] with a well-predicted flavor composition at the source could provide additional knowledge on the neutrino mixing parameters [17, 18, 19] Although the existence of such astrophysical neutrinos is not yet proven, the detection of very high energy cosmic rays indicates that cosmic accelerators producing high energy neutrinos exist. There are many candidates for neutrino sources, such as gamma ray bursts, active galactic nuclei,or starburst galaxies.

The astrophysical neutrino sources to analyze for oscillation we consider are neutrino beam source, pion beam source and muon damped source as discussed in [20]. If the distance to the source is long enough, neutrino oscillations on the way from the source to the Earth is averaged out and then the probability is simply given by

$$P^t_{\alpha\beta} \to \delta_{\alpha\beta} - 2\sum_{i=1}^{2}\sum_{j=i+1}^{3} Re[U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j}] = \sum_{i=1}^{3} |U_{\alpha i}|^2 |U_{\beta i}|^2, \tag{3}$$

where the relatively fast oscillation compared with the distance to the source is averaged out. From this result, we see that the predicted flavor composition at the Earth depends on the mixing parameters including CP phase and the CP-even part of the mixing only, whereas it is insensitive to the neutrino mass hierarchy.

On the detection of the astrophysical neutrinos, we do not need to distinguish between neutrinos and antineutrinos, so both neutrino and antineutrino fluxes are simply added. While a neutrino telescope can identify muons by their tracks, electron and tau neutrino events are harder to disentangle as they both produce showers of particles with a larger threshold. Depending on the neutrino energy, the electromagnetic showers and hadronic showers may be discriminated by their muon content as well as the tau track may be measured, which means that there could be a possibility to disentangle electron and tau neutrino events [21, 22]. An additional process to identify the electron antineutrino is the Glashow resonant process $\bar{\nu}_e + e^- \to W^-$ at 6.3 PeV, which can also be used for neutrino oscillation parameter measurements [19].

To investigate the impact of astrophysical neutrino flux measurements on the determination of the individual neutrino parameters unknown to date, it is useful to introduce a parameter R defined by the flux ratio $R = \Phi(\nu_\mu)/(\Phi(\nu_e) + \Phi(\nu_\tau))$ [17, 18], in which it is assumed that electron and tau events cannot be disentangled. This observable R can be extracted from the ratio of muon tracks to showers [22], but note that there is an additional hadronic shower background from neutral current events for all flavors which needs to be subtracted. We assume that we have a precision measurement of R with an effective relative error exploiting all available information and containing all systematics and backgrounds. For the effective error on R, we typically use 5%, 10%, and 20% errors on R in order to discuss the requirements to an astrophysical measurement in the following sections. For the study with astrophysical neutrino fluxes, it is assumed that the type of the astrophysical source can be identified.

It is known that the neutrino flux ratio at a telescope obtained by $\Phi^t(\nu_\alpha) = \sum_\beta P^t_{\alpha\beta}\Phi^0(\nu_\beta)$ is $1:1:1$ for pion source and $1:1.8:1.8$ for muon-damped source, if the best-fit values of the elements of MNS matrix are adopted [21]. Here, $\Phi(\nu_\alpha)$ denotes the flux of both neutrinos and antineutrinos. A neutrino telescope experiment is possible to measure simultaneously individual flavor fluxes, $\Phi^t(\nu_e)$, $\Phi^t(\nu_\mu)$, and $\Phi^t(\nu_\tau)$. Unless neutrinos decay, $\Phi^0(\nu_e) + \Phi^0(\nu_\mu) + \Phi^0(\nu_\tau) = \Phi^t(\nu_e) + \Phi^t(\nu_\mu) + \Phi^t(\nu_\tau)$, and $\sum_\beta P^0_{\alpha\beta} = \sum_\beta P^t_{\alpha\beta} = 1$. The sum of fluxes at a telescope can be normalized without loss of generality so that one can impose $\sum_\alpha \Phi^0(\nu_\alpha) = \sum_\alpha \Phi^t(\nu_\alpha) = 1$. For muon-damped source $\Phi^t(\nu_\alpha) = P_{\alpha\mu}$, while for pion source $\Phi^t(\nu_\alpha) = 1/3(P_{\alpha e} + 2P_{\alpha\mu})$ (see [23]).

