

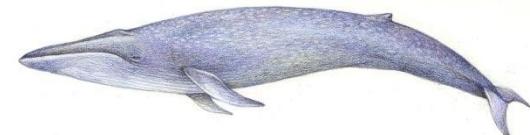
What can we hope on neutrino physics at the LHC

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Introduction

Neutrino masses are extremely smaller than other fermion masses.

neutrino $\lesssim 1 \text{ eV}$ electron = 0.5 MeV tau = 1.8 GeV top = 172 GeV



1 MeV "≈" 1 kg

→ Neutrino-specific mechanism to generate their masses ?

Yukawa with SM Higgs?

Yukawa with new scalar ?

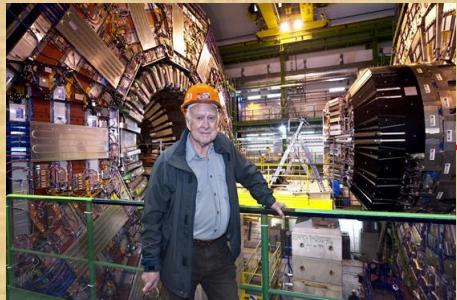
Seesaw with very heavy particle ?

Light new scalar ?

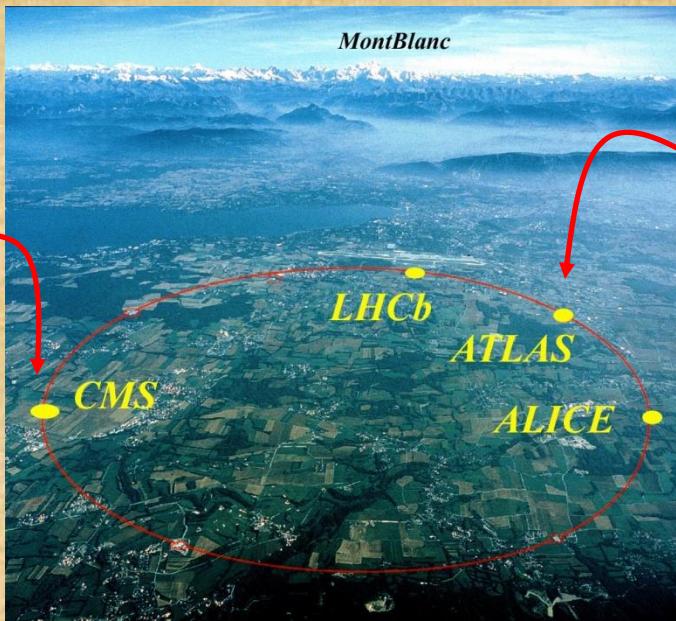
Far from experimental reach

LHC ?

Large Hadron Collider



**first event of Higgs
at CMS (4 Apr. 2008)**



**first event of Higgs
at ATLAS (4 Apr. 2008)**

pp collider at $\sqrt{s} = 7, 8, 14 \text{ TeV}$ (cf. Tevatron: $p\bar{p}$ at $\sqrt{s} = 1.96 \text{ TeV}$)

26.7km ring (cf. 26.4km loop subway in Nagoya, Japan)

ATLAS: Higgs, new phys.

ALICE: quark-gluon plasma

TOTEM (near CMS): elastic and diffractive cross sections

LHCf (near ATLAS): cosmic ray

CMS: Higgs, new phys.

LHCb: B phys.

New scalars at the LHC



Leptophilic Scalars !

Origin of neutrino masses

Yukawa Interactions with Leptophilic Scalars

SU(2)_L singlet

$\overline{L_\ell^c} i\sigma_2 L_{\ell'} s^+$: **Singly charged** (e.g., in Zee model)

$\overline{(\ell_R)^c} \ell'_R s^{++}$: **Doubly charged** (e.g., in Zee-Babu model)

