What is the impact of the observation of θ_{13} on neutrino flavor structure ?

第25回宇宙ニュートリノ研究会
宇宙線研究所
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Morimitsu Tanimoto (Niigata University)

Plan of my talk

1 Introduction

Toward observation for θ_{13}

2 Tri-bimaximal mixing Paradigm

and Discrete Flavor Symmetry

3 Breaking with Tri-bimaximal mixing

4 Large θ_{13} and the neutrino masses

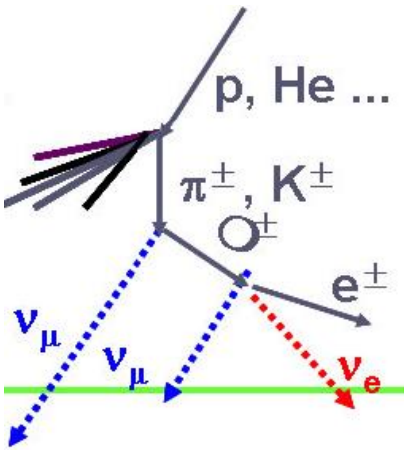
5 Summary

1 Introduction

Discovery of the Lepton Mixing Angles

Evidence of neutrino oscillation in 1998

Atmospheric Neutrinos



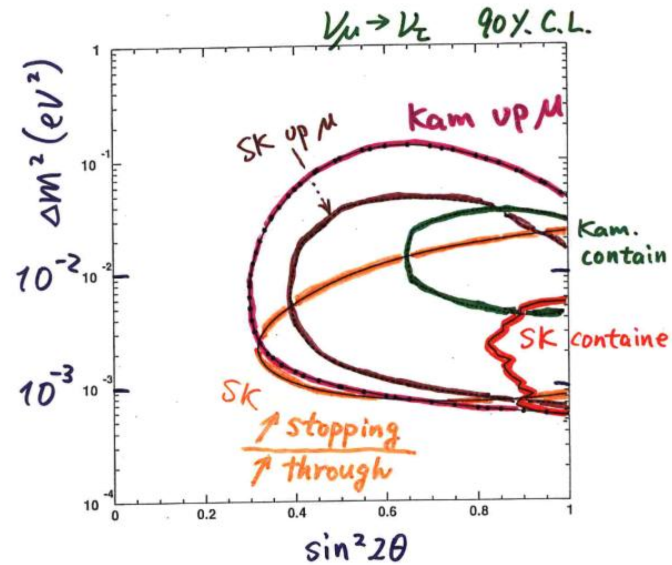
$$R = \frac{(\nu_\mu + \bar{\nu}_\mu)/(\nu_e + \bar{\nu}_e)|_{DATA}}{(\nu_\mu + \bar{\nu}_\mu)/(\nu_e + \bar{\nu}_e)|_{MC}} =$$

$$0.65 \pm 0.05 \pm 0.08$$

Multi-GeV

Summary

Evidence for ν_μ oscillations



Talk at Takayama 1998 by Kajita

• $\begin{cases} \sin^2 2\theta > 0.8 \\ \Delta m^2 \sim 10^{-3} \sim 10^{-2} \end{cases}$

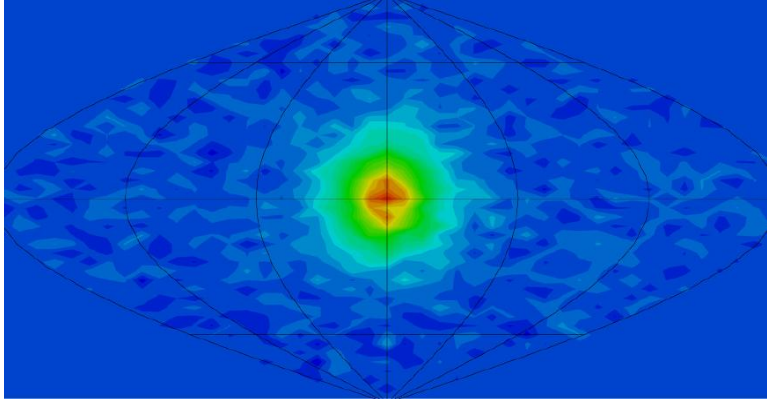
(• $\nu_\mu \to \nu_\tau$ or $\nu_\mu \to \nu_s$?)

Large Mixing Angle, almost maximal mixing !
Theorists could not predict such large one.