The oscillation probability to be determined at a neutrino telescope is not free from the degeneracy either. Since the oscillating aspect is averaged out, the ambiguity in the sign of Δm^2_{ji} is hidden. Although Δm^2 is not exposed in estimating the fluxes at a telescope, it can be an implicit parameter to $\Phi^t(\nu)$ if another type of oscillation probability constrains all the involved parameters including Δm^2_{ji}. So, if the probability $P_{\mu e}$ at long baseline oscillation and a neutrino flux at a telescope are examined together, there are different curves depending on the sign of Δm^2_{31} as shown in Fig. 2 and Fig. 3 [23].

In the figures, CP δ runs from 0 to 2π for a given θ_{13} in space of the probability $P_{\mu e}$ on vertical axis versus the flux of a flavor of neutrinos and antineutrinos $\Phi(\nu_\alpha)$ on horizontal axis. Once both data are obtained, there are eight δ curves passing the data point so that the point can be expressed in terms of eight pairs of (θ_{13}, δ)'s. The eight pairs can be divided into two groups; one(a,b,c,d) is compatible with a value of θ_{23} smaller than $\pi/4$, while the other(e,f,g,h) is compatible with θ_{23} larger than $\pi/4$. Then, each group can be divided into two subgroups depending on the sign of Δm^2_{31}. Thus, if different signs of Δm^2_{31}, different values of θ_{23}'s and different pairs of (θ_{13}, δ)'s are all allowed simultaneously, a point of $(\Phi^t(\nu_\alpha), P_{\mu e})$ can be eight-fold degenerated [23].

4. Complementarity Terrestrial-Astrophysical

From now on, we demonstrate some idea on the impact of astrophysical neutrino observations on resolving the neutrino parameter degeneracies discussed in [20]. First, let us discuss the complementarity between astrophysical sources and terrestrial neutrino oscillation experiments, *ie*, neutrino beams and reactor experiments. First, we consider the dependence of R on the neutrino parameters. Depending

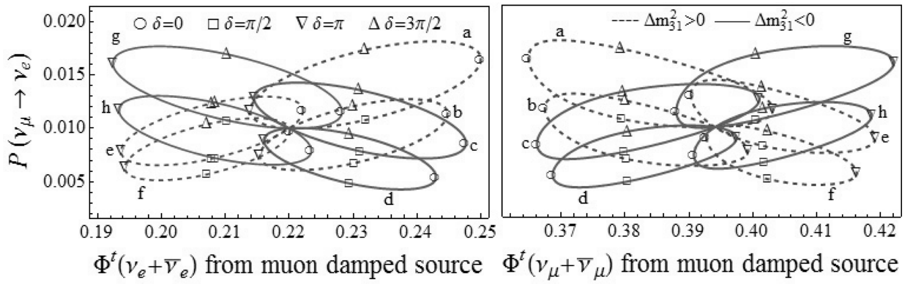

Fig. 2 Eight curves pass a point which is the measurements of $P_{\mu e}$ at LBL and $\Phi^t(\nu_e)$(left) or $\Phi^t(\nu_\mu)$(right) at a telescope from muon-damped source. It is eight-fold degenerated due to different signs of Δm^2_{31}, $\pi/4-\theta_{23}$ and different pairs of (θ_{13},δ). Each δ curve runs from 0 to 2π for a given θ_{13}. The value of θ_{23} is fixed as either $\pi/4-0.03$(a,b,c,d) or $\pi/4+0.03$(e,f,g,h). The blue dashed and the purple solid represent NH($\Delta m^2_{31}>0$) and IH($\Delta m^2_{31}<0$), respectively. [23]