Radiative neutrino mass

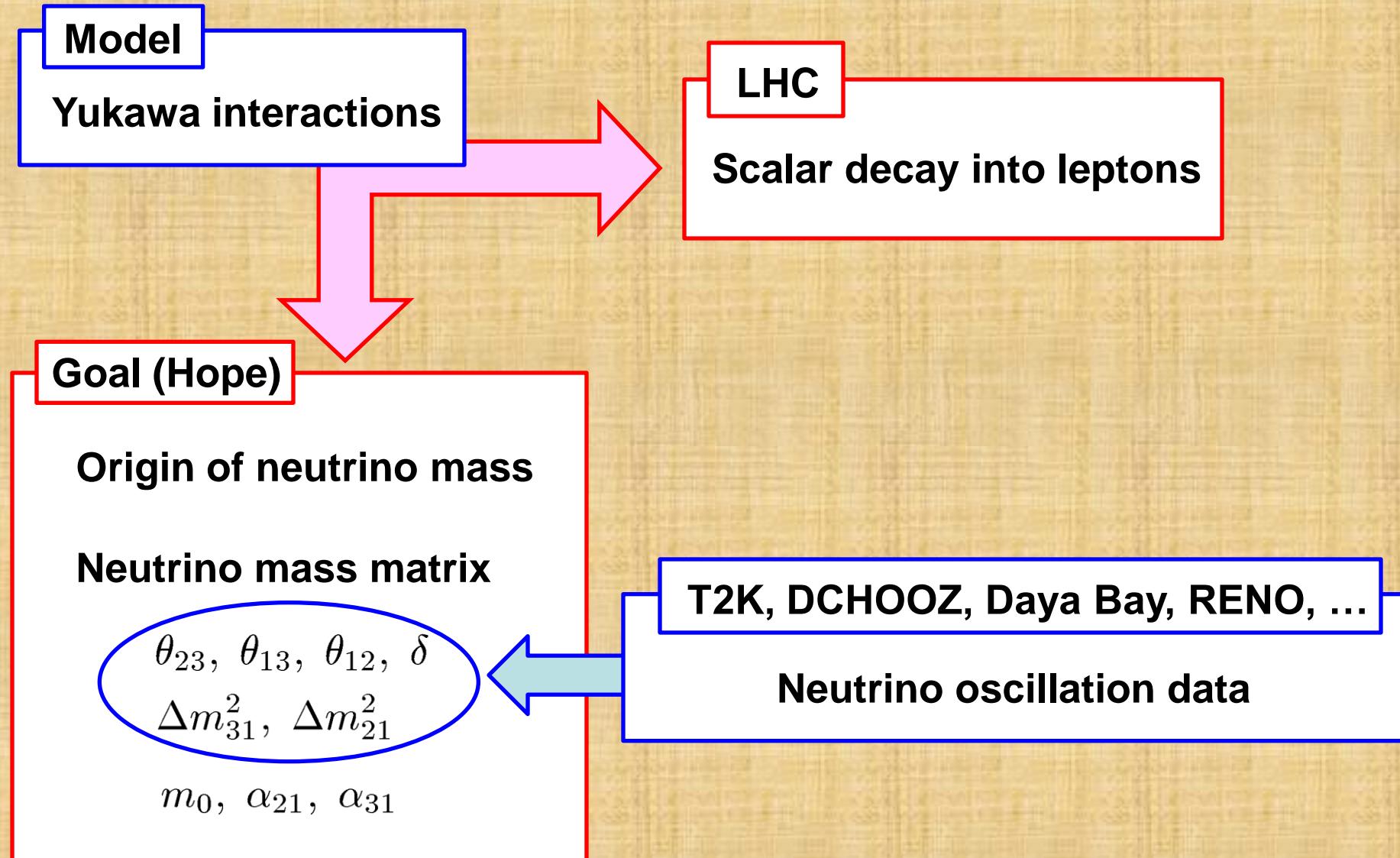
SU(2)_L doublet

$\overline{\nu_{Ri}} \Phi_\nu^T i\sigma_2 L_\ell$: **Neutrinophilic doublet**

SU(2)_L triplet

$\overline{L_\ell^c} i\sigma_2 \Delta L_{\ell'}$ (e,g., in Higgs Triplet Model)

Tree neutrino mass



Doubly Charged Scalar

Doubly Charged Higgs - Triplet -



Target : $h_{\ell\ell'} \left[\overline{L}_\ell^c i\sigma_2 \Delta L_{\ell'} \right]$

$$\Delta \equiv \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}$$

Symmetric

Higgs Triplet Model

W. Konetschny and W. Kumer, PLB**70**, 433 (1977)
 M. Magg and C. Wetterich, PLB**94**, 61 (1980)
 T.P. Cheng and L.F. Li, PRD**22**, 2860 (1980)
 J. Schechter and J.W.F. Valle, PRD**22**, 2227 (1980)

Yukawa : $h_{\ell\ell'} \left[\overline{L}_\ell^c i\sigma_2 \Delta L_{\ell'} \right]$

Neutrino masses : $(m_\nu)_{\ell\ell'} = h_{\ell\ell'} \langle \Delta^0 \rangle$

Scalar potential :

$$V = -m_\Phi^2 [\Phi^\dagger \Phi] + m_\Delta^2 [\text{Tr}(\Delta^\dagger \Delta)] - \left(\mu [\Phi^T i\sigma_2 \Delta^\dagger \Phi] + \text{h.c.} \right) + \text{quartic}$$

$$\langle \Delta^0 \rangle \sim \frac{\mu \langle \phi^0 \rangle^2}{m_\Delta^2}$$

Soft breaking of $L\#$



Set up

Target : $h_{\ell\ell'} \left[\overline{L_\ell^c} i\sigma_2 \Delta L_{\ell'} \right]$

$$\begin{cases} (m_\nu)_{\ell\ell'} = h_{\ell\ell'} \langle \Delta^0 \rangle \\ = \left(U_{\text{MNS}}^* \text{diag}(m_1, m_2 e^{i\alpha_{21}}, m_3 e^{i\alpha_{31}}) U_{\text{MNS}}^\dagger \right)_{\ell\ell'} \\ 10 \text{ eV} \lesssim \langle \Delta^0 \rangle \lesssim 10 \text{ KeV} \end{cases}$$

$$m_{\Delta^{\pm\pm}} \leq m_{\Delta^\pm}$$

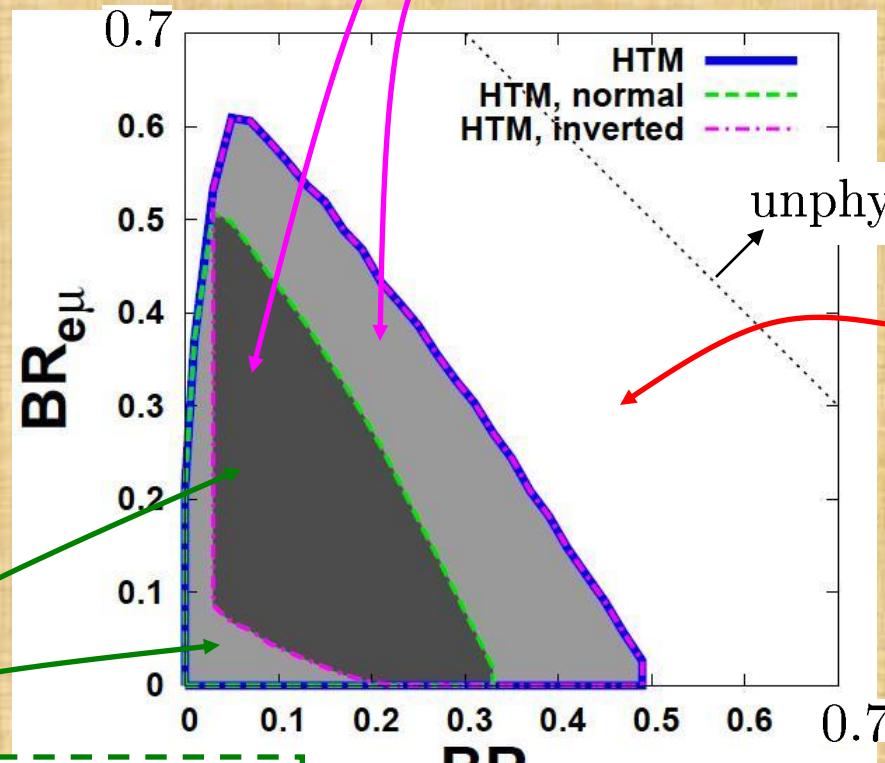
$m_{\Delta^{\pm\pm}} = 500 \text{ GeV} \Rightarrow \sigma(pp \rightarrow \Delta^{++}\Delta^{--}) = 3.2 \times 10^{-1} [\text{fb}] @ 8 \text{ TeV LHC}$
 $1.7 [\text{fb}] @ 14 \text{ TeV LHC}$
w/o QCD K -factor

$$\text{BR}_{\ell\ell'} \equiv \text{BR}(\Delta^{\pm\pm} \rightarrow \ell^\pm \ell'^\pm) = \frac{2}{1 + \delta_{\ell\ell'}} \frac{|(m_\nu)_{\ell\ell'}|^2}{\sum_j m_j^2}$$

Information on $\text{sign}(\Delta m_{31}^2)$



possible with $\Delta m_{31}^2 < 0$



possible with $\Delta m_{31}^2 > 0$

$H^{\pm\pm} \neq SU(2)_L$ triplet ?
(ex. $Y = 4$ singlet,
 $SU(2)_R$ triplet)
other mechanism for m_ν ?
(ex. see-saw with ν_R)

