第1, 2世代ニュートリノの質量縮退から探る世代構造の謎

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Based on arXiv:1211.4452 [hep-ph].

Partially-(quasi-)Degenerate Mass-Spectrum of Neutrinos and Flavor Puzzles

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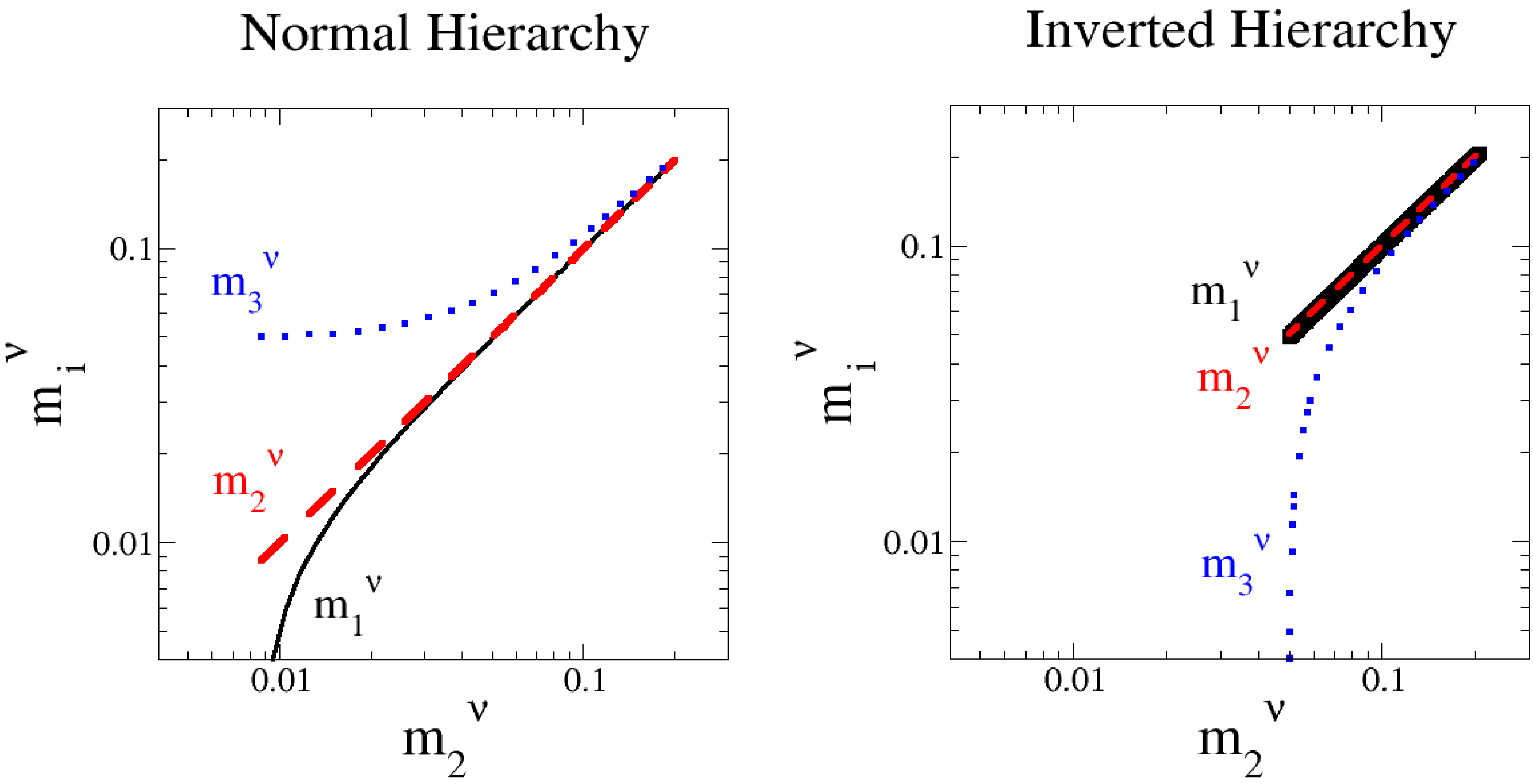
Based on arXiv:1211.4452 [hep-ph].

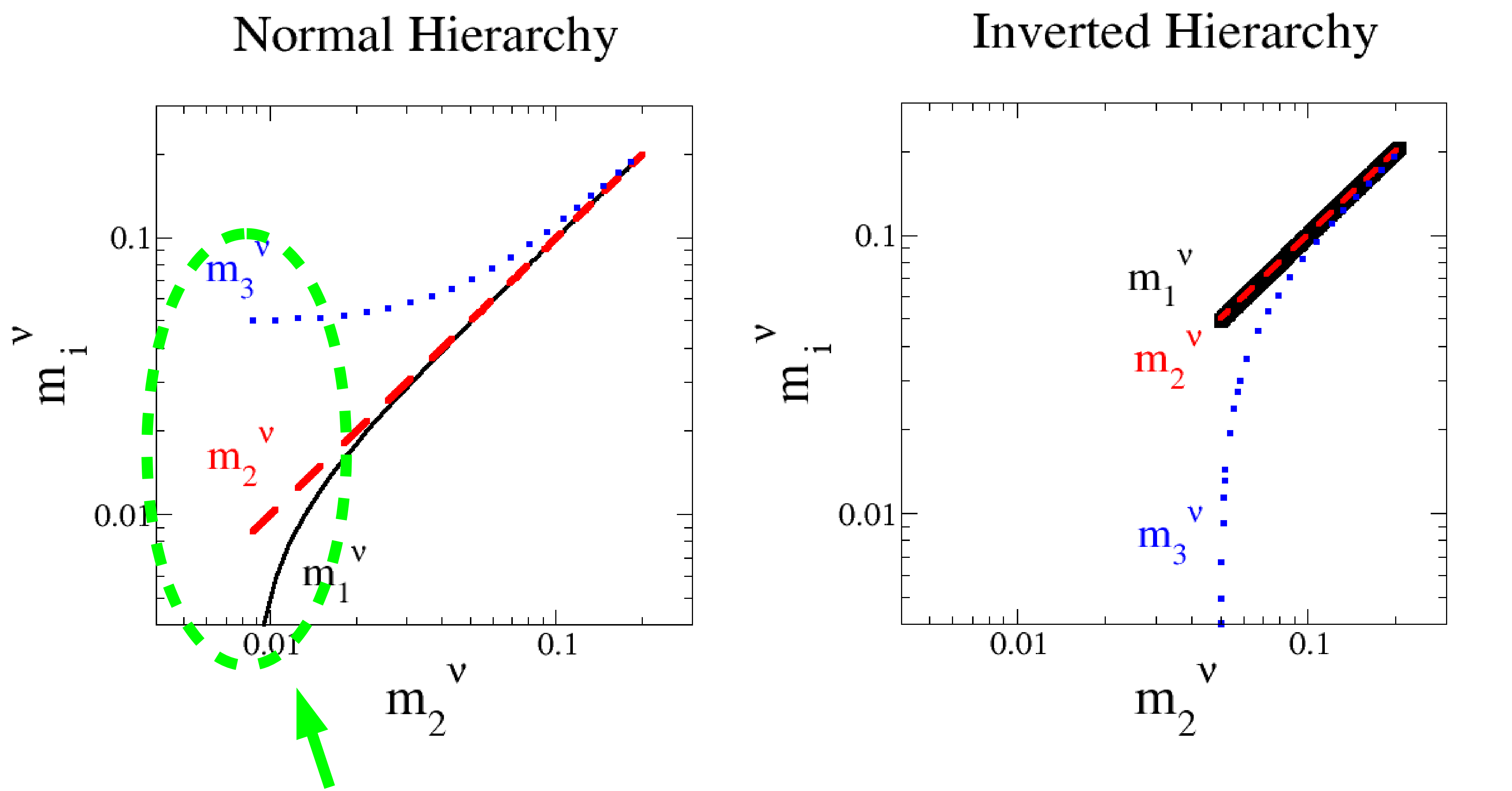
Neutrino mass hierarchy



Fixed: $\Delta m^2_{12} = 7.54 \times 10^{-5}\ eV^2 \quad \Delta m^2_{23} = 2.43(2) \times 10^{-3}\ eV^2$