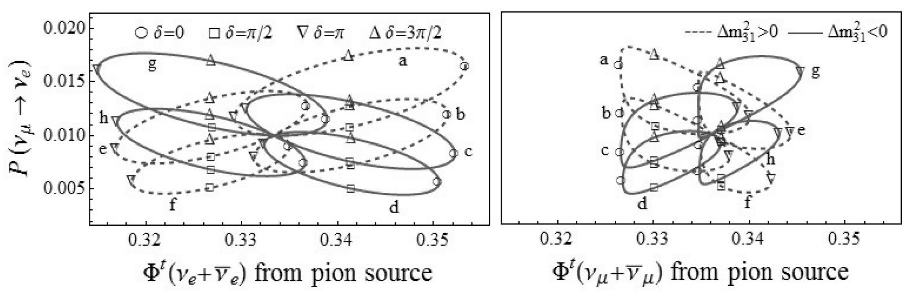

Fig. 3 Similar to Fig. 2, but for pion source. [23]

on the type of neutrino sources, the formulae for R are given by (see [20])

$$
\begin{aligned}
R^{\text{Neutron beam}} &= \frac{P_{e\mu}}{P_{ee}+P_{e\tau}} = \frac{P_{e\mu}}{1-P_{e\mu}}\,,\\
R^{\text{Muon damped}} &= \frac{P_{\mu\mu}}{P_{\mu e}+P_{\mu\tau}} = \frac{P_{\mu\mu}}{1-P_{\mu\mu}}\,,
\end{aligned}
\tag{4}
$$

whereas for the pion beam

$$
R^{\text{Pion beam}} = \frac{2P_{\mu\mu}+P_{e\mu}}{2P_{\mu e}+P_{ee}+2P_{\mu\tau}+P_{e\tau}}\,. \tag{5}
$$

Using the MNS neutrino mixing matrix and our standard values of the other oscillation parameters, one can then calculate R as function of the oscillation parameters for the different sources. Expanding R for the different astrophysical sources to first order in θ_{13}, we obtain [20]

$$
\begin{aligned}
R^{\text{Neutron beam}} &\sim 0.26 + 0.30\ \theta_{13}\ \cos\delta_{cp}\,, &(6)\\
R^{\text{Muon damped}} &\sim 0.66 - 0.52\ \theta_{13}\ \cos\delta_{cp}\,, &(7)\\
R^{\text{Pion beam}} &\sim 0.50 - 0.14\ \theta_{13}\ \cos\delta_{cp}\,. &(8)
\end{aligned}
$$

For neutrino beams, the δ_{cp}-dependent terms in $P_{\mu e}$ are suppressed by the mass hierarchy, which means that the θ_{13}^2-term is the leading term for large θ_{13}. At the first oscillation maximum and in vacuum, we find [20]:

$$P_{\mu e} \sim 2\,\theta_{13}^2 \pm 0.09\ \theta_{13} \sin\delta_{cp}\,, \tag{9}$$

where the plus is for antineutrinos and the minus for neutrinos. Because most of the first-generation superbeams are operated close to the first oscillation maximum and have a very narrow beam spectrum, this approximation should be useful for qualitative discussions. Most importantly, neutrino beams are dependent on the CP-odd $\sin\delta_{cp}$, whereas astrophysical sources are dependent on the CP-even $\cos\delta_{cp}$.

Fig. 4 [20] presents the exact dependence of the observables R (astrophysical neutrinos) and $P_{\mu e}$ (neutrino beams) on δ_{cp} for different sources. While astrophysical sources have the largest modulation of the amplitude at $\delta_{cp} = 0$ and π because of the $\cos\delta_{cp}$-dependence, neutrino beams are strongly influenced at $\delta_{cp} = \pm\pi/2$ because of the $\sin\delta_{cp}$-dependence at the oscillation maximum. In addition, as discussed above, neutrino beams show a different behavior for the neutrino and antineutrino operation modes. In order to illustrate the measurement precision of δ_{cp}, we show possible error bars for 5%, 10%, and 20% measurement errors as the shaded bars. From the projection of the curves onto these bars, we can immediately read off the required precisions for R and the relevant parameter regions in δ_{cp} (for large $\sin^2 2\theta_{13}$) for the different sources.

As far as different measurements are concerned, we expect an impact of the astrophysical sources on CP precision measurements (especially for δ_{cp} close to 0 and π), and for the mass hierarchy measurements at the superbeams because the $\mathrm{sgn}(\Delta m^2)$-degeneracy is located at a different value of (fake) δ_{cp} than the original solution.