A.G. Akeroyd, M. Aoki, HS, PRD 77, 075010

See also J. Garayoa, T. Schwetz, JHEP 0803, 009

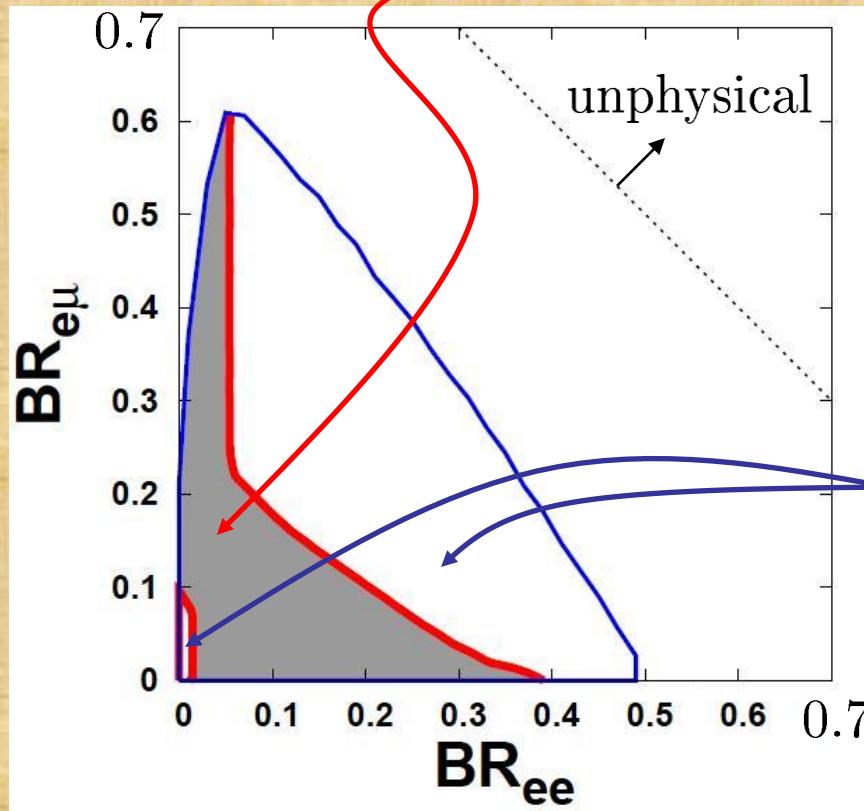
M. Kadastki, M. Raidal, L. Rebane, PRD 77, 115023

Information on Non-Zero m_0

the lightest mass



shaded $\Rightarrow m_0 \neq 0$

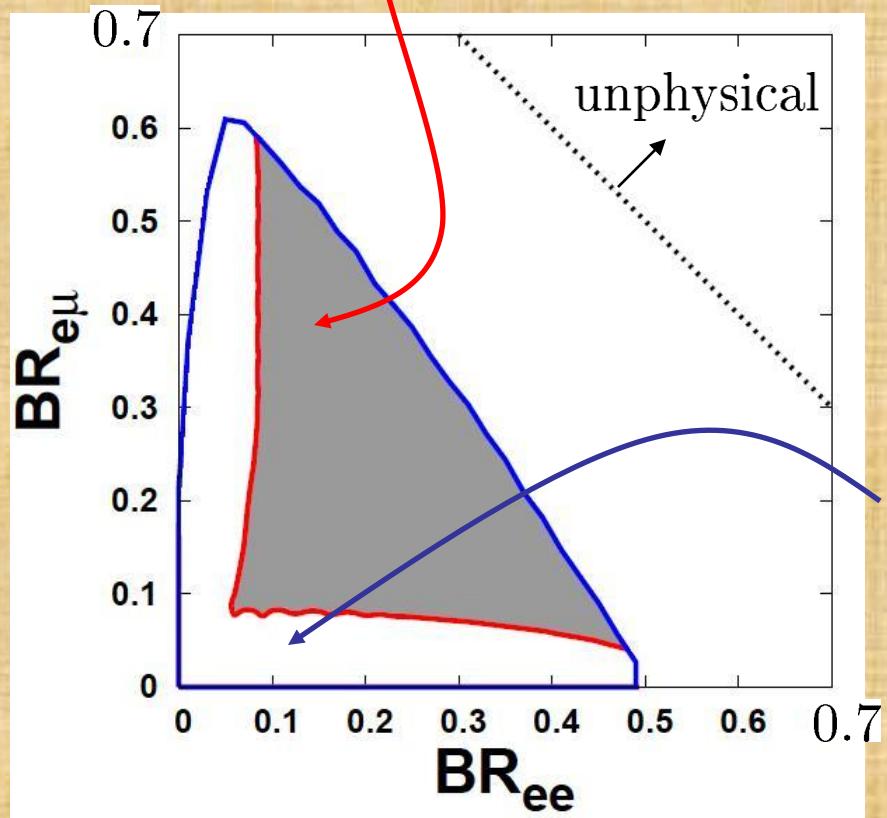


Note: Knowledge on $\text{sign}(\Delta m_{31}^2)$ is not assumed



Information on CP-Violation with Majorana Phases

shaded \Rightarrow CP violation by Majorana phases



Note: Knowledge on $\text{sign}(\Delta m_{31}^2)$ is not assumed

Doubly Charged Scalar - Singlet -

Target : $h_{\ell\ell'} \left[\overline{(\ell_R)^c} \ell'_R s^{++} \right]$

Symmetric



Zee-Babu Model

A. Zee, NPB264, 99 (1986)
K.S. Babu, PLB203, 132 (1988)

Yukawa : $h_{\ell\ell'} \left[\overline{(\ell_R)^c} \ell'_R s^{++} \right], f_{\ell\ell'} \left[\overline{L_\ell^c} i\sigma_2 L_{\ell'} s^+ \right]$

