Neutrino diffraction

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- Is neutrino interference (diffraction) observable?

Neutrino is a quantum mechanical wave and interacts with matter extremely weakly.

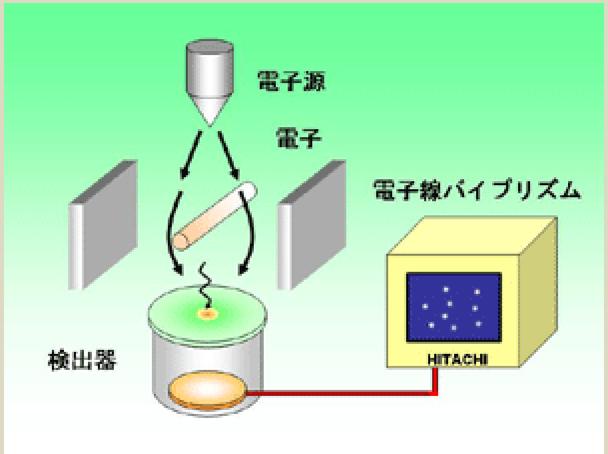
Neutrino should show a phenomenon of interference or diffraction, in a double slit-like experiment.

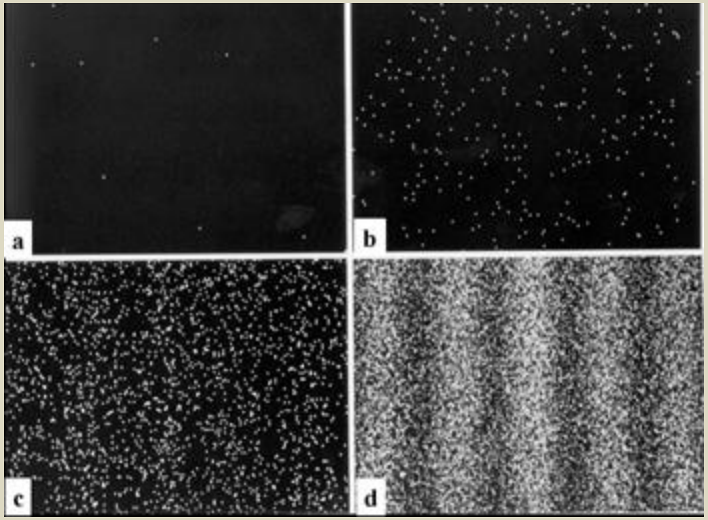
However, it is very hard to control the neutrino ,since it interacts with any material so weakly.

Hence a new method must be considered.

Our answer and proposal ” Neutrino diffraction is observable . Use a pion (and other particle) decay process !”

Electron interference (bi-blism;Tonomura)





Number of events
N_a<N_b<N_c<N_d

interference pattern becomes clear and visible in d

Unknown facts on neutrinos

- Absolute neutrino mass
- Tritium decay electron spectrum
- ¥nu-less double Beta decay
- Cosmology gives bound.

Unclear now !

- How many ? 3?
- Mixing matrix

Scattering ,interference, and diffraction of neutrino (our works)

1. Neutrino is a wave , so it varies in space and time, and follows “Superposition principle”
2. Neutrino is identified from physical reactions that are caused by the neutrino.
3. 1 and 2 are combined. This method is different from an ordinary oscillation analysis and supply a new physical quantity.
4. Diffraction was found theoretically and is observed easily with a large number of events.

An example of a wave phenomenon

On mass shell neutrino is in the intermediate state.
External particles are plain waves.

P/T^2

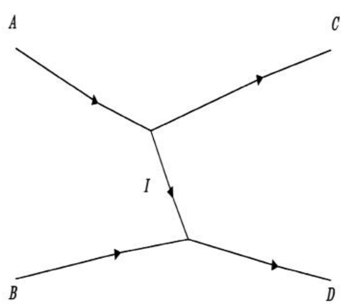
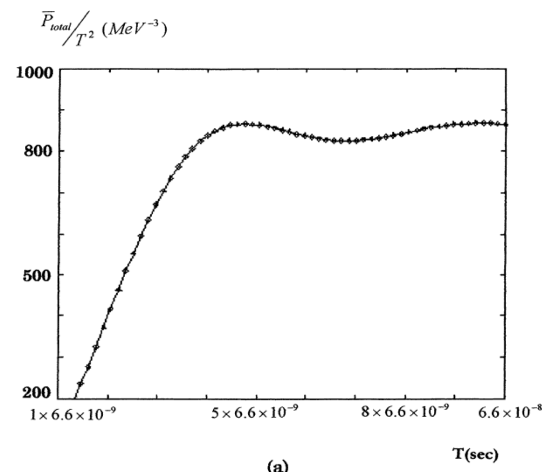
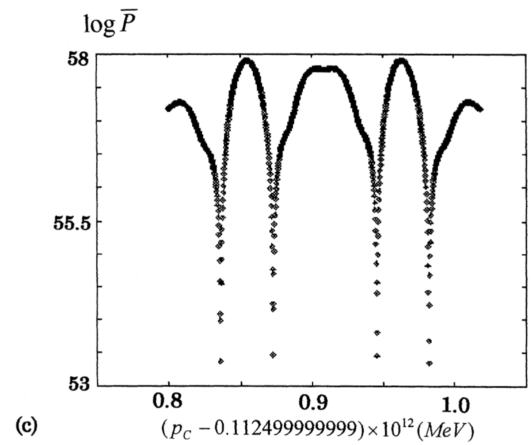