5. Measuring CP Violation

In fact, it is difficult to obtain information on δ_{cp} from an astrophysical source alone as well after the other oscillation parameters have been marginalized over. However, if an astrophysical source is able to provide this information on a similar timescale as the reactor experiment, one will actually be able to learn something on δ_{cp} already before the superbeams provide results. In some case, it may be possible to get some information on δ_{cp} by combining results from reactor experiments and a measurement of astrophysical neutrino flux without the help of superbeams. Fig. 5 [20] shows some examples for the combination of an astrophysical and reactor (Double Chooz) experimental results for fit values of δ_{cp} as a function of $\sin^2 2\theta_{13}$. As far as the dependence on the true δ_{cp} is concerned, $\delta_{cp} = 0$ and π for neutron beam and muon damped sources yield similar qualitative results. For δ_{cp} close to maximal CP violation, only muon damped and pion sources can provide some hint for excluded values of δ_{cp} at the 1σ confidence level for high precisions of R (about 5%). If the precision of the astrophysical measurement is only 20%, there will be some hints for some points at the 1σ confidence level for neutron beam and muon damped sources. A 3σ exclusion is only possible for a muon damped source if $\delta_{cp} = 0$. In this case, one can actually exclude $\delta_{cp} = \pi$ at the 3σ C.L.

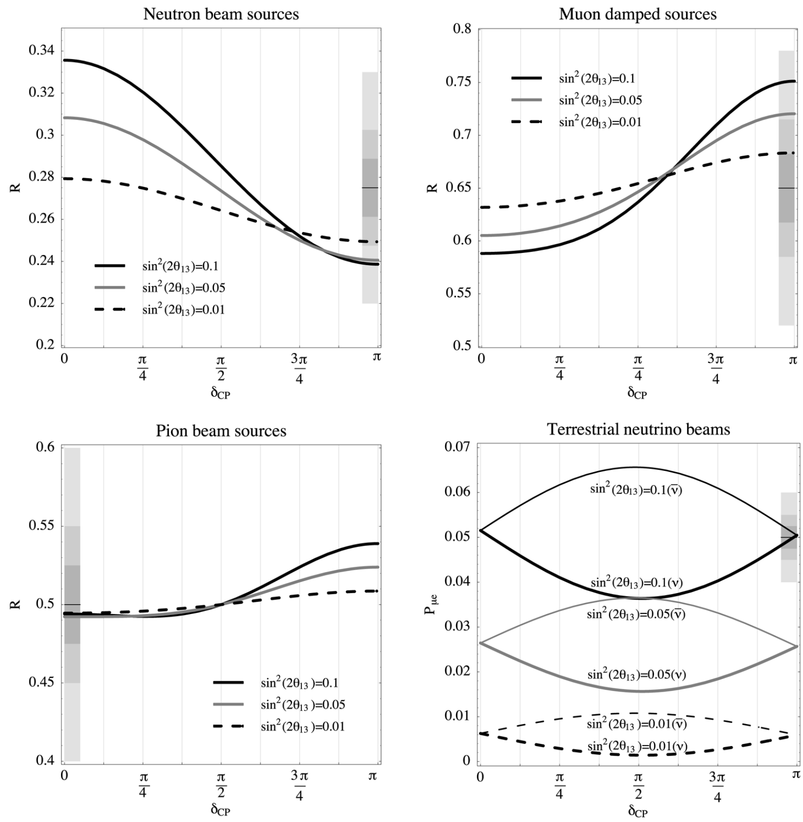

Fig. 4 Sources used in this study for which the signal depends on δ_{cp}. We show the quantities R and $P_{\mu e}$ ($P_{\bar{\mu}\bar{e}}$), respectively, as function of δ_{cp} for different values of $\sin^2 2\theta_{13}$. The shaded bars illustrate the size of the 5%, 10%, and 20% errors for the chosen central values (horizontal lines). [20]