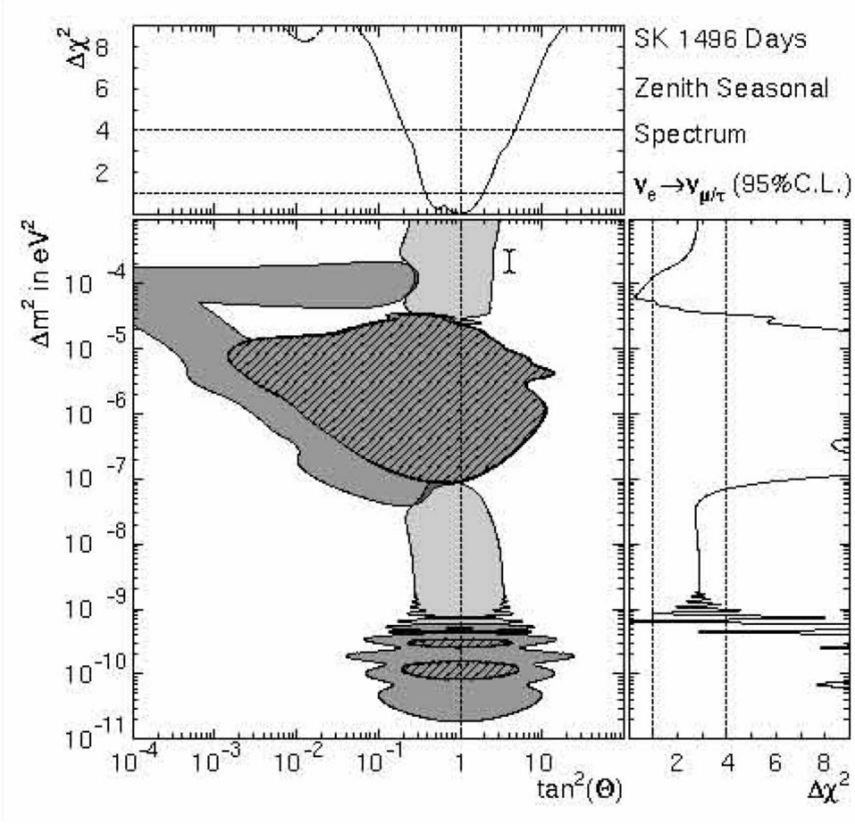
θ_{23}

Solar Neutrinos

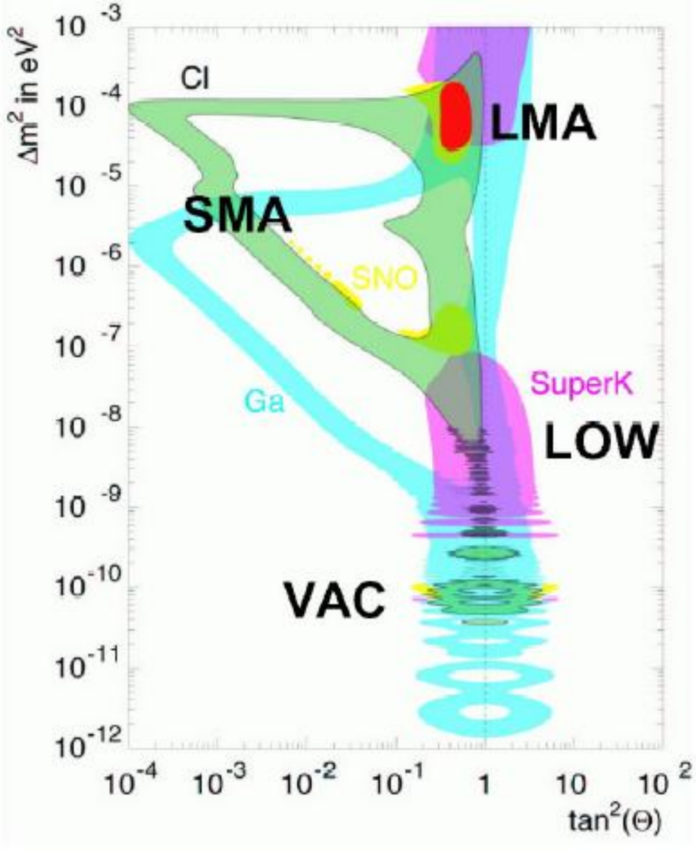
θ_{12}



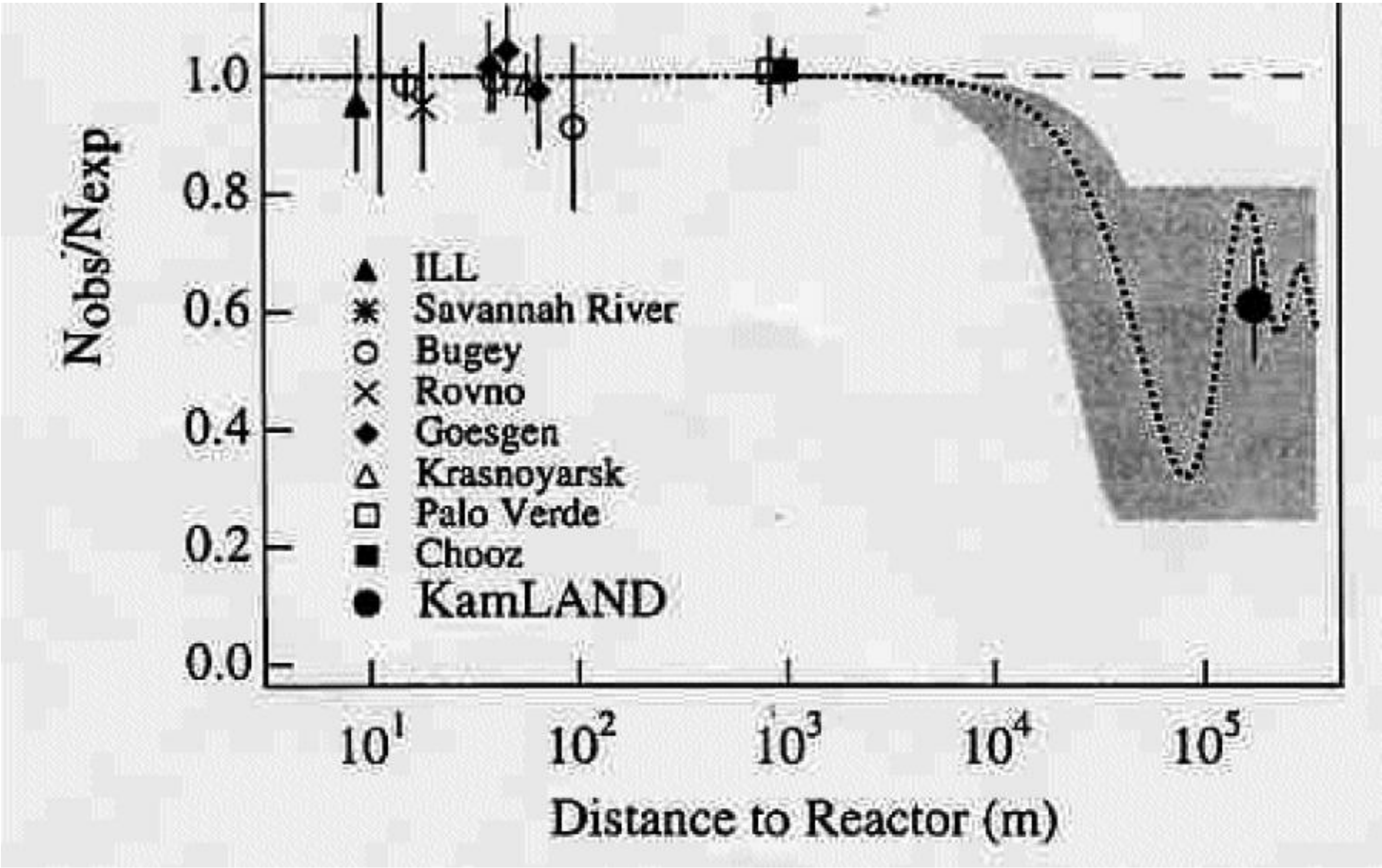
SuperK I



combined



Kamaland 2003 θ_{12}



Evidence for Reactor anti-neutrino disappearence

There are three mixing angles for leptons.

Two large mixing angles $\theta_{23}, \quad \theta_{12}$

How large is the last mixing angle ? θ_{13}

Lepton Mixing Pattern

Bi-maximal mixing
Tri-bi maximal mixing
$\theta_{13} = 0$

However, there is no reason why $\theta_{13}=0$ exactly.

Mission : Observe θ_{13} ! Predict θ_{13} !

Toward observation for θ 13

Neutrino Mixing Matrix
(MNS Matrix)

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$c_{ij} \equiv \cos\theta_{ij},\ s_{ij} \equiv \sin\theta_{ij}$$

Including Majorana Phases

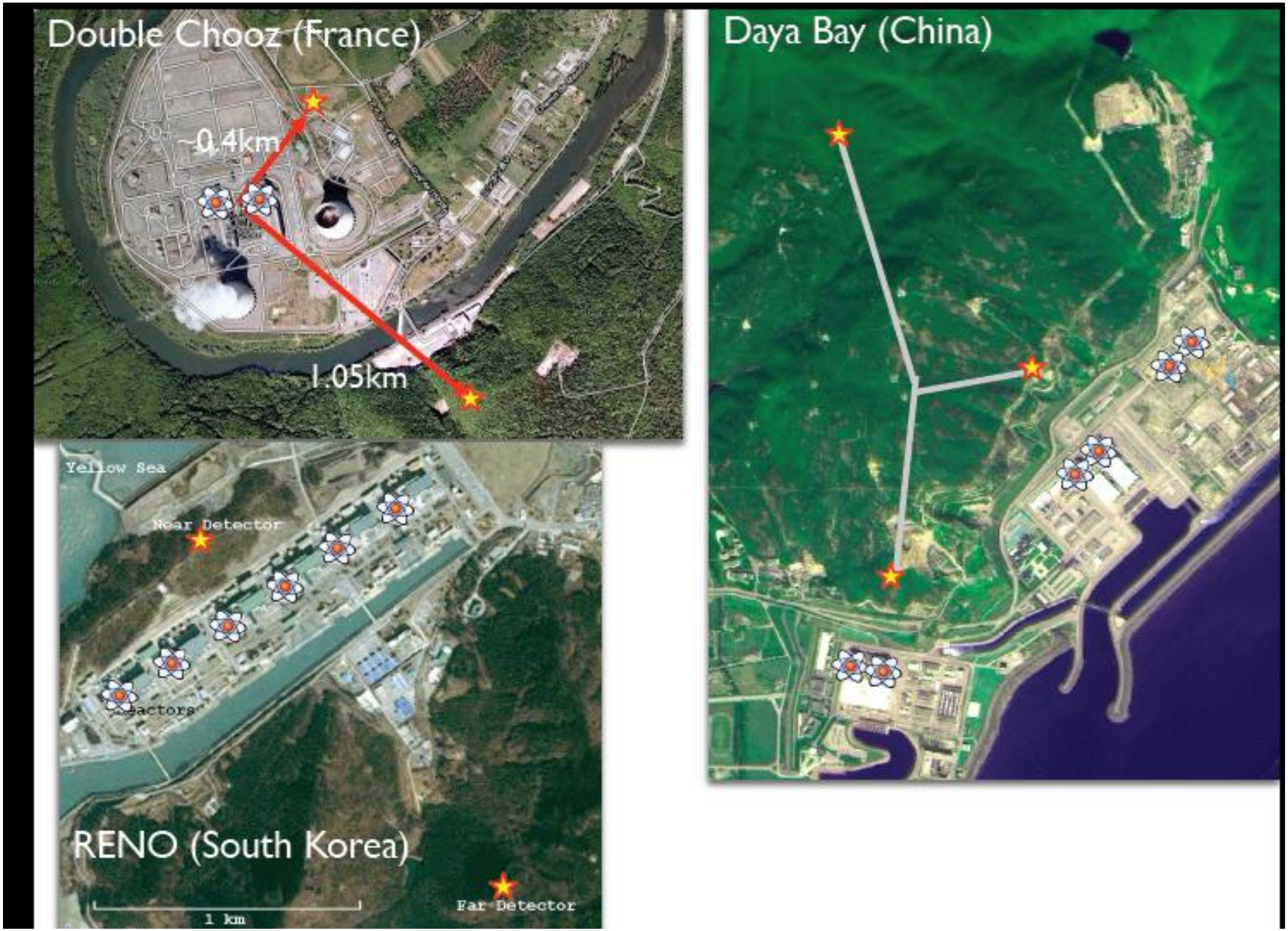
$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_1} & 0 \\ 0 & 0 & e^{i\alpha_2} \end{pmatrix}$$

T2K (Tokai-to-Kamioka) experiment

T2K Main Goals:

★ <u>Discovery of $\nu_\mu \rightarrow \nu_e$ oscillation (ν_e appearance)</u>

★ Precision measurement of ν_μ disappearance


Double Chooz (France)
~0.4km
1.05km
Daya Bay (China)
Yellow Sea
Near Detector
RENO (South Korea)
1 km
Far Detector