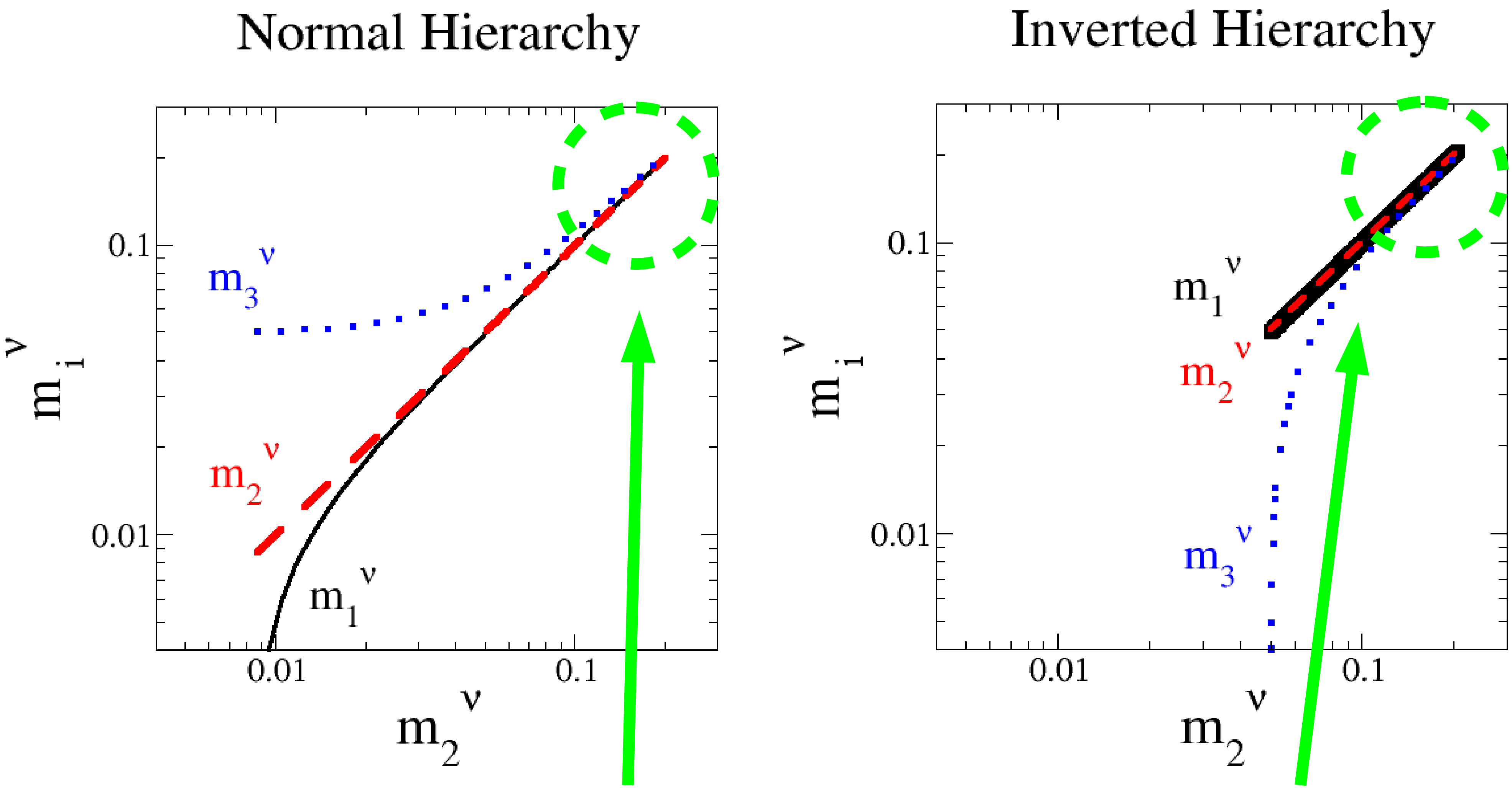
[Fogli, *etal*, PRD86(2012)]

Neutrino mass hierarchy



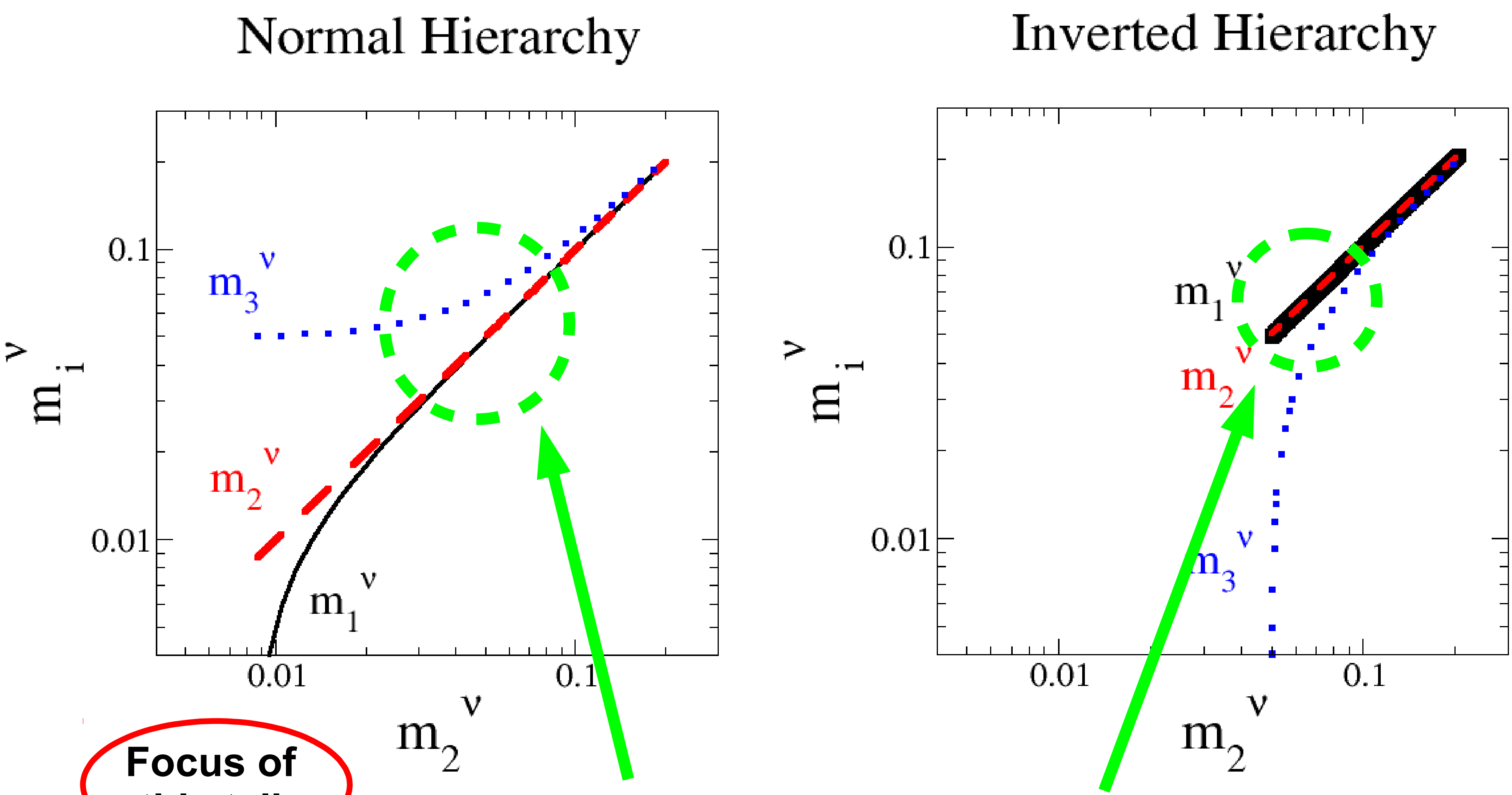
Hierarchical, like the quarks and charged leptons.

Neutrino mass hierarchy



All three neutrinos are quasi-degenerate in mass.

Neutrino mass hierarchy



Focus of this talk

Only the first two neutrinos are degenerate.

Contents

- Introduction

  A few more words about

  the neutrino masses and mixings
- Motivation
- Model

  D_N family symmetric model
- Summary and Future works

Neutrino oscillation

TABLE I: Results of the global 3ν oscillation analysis, in terms of best-fit values and allowed 1, 2 and 3σ ranges for the 3ν mass-mixing parameters. We remind that Δm^2 is defined herein as $m_3^2 - (m_1^2 + m_2^2)/2$, with $+\Delta m^2$ for NH and $-\Delta m^2$ for IH.

Parameter	Best fit	1σ range	2σ range	3σ range
$\delta m^2/10^{-5}$ eV^2 (NH or IH)	7.54	7.32 – 7.80	7.15 – 8.00	6.99 – 8.18
$\sin^2\theta_{12}/10^{-1}$ (NH or IH)	3.07	2.91 – 3.25	2.75 – 3.42	2.59 – 3.59
$\Delta m^2/10^{-3}$ eV^2 (NH)	2.43	2.33 – 2.49	2.27 – 2.55	2.19 – 2.62
$\Delta m^2/10^{-3}$ eV^2 (IH)	2.42	2.31 – 2.49	2.26 – 2.53	2.17 – 2.61
$\sin^2\theta_{13}/10^{-2}$ (NH)	2.41	2.16 – 2.66	1.93 – 2.90	1.69 – 3.13
$\sin^2\theta_{13}/10^{-2}$ (IH)	2.44	2.19 – 2.67	1.94 – 2.91	1.71 – 3.15
$\sin^2\theta_{23}/10^{-1}$ (NH)	3.86	3.65 – 4.10	3.48 – 4.48	3.31 – 6.37
$\sin^2\theta_{23}/10^{-1}$ (IH)	3.92	3.70 – 4.31	3.53 – 4.84 ⊕ 5.43 – 6.41	3.35 – 6.63
δ/π (NH)	1.08	0.77 – 1.36	—	—
δ/π (IH)	1.09	0.83 – 1.47	—	—

[Fogli, *etal*, PRD86(2012)]

Two large mixing angles and one small but non-zero angle:

$$\theta_{23}^{\text{Best}} = 38.4^\circ, \quad \theta_{12}^{\text{Best}} = 33.6^\circ \gg \theta_{13}^{\text{Best}} = 8.9^\circ,$$