6. Mass Hierarchy Determination

The mass hierarchy determination using astrophysical flavor ratios alone will not be possible because the observables do not depend on the mass hierarchy for averaged oscillations. Fig. 6 [20] shows the sensitivity to the normal mass hierarchy as function of true $\sin^2 2\theta_{13}$ and true δ_{cp} for MINOS, Double Chooz, T2K, and NOνA combined with an astrophysical neutrino source. The interpretation of this figure is as follows: For large $\sin^2 2\theta_{13}$, the terrestrial experiments alone will only be able to determine the mass hierarchy for about 50% of all possible values of δ_{cp} which could be realized by nature. However, using, for instance, a neutrino beam source flux measured with a precision of 20% increases this fraction to 80% of all values of δ_{cp}. Therefore, depending on chosen confidence level and precision of the astrophysical flux, the chance to discover the mass hierarchy will be improved from about half of all possible cases of δ_{cp} to almost certain. In addition, we expect qualitatively similar results for the inverted hierarchy, where the role of $\delta_{cp} = \pi/2$ and $3\pi/2$ is exchanged for the superbeams, while the astrophysical sources still provide the relevant information close to $\delta_{cp} = 0$ and π.

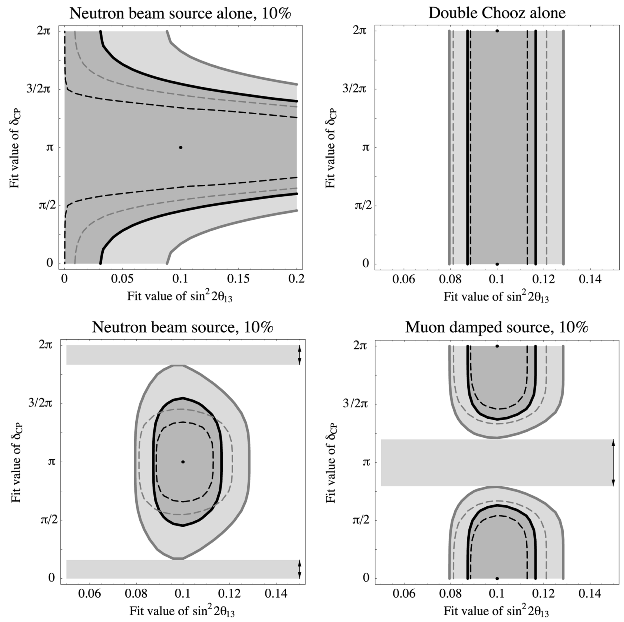

Fig. 5 Fit regions as function of $\sin^2 2\theta_{13}$ and δ_{cp} for different experiments as given in the plot captions (the lower row is always in combination with Double Chooz). The simulated values are chosen as marked by the dots. The contours are shown for the 1σ (black curves, dark regions) and 90% (gray curves, light regions) confidence level (1 d.o.f.). Dashed curves represent the results when the other (not shown) oscillation parameters are fixed, *ie*, not marginalized over. The arrows in the lower row mark the ranges in δ_{cp} which can be excluded at the 90% confidence level. [20]

7. Resolving the Octant Degeneracy

As discussed in [18], the flux ratio R measurable at neutrino telescopes has a distinctive dependence on θ_{23} which is sensitive to the $(\theta_{23}, \pi/2 - \theta_{23})$-degeneracy. Note, however, that without additional knowledge on the other mixing parameters, this information can only be extracted from R in very specific cases, but in a rather model-independent way [18]. We expect that the complementarity among superbeams, reactor experiments, and astrophysical sources allows for an exclusion of the octant degeneracy for any substantial deviation from maximal mixing. Fig. 7 [20] indicates the observables (R for astrophysical sources, total event rates for the beam) for the octant degeneracy resolution as function of $\sin^2\theta_{23}$ for $\sin^2 2\theta_{13} = 0$ (left) and $\sin^2 2\theta_{13} = 0.1$ (right). The bands reflect the unknown value of δ_{cp}. In order to discuss the potential to resolve the octant degeneracy, compare the left branch of each source ($\sin^2\theta_{23} < 0.5$) with the right branch ($\sin^2\theta_{23} > 0.5$). If we assume that a reactor experiment determines $\sin^2 2\theta_{13}$ fairly well and the impact of the unknown δ_{cp} is one of the main uncertainties, this picture should be quite accurate. For $\sin^2 2\theta_{13} = 0$, the situation is quite simple because, in this limit, δ_{cp} is meaningless. Even if the superbeam disappearance channel measures $\sin^2 2\theta_{23}$ very precisely, it does not have information on the octant. The rates in the appearance channel are too low to imply any information, and reactor experiment is not affected by θ_{23}. However, the flux ration R of the astrophysical sources is