Fig. 2. Lowest order Feynmann diagram for the I production process $A \to C + I$ and the I detection process $I + B \to D$.

ν

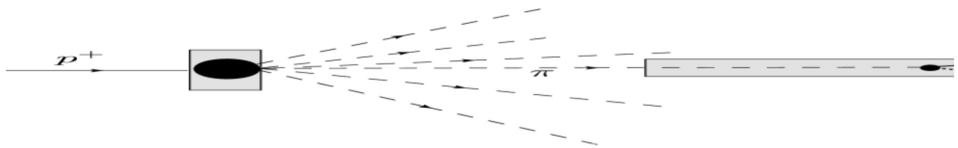




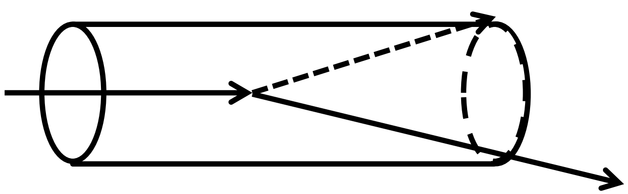
Large correlation and amplification

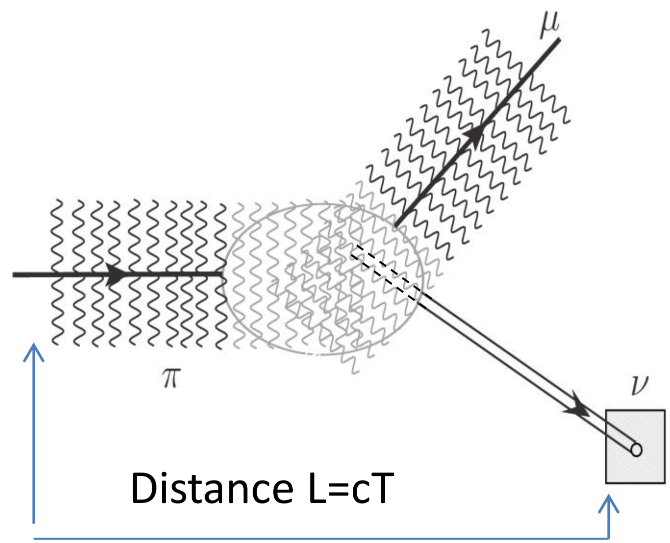
Yabuki and Ishikawa, PTP(2002)

Real pion decays



experiment





which?

Particle decay
(traditional ?)

Decay of wave ,wave packet（ishikawa-shimomura）

Transition in a finite time-interval

- S[T]

$$H = H_0 + H_1 \quad (1)$$

$$U(t) = e^{-iHt}, U_0 = e^{-iH_0 t} \quad (2)$$

$$\Omega_{\pm}(T) = lim_{t \to -\pm T} U(t)^{\dagger} U^{(0)}(t) \quad (3)$$

$$S - matrix : S[T] = \Omega_{-}^{\dagger}(T)\Omega_{+}(T) \quad (1)$$

$$energy\ non-conservation;$$

$$[S[T], H_0] = i((\frac{\partial}{\partial T}\Omega_{-}(T))^{\dagger}\Omega_{+}(T) - i\Omega_{-}^{\dagger}\frac{\partial}{\partial T}\Omega_{+}(T) \quad (2)$$

$$transition\ amplitude;\ \langle\beta|S[T]|\alpha\rangle = \delta(E_\alpha - E_\beta) f_{\alpha,\beta} + \delta f \quad (1)$$

$$probability; \quad (2)$$

$$P = P_{normal} + P_{non-conserving}, P_{non-conserving} = \sum |\delta f|^2 \geq 0$$

Finite-size correction

What is the finite-size effect ？

1．Quantum mechanical transition in a finite time interval T.

$$i + \omega \to j \tag{1}$$

$$P_{ij} = |T_{ij}|^2 \frac{\sin^2(E_i + \hbar\omega - E_j)T/2}{(E_i + \hbar - E_j)^2} \tag{2}$$

2．Violates the energy　conservation

3．States at ultra-violet energy region, J　, gives a universal correction.