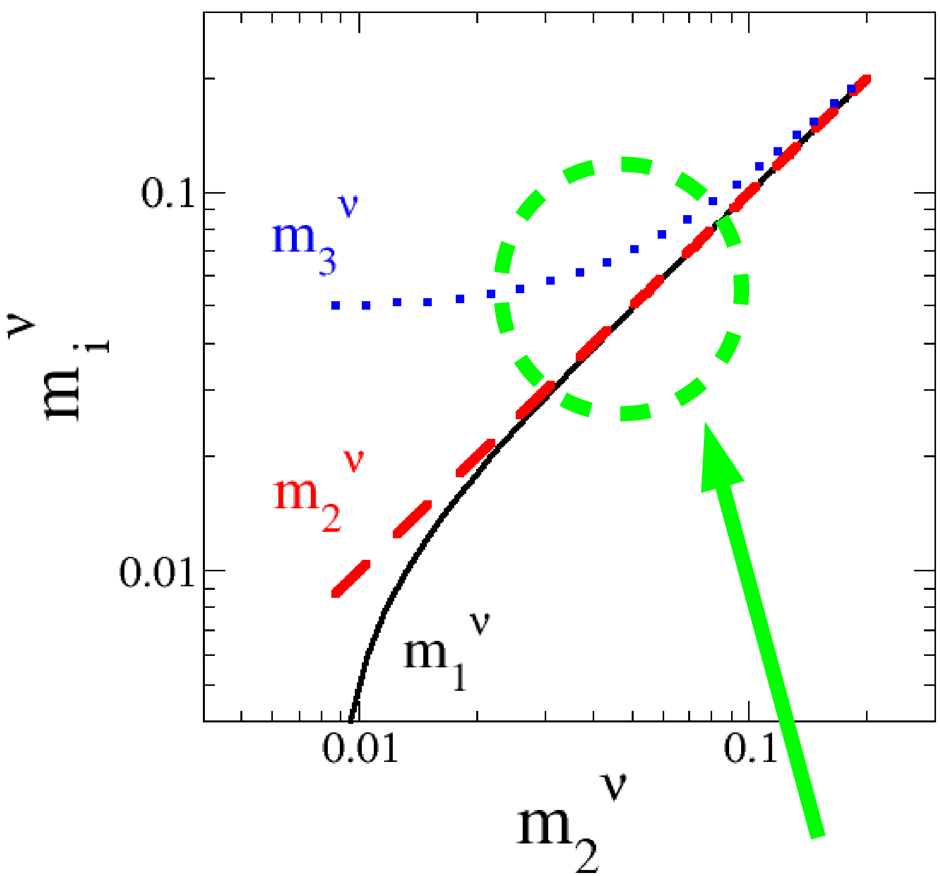
only two squared-mass differences are known:

$$\Delta m_{23}^2 \simeq 10^{-3} \gg \Delta m_{12}^2 \simeq 10^{-5},$$

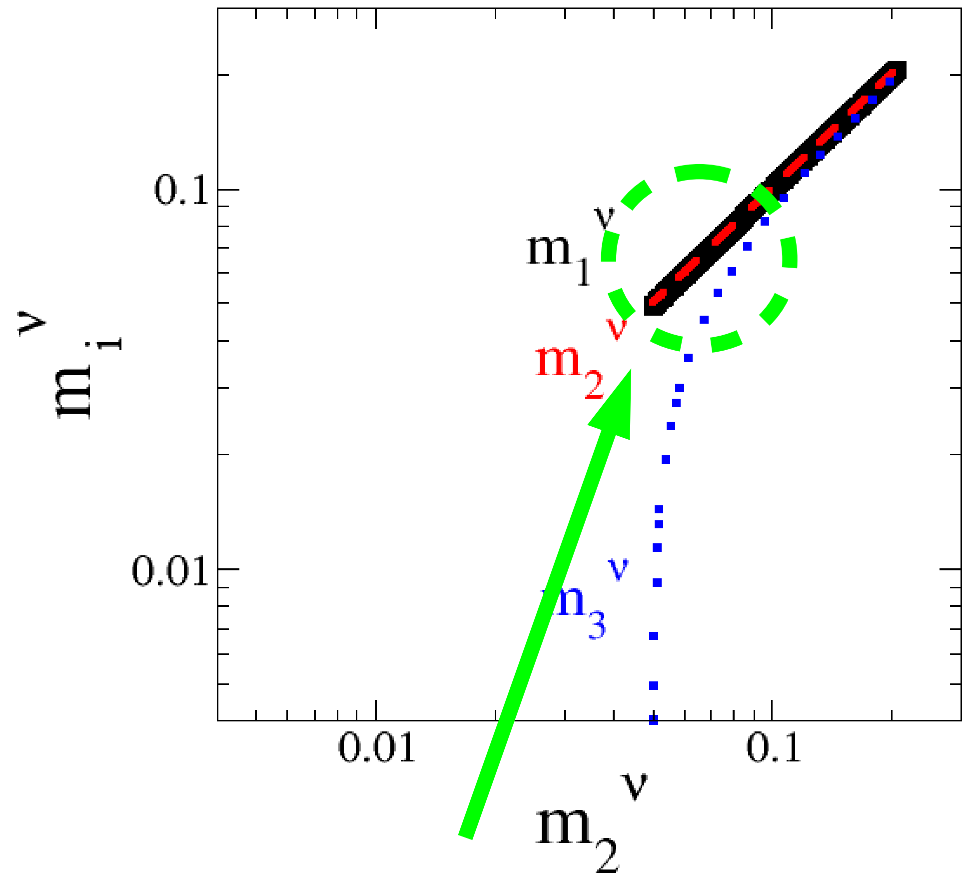
no information on the leptonic CP violation.

Partially-degenerate region

Normal Hierarchy



Inverted Hierarchy



NH ‡ $m_2^\nu = 0.03 \sim 0.1$ eV

$\left(\sum m^\nu = 0.12 \sim 0.31\right)$

IH ‡ $m_2^\nu = 0.05 \sim 0.1$ eV

$\left(\sum m^\nu = 0.10 \sim 0.29\right)$

Motivation

On the experimental side

Would be observed? by 0nbb experiments and CMB observations.

EXO(90% C.L.)

$$\langle m_{ee} \rangle < 0.14 - 0.38 \text{ eV}$$

[EXO, PRL109(2012)]

KamLAND-Zen(90% C.L.)

$$\langle m_{ee} \rangle < 0.26 - 0.54 \text{ eV}$$

[KamLAND-Zen, PRC89(2012)]

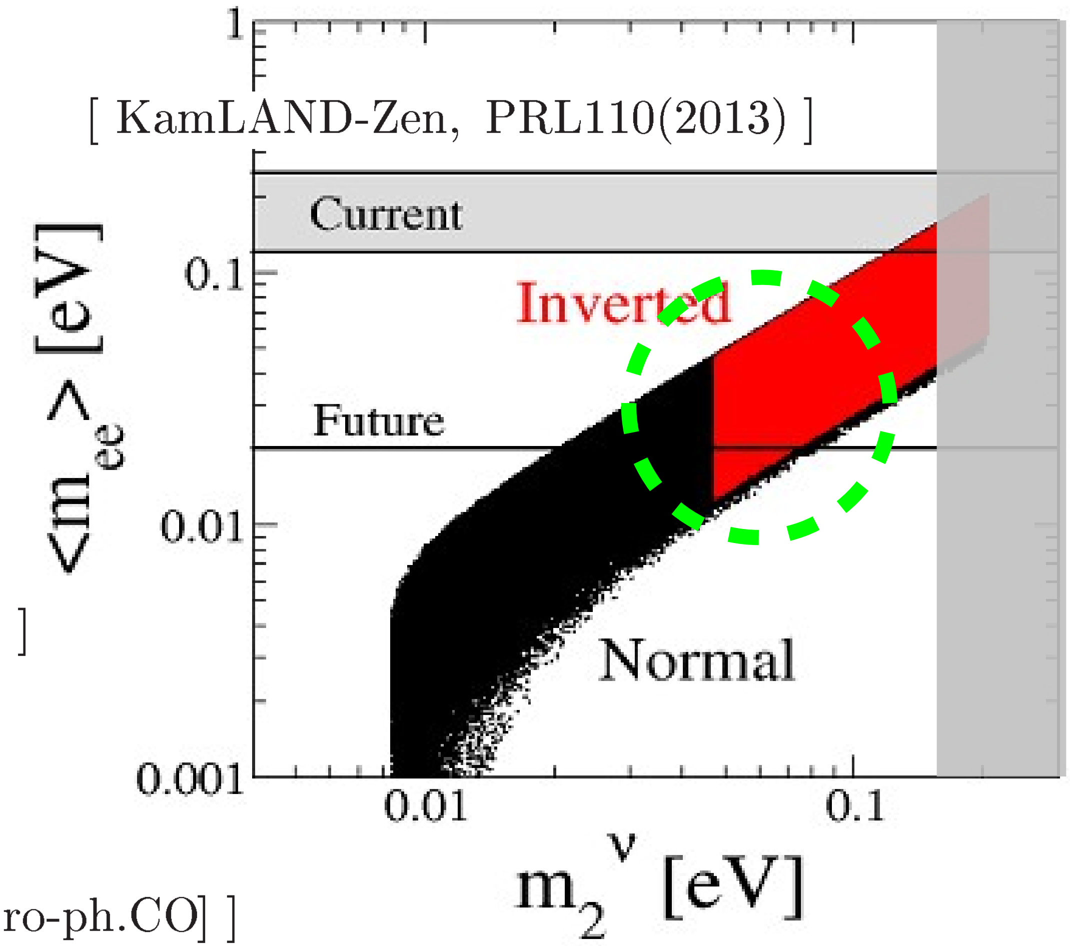
WMAP9(95% C.L.)

$$\sum m_i^{\nu} < 0.44 \text{ eV}$$

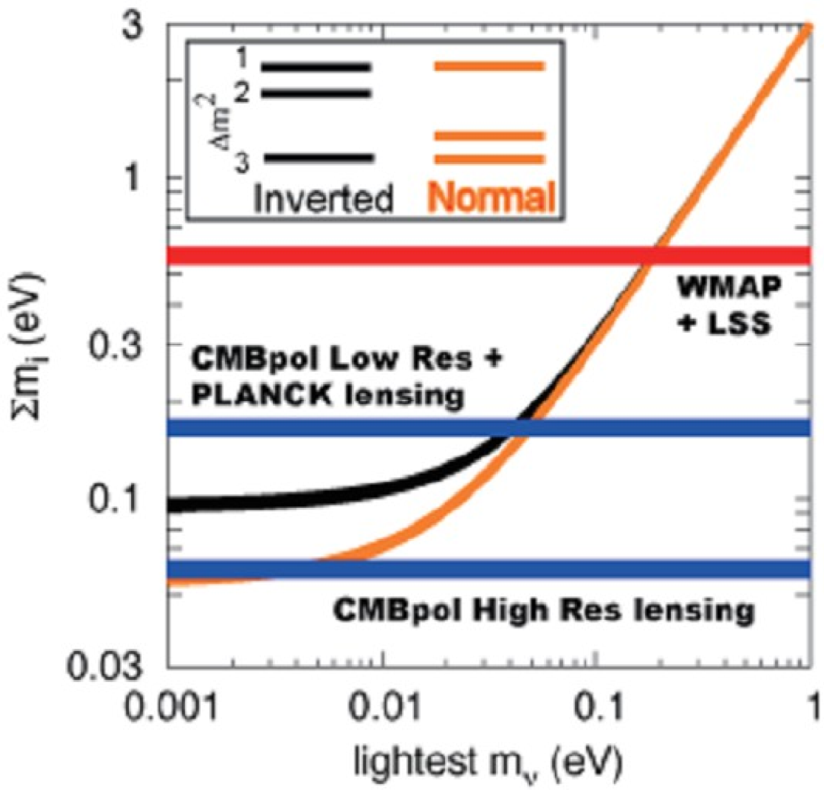
[9yearWMAP, arXiv :1212.5226[astro-ph.CO]]

WMAP

[KamLAND-Zen, PRL110(2013)]



Neutrino masses: normal or inverted? Majorana or Dirac?