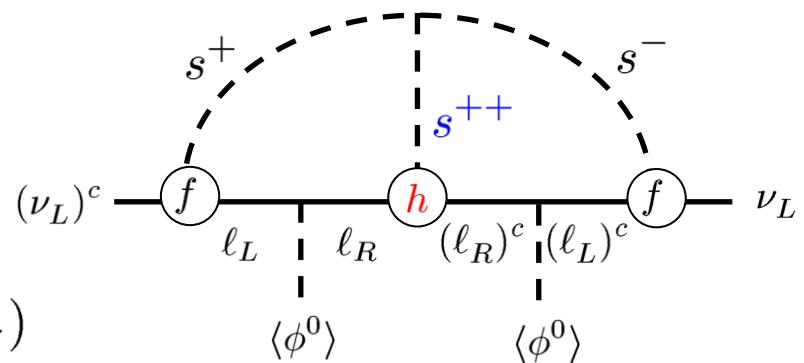
Mass eigenstate



Neutrino masses :

$$(m_\nu)_{\ell\ell'} \propto \left(f m_\ell^{\text{diag}} h m_{\ell'}^{\text{diag}} f^\dagger \right)_{\ell\ell'}$$

$$m_\ell^{\text{diag}} \equiv \text{diag}(m_e, m_\mu, m_\tau)$$



$$m_{s^{\pm\pm}} = 500 \text{ GeV} \Rightarrow \sigma(pp \rightarrow s^{++} s^{--}) = 1.4 \times 10^{-1} [\text{fb}] @ 8 \text{ TeV LHC}$$

$$7.2 \times 10^{-1} [\text{fb}] @ 14 \text{ TeV LHC}$$

$$\text{Det}(m_\nu) \propto \text{Det}(f) = 0 \longrightarrow m_1 = 0 \text{ or } m_3 = 0$$

$$(m_\nu)_{\ell\ell'} \propto \left(f m_\ell^{\text{diag}} h m_{\ell'}^{\text{diag}} f \right)_{\ell\ell'}$$

No e contribution (assumption)



$$\Rightarrow \text{BR}_{\mu\mu} : \text{BR}_{\mu\tau} : \text{BR}_{\tau\tau} \sim 1 : 0 : 0$$

$$\text{BR}_{\ell\ell'} \equiv \text{BR}(\textcolor{blue}{s}^{\pm\pm} \rightarrow \ell^\pm \ell'^\pm) \propto |h_{\ell\ell'}|^2$$

$$h_{\mu\mu} : h_{\mu\tau} : h_{\tau\tau} \sim 1 : \frac{m_\mu}{m_\tau} : \frac{m_\mu^2}{m_\tau^2} \simeq 1 : 0.06 : 0.003$$

- K.S. Babu and C. Macesanu, PRD**67**, 073010 (2003)
 D. Aristizabal Sierra and M. Hirsch, JHEP**0612**, 052 (2006)
 M. Nebot *et al.*, PRD**77**, 093013 (2008)

Singly Charged Scalar

Singly Charged Higgs - Doublet or Triplet -

Neutrinophillic Doublet

S. Gabriel and S. Nandi, PLB655, 141 (2007)

S.M. Davidson and H.E. Logan, PRD80, 905008 (2009)

Neutrino Yukawa : $(y_\nu)_{i\ell} \left[\overline{\nu_{Ri}} \Phi_\nu i\sigma_2 L_\ell \right]$

Only for neutrinos

$\nu_{Ri}, \Phi_\nu : Z_2$ odd



Dirac neutrinos : $(m_\nu)_{i\ell} = (y_\nu)_{i\ell} \langle \phi_\nu^0 \rangle$

$$\langle \phi_\nu^0 \rangle \ll \langle \phi_{SM}^0 \rangle$$

(Spontaneous Symmetry Breaking $\Rightarrow m_{H_\nu^0} \propto \langle \phi_\nu^0 \rangle$)

Scalar potential :

$$V = -m_{11}^2 [\Phi_{SM}^\dagger \Phi_{SM}] + m_{22}^2 [\Phi_\nu^\dagger \Phi_\nu] - \left(m_{12}^2 [\Phi_{SM}^\dagger \Phi_\nu] + h.c. \right) + \text{quartic}$$

Soft breaking of Z_2

$$\langle \phi_\nu^0 \rangle \simeq \frac{m_{12}^2 \langle \phi_{SM}^0 \rangle}{m_{22}^2}$$



Set up for Dirac neutrinos



Target : $(y_\nu)_{i\ell} \left[\overline{\nu_{Ri}} \Phi_\nu^T i\sigma_2 L_\ell \right]$

$$\begin{cases} (m_\nu)_{i\ell} = (y_\nu)_{i\ell} \langle \phi_\nu^0 \rangle \\ = \left(\text{diag}(m_1, m_2, m_3) U_{\text{MNS}}^\dagger \right)_{i\ell} \\ 10 \text{ eV} \lesssim \langle \phi_\nu^0 \rangle \lesssim 10 \text{ KeV} \end{cases}$$

$m_{\phi_\nu^\pm} \leq m_{H_\nu^0}$

$$m_{\phi_\nu^\pm} = 500 \text{ GeV} \Rightarrow \sigma(pp \rightarrow \phi_\nu^+ \phi_\nu^-) = 8.0 \times 10^{-2} [\text{fb}] @ 8 \text{ TeV LHC}$$

$$4.2 \times 10^{-1} [\text{fb}] @ 14 \text{ TeV LHC}$$



Set up for Majorana neutrinos (“HTM-like” case)



Target : $h_{\ell\ell'} \left[\overline{L_\ell^c} i\sigma_2 \Delta L_{\ell'} \right]$