4．Relativistic invariance >> large momentum states

Neutrino diffraction(ishikawa and tobita)

decay amplitude

Neutrino's momentum and position

$$T = \int d^4x d\vec{k}_\nu \langle 0 | J^\mu_{V-A} | \pi \rangle \bar{u}(p_l) \gamma_\mu (1-\gamma_5) \nu(k_\nu) e^{ip_l x + ik_\nu (x - X_\nu) - \frac{\sigma_\nu}{2}(k_\nu - p_\nu)^2}$$

Probability is expressed

$$\int \frac{d\vec{p}_l}{(2\pi)^3} \sum_{s_1, s_2} |T|^2 = \frac{N_2}{E_\nu} \int d^4x_1 d^4x_2 \Delta_{\pi,l}(\delta x) e^{i\phi(\delta x)}$$

Correlation function

$$\Delta_{\pi,l}(\delta x) = \frac{1}{(2\pi)^3} \int \frac{d^3 p_l}{E_l} (2 p_\pi p_\nu p_\pi p_l - m_\pi^2 p_l p_\nu) e^{-i(p_\pi - p_l)\delta x}$$

$$= m_l^2 p_l p_\nu$$

(Only If energy-momentum is conserved)

1.Correlation function has a light-cone singularity which is generated by a superposition of relativistic waves.
2.The energy non-conserving term gives the finite-size correction, which does not suppress the electron mode.
3 Asymptotic boundary conditions are satisfied with a wave packet（LSZ）.

Correlation function

- Integration variable is change to $q = p_l - p_\pi$

$$\Delta_{\pi,\mu}(\delta x = x_1 - x_2)$$
$$= \frac{1}{4\pi^4}\int d^4q\,\mathrm{Im}\left[\frac{1}{q^2 + 2p_\pi\cdot q + \tilde{m}^2 - i\epsilon}\right]\theta(p_\pi^0 + q^0)$$
$$\times\,\{(2p_\pi\cdot p_\nu)p_\pi\cdot(p_\pi + q) - m_\pi^2(q + p_\pi)\cdot p_\nu\}e^{-iq\cdot\delta x} \qquad \tilde{m}^2 = m_\pi^2 - m_l^2$$

- Integral region is separated in two parts

$$\boxed{\Delta_{\pi,\mu}(\delta x) = I_1 + I_2 \qquad q^0 > 0 \qquad 0 > q^0 > -p_\pi^0}$$

Finite size correction
(energy non-conserving)

Normal term

Extract the light-cone singularity

About I_1

$$I_1 = \left[p_\pi \cdot p_\nu - i p_\nu \cdot \left(\frac{\partial}{\partial \delta x} \right) \right] \tilde{I}_1$$

To extract light-cone singularity expanding in $2p_\pi \cdot q$

convergence condition: $2p_\pi \cdot p_\nu \leq m_\pi^2 - m_\mu^2$

$$\tilde{I}_1 = \int d^4 q \frac{\theta(q^0)}{4\pi^4} \mathrm{Im} \left[\frac{1}{q^2 + 2p_\pi \cdot q + \tilde{m}^2 - i\epsilon} \right] e^{iq\cdot\delta x}$$

$$= \boxed{\int d^4 q \frac{\theta(q^0)}{4\pi^4} \mathrm{Im} \left[\frac{1}{q^2 + \tilde{m}^2 - i\epsilon} \right] e^{iq\cdot\delta x}}$$

$$+ \sum_{n=1} \frac{1}{n!} \left(2p_\pi \cdot \left(-i \frac{\partial}{\partial \delta x} \right) \frac{\partial}{\partial \tilde{m}^2} \right)^n \int d^4 q \frac{\theta(q^0)}{4\pi^4} \mathrm{Im} \left[\frac{1}{q^2 + \tilde{m}^2 - i\epsilon} \right] e^{iq\cdot\delta x}$$