[EPIC report ('08)]

⇒ Astrophysical constraint will become comparable to that of on-going $0\nu\beta\beta$ experiments

⇒ Let's wait PLANCK results (March 21)

On the theoretical side

In the limit of $\Delta m_{12}^2 = 0 \ (m_2^\nu = m_1^\nu)$, a Majorana neutrino mass matrix comes to enjoy the O(2) symmetry:

$$R^T \begin{pmatrix} m_1^\nu & 0 & 0 \\ 0 & m_1^\nu & 0 \\ 0 & 0 & m_3^\nu \end{pmatrix} R = P^T \begin{pmatrix} m_1^\nu & 0 & 0 \\ 0 & m_1^\nu & 0 \\ 0 & 0 & m_3^\nu \end{pmatrix} P = \begin{pmatrix} m_1^\nu & 0 & 0 \\ 0 & m_1^\nu & 0 \\ 0 & 0 & m_3^\nu \end{pmatrix}$$

where

$$R = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

$$\frac{f_{ij}}{\Lambda_\nu} L_i L_j H H$$

In other words, $\begin{pmatrix} L_1 & L_2 \end{pmatrix}^T \ddagger \mathbf{2}_n$ and $L_3 \ddagger \mathbf{1}$ of the O(2).

Slight breaking of O(2) may be responsible for the slight mass splitting: ~~O(2)~~ → $m_2^\nu \neq m_1^\nu$.

On the theoretical side

This partial degenerate limit seems to conflict with the hierarchical mass spectrum of the charged leptons.

$$\begin{pmatrix} m_{e(\mu)} & 0 & 0 \\ 0 & m_{e(\mu)} & 0 \\ 0 & 0 & m_\tau \end{pmatrix} \quad ? \quad \boxed{\frac{m_e}{m_\mu} = 0.005}$$

We can assign different reps. to the right-handed leptons.

$$\left[\begin{pmatrix} \ell_{R_1} \\ \ell_{R_2} \end{pmatrix} \ddagger \mathbf{2}_m \quad \ell_{R_3} \ddagger \mathbf{1} \quad \text{or} \quad \ell_{R_{1,2,3}} \ddagger \mathbf{1} \right] + \begin{pmatrix} L_1 \\ L_2 \end{pmatrix} \ddagger \mathbf{2}_n \quad L_3 \ddagger \mathbf{1}$$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & m_\tau \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ m_1 & m_2 & m_3 \end{pmatrix} \rightarrow MM^\dagger = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & m_\tau^2 \end{pmatrix}$$

Slight breaking of O(2) may be responsible for the electron and muon masses: $\not{O}(2) \longrightarrow m_e,\ m_\mu \neq 0.$

On the theoretical side

On one hand, small corrections generate small mixing in the charged lepton sector:

(Note V_12 can be large.)

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & m_3^\ell \end{pmatrix} + \delta^\ell \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix} \Rightarrow V^\ell \propto \frac{\delta^\ell}{m_3^\ell}$$

On the other hand, the small corrections can generate large mixing in the neutrino sector:

$$\begin{pmatrix} m_1^\nu & 0 & 0 \\ 0 & m_1^\nu & 0 \\ 0 & 0 & m_3^\nu \end{pmatrix} + \delta^\nu \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix} \Rightarrow V^\nu \not\propto \frac{\delta^\nu}{m_3^\nu}$$

because $m_1^\nu \simeq m_3^\nu$.

Thus, large PMNS mixing.

Note: small CKM mixing may be possible.

On the theoretical side

To summarize, the partial degenerate limit may be a good starting point for understanding

$$1. \quad \Delta m^2_{12} \ll \Delta m^2_{23}$$

$$2. \quad m^f_{1,2} \ll m^f_3$$

$$(3. \quad \theta^{\mathrm{CKM}}_{ij} \ll \theta^{\mathrm{PMNS}}_{ij})$$

and suggests an **O(2) family symmetry.** (or its subgroup)

Purpose

Constructing a comprehensive framework for the flavor structure in view of the partial mass-degeneracy.

Model

O(2) symmetry

The O(2) consists of two singlet and an infinite number of doublet irreducible representations:

$$\mathbf{1} \otimes \mathbf{1}' = \mathbf{1}', \quad \mathbf{1}' \otimes \mathbf{1}' = \mathbf{1},$$

$$\begin{array}{ccccccccc} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} & \otimes & \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} & = & (x_1 y_1 + x_2 y_2) & \oplus & (x_1 y_2 - x_2 y_1) & \oplus & \begin{pmatrix} x_1 y_1 - x_2 y_2 \\ x_1 y_2 + x_2 y_1 \end{pmatrix} \\ \mathbf{2}_n & \otimes & \mathbf{2}_n & = & \mathbf{1} & \oplus & \mathbf{1}' & \oplus & \mathbf{2}_{2n} \end{array}$$

$$\begin{array}{ccccccc} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} & \otimes & \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} & = & \begin{pmatrix} x_1 y_1 + x_2 y_2 \\ x_1 y_2 - x_2 y_1 \end{pmatrix} & \oplus & \begin{pmatrix} x_1 y_1 - x_2 y_2 \\ x_1 y_2 + x_2 y_1 \end{pmatrix}. \\ \mathbf{2}_n & \otimes & \mathbf{2}_{m \neq n} & = & \mathbf{2}_{m-n} & \oplus & \mathbf{2}_{m+n} \end{array}$$

(n ∈ **N**)

We will use the D_N group because we are free from gauge amomalies, massless NGBs and so on.

Model with D_N symmetry

	L_I	L_3	ℓ_i	H	S_I
D_N	$\mathbf{2}_2$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}_1$

$I = 1, 2$
$i = 1, 2, 3$

gauge singlet

The charged lepton sector:

$$[\mathrm{rank} = 1]$$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ y_1^0 & y_2^0 & y_3^0 \end{pmatrix}$$

$$y_i^0 \overline{L}_3 H \ell_i$$

$$\Rightarrow m_\tau$$

Model with D_N symmetry

	L_I	L_3	ℓ_i	H	S_I
D_N	$\mathbf{2}_2$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}_1$

$I = 1, 2$
$i = 1, 2, 3$

gauge singlet

The charged lepton sector:

$$\underbrace{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ y_1^0 & y_2^0 & y_3^0 \end{pmatrix}}_{[\text{rank}=1]} + \frac{1}{\Lambda_F^2} \underbrace{\begin{pmatrix} y_1\delta_1 & y_2\delta_1 & y_3\delta_1 \\ y_1\delta_2 & y_2\delta_2 & y_3\delta_2 \\ 0 & 0 & 0 \end{pmatrix}}_{[\text{rank}=1]} + \frac{1}{\Lambda_F^4} \underbrace{\begin{pmatrix} y_1'\epsilon_1 & y_2'\epsilon_1 & y_3'\epsilon_1 \\ y_1'\epsilon_2 & y_2'\epsilon_2 & y_3'\epsilon_2 \\ 0 & 0 & 0 \end{pmatrix}}_{[\text{rank}=1]} + \cdots$$

$y_i^0 \overline{L}_3 H \ell_i$ → m_τ

$\frac{y_i}{\Lambda_F^2} \overline{L}_I H \ell_i (S^2)_I$ → m_μ

$\frac{y_i'}{\Lambda_F^4} \overline{L}_I H \ell_i (S^4)_I$ → m_e

Thanks to a FN-like mechanism, a hierarchical mass texture is obtained.

Especially, each matrix is responsible for each mass.

Model with D_N symmetry

	L_I	L_3	ℓ_i	H	S_I
D_N	$\mathbf{2}_2$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}_1$

$I = 1,2$
$i = 1,2,3$

gauge singlet

The charged lepton sector:

$$\underset{y_i^0 \overline{L}_3 H \ell_i}{\overset{[\mathrm{rank}=1]}{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ y_1^0 & y_2^0 & y_3^0 \end{pmatrix}}} + \frac{1}{\Lambda_F^2}\begin{pmatrix} y_1\delta_1 & y_2\delta_1 & y_3\delta_1 \\ y_1\delta_2 & y_2\delta_2 & y_3\delta_2 \\ 0 & 0 & 0 \end{pmatrix} + \frac{1}{\Lambda_F^4}\begin{pmatrix} y_1'\epsilon_1 & y_2'\epsilon_1 & y_3'\epsilon_1 \\ y_1'\epsilon_2 & y_2'\epsilon_2 & y_3'\epsilon_2 \\ 0 & 0 & 0 \end{pmatrix} + \cdots$$

$$\frac{y_i}{\Lambda_F^2}\overline{L}_I H \ell_i (S^2)_I \qquad \frac{y_i'}{\Lambda_F^4}\overline{L}_I H \ell_i (S^4)_I$$

However,

Actually, $[\mathrm{rank}=1]$.

$$\delta_1 = s_1^2 - s_2^2 \qquad \epsilon_1 = (s_1^2 + s_2^2)\delta_1$$
$$\delta_2 = 2 s_1 s_2 \qquad \epsilon_2 = (s_1^2 + s_2^2)\delta_2$$

The electron remains massless, $m_e = 0$, even if one takes further higher-order terms into account.