Global Fit of Neutrino Parameters

(including T2K, MINOS before Daya Bay)

parameter	best fit $\pm 1\sigma$	2σ	3σ
Δm^2_{21} $[10^{-5}\mathrm{eV}^2]$	$7.59^{+0.20}_{-0.18}$	7.24–7.99	7.09–8.19
Δm^2_{31} $[10^{-3}\mathrm{eV}^2]$	$2.50^{+0.09}_{-0.16}$ $-(2.40^{+0.08}_{-0.09})$	$2.25 - 2.68$ $-(2.23 - 2.58)$	$2.14 - 2.76$ $-(2.13 - 2.67)$
$\sin^2\theta_{12}$	$0.312^{+0.017}_{-0.015}$	0.28–0.35	0.27–0.36
$\sin^2\theta_{23}$	$0.52^{+0.06}_{-0.07}$ 0.52 ± 0.06	0.41–0.61 0.42–0.61	0.39–0.64
$\sin^2\theta_{13}$	$0.013^{+0.007}_{-0.005}$ $0.016^{+0.008}_{-0.006}$	0.004–0.028 0.005–0.031	0.001–0.035 0.001–0.039
δ	$\left(-0.61^{+0.75}_{-0.65}\right)\pi$ $\left(-0.41^{+0.65}_{-0.70}\right)\pi$	$0 - 2\pi$	$0 - 2\pi$

Daya Bay 0.012-0.038

Upper (Lower) : Normal (Inverted) Mass Hierarchy

Schwetz, Tortola, Valle, New J, Phys.13:109401, 2011

2 Tri-bimaximal mixing Paradigm and Discrete Flavor Symmetry
(Before T2K, DChooz and Daya Bay)

Neutrino Data suggested
Tri-bimaximal Mixing of Neutrinos

$$\sin^2\theta_{12} = 0.312^{+0.017}_{-0.015}\,, \quad \sin^2\theta_{23} = 0.52^{+0.06}_{-0.07}$$

$$\sin^2\theta_{12} = 1/3,\ \sin^2\theta_{23} = 1/2,\ \sin^2\theta_{13} = 0,$$

$$U_{\text{tri-bimaximal}} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}$$

Harrison, Perkins, Scott (2002)

Consider the structure of Neutrino Mass Matrix, which gives Tri-bi maximal mixing

$$M_\nu^{\rm exp} \simeq V^*_{\rm tri-bi}\begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix} V^\dagger_{\rm tri-bi}$$

$$= \boxed{\frac{m_1+m_3}{2}\begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} + \frac{m_2-m_1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{m_1-m_3}{2}\begin{pmatrix} 1 & & \\ & & 1 \\ & 1 & \end{pmatrix}}$$

- integer (inter-family related) matrix elements

  $\Longleftrightarrow$ non-abelian discrete flavor sym

Mixing angles are independent of mass eigenvalues

$$\left(\theta_{ij} \cancel{\sim} \sqrt{\frac{m_i}{m_j}}\right)$$

Different from quark mixing angles

For example, consider A_4 group

Four irreducible representations in A_4 symmetry

1 1' 1" 3

Consider A_4 triplet, 3 $(l_e, \ l_\mu, \ l_\tau)_L$

Tensor Product of $\mathbf{3}$ (a_1, a_2, a_3) and $\mathbf{3}$ (b_1, b_2, b_3)

$$\mathbf{3} \times \mathbf{3} \Rightarrow \mathbf{1} = a_1 * b_1 + a_2 * b_3 + a_3 * b_2$$

$l\ l\ h\ h/\Lambda$ $(3 \times 3 \times 1 \times 1)$ gives

$$(le\ le + l_\mu l_\tau + l_\mu l_\tau)\ v^2/\Lambda$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

A_4 symmetric

$$M_\nu^{\exp} = \frac{m_1+m_3}{2}\begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} + \frac{m_2-m_1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{m_1-m_3}{2}\begin{pmatrix} 1 & & \\ & & 1 \\ & 1 & \end{pmatrix}$$

The third matrix is A_4 symmetric !

The first and second matrices are well known to be of S_3 symmetric.

In order to get the first and second matrix in A_4 model, non-trivial flavons are required.

Typical Non-Abelian Discrete Groups

S_3 S_4 A_4 T' D_4 $\Delta(27)$ $\Delta(54)$ ……

○ : includes both 2 and 3

Singlet 1 1' 1" …..

Doublet 2 ….. **2 families**

Triplets 3 …. **3 families**

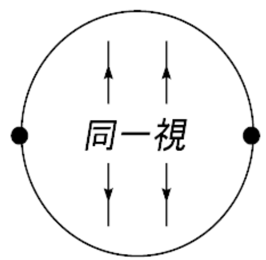
Remark: S_3 and A_4 are sub-groups of S_4 .

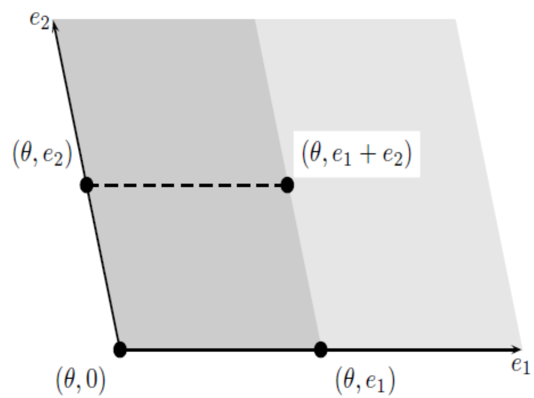
H. Ishimori, T. Kobayashi, H. Ohki, Y. Shimizu, H. Okada, M. Tanimoto, Prog. Theor. Phys. Suppl. **183** (2010) 1-163. [arXiv:1003.3552 [hep-th]].

Text book to be published at Springer for Physicist (2012)

高次元をオービフォールド化したとき離散対称性が現れる。

(A. Adulpravitchai, A. Blum, M. Lindner, JHEP 0907: 053, 2009)





オービフォールド	格子の成す角	フレーバー対称性
$\mathbf{S}^1/\mathbf{Z}_2$		S_2
$\mathbf{T}^2/\mathbf{Z}_2$	60°	S_4 A_4(proper Lorenz tr.)
	90°	D_4
	上記以外	$Z_2 \times Z_2$

Stringy origin of non-Abelian discrete flavor symmetries

T. Kobayashi, H. Niles, F. PloegerS, S. Raby, M. Ratz, hep-ph/0611020

Non-Abelian Discrete Flavor Symmetries from Magnetized/Intersecting Brane Models

H. Abe, K-S. Choi, T. Kobayashi, H. Ohki, 0904.2631

Remark:

Tri-bimaximal Mixing realized in

$$M_{\rm TBM} = \frac{m_1+m_3}{2}\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} + \frac{m_2-m_1}{3}\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix} + \frac{m_1-m_3}{2}\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}$$

Additional Matrices break Tri-bimaximal mixing

$$\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix}, \quad \begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix}$$

$$\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix} = \begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix} - \begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix} - \begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}$$

which could appear in A_4, S_4, $\Delta(27)$ flavor symmetries.

Famous Model of Tri-bimaximal Mixing
A_4 Symmetry

Tetrahedral Symmetry

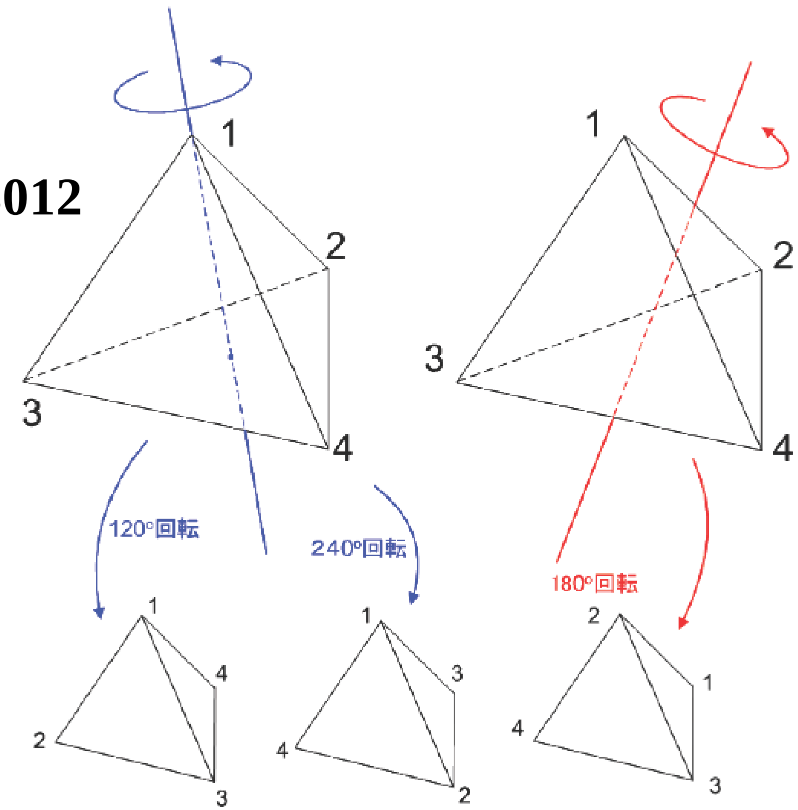
Four irreducible representations in A_4 symmetry

1 1' 1'' 3

E. Ma and G. Rajasekaran, PRD64(2001)113012

the even permutation of 4 objects

class	n	h	χ_1	$\chi_{1'}$	$\chi_{1''}$	χ_3
C_1	1	1	1	1	1	3
C_2	4	3	1	ω	ω^2	0
C_3	4	3	1	ω^2	ω	0
C_4	3	2	1	1	1	-1



At first

let us understand how to get the tri-bimaximal mixing in the example of A_4 flavor model.