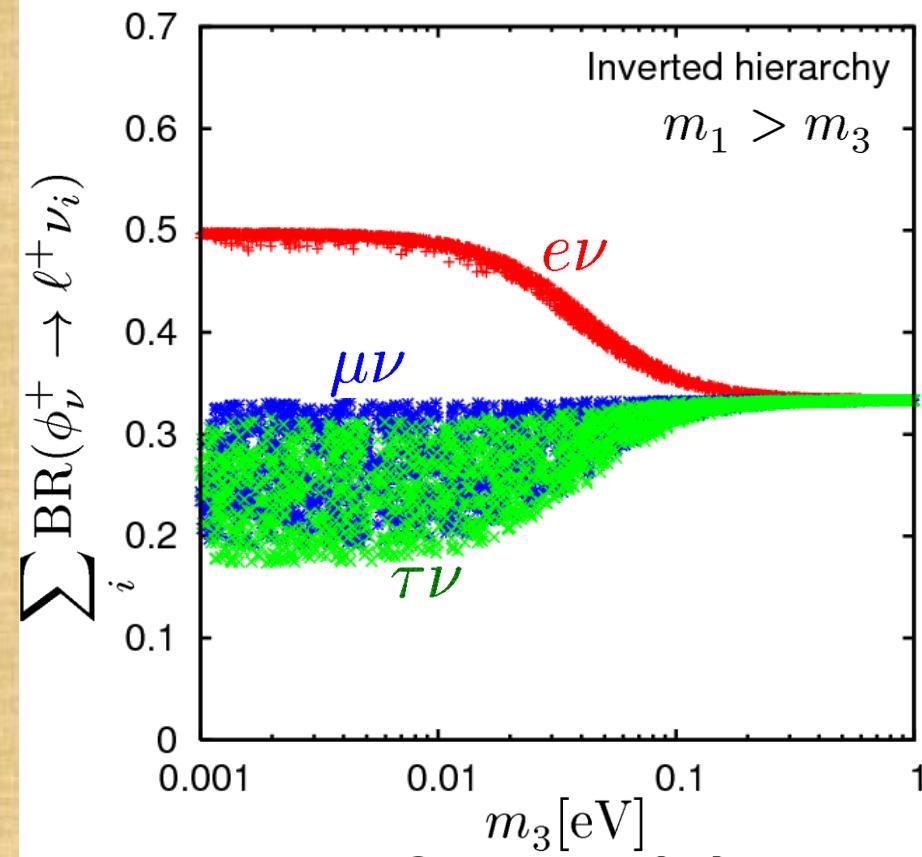
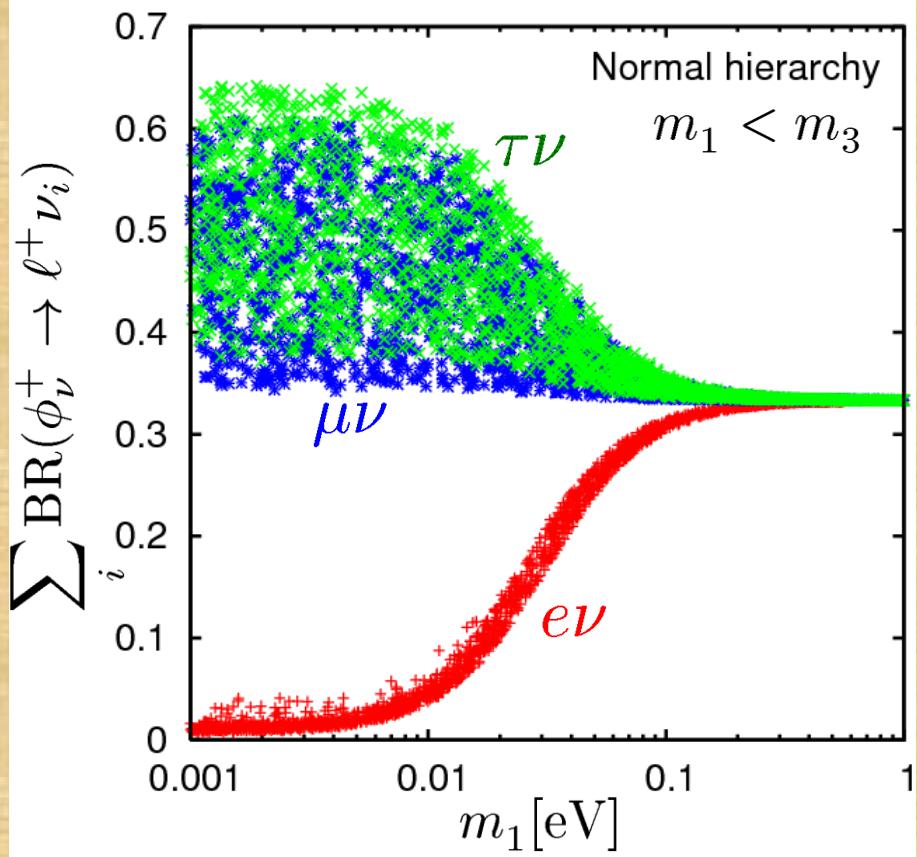
$$\begin{cases} (m_\nu)_{\ell\ell'} = h_{\ell\ell'} \langle \Delta^0 \rangle \\ = \left(U_{\text{MNS}}^* \text{diag}(m_1, m_2 e^{i\alpha_{21}}, m_3 e^{i\alpha_{31}}) U_{\text{MNS}}^\dagger \right)_{\ell\ell'} \\ 10 \text{ eV} \lesssim \langle \Delta^0 \rangle \lesssim 10 \text{ KeV} \end{cases}$$

$m_{\Delta^{\pm\pm}} \simeq m_{\Delta^\pm} \simeq m_{\Delta^0}$ (See also, S. Kanemura and K. Yagyu, arXiv:1201.6287)

$$m_{\Delta^\pm} = 500 \text{ GeV} \Rightarrow \sigma(pp \rightarrow \Delta^+ \Delta^-) = 3.4 \times 10^{-2} [\text{fb}] @ 8 \text{ TeV LHC}$$

$$1.8 \times 10^{-1} [\text{fb}] @ 14 \text{ TeV LHC}$$

$$\text{BR}_\ell \equiv \sum_i \text{BR}(\phi_\nu^+ \rightarrow \ell^+ \nu_i) = \sum_i \text{BR}(\Delta^+ \rightarrow \ell^+ \bar{\nu}_i) = \frac{\sum_i m_i^2 |(U_{\text{MNS}})_{\ell i}|^2}{\sum_j m_j^2}$$