Green's function

$$\int d^4 q \frac{\theta(q^0)}{4\pi^4} \mathrm{Im} \left[\frac{1}{q^2 + \tilde{m}^2 - i\epsilon} \right] e^{iq\cdot\delta x} = \frac{2}{(2\pi)^3} \int d^4 q \theta(q^0) \delta(q^2 + \tilde{m}^2) e^{iq\cdot\delta x}$$

$$= 2i \left[\boxed{\frac{\epsilon(\delta t)}{4\pi} \delta(\lambda)} + f_{\text{short}} \right]$$

Light-cone singularity

$$f_{\text{short}} = -\frac{i\tilde{m}^2}{8\pi\sqrt{-\lambda}} \theta(-\lambda) \left\{ N_1\left(\tilde{m}\sqrt{-\lambda}\right) - i\epsilon(\delta t) J_1\left(\tilde{m}\sqrt{\lambda}\right) \right\} - \theta(\lambda) \frac{i\tilde{m}}{4\pi^2\sqrt{\lambda}} K_1\left(\tilde{m}\sqrt{\lambda}\right), \ \lambda = \delta t^2 - \delta\vec{x}^2$$

Integration in space-time coordinates

$$\int \frac{d^3p_l}{(2\pi)^3} \sum_{spin} |T|^2 = \frac{N_2}{E_\nu} \int d^4x_1 d^4x_2 e^{-\frac{1}{2\sigma_\nu}\sum_i(\vec{x}_i - \vec{\mathrm{X}}_\nu - \vec{v}_\nu(t_i - \mathrm{T}_\nu))^2}$$

$$\times \Delta_{\pi,l}(x_1 - x_2) e^{ip_\nu \cdot (x_1 - x_2)}$$

$$= \frac{N_3}{E_\nu} \int d^4x_1 d^4x_2 e^{-\frac{1}{2\sigma_\nu}\sum_i(\vec{x}_i - \vec{\mathrm{X}}_\nu - \vec{v}_\nu(t_i - \mathrm{T}_\nu))^2} \left[i\frac{\epsilon(\delta t)}{4\pi}\delta(\lambda) + f'_{\text{short}} + I_2 \right] e^{ip_\nu \cdot \delta x}$$

$$N_3 = 8g^2 \left\{ p_\pi \cdot p_\nu (m_\pi^2 - 2p_\pi \cdot p_\nu) \right\} (\pi^2/\sigma_\nu)^{\frac{3}{2}}$$

Long-range term

$$\int d^4x_1 d^4x_2 e^{-\frac{1}{2\sigma_\nu}\sum_i(\vec{x}_i - \vec{\mathrm{X}}_\nu - \vec{v}_\nu(t_i - \mathrm{T}_\nu))^2} e^{ip_\nu \cdot \delta x} \times i\frac{\epsilon(\delta t)}{4\pi}\delta(\lambda)$$

$$= (\sigma_\nu \pi)^{\frac{3}{2}} \frac{\sigma_\nu}{2} i \int_0^{\mathrm{T}} dt_1 dt_2 \frac{\epsilon(\delta t)}{|\delta t|} e^{i\frac{m_\nu^2}{2E_\nu}\delta t}$$

$\mathrm{L} = c\mathrm{T}$ is length of decay volume

Three flavor neutrino

$$P = P^{(0)}(long) + P_{diffraction}(short), \bar{m}_i^2 \gg \delta m^2$$

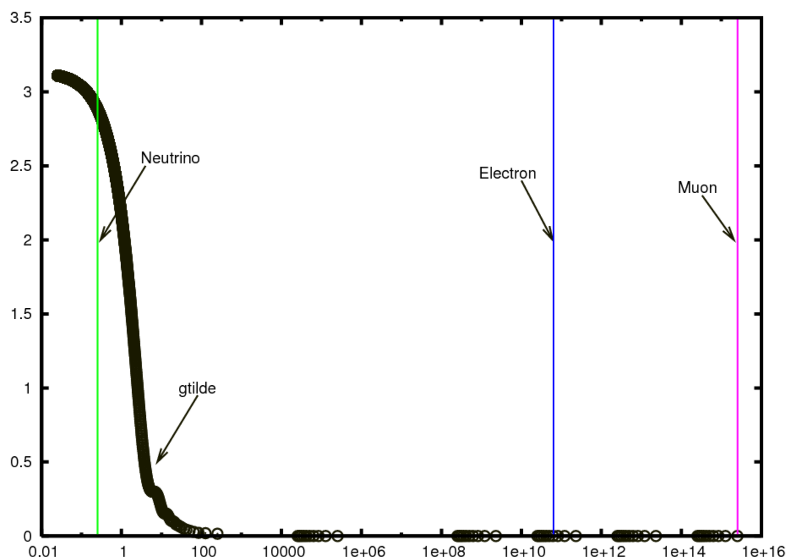
$$P^{(0)}(long) = flavour\, oscillation, \quad P_{diffraction}(short) = neutrino\, diffraction$$