G. Altarelli, F. Feruglio, Nucl.Phys. B720 (2005) 64

$A_4 \times Z_3$ charge assignment

A_4 Flavor model

	(L_e, L_μ, L_τ)	R_e^c	R_μ^c	R_τ^c	$H_{u,d}$	χ_ℓ	χ_ν	χ
A_4	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{1}''$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$
Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω

χ_ℓ, χ_ν, χ are new scalars of gauge singlets.

$\omega^3 = 1$

A_4 invariant superpotential can be written by:

for charged leptons

$1' \times 1'' \rightarrow 1$

$$W_L = \frac{y_e}{\Lambda}(L_e \chi_{\ell_1} + L_\mu \chi_{\ell_3} + L_\tau \chi_{\ell_2}) R_e H_d \qquad \mathbf{3_L} \times \mathbf{3_{flavon}} \rightarrow \mathbf{1}$$
$$+ \frac{y_\mu}{\Lambda}(L_e \chi_{\ell_2} + L_\mu \chi_{\ell_1} + L_\tau \chi_{\ell_3}) R_\mu H_d \qquad \mathbf{3_L} \times \mathbf{3_{flavon}} \rightarrow \mathbf{1''}$$
$$+ \frac{y_\tau}{\Lambda}(L_e \chi_{\ell_3} + L_\mu \chi_{\ell_2} + L_\tau \chi_{\ell_1}) R_\tau H_d + h.c., \qquad \mathbf{3_L} \times \mathbf{3_{flavon}} \rightarrow \mathbf{1'}$$

for neutrinos

$$W_\nu = \frac{y_1}{\Lambda^2}(L_e L_e + L_\mu L_\tau + L_\tau L_\mu) H_u H_u \chi \qquad \mathbf{3_L} \times \mathbf{3_L} \times \mathbf{1_{flavon}} \rightarrow \mathbf{1}$$
$$+ \frac{y_2}{3\Lambda^2}[(2L_e L_e - L_\mu L_\tau - L_\tau L_\mu)\chi_{\nu_1}$$
$$+(-L_e L_\tau + 2L_\mu L_\mu - L_\tau L_e)\chi_{\nu_2} \qquad \mathbf{3_L} \times \mathbf{3_L} \times \mathbf{3_{flavon}} \rightarrow \mathbf{1}$$
$$+(-L_e L_\mu - L_\mu L_e + 2L_\tau L_\tau)\chi_{\nu_3}] H_u H_u + h.c.,$$

$$\mathbf{3} \times \mathbf{3} \Rightarrow \mathbf{1} = a_1 * b_1 + a_2 * b_3 + a_3 * b_2$$

After $A_4 \times Z_3$ symmetry is spontaneously broken by VEVs of χ_ℓ, χ_ν, and χ, mass matrices are obtained as

$$M_l = \frac{v_d}{\Lambda}\begin{pmatrix} y_e \langle \chi_{\ell_1} \rangle & y_e \langle \chi_{\ell_3} \rangle & y_e \langle \chi_{\ell_2} \rangle \\ y_\mu \langle \chi_{\ell_2} \rangle & y_\mu \langle \chi_{\ell_1} \rangle & y_\mu \langle \chi_{\ell_3} \rangle \\ y_\tau \langle \chi_{\ell_3} \rangle & y_\tau \langle \chi_{\ell_2} \rangle & y_\tau \langle \chi_{\ell_1} \rangle \end{pmatrix}$$

$$M_\nu = \frac{v_u^2}{3\Lambda}\begin{pmatrix} 3y_1 \langle \chi \rangle + 2y_2 \langle \chi_{\nu_1} \rangle & -y_2 \langle \chi_{\nu_3} \rangle & -y_2 \langle \chi_{\nu_2} \rangle \\ -y_2 \langle \chi_{\nu_3} \rangle & 2y_2 \langle \chi_{\nu_2} \rangle & 3y_1 \langle \chi \rangle - y_2 \langle \chi_{\nu_1} \rangle \\ -y_2 \langle \chi_{\nu_2} \rangle & 3y_1 \langle \chi \rangle - y_2 \langle \chi_{\nu_1} \rangle & 2y_2 \langle \chi_{\nu_3} \rangle \end{pmatrix}$$

where $v_d = \langle H_d \rangle$, $v_u = \langle H_u \rangle$.

The mass matrices do not yet predict tri-bimaximal mixing !

Can one get Desired Vacuum
in Spontaneous Symmetry Breaking ?

We need Scalar Potential Analysis.

If vacuum expectation values are aligned,

$\langle \chi_\ell \rangle = (V_\ell, 0, 0)$ and $\langle \chi_\nu \rangle = (V_\nu, V_\nu, V_\nu)$,

which are obtained by potential analysis, then

$$M_l = \frac{v_d v_T}{\Lambda} \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$

$$M_\nu = \frac{v_u^2}{\Lambda} \begin{pmatrix} a + 2b/3 & -b/3 & -b/3 \\ -b/3 & 2b/3 & a - b/3 \\ -b/3 & a - b/3 & 2b/3 \end{pmatrix} .$$

where $a = y_1 V/\Lambda$, $b = y_2 V_\nu/\Lambda$.

$\mathbf{3}_L \times \mathbf{3}_L \times \mathbf{3}_{flavon}$ $\qquad$ $\mathbf{3}_L \times \mathbf{3}_L \times \mathbf{1}_{flavon}$

$$M_\nu = \frac{v_u^2 b}{\Lambda} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \frac{v_u^2 b}{3\Lambda} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{v_u^2 a}{\Lambda} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Therefore, mixing matrix is tri-bimaximal matrix, and masses are

$$m_1 = \frac{v_u^2 (a+b)}{\Lambda}, \quad m_2 = \frac{v_u^2 a}{\Lambda}, \quad m_3 = -\frac{v_u^2 (a-b)}{\Lambda}.$$

In order to get Tri-bimaximal Mixing, one needs

◎ **Non-trivial Flavons 3**

◎ **Additional U(1) or Zn**

◎ **VEV Alignment (1,0,0), (1,1,1) ……**

3 Breaking with tri-bimaximal mixing

T2K, DChooz, Daya Bay suggest us

the breaking with tri-bimaximal paradigm !

Let us show
how to go beyond the tri-bi maximal mixing.

Consider Modified A_4 Model to get non-zero θ_{13}

	(l_e, l_μ, l_τ)	e^c	μ^c	τ^c	$h_{u,d}$	ϕ_l	ϕ_ν	ξ	ξ'
$SU(2)$	2	1	1	1	2	1	1	1	1
A_4	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1''}$	$\mathbf{1'}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1'}$
Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω	ω

$$\mathbf{3} \times \mathbf{3} = \mathbf{3} + \mathbf{3} + \mathbf{1} + \mathbf{1'} + \mathbf{1''}$$

$$\mathbf{3} \times \mathbf{3} \Rightarrow \mathbf{1} \quad = a_1 * b_1 + a_2 * b_3 + a_3 * b_2$$

$$\mathbf{3} \times \mathbf{3} \Rightarrow \mathbf{1'} \quad = a_1 * b_2 + a_2 * b_1 + a_3 * b_3$$

$$\mathbf{3} \times \mathbf{3} \Rightarrow \mathbf{1''} \quad = a_1 * b_3 + a_2 * b_2 + a_3 * b_1$$

ξ $\qquad$ ξ'

$$\mathbf{1} \times \mathbf{1} \Rightarrow \mathbf{1} \quad , \qquad \mathbf{1''} \times \mathbf{1'} \Rightarrow \mathbf{1}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

1' × 1''

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Y. Simizu, M. Tanimoto, A. Watanabe, PTP **126**, 81(2011)

$$M_\nu = a\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} + b\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix} + c\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix} + d\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix}$$