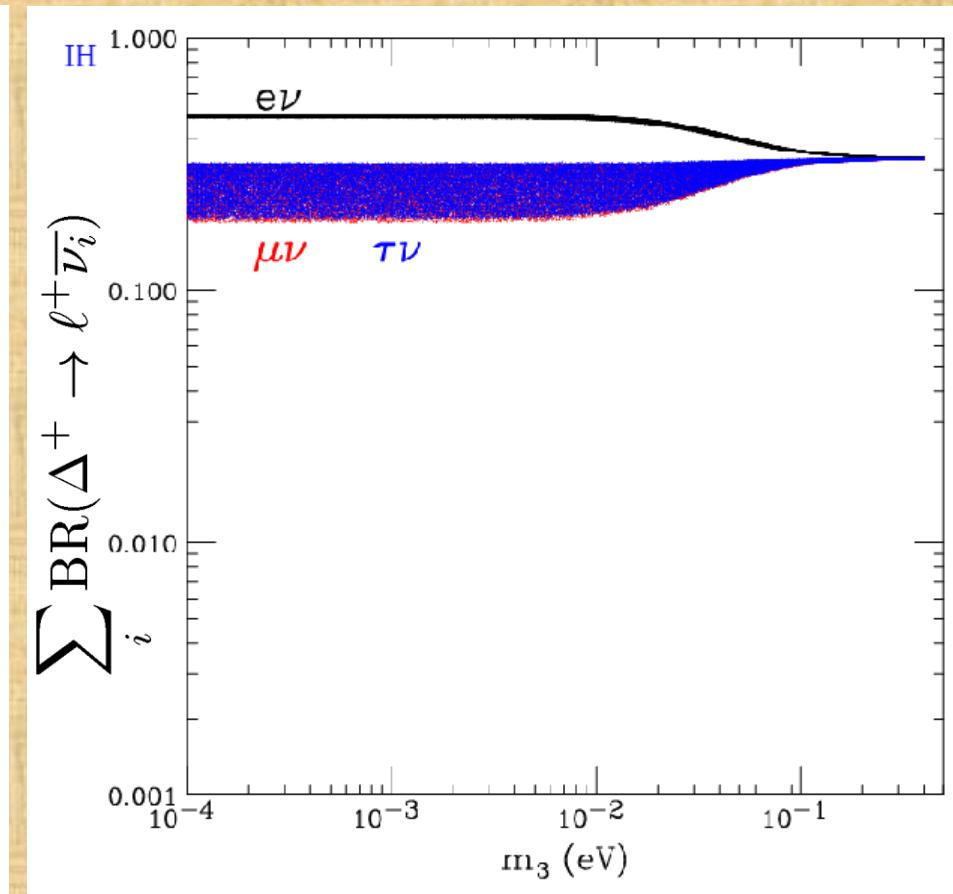
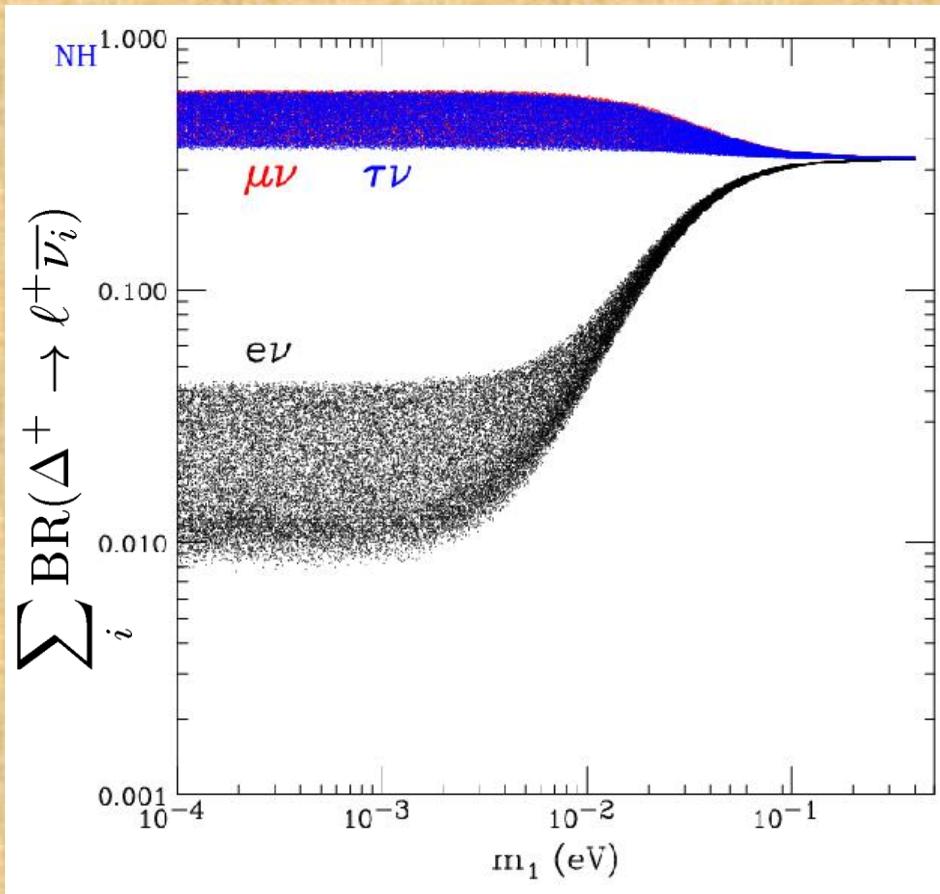


S.M. Davidson and H.E. Logan, PRD **80**, 905008 (2009)



$$\text{BR}_\mu \simeq \text{BR}_\tau$$

$$\frac{\text{BR}_e}{\text{BR}_\mu} \longrightarrow \begin{cases} \text{sign}(m_3 - m_1) \\ m_0 (= \min(m_1, m_3)) \end{cases}$$



P. Fileviez Perez *et al.*, PRD78, 015018 (2008)

Singly Charged Scalar - Singlet 1 -



Target : $f_{\ell\ell'} \left[\overline{L_\ell^c} i\sigma_2 L_{\ell'} s^+ \right]$ \simeq Mass eigenstate (assumption)

Antisymmetric

Simplest Zee Model

A. Zee, PLB93, 389 (1980)

L. Wolfenstein, NPB175, 93 (1980)

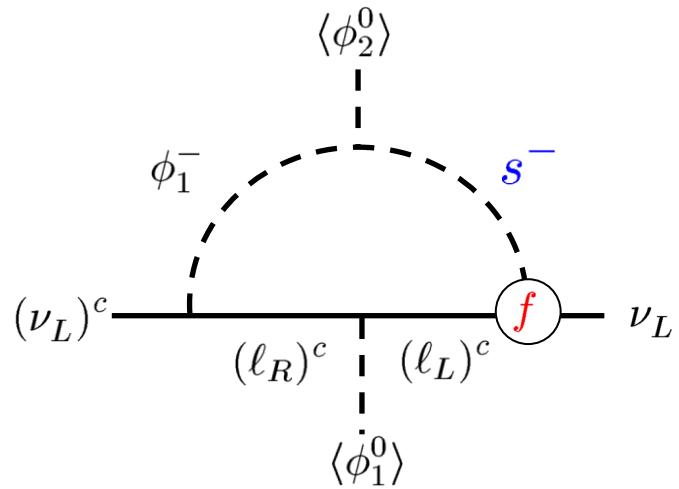
Doublet Yukawa : $\frac{m_\ell}{\langle \phi_1^0 \rangle} \left[\overline{L_\ell} \Phi_1 \ell_R \right]$

No Flavor-Changing-Neutral-Current

$$(m_\nu)_{\ell\ell'} \propto (m_\ell^2 - m_{\ell'}^2) f_{\ell\ell'}$$

Excluded by Oscillation data

e.g., X.G. He, EPJC34, 371 (2004)



General (Original) Zee Model

A. Zee, PLB93, 389 (1980)

$$\text{Doublet Yukawa : } \frac{m_\ell}{\langle \phi_1^0 \rangle} \left[\overline{L}_\ell \Phi_1 \ell_R \right] + (y_2)_{\ell\ell'} \left[\overline{L}_\ell \left(\Phi_2 - \frac{\langle \phi_2^0 \rangle}{\langle \phi_1^0 \rangle} \Phi_1 \right) \ell'_R \right]$$