Diffraction effect (muon ¥nu)

$$P = P_{normal} + P_{diffraction}$$

$$P_{diffraction} = C\tilde{g}(T\omega), \omega = \frac{m_\nu^2}{2E_\nu}$$

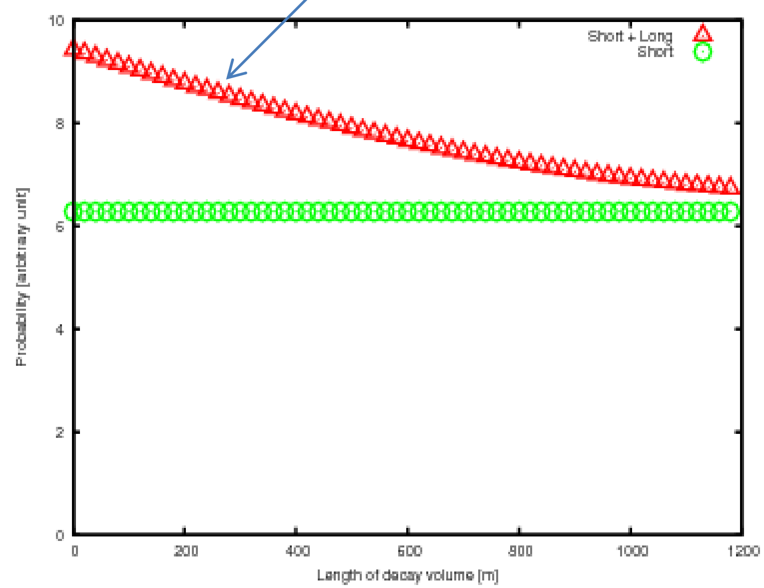
$\tilde{g}(T\omega)$



$T\omega$

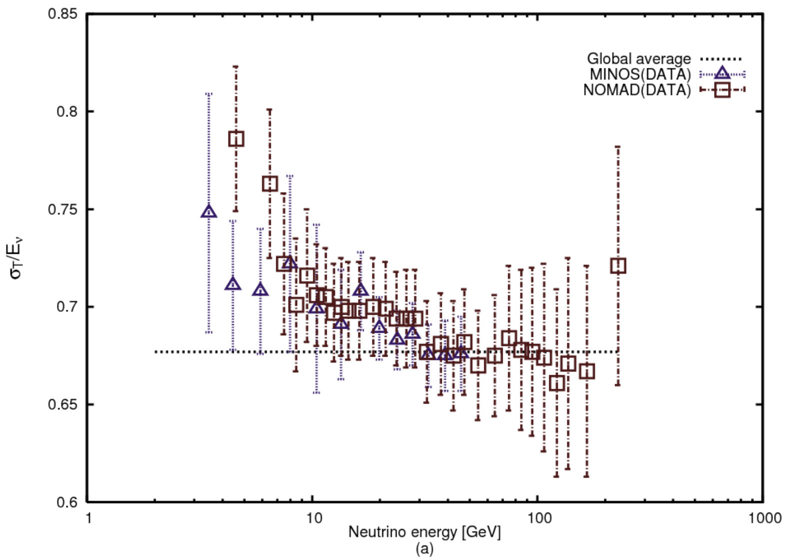
Large finite-size correction

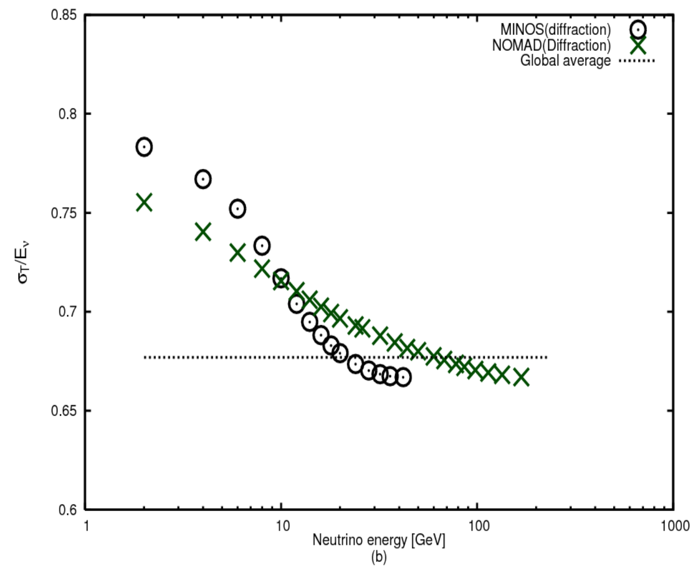
P



Length(M)