Model with D_N symmetry

	L_I	L_3	ℓ_i	H	S_I
D_N	$\mathbf{2}_2$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}_1$

$I = 1, 2$
$i = 1, 2, 3$

gauge singlet

The charged lepton sector:

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ y_1^0 & y_2^0 & y_3^0 \end{pmatrix} + \underline{\frac{1}{\Lambda_F^2} \begin{pmatrix} y_1\delta_1 & y_2\delta_1 & y_3\delta_1 \\ y_1\delta_2 & y_2\delta_2 & y_3\delta_2 \\ 0 & 0 & 0 \end{pmatrix}} + \frac{1}{\Lambda_F^4} \begin{pmatrix} y_1'\epsilon_1 & y_2'\epsilon_1 & y_3'\epsilon_1 \\ y_1'\epsilon_2 & y_2'\epsilon_2 & y_3'\epsilon_2 \\ 0 & 0 & 0 \end{pmatrix} + \cdots$$

Suppose complex VEVs:

$$\langle S_I \rangle = (s_1 e^{i\phi_1}, s_2 e^{i\phi_2})$$

$[\mathrm{rank} = 2]$

$$\frac{1}{\Lambda_F^2} \left[\begin{pmatrix} y_1\delta_1 & y_2\delta_1 & y_3\delta_1 \\ y_1\delta_2 & y_2\delta_2 & y_3\delta_2 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} y_1'\delta_1^* & y_2'\delta_1^* & y_3'\delta_1^* \\ y_1'\delta_2^* & y_2'\delta_2^* & y_3'\delta_2^* \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} y_1''\delta_a & y_2''\delta_a & y_3''\delta_a \\ y_1''\delta_b & y_2''\delta_b & y_3''\delta_b \\ 0 & 0 & 0 \end{pmatrix} \right]$$

$\phi_1 = \phi_2$ corresponds to the massless limit of the electron.

Model with D_N symmetry

	L_I	L_3	ℓ_i	H	S_I
D_N	$\mathbf{2}_2$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}_1$

$I = 1, 2$
$i = 1, 2, 3$

gauge singlet

The charged lepton sector:

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ y_1^0 & y_2^0 & y_3^0 \end{pmatrix} + \underline{\frac{1}{\Lambda_F^2}\begin{pmatrix} y_1\delta_1 & y_2\delta_1 & y_3\delta_1 \\ y_1\delta_2 & y_2\delta_2 & y_3\delta_2 \\ 0 & 0 & 0 \end{pmatrix}} + \frac{1}{\Lambda_F^4}\begin{pmatrix} y_1'\epsilon_1 & y_2'\epsilon_1 & y_3'\epsilon_1 \\ y_1'\epsilon_2 & y_2'\epsilon_2 & y_3'\epsilon_2 \\ 0 & 0 & 0 \end{pmatrix} + \cdots$$

Suppose complex VEVs:

$$\langle S_I \rangle = (s_1 e^{i\phi_1}, s_2 e^{i\phi_2})$$

[rank = 2]

$$\frac{1}{\Lambda_F^2}\begin{pmatrix} Y_1(s_1^2 - s_2^2) & Y_2(s_1^2 - s_2^2) & Y_3(s_1^2 - s_2^2) \\ Y_1 2 s_1 s_2 & Y_2 2 s_1 s_2 & Y_3 2 s_1 s_2 \\ 0 & 0 & 0 \end{pmatrix} + \frac{i\delta\phi}{\Lambda_F^2}\begin{pmatrix} -Y_1' 2 s_2^2 & -Y_2' 2 s_2^2 & -Y_3' 2 s_2^2 \\ Y_1' 2 s_1 s_2 & Y_2' 2 s_1 s_2 & Y_3' 2 s_1 s_2 \\ 0 & 0 & 0 \end{pmatrix}$$

$\Rightarrow m_\mu$ $\Rightarrow m_e$

$\delta\phi = 0$ corresponds to the massless limit of the electron.

Model with D_N symmetry

	L_I	L_3	ℓ_i	H	S_I
D_N	$\mathbf{2}_2$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}_1$

$I = 1, 2$
$i = 1, 2, 3$
$\langle S_I \rangle = (s_1 e^{i\phi_1}, s_2 e^{i\phi_2})$

The neutrino sector:

$$\begin{pmatrix} f_1 & 0 & 0 \\ 0 & f_1 & 0 \\ 0 & 0 & f_3 \end{pmatrix} + \frac{1}{\Lambda_F^2} \begin{pmatrix} 0 & 0 & \alpha\delta_1 + \alpha'\delta_1^* + \alpha''\delta_a \\ 0 & 0 & \alpha\delta_2 + \alpha'\delta_2^* + \alpha''\delta_b \\ \alpha\delta_1 + \alpha'\delta_1^* + \alpha''\delta_a & \alpha\delta_2 + \alpha'\delta_2^* + \alpha''\delta_b & 0 \end{pmatrix}$$

$$\frac{f}{\Lambda_\nu} LLHH \qquad \frac{\alpha}{\Lambda_\nu \Lambda_F^2} L_I L_3 HHS^2$$

$$+\frac{1}{\Lambda_F^4} \begin{pmatrix} \beta\epsilon_1 + \beta'\epsilon_1^* + \beta''\epsilon_a + \beta'''\epsilon_x + \beta''''\epsilon_x^* & \beta\epsilon_2 + \beta'\epsilon_2^* + \beta''\epsilon_b + \beta'''\epsilon_y + \beta''''\epsilon_y^* & 0 \\ \beta\epsilon_2 + \beta'\epsilon_2^* + \beta''\epsilon_b + \beta'''\epsilon_y + \beta''''\epsilon_y^* & -(\beta\epsilon_1 + \beta'\epsilon_1^* + \beta''\epsilon_a + \beta'''\epsilon_x + \beta''''\epsilon_x^*) & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\frac{\beta}{\Lambda_\nu \Lambda_F^4} L_I L_J HHS^4$$

Model with D_N symmetry

	L_I	L_3	ℓ_i	H	S_I
D_N	$\mathbf{2}_2$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}_1$

$I = 1, 2$
$i = 1, 2, 3$
$\langle S_I \rangle = (s_1 e^{i\phi_1}, s_2 e^{i\phi_2})$

$$\delta_1 = (s_1^2 e^{2i\phi_1} - s_2^2 e^{2i\phi_2}), \quad \delta_2 = 2 s_1 s_2 e^{i(\phi_1 + \phi_2)}$$

$$\delta_a = s_1^2 - s_2^2, \quad \delta_b = 2 s_1 s_2 \cos(\phi_1 - \phi_2)$$

$$\epsilon_1 = \delta_1^2 - \delta_2^2, \quad \epsilon_2 = 2\delta_1\delta_2$$

$$\epsilon_a = \delta_a^2 - \delta_b^2, \quad \epsilon_b = 2\delta_a\delta_b$$

$$\epsilon_x = \delta_1\delta_a - \delta_2\delta_b, \quad \epsilon_y = \delta_1\delta_b + \delta_2\delta_a$$

Model with D_N symmetry

	L_I	L_3	ℓ_i	H	S_I
D_N	$\mathbf{2}_2$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}_1$

$I = 1, 2$
$i = 1, 2, 3$
$\langle S_I \rangle = (s_1 e^{i\phi_1}, s_2 e^{i\phi_2})$

The charged lepton sector.

$$M^\ell = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ y_1^0 & y_2^0 & y_3^0 \end{pmatrix} + \frac{1}{\Lambda_F^2} \begin{pmatrix} y_1\delta_1 & y_2\delta_1 & y_3\delta_1 \\ y_1\delta_2 & y_2\delta_2 & y_3\delta_2 \\ 0 & 0 & 0 \end{pmatrix}$$
$$+ \frac{1}{\Lambda_F^2} \begin{pmatrix} y_1'\delta_1^* & y_2'\delta_1^* & y_3'\delta_1^* \\ y_1'\delta_2^* & y_2'\delta_2^* & y_3'\delta_2^* \\ 0 & 0 & 0 \end{pmatrix} + \frac{1}{\Lambda_F^2} \begin{pmatrix} y_1''\delta_a & y_2''\delta_a & y_3''\delta_a \\ y_1''\delta_b & y_2''\delta_b & y_3''\delta_b \\ 0 & 0 & 0 \end{pmatrix}$$

$$y_3^0 = 1.0, \quad y_1^0 = y_2^0 = 1.2, \quad y_1 = -y_2 = y_3 = 0.8,$$
$$y_1' = y_2' = -y_3' = 0.8, \quad y_1'' = -y_2'' = -y_3'' = 0.8 - 1.3,$$
$$\frac{s_1}{\Lambda_F} = 0.15 - 0.30, \quad \frac{s_2}{\Lambda_F} = 0.15 - 0.30, \quad \phi_1 = 0 - 2\pi, \quad \phi_2 = 0 - 2\pi.$$

Model with D_N symmetry

	L_I	L_3	ℓ_i	H	S_I
D_N	$\mathbf{2}_2$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}_1$

$I = 1, 2$
$i = 1, 2, 3$
$\langle S_I \rangle = (s_1 e^{i\phi_1}, s_2 e^{i\phi_2})$

The neutrino sector.