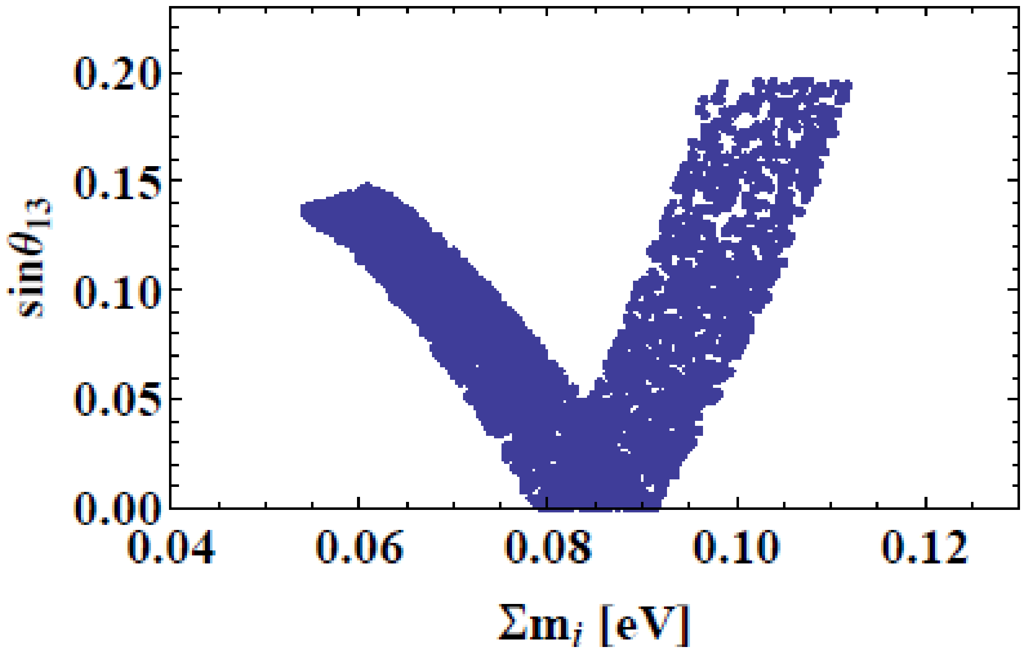
$$a = \frac{y^\nu_{\phi_\nu}\alpha_\nu v_u^2}{\Lambda}, \qquad b = -\frac{y^\nu_{\phi_\nu}\alpha_\nu v_u^2}{3\Lambda}, \qquad c = \frac{y^\nu_\xi \alpha_\xi v_u^2}{\Lambda}, \qquad d = \frac{y^\nu_{\xi'}\alpha_{\xi'} v_u^2}{\Lambda}$$

There is one relation $a = -3b$

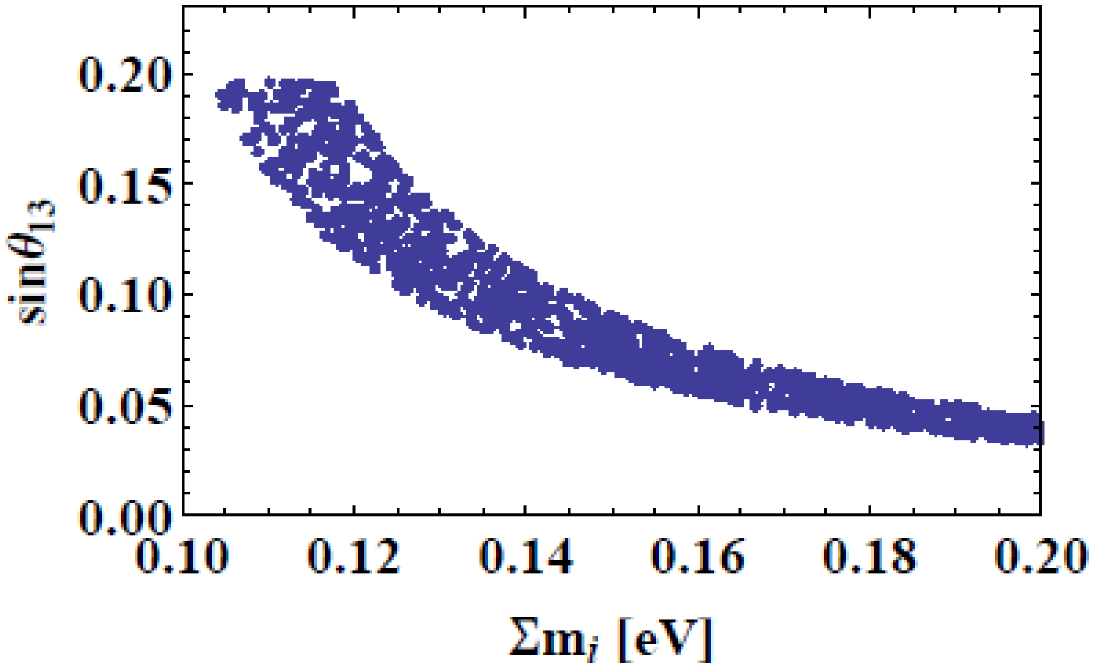
$$m_1 = a + \sqrt{c^2 + d^2 - cd}$$
$$m_2 = c + d$$
$$m_3 = -a + \sqrt{c^2 + d^2 - cd}$$

$$\Delta m^2_{\rm atm} \simeq 4(c^2 + d^2 - cd), \quad \Delta m^2_{\rm sol} \simeq (c+d)^2$$

Normal



Inverted



4 Large θ$_{13}$ and the neutrino masses

A_4 model could give non-zero θ$_{13}$
by adding a new flavon A4 , 1’ and/or 1” .
However, mass dependence of θ$_{13}$ is not clear.

Daya Bay results (3σ)

$$\sin\theta_{13} = 0.153 \pm 0.014$$

Order of Cabbibo angle !?

What does this value of θ$_{13}$ indicate ?

$$\text{Recall} \quad \sin\theta_C \simeq \sqrt{\frac{m_d}{m_s}} \simeq 0.2$$

Θ_{13} could be related to neutrino masses clearly!

Ratio of Neutrino Mass Squared differences

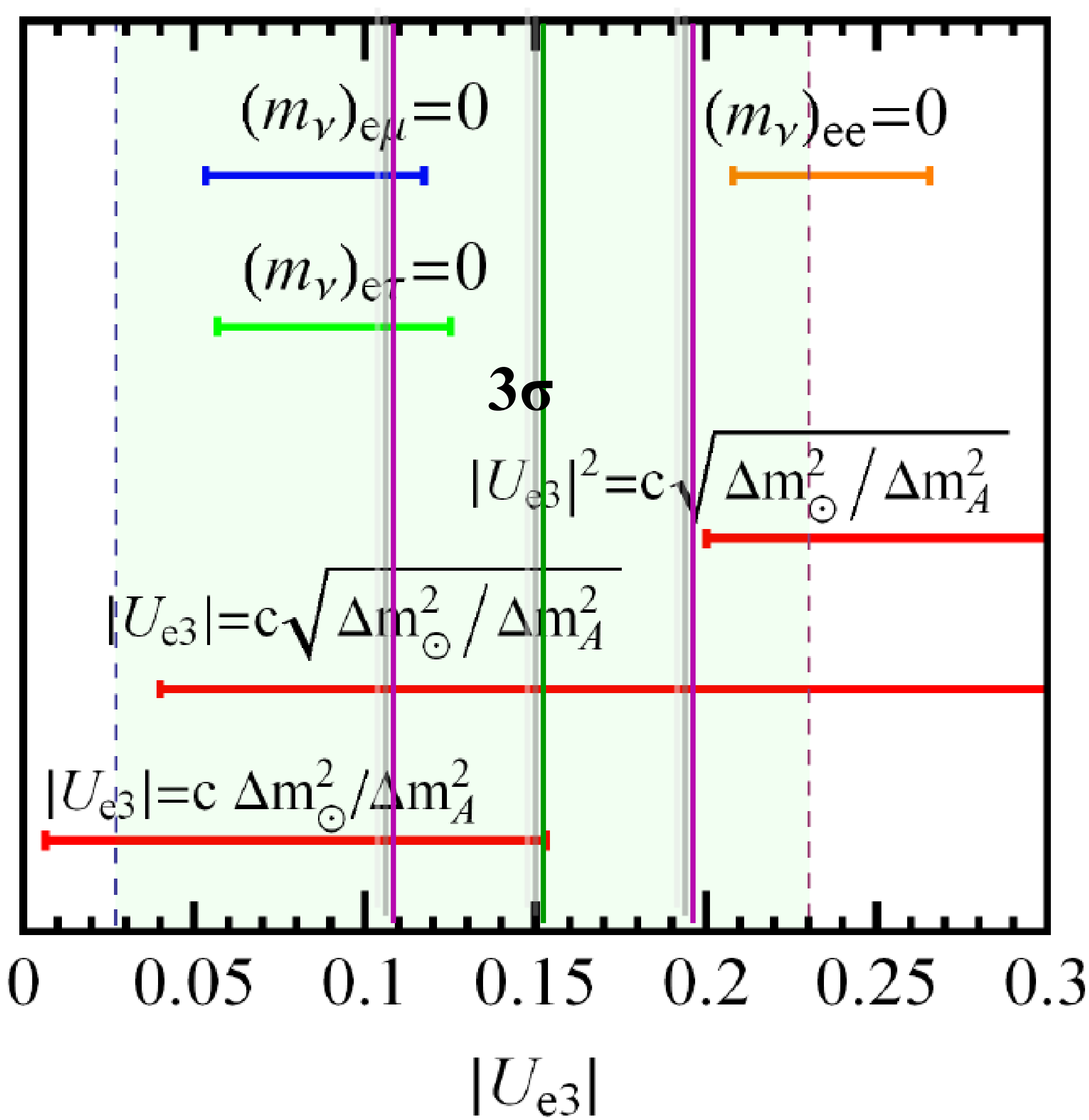
$$\frac{\Delta m^2_{21}}{\Delta m^2_{31}} = 0.026 - 0.038 \sim \mathcal{O}(\lambda^2)$$