Set up

Incl. FCNC

$$\text{Target : } f_{\ell\ell'} \left[\overline{L}_\ell^c i\sigma_2 L_{\ell'} s^+ \right] \simeq \text{Mass eigenstate (assumption)}$$

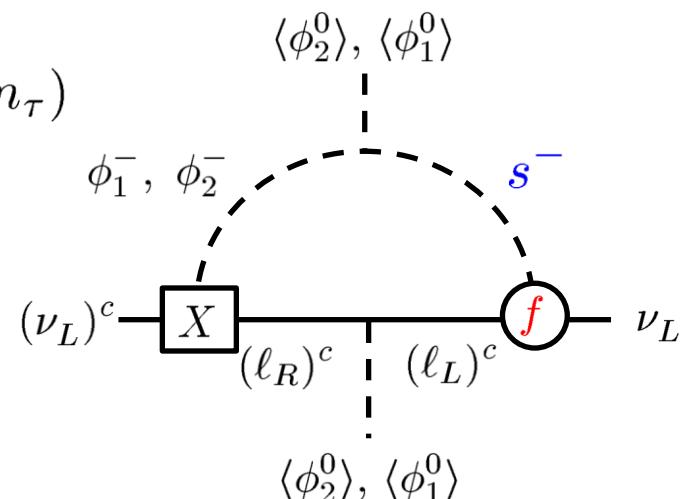
Antisymmetric

Neutrino mass :

$$(m_\nu)_{\ell\ell'} = \left(X m_\ell^{\text{diag}} \mathbf{f} + (X m_\ell^{\text{diag}} \mathbf{f})^T \right)_{\ell\ell'}$$

$$\sim m_\tau \left(X_{\ell\tau} \mathbf{f}_{\tau\ell'} + (\mathbf{f}^T)_{\ell\tau} (X^T)_{\tau\ell'} \right)$$

τ dominance (assumption)



$$m_{s^\pm} = 500 \text{ GeV} \Rightarrow \sigma(pp \rightarrow s^+ s^-) = 3.4 \times 10^{-2} [\text{fb}] @ 8 \text{ TeV LHC}$$

$$1.8 \times 10^{-1} [\text{fb}] @ 14 \text{ TeV LHC}$$

$$\text{Det}(m_\nu) = m_\tau \text{Det} \left(X_{\ell\tau} \mathbf{f}_{\tau\ell'} + (\mathbf{f}^T)_{\ell\tau} (X^T)_{\tau\ell'} \right) = 0 \longrightarrow m_1 = 0 \text{ or } m_3 = 0$$

$$m_1 = 0$$

$$\left| \frac{m_2}{m_3} \right| = \frac{c_{23}^2 c_{13}^2}{|c_{23}s_{12}s_{13} + c_{12}s_{23}e^{i\delta}|^2} \gtrsim 1$$

Not acceptable



$$m_3 = 0$$

$$\frac{\text{BR}_\tau}{\text{BR}_\mu - \text{BR}_e} \simeq \frac{17}{15}$$

$$\begin{cases} \sin^2 2\theta_{13} \simeq 0.118 \\ \delta \simeq \pi \\ \alpha_{21} \simeq \pi \end{cases}$$

$$\text{BR}_\ell \equiv \sum_i \text{BR}(s^- \rightarrow \ell \nu_i)$$

$$\begin{cases} \frac{f_{\mu\tau}}{f_{e\tau}} = -\frac{\tan \theta_{13}}{s_{23}} e^{-i\delta} \\ f_{e\mu} : \text{Arbitlary} \end{cases}$$

$$s_{23} = \frac{1}{\sqrt{2}}, \quad s_{12} = \frac{1}{\sqrt{3}}, \quad \left| \frac{m_1}{m_2} \right| \simeq 1 \quad \Rightarrow$$

$$\begin{cases} s_{13} \simeq 3 - \sqrt{2} \simeq 0.17 \\ \delta \simeq \pi \\ \alpha_{21} \simeq \pi \end{cases}$$

Singly Charged Scalar - Singlet 2 -



Set up

Target : $f_{\ell\ell'} \left[\overline{L_\ell^c} i\sigma_2 L_{\ell'} s^+ \right]$

\simeq Mass eigenstate (assumption)

Antisymmetric

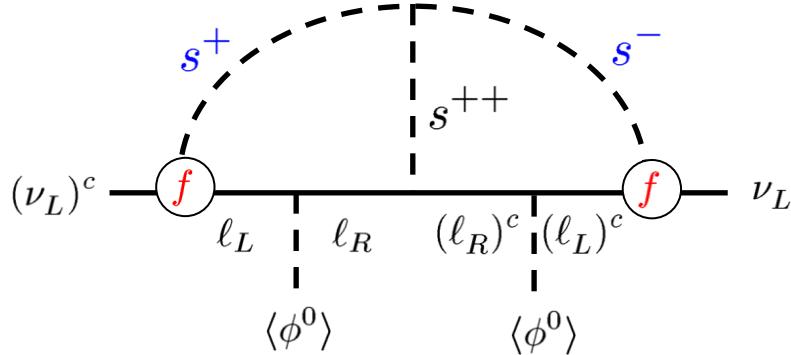


Majorana neutrinos :

$$\frac{1}{2} (m_\nu)_{\ell\ell'} \left[\overline{(\nu_{L\ell})^c} \nu_{L\ell'} \right]$$

$$= \frac{1}{2} (f X f)_{\ell\ell'} \left[\overline{(\nu_{L\ell})^c} \nu_{L\ell'} \right]$$

Example : Zee-Babu model



A. Zee, NPB**264**, 99 (1986)
K.S. Babu, PLB**203**, 132 (1988)

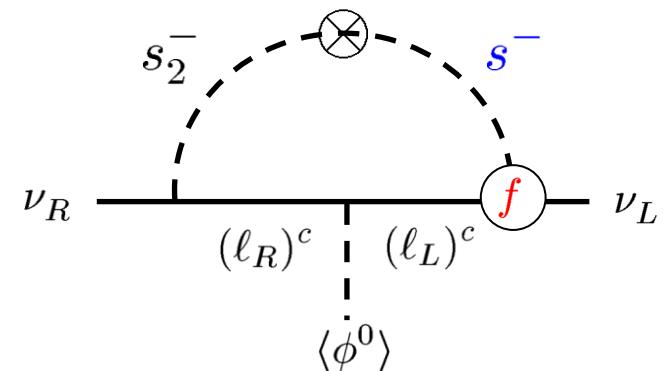


Dirac neutrinos :