$$\begin{pmatrix} f_1 & 0 & 0 \\ 0 & f_1 & 0 \\ 0 & 0 & f_3 \end{pmatrix} + \frac{1}{\Lambda_F^2} \begin{pmatrix} 0 & 0 & \alpha\delta_1 + \alpha'\delta_1^* + \alpha''\delta_a \\ 0 & 0 & \alpha\delta_2 + \alpha'\delta_2^* + \alpha''\delta_b \\ \alpha\delta_1 + \alpha'\delta_1^* + \alpha''\delta_a & \alpha\delta_2 + \alpha'\delta_2^* + \alpha''\delta_b & 0 \end{pmatrix}$$

$$+ \frac{1}{\Lambda_F^4} \begin{pmatrix} \beta\epsilon_1 + \beta'\epsilon_1^* + \beta''\epsilon_a + \beta'''\epsilon_x + \beta''''\epsilon_x^* & \beta\epsilon_2 + \beta'\epsilon_2^* + \beta''\epsilon_b + \beta'''\epsilon_y + \beta''''\epsilon_y^* & 0 \\ \beta\epsilon_2 + \beta'\epsilon_2^* + \beta''\epsilon_b + \beta'''\epsilon_y + \beta''''\epsilon_y^* & -(\beta\epsilon_1 + \beta'\epsilon_1^* + \beta''\epsilon_a + \beta'''\epsilon_x + \beta''''\epsilon_x^*) & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

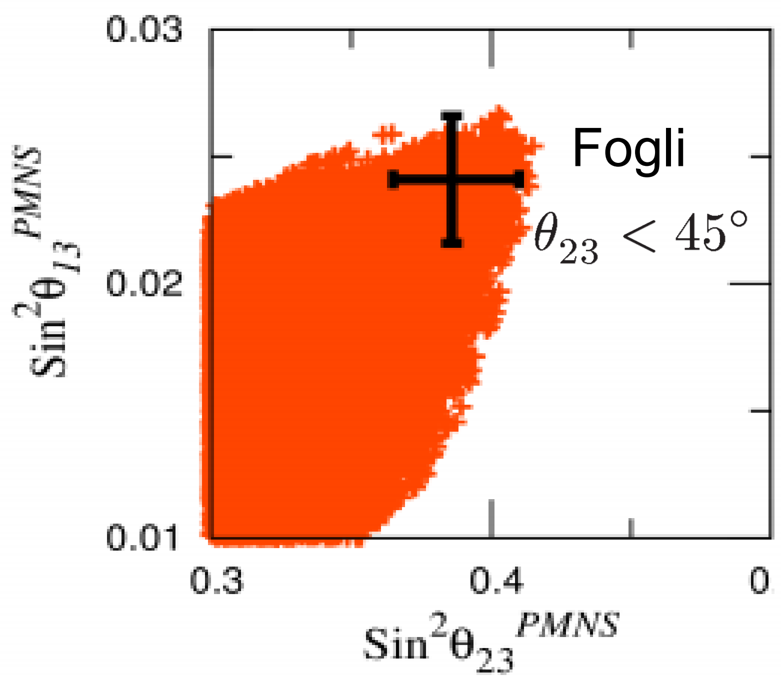
$$f_3 = 1.0, \quad f_1 = 0.90 - 0.95, \ \alpha = \alpha'' = 0.9,$$

$$\alpha' = 0.8 - 1.3, \ \beta = \beta' = \beta'' = -\beta''' = -\beta'''' = 0.8 - 1.3$$

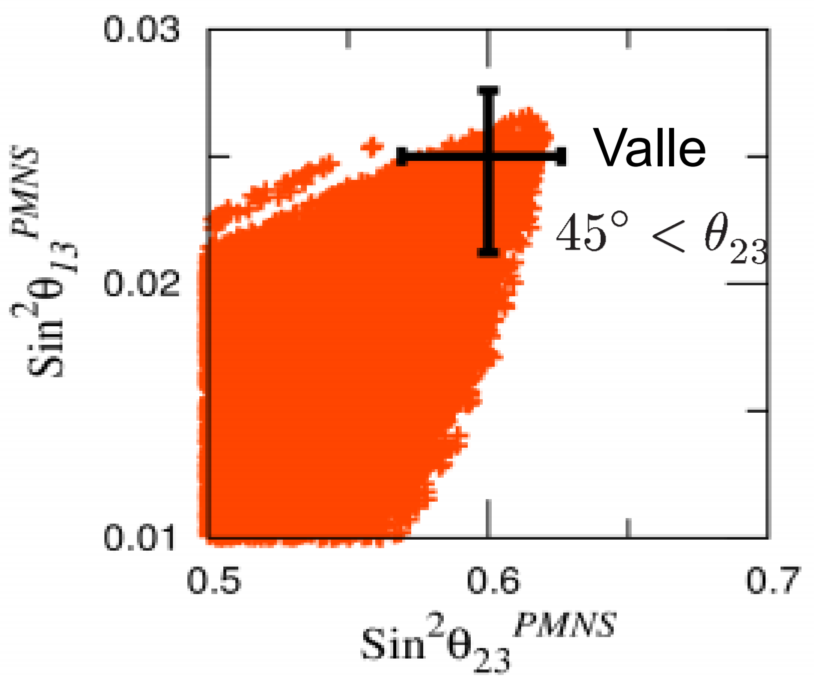
$$\frac{s_1}{\Lambda_F} = 0.15 - 0.30, \quad \frac{s_2}{\Lambda_F} = 0.15 - 0.30, \ \phi_1 = 0 - 2\pi, \ \phi_2 = 0 - 2\pi.$$

Model with D_N symmetry

Normal Hierarchy



Inverted Hierarchy



Constrains

$$\theta_{12}^{\mathrm{PMNS}},\ \Delta m^2_{12},\ \Delta m^2_{23}\ ‡\ 1\sigma \text{ range}$$

$$\frac{m_e}{m_\mu} = 0.00473, \quad \frac{m_\mu}{m_\tau} = 0.0588.$$

Model with D_N symmetry

- Correlation between m_e and θ_{13}.

The electron mass is proportional to $\delta\phi\,(\delta\phi = \phi_1 - \phi_2)$:

$$\frac{m_e}{m_\mu} \simeq \delta\phi\sqrt{\frac{4s_1^2 s_2^2 \sum_i |Y_i'|^2}{(s_1^2+s_2^2)^2 \sum_i |Y_i|^2}} + \mathcal{O}(\delta\phi^2).$$

In the diagonal basis of the charged leptons,

$$\frac{\Lambda_\nu}{v^2}(\mathcal{M}_\nu')_{13} \simeq 2i\delta\phi\frac{s_1 s_2}{\Lambda_F^2}\left[G_\nu \frac{\sum_i Y_i^* Y_i'}{\sum_i |Y_i|^2} - G_\nu'\right] + \mathcal{O}(\delta\phi^2).$$

Thus, the electron mass and θ_{13} are correlated via the CP violating phases.

Model with D_N symmetry

The size of $\theta_{13}^{\rm PMNS}$ is constrained by the electron mass as long as couplings are not tuned.

$$0.012 < \sin^2\theta_{13}^{\rm PMNS}$$

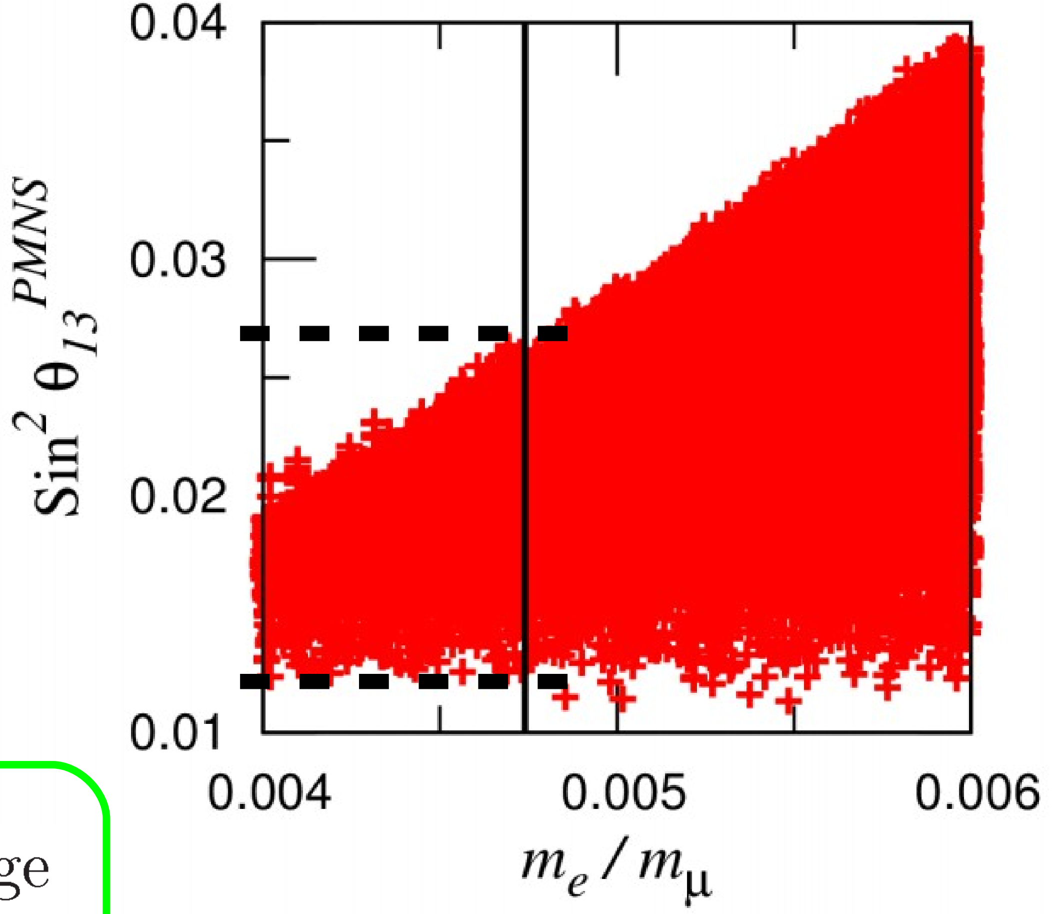
$$\sin^2\theta_{13}^{\rm PMNS} < 0.027$$

Constrains

$$\theta_{12,23}^{\rm PMNS},\ \Delta m_{12}^2,\ \Delta m_{23}^2 \ ‡\ 1\sigma \text{ range}$$

$$\frac{m_\mu}{m_\tau} = 0.0588.$$

Normal Hierarchy



Summary

- The partial-mass-degenerate limit may be a good starting point for understanding the observed fermion mass spectra and mixing patterns.
- Such a mass region would be tested by 0nbb experiments and CMB observations.
- The limit suggests an O(2) (or D_N) flavor symmetry.
- We have proposed a simple model for the lepton sector and shown that the model can reproduce experimental results without making coupling constants hierarchical.
- We have found a novel correlation between the electron mass and $\theta_{13}^{\mathrm{PMNS}}$.