$$\sqrt{\frac{\Delta m^2_{21}}{\Delta m^2_{31}}} = 0.160 - 0.196 \sim \mathcal{O}(\lambda)$$

$$\sqrt[4]{\frac{\Delta m^2_{21}}{\Delta m^2_{31}}} = 0.40 - 0.44 \sim \mathcal{O}(\sqrt{\lambda})$$

$\lambda=0.2$

W.Rodejohann, M.Tanimoto, A.Watanabe, arXiv:1201.4936



where c is varied between 0.25 and 4.

The relation of masses and mixing angles is given by the texture zeros !

$$m_\nu = U\,\mathrm{diag}(m_1, m_2, m_3)\,U^T$$

$$(m_\nu)_{\alpha\beta} = \sum_i U_{\alpha i} m_i U_{\beta i}$$

Well Known Simple approach:
Texture one zero or two zeros!

Example

$$\begin{pmatrix} 0 & x & x \\ x & x & x \\ x & x & x \end{pmatrix}$$

Suppose normal mass hierachy and neglect smallest m_1, then we have

$$U_{\alpha 2} U_{\beta 2} m_2 + U_{\alpha 3} U_{\beta 3} m_3 = 0$$

$$\frac{|U_{\alpha 3} U_{\beta 3}|}{|U_{\alpha 2} U_{\beta 2}|} = \frac{m_2}{m_3} = \sqrt{\frac{\Delta m_{\odot}^2}{\Delta m_{\mathrm{A}}^2}}$$

$$\arg(U_{\alpha 3} U_{\beta 3} U_{\alpha 2}^* U_{\beta 2}^*) = \pi$$

m_1=0

In the case of $\alpha = \beta = e\,, \quad (m_\nu)_{ee} = 0$

$$\tan^2\theta_{13} = \sqrt{\frac{\Delta m^2_\odot}{\Delta m^2_{\rm A}}}\,\sin^2\theta_{12} \qquad \alpha_1 - \alpha_2 \simeq -\frac{1}{2}\delta$$

U_{e3} is given by the fourth root !!

In the case of $\alpha = e, \quad \beta = \mu, \quad (m_\nu)_{e\mu} = 0$

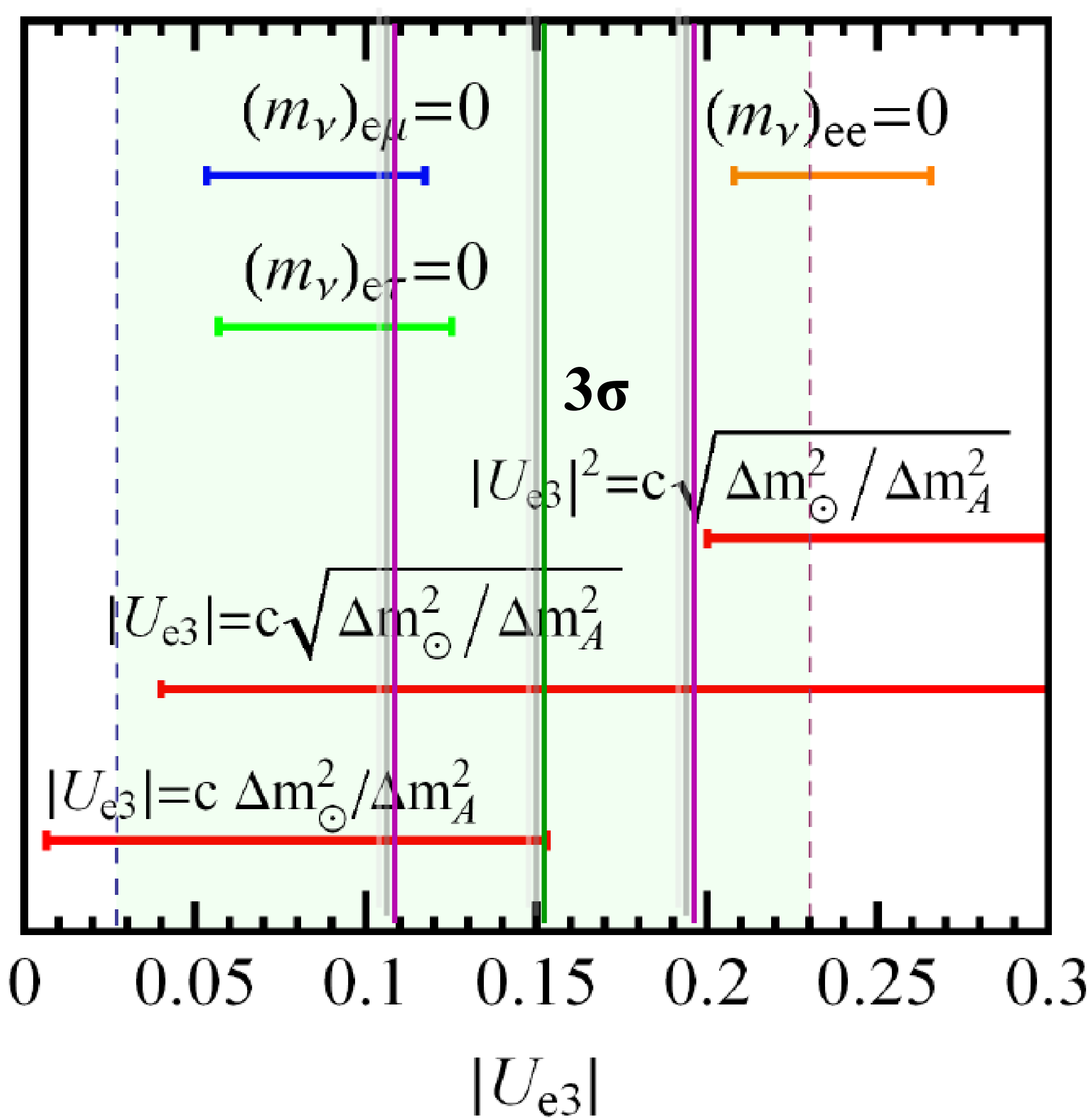
$$|U_{e3}| \simeq \frac{1}{2}\sqrt{\frac{\Delta m^2_\odot}{\Delta m^2_{\rm A}}}\,\sin 2\theta_{12}\cot\theta_{23} \qquad \alpha_1 - \alpha_2 \simeq \frac{1}{2}(\pi - \delta)$$

U_{e3} is given by the square root !!

In the case of $\alpha = e, \quad \beta = \tau, \quad (m_\nu)_{e\tau} = 0, \quad \cot\theta_{23} \to \tan\theta_{23}$

$\mu\mu$, $\mu\tau$ and $\tau\tau$ elements cannot vanish.

W.Rodejohann, M.Tanimoto, A.Watanabe, arXiv:1201.4936



$m_1=0$ limit

where c is varied between 0.25 and 4.