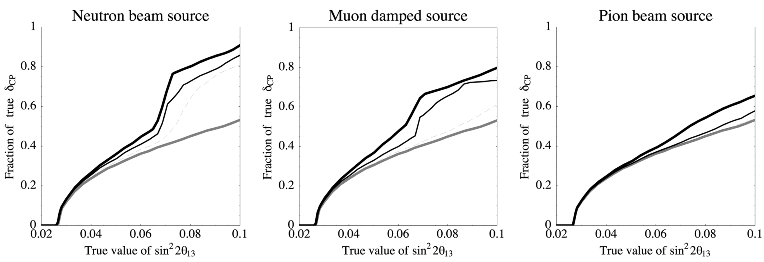

Fig. 6 Sensitivity to the normal mass hierarchy as function of true $\sin^2 2\theta_{13}$ and true δ_{cp} for MINOS, Double Chooz, T2K, and NOνA combined with an astrophysical neutrino source at the 2σ confidence level. The curves are for the following errors on the astrophysical flux ratio: no astrophysical flux observed (thick gray curves), 20% error on R (dashed curves), 10% error on R (thin black curves), a nd 5% error on R (thick black curves). [20]

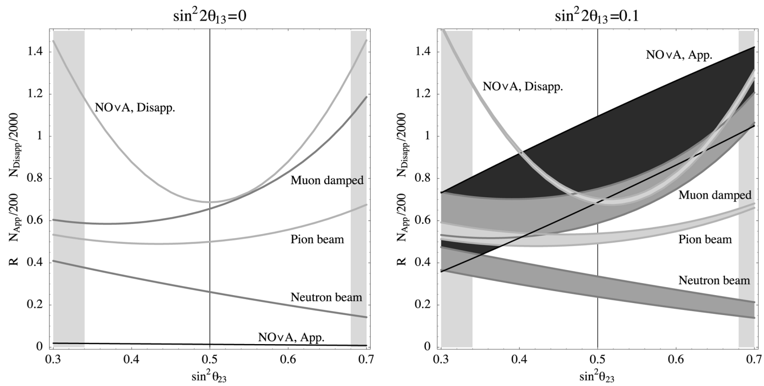

Fig. 7 Illustration of the observables (R for astrophysical sources, total event rates for the beam) for the octant degeneracy resolution as function of $\sin^2\theta_{23}$ for $\sin^2 2\theta_{13} = 0$ (left) and $\sin^2 2\theta_{13} = 0.1$ (right). The bands reflect the unknown value of δ_{cp}. The gray-shaded areas mark the 3σ excluded region [24]. For NOνA, we assume five years of neutrino running for this figure.[20]

very different for $\sin^2\theta_{23} < 0$ and $\sin^2\theta_{23} > 0$, which means that a resolution of the octant degeneracy should be possible. It turned out that the measurement for small $\sin^2 2\theta_{13}$ should be dominated by the astrophysical source, whereas the measurement for large $\sin^2 2\theta_{13}$ could be dominated by the beam (plus reactor experiment). The latter hypothesis needs to be quantified, because it it unclear how the correlations with the other oscillation parameters affect the degeneracy.

8. Conclusion

We reviewed on how the neutrino parameter degeneracies occur and some possible way to resolve those degeneracies in order to determine the individual neutrino parameters uniquely. We discussed the the ability of neutrino telescopes to extract the information on neutrino oscillation physics when terrestrial experimental results are combined, since this information may have a major impact on the neutrino oscillation program for the coming decade. But, as expected, the exact procedure and obtainable precision may require further research and the actual detection of a source. In addition, the interpretation of the data from all the discussed experiments will depend on the type of the astrophysical sources. However, important hints for the planning of future experiments may be obtained early from the combination of a set of experiments with poor or moderate statistics each, but great synergistic potential.

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