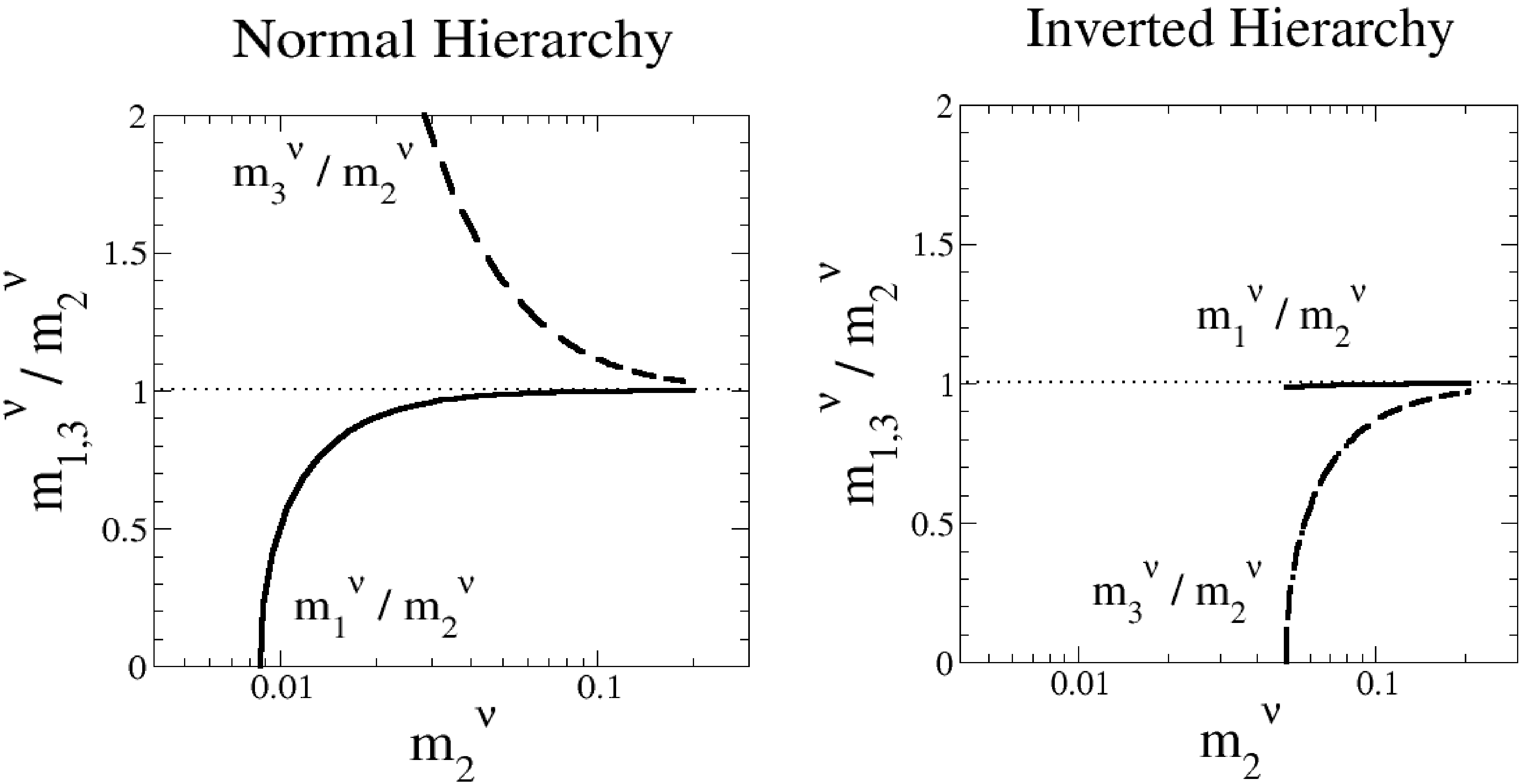
Future works

- Something distinguishable resulting from the partial mass-degeneracy.
- Inclusion of the quark sector.
- CP violation and masses of the other 1st generations.
- Enlarging to O(2) including a new gauge boson and gauge anomalies.
- Mass hierarchy among the 3rd generations:

$$m_\tau, m_b \ll m_t.$$

Backup slides

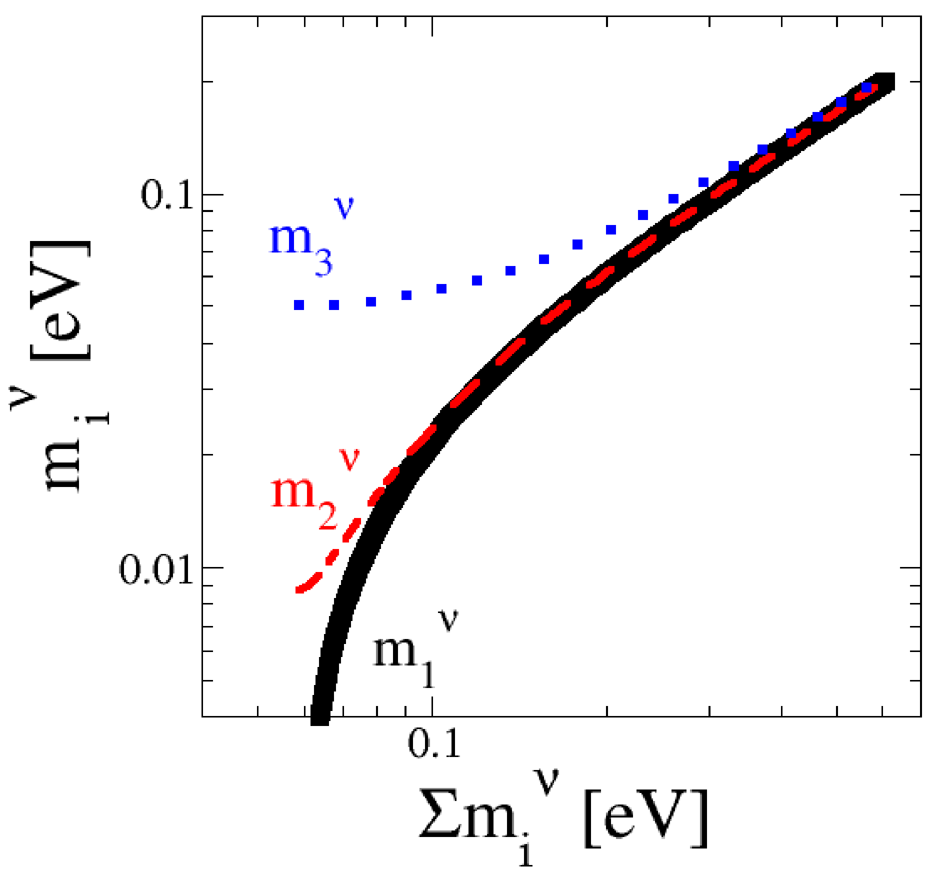
Neutrino mass hierarchy

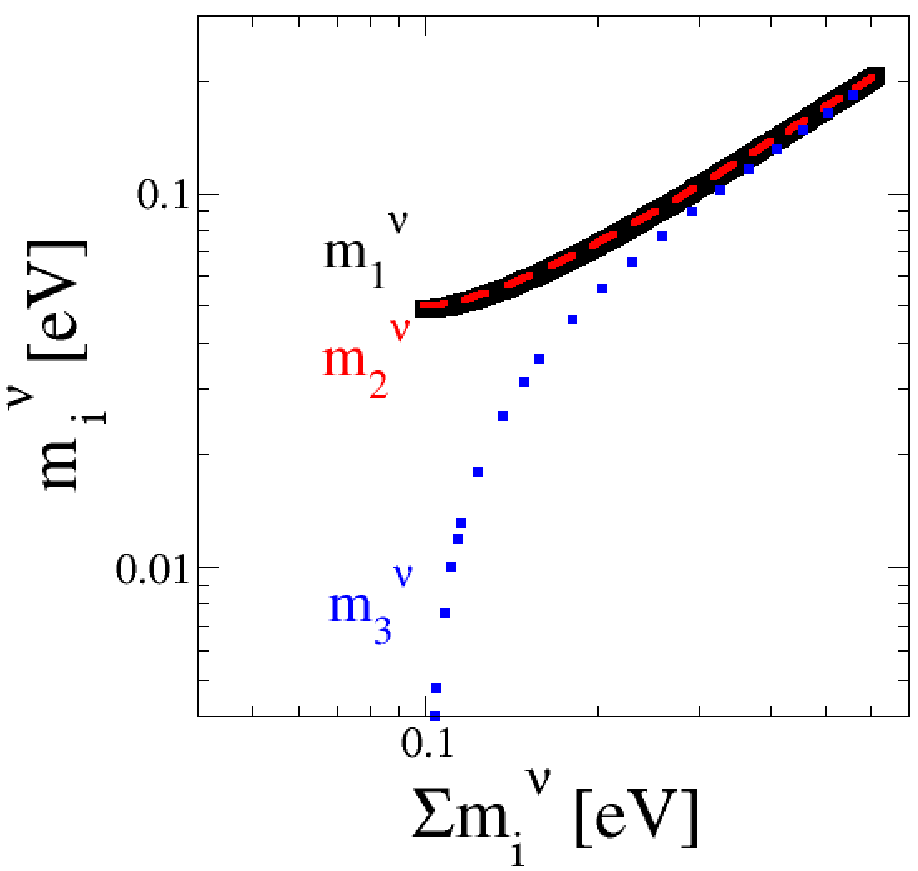


Fixed: $\Delta m^2_{12} = 7.54 \times 10^{-5}\ eV^2 \quad \Delta m^2_{23} = 2.43(2) \times 10^{-3}\ eV^2$