Neutrino Masses

Model S_3

$$m_D \simeq \begin{pmatrix} a & 0 \\ b & a \\ 0 & c \end{pmatrix}, \quad M_R \simeq \begin{pmatrix} M_A & M_B \\ M_B & 0 \end{pmatrix}$$

$$m_\nu \simeq \begin{pmatrix} 0 & a^2 & ac \\ a^2 & 2ab & bc \\ ac & bc & 0 \end{pmatrix} \frac{1}{M_B} - \begin{pmatrix} 0 & 0 & 0 \\ 0 & a^2 & ac \\ 0 & ac & c^2 \end{pmatrix} \frac{M_A}{M_B^2}$$

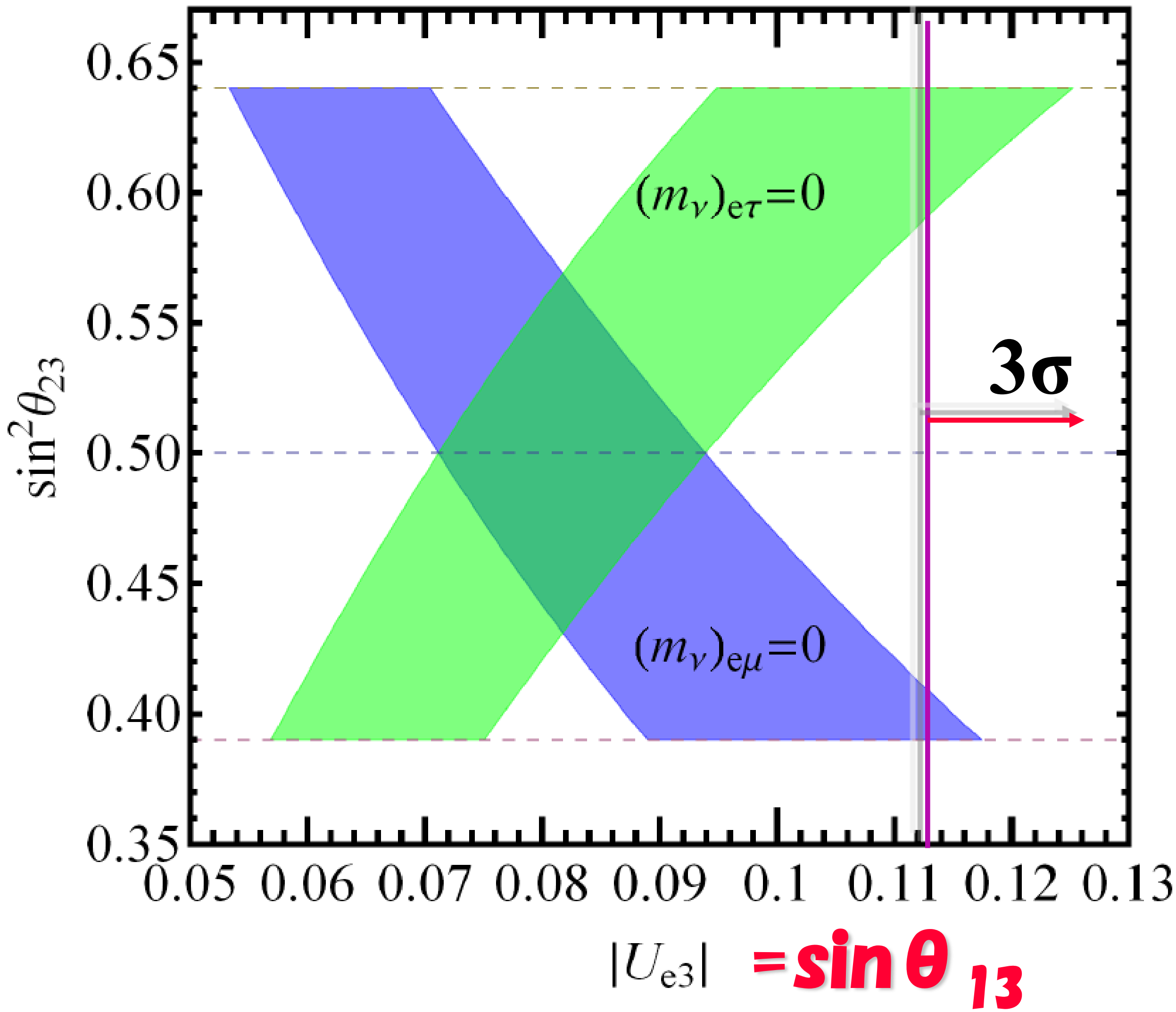
$(m_\nu)_{ee}=0$!!

Charged Lepton Masses

$$-\mathcal{L}_l = y_e(\overline{L_1}\phi_1 + \overline{L_2}\phi_2)e_R H, + y_\mu(\overline{L_1}\phi_2^* + \overline{L_2}\phi_1^*)\mu_R H + y_\tau \overline{L_3}\tau_R H + \text{h.c.}$$

$$M_l \simeq \begin{pmatrix} y_e\langle\phi_1\rangle & 0 & 0 \\ 0 & y_\mu\langle\phi_1\rangle^* & 0 \\ 0 & 0 & y_\tau \end{pmatrix} v$$

Deviation from the maximal θ_{23}



Suppose inverted mass hierachy and neglect smallest m_3 , then we have

$$\alpha = e, \quad \beta = \mu, \quad (m_\nu)_{e\mu} = 0$$

$$|U_{e3}| \cos\delta \simeq \frac{1}{4} \frac{\Delta m_\odot^2}{\Delta m_\mathrm{A}^2} \frac{\sin 2\theta_{12}}{\tan\theta_{23}}$$

$$\alpha = \mu, \quad \beta = \tau, \quad (m_\nu)_{\mu\tau} = 0$$

$$|U_{e3}| \cos\delta \simeq \frac{1}{4} \frac{\Delta m_\odot^2}{\Delta m_\mathrm{A}^2} \frac{\cos 2\theta_{12} \tan\theta_{12}}{\cos^2\theta_{12} \tan\theta_{23}}$$

Simple Model to get Large θ_{13}

Fukugita, Tanimoto, Yanagida,
PLB 562(2003) 273 [arXiv:hep-ph/0303177].

$$m_E = \begin{pmatrix} 0 & A_\ell & 0 \\ A_\ell & 0 & B_\ell \\ 0 & B_\ell & C_\ell \end{pmatrix}, \qquad m_{\nu D} = \begin{pmatrix} 0 & A_\nu & 0 \\ A_\nu & 0 & B_\nu \\ 0 & B_\nu & C_\nu \end{pmatrix}$$

$$M_R = M_0 \mathbf{I}$$

$$m_i = \left(m_{\nu D}^T M_R^{-1} m_{\nu D} \right)_i$$

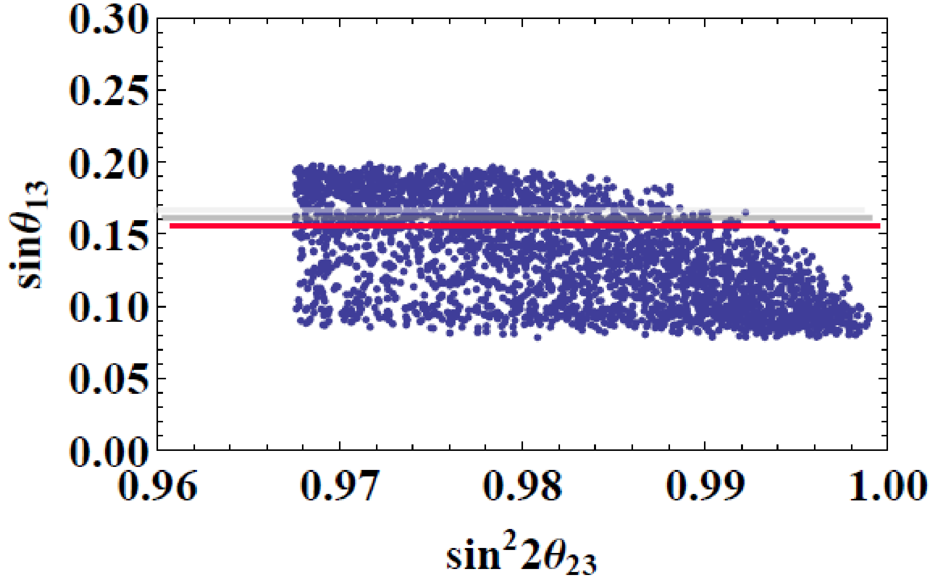
$$Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & e^{i\tau} \end{pmatrix}$$

$$U = U_\ell^\dagger \, Q \, U_\nu$$

Fritzsch texture failed in Quark sector!