¥nu-nucleon total cross section





Exp.:
NOMAD & MINOS

theory :
normal+diffraction

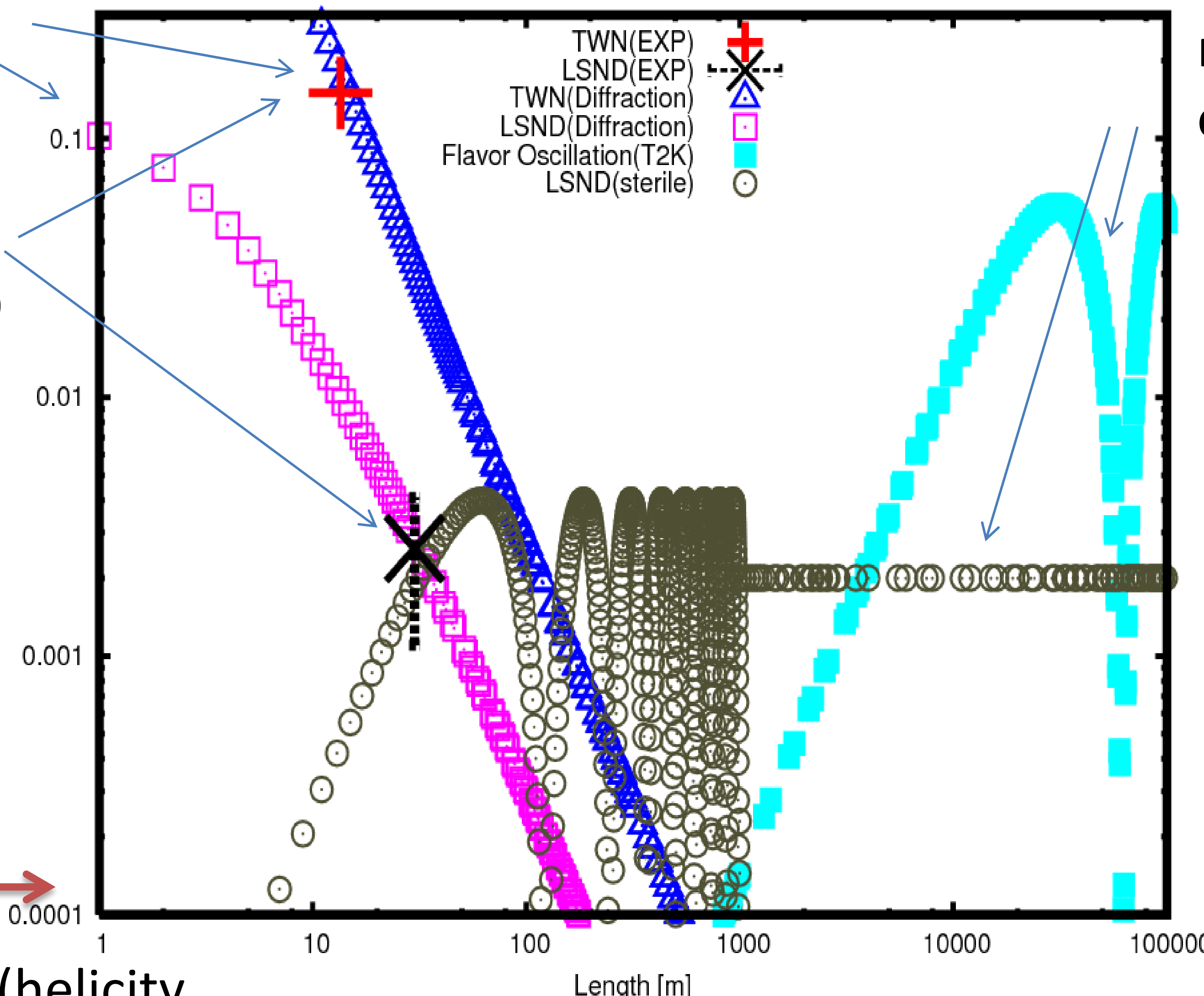
Helicity suppression

- The finite-size correction comes from the energy non-conserving term.
- The pi > e+neutrino_e mode is suppressed by the angular momentum conservation and the energy-momentum conservation.
- Since the finite-size correction does not conserve the energy, it violates the helicity suppression .
- When the neutrino is detected, the electron mode is enhanced.