[Fogli, *etal*, PRD86(2012)]

Neutrino mass hierarchy

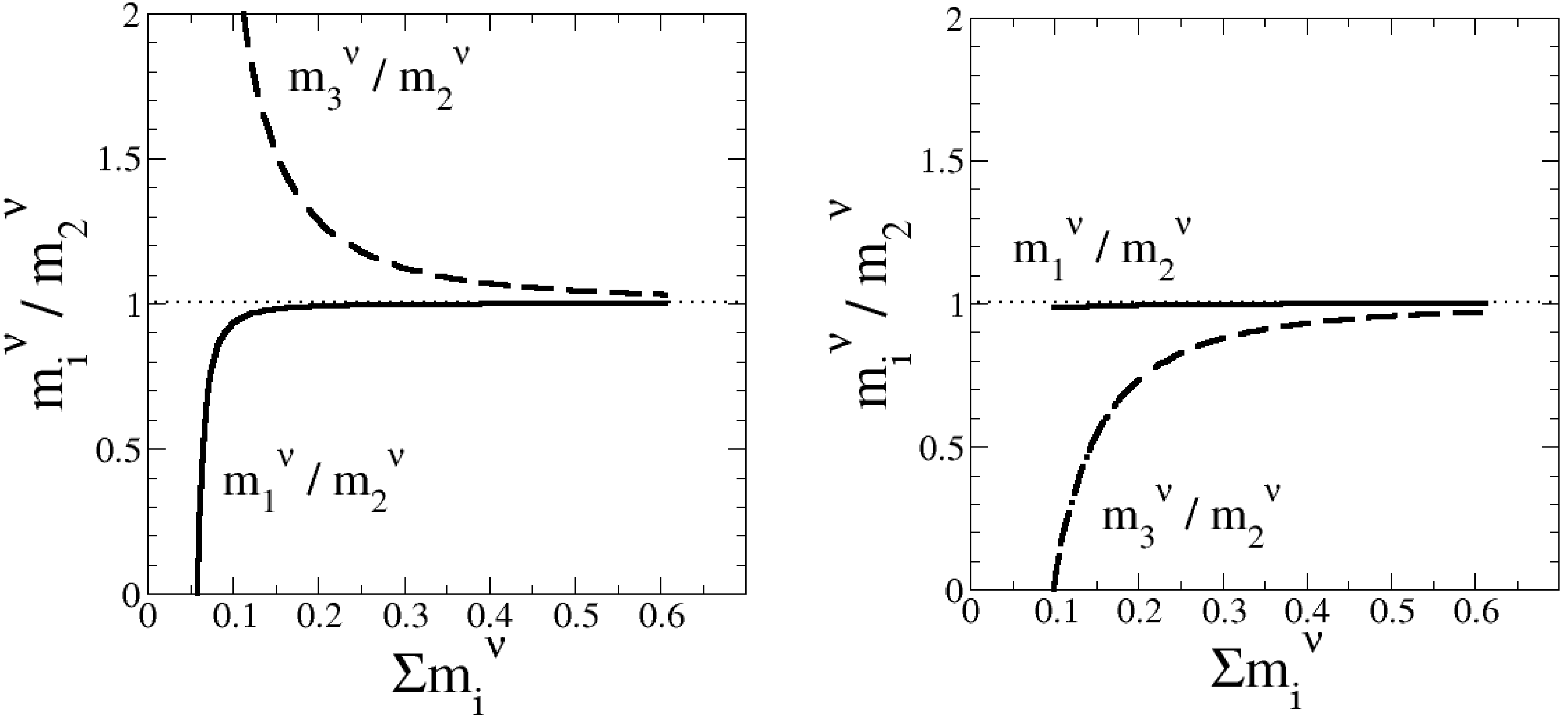




Fixed: $\Delta m^2_{12} = 7.54 \times 10^{-5}\ eV^2 \quad \Delta m^2_{23} = 2.43(2) \times 10^{-3}\ eV^2$

[Fogli, *etal*, PRD86(2012)]

Neutrino mass hierarchy



Fixed: $\Delta m_{12}^2 = 7.54 \times 10^{-5}\ eV^2 \quad \Delta m_{23}^2 = 2.43(2) \times 10^{-3}\ eV^2$

[Fogli, *etal*, PRD86(2012)]

On the experimental side

Such a mass region would be excluded or confirmed by 0nbb experiments and CMB observations.

EXO(90% C.L.)

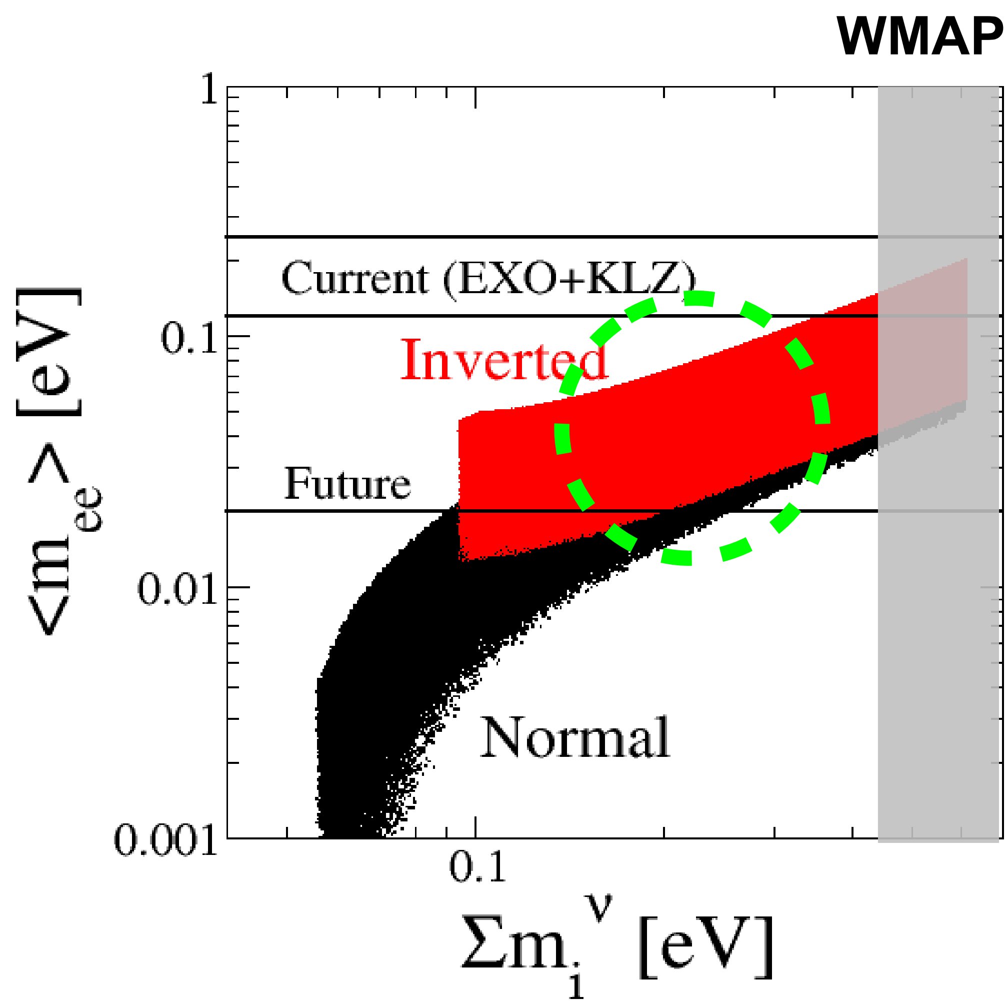
$$\langle m_{ee} \rangle < 0.14 - 0.38 \text{ eV}$$

KamLAND-Zen(90% C.L.)

$$\langle m_{ee} \rangle < 0.26 - 0.54 \text{ eV}$$

WMAP9(95% C.L.)

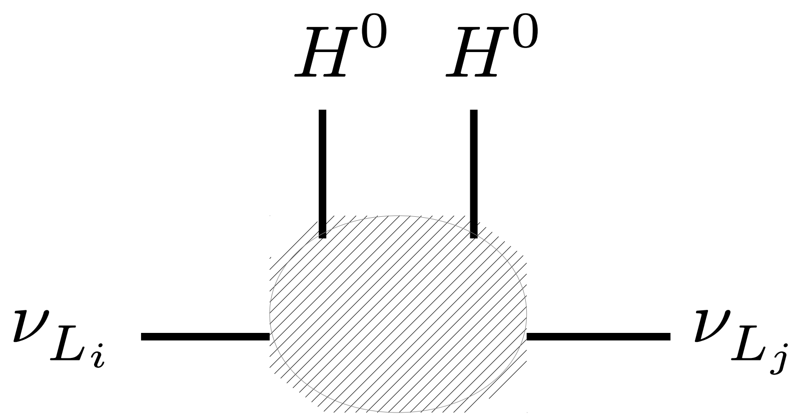
$$\sum_i m_i^\nu < 0.44 \text{ eV}$$



What and How?

Example: (Majorana) neutrino masses and lepton # violation.

A Majorana neutrino mass term can be constructed.



$$\frac{f_{ij}}{\Lambda_\nu} L_i L_j H H$$
$$= \frac{f_{ij}}{\Lambda_\nu} (\nu_{L_i} H^0 - \ell_{L_i} H^+)^2$$

[S.Weinberg, PRL43(1979)]

Setting neutrino masses to zero, the lepton # is restored

$$\lim_{m_\nu \to 0} \Longleftrightarrow U(1)_L$$

The observed tiny neutrino masses can be regarded as breaking terms of $U(1)_L$ broken at a high-energy scale.