$$(m_\nu)_{i\ell} \left[\overline{\nu_{Ri}} \nu_{L\ell} \right]$$

$$= (X f)_{i\ell} \left[\overline{\nu_{Ri}} \nu_{L\ell} \right]$$

Example : 1-loop model



S. Nasri and S. Moussa, MPLA**17**, 771 (2002)
S. Kanemura, T. Nabeshima, HS, PLB**703**, 66 (2011)

$$\text{Det}(m_\nu) \propto \text{Det}(\textcolor{red}{f}) = 0 \longrightarrow m_1 = 0 \text{ or } m_3 = 0$$



$$m_1 = 0$$

$$\text{BR}_e : \text{BR}_\mu : \text{BR}_\tau \simeq 2 : [5 + 2\sqrt{2} s_{13} \cos \delta] : [5 - 2\sqrt{2} s_{13} \cos \delta]$$

$$\text{BR}_\ell \equiv \sum_i \text{BR}(s^- \rightarrow \ell \nu_i) = \frac{1}{3} \sum_{\ell'} |f_{\ell \ell'}|^2$$

$$\frac{f_{e\tau}}{f_{\mu\tau}} = \frac{\tan \theta_{12}}{\cos \theta_{13}} \cos \theta_{23} + \sin \theta_{23} \tan \theta_{13} e^{i\delta} \simeq \frac{1}{2} + \frac{1}{\sqrt{2}} s_{13} e^{i\delta}$$

$$\frac{f_{e\mu}}{f_{\mu\tau}} = \frac{\tan \theta_{12}}{\cos \theta_{13}} \sin \theta_{23} - \cos \theta_{23} \tan \theta_{13} e^{i\delta} \simeq \frac{1}{2} - \frac{1}{\sqrt{2}} s_{13} e^{i\delta}$$

- K.S. Babu and C. Macesanu, PRD**67**, 073010 (2003)
 D. Aristizabal Sierra and M. Hirsch, JHEP**0612**, 052 (2006)
 M. Nebot *et al.*, PRD**77**, 093013 (2008)
 S. Kanemura, T. Nabeshima, HS, PLB**703**, 66 (2011)

$$\text{Det}(m_\nu) \propto \text{Det}(\textcolor{red}{f}) = 0 \longrightarrow m_1 = 0 \text{ or } m_3 = 0$$



$$m_3 = 0$$

$$\text{BR}_e : \text{BR}_\mu : \text{BR}_\tau = 2 : [1 + 0.5 \sin^2 2\theta_{13}] : [1 + 0.5 \sin^2 2\theta_{13}]$$

$$\text{BR}_\ell \equiv \sum_i \text{BR}(s^- \rightarrow \ell \nu_i) = \frac{1}{3} \sum_{\ell'} |f_{\ell \ell'}|^2$$

$$\frac{f_{e\tau}}{f_{e\mu}} = -\tan \theta_{23} \simeq -1$$

$$\frac{f_{\mu\tau}}{f_{e\mu}} = \frac{\tan \theta_{13}}{\cos \theta_{23}} e^{-i\delta} \simeq \sqrt{2} s_{13} e^{-i\delta} \lesssim 0.3 e^{-i\delta}$$

- K.S. Babu and C. Macesanu, PRD**67**, 073010 (2003)
 D. Aristizabal Sierra and M. Hirsch, JHEP**0612**, 052 (2006)
 M. Nebot *et al.*, PRD**77**, 093013 (2008)
 S. Kanemura, T. Nabeshima, HS, PLB**703**, 66 (2011)

Summary of My Hope - 1

SU(2)_L triplet : $h_{\ell\ell'} \left[\overline{L_\ell^c} i\sigma_2 \Delta L_{\ell'} \right]$



Doubly charged : $\text{BR}_{\ell\ell'} \longrightarrow \begin{cases} (m_\nu)_{\ell\ell'} = h_{\ell\ell'} \langle \Delta^0 \rangle \\ \Delta m_{31}^2 > 0 \text{ or not ? } \quad m_0 = 0 \text{ or not ?} \\ \sin \alpha_{21} = 0, \sin \alpha_{31} = 0 \text{ or not ?} \end{cases}$

Singly charged : $\text{BR}_\mu \simeq \text{BR}_\tau, \frac{\text{BR}_e}{\text{BR}_\mu} \longrightarrow \begin{cases} (m_\nu)_{\ell\ell'} = h_{\ell\ell'} \langle \Delta^0 \rangle \\ \text{or } (m_\nu)_{i\ell} = (y_\nu)_{i\ell} \langle \phi_\nu^0 \rangle \\ \Delta m_{31}^2 > 0 \text{ or not ?} \\ m_0 \end{cases}$

SU(2)_L doublet : $(y_\nu)_{i\ell} \left[\overline{\nu_{Ri}} \Phi_\nu i\sigma_2 L_\ell \right]$



Summary of My Hope - 2

singly charged $SU(2)_L$ singlet : $f_{\ell\ell'} \left[\overline{L_\ell^c} i\sigma_2 L_{\ell'} s^+ \right]$



$$\text{BR}_e : \text{BR}_\mu : \text{BR}_\tau \simeq 2 : 5 : 5 \Rightarrow \begin{cases} m_1 = 0 \\ (m_\nu)_{\ell\ell'} \propto (\cancel{f} X \cancel{f})_{\ell\ell'} \quad \text{(Majorana)} \quad \text{or} \quad (m_\nu)_{i\ell'} \propto (X \cancel{f})_{i\ell'} \quad \text{(Dirac)} \end{cases}$$

$$\text{BR}_e : \text{BR}_\mu : \text{BR}_\tau \simeq 2 : 1 : 1 \Rightarrow \begin{cases} m_3 = 0 \end{cases}$$

$$\frac{\text{BR}_\tau}{\text{BR}_\mu - \text{BR}_e} \simeq \frac{17}{15} \Rightarrow \begin{cases} (m_\nu)_{\ell\ell'} \sim m_\tau \left(X_{\ell\tau} \cancel{f}_{\ell'} + (\cancel{f}^T)_{\ell\tau} (X^T)_{\tau\ell'} \right) \\ m_3 = 0, \sin^2 2\theta_{13} \simeq 0.118, \delta \simeq \pi, \alpha_{21} \simeq \pi \end{cases}$$

doubly charged $SU(2)_L$ singlet : $h_{\ell\ell'} \left[(\overline{\ell_R})^c \ell'_R s^{++} \right]$



$\text{BR}_{\mu\mu} : \text{BR}_{\mu\tau} : \text{BR}_{\tau\tau} \sim 1 : 0 : 0 \Rightarrow \text{Zee-Babu model ??}$

$$(m_\nu)_{\ell\ell'} \propto (f m_\ell^{\text{diag}} \cancel{h} m_{\ell'}^{\text{diag}} f)_{\ell\ell'}$$