Diffraction Events(enhances electron neutrino)

Diffraction

Exp.
(lSND,TWN)

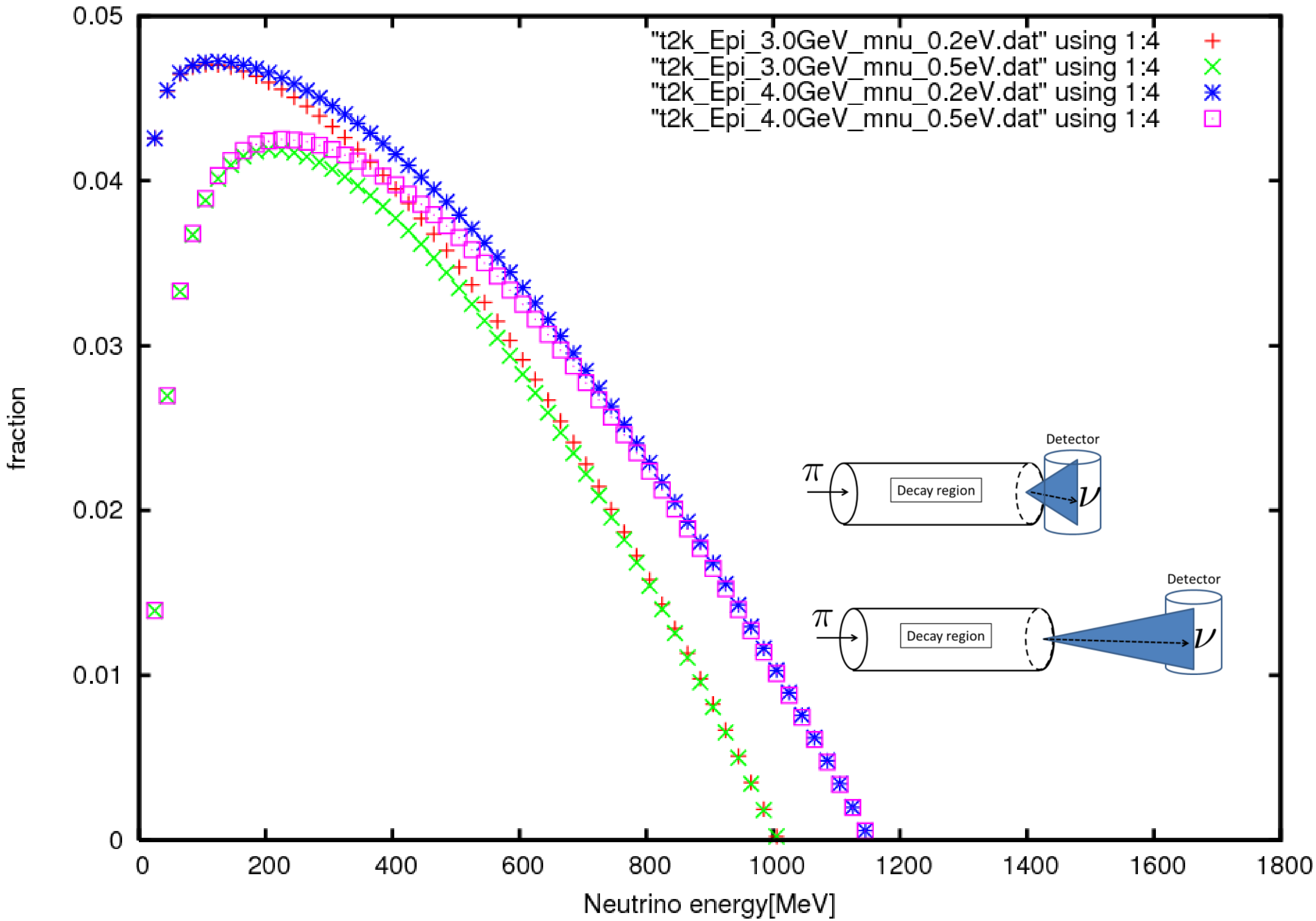


neutrino oscillation

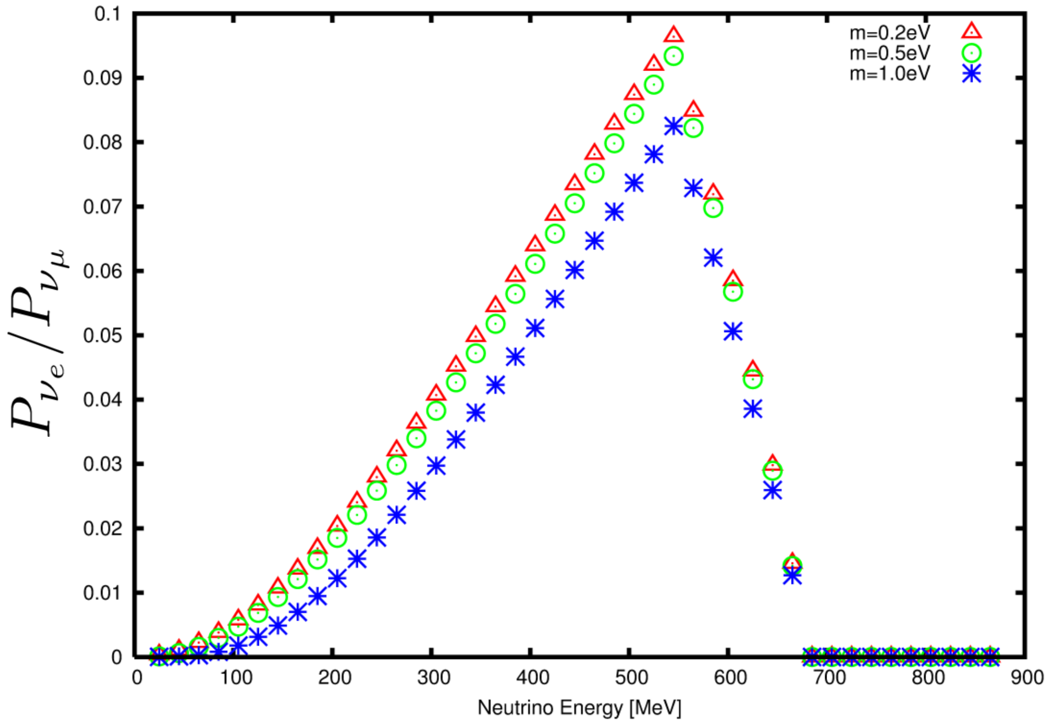
Normal calculation(helicity suppression)

Distance[M]

Diffraction prediction of the electron neutrino diffraction(T2K、on axis)



ν_e appearance at near detector



$|\vec{p}_\pi| = 2[\mathrm{GeV}],\ \mathrm{L} = 110[\mathrm{m}]$

L_{decay-detector} = 170[m], axis = 2.5 degree, Detector 3[m]x3[m]

～10% excess (maximum)

Comparisons with previous experiments

- Diffraction effect has been observed but that has not been recognized. So, unusual events have been regarded as anomalous events . They are explained with the neutrino diffraction.

- High energy neutrino nucleon scattering cross section decreases with the energy slowly. This is understood by the diffraction effect of the neutrino process.

- High precision experiment may provide the neutrino absolute mass.

Anomalous properties of the diffraction

1 neutrino diffraction is easily observed once the statistics becomes large. quantum interference

2 energy conservation is violated : finite-size effect

3 lepton number appears to be non-conserved.

P(L) decreases with L , so unitarity appears to be violated. But they are not. : finite-size effect and retarded effect.

4 pion life time varies due to the measurement, quantum Zeno effect . However the majority of the pion are unchanged because the neutrino interacts with matter so weakly.

The neutrino diffraction is a finite-size correction

- Why does the neutrino diffraction emerge?

1.Transition in a finite time interval T,

$$i + \omega \rightarrow j \tag{1}$$

$$P_{ij} = |T_{ij}|^2 \frac{\sin^2(E_i + \hbar\omega - E_j)T/2}{(E_i + \hbar - E_j)^2} \tag{2}$$

, violates the energy conservation .

2.States at ultra-violet energy region

$$E_{\mathrm{J}} \rightarrow \infty$$

give a universal correction.

3. Relativistic invariance >> large momentum states

Other channels

- 1. muon decay $\mu \to e + \nu_\mu + \bar{\nu}_e$

- 2. neutron decay $n \to p + e + \bar{\nu}_e$

- 3.nucleus decays $A \to A' + e + \bar{\nu}_e$

  $A + e \to A' + \nu$

- 4.neutrino scattering $\nu + A \to \nu + A$

(In progress)

New phenomena caused by finite-size effects

1．Emission of light particles .

Energy non-conserving transition lead background noises that has universal properties.

“theory of universal noises “

2．Interference and diffraction.

interference of a new scale that is very different from wave length “physics of a new scale ”

3．Absorption ：

“Coherent absorption phenomena”

conclusion

- With enough number of neutrino events,

the neutrino diffraction is easy to observe and may provide the absolute neutrino mass.