Lepton Mixing Matrix

Before Daya Bay



$$U_{e2} \simeq -\left(\frac{m_1}{m_2}\right)^{1/4} + \left(\frac{m_e}{m_\mu}\right)^{1/2} e^{i\sigma}$$

$$U_{\mu 1} \simeq \left(\frac{m_1}{m_2}\right)^{1/4} e^{i\sigma} - \left(\frac{m_e}{m_\mu}\right)^{1/2}$$

$$U_{\mu 3} \simeq \left(\frac{m_2}{m_3}\right)^{1/4} e^{i\sigma} - \left(\frac{m_\mu}{m_\tau}\right)^{1/2} e^{i\tau}$$

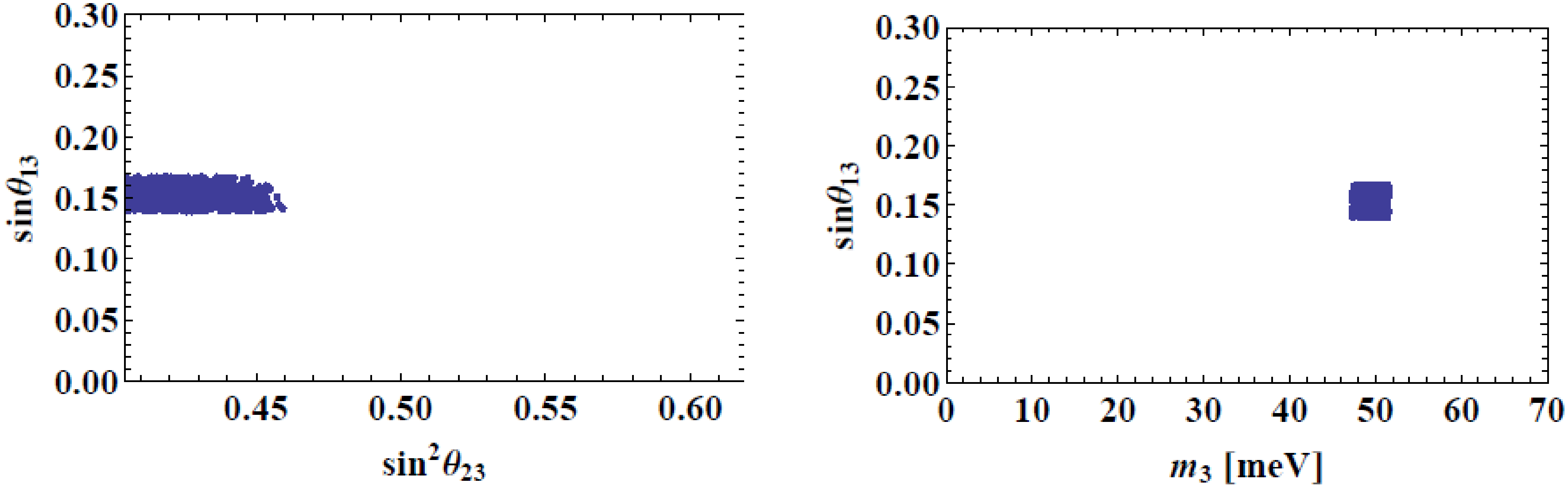
$$U_{\tau 2} \simeq -\left(\frac{m_2}{m_3}\right)^{1/4} e^{i\tau} + \left(\frac{m_\mu}{m_\tau}\right)^{1/2} e^{i\sigma}$$

$$U_{e3} \simeq \left(\frac{m_e}{m_\mu}\right)^{1/2} U_{\mu 3} + \left(\frac{m_2}{m_3}\right)^{1/2} \left(\frac{m_1}{m_3}\right)^{1/4}$$

$$U_{\tau 1} \simeq \left(\frac{m_1}{m_2}\right)^{1/4} U_{\tau 2}$$

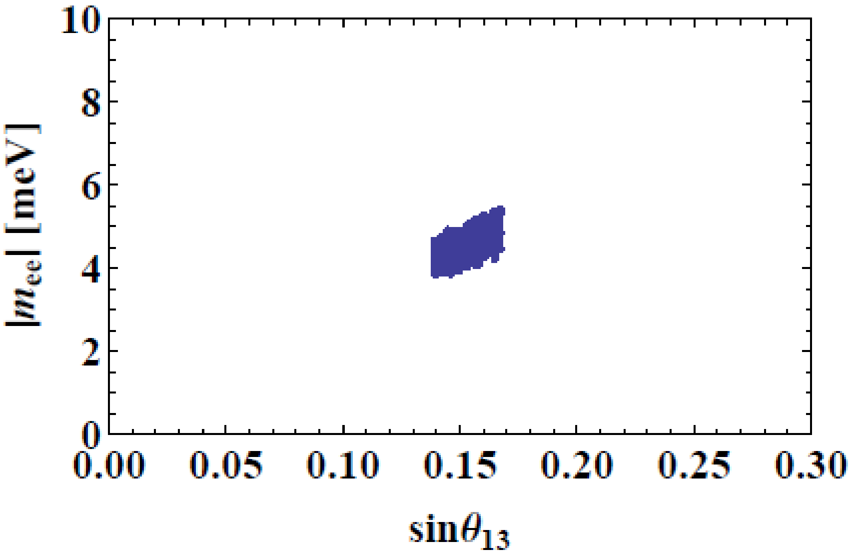
$$|U_{e3}| \simeq |U_{\mu 3}|^3 |U_{e2}| \simeq 0.2$$

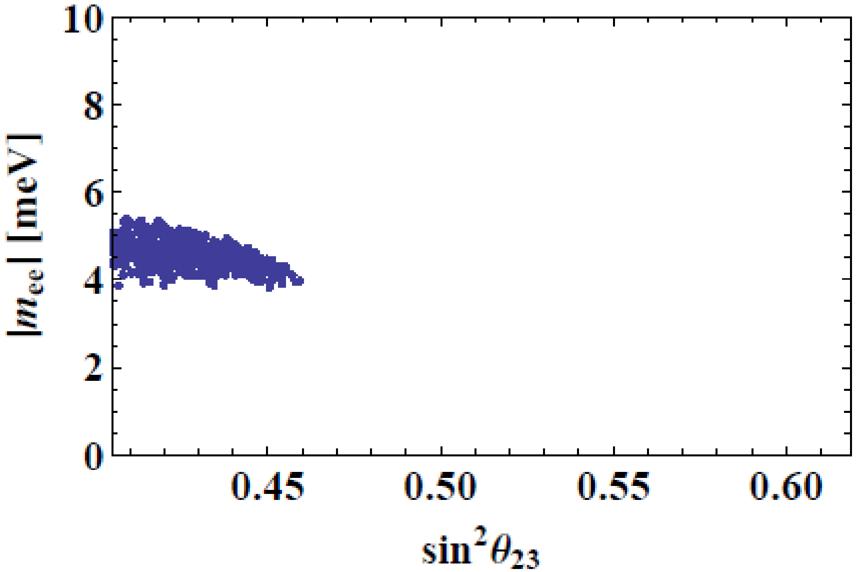
$\sin\theta_{13}$: input
maximal θ_{23} is excluded
Normal Mass Hierarchy



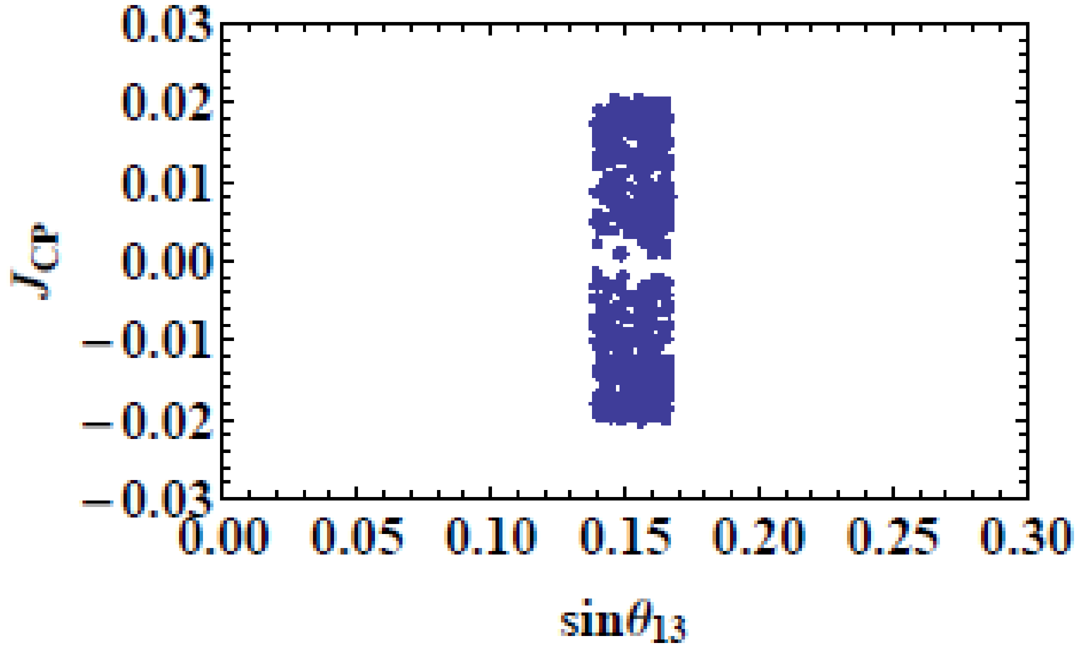
Fukugita, Shimizu, Tanimoto, Yanagida (2012)

Neutrinoless Double Beta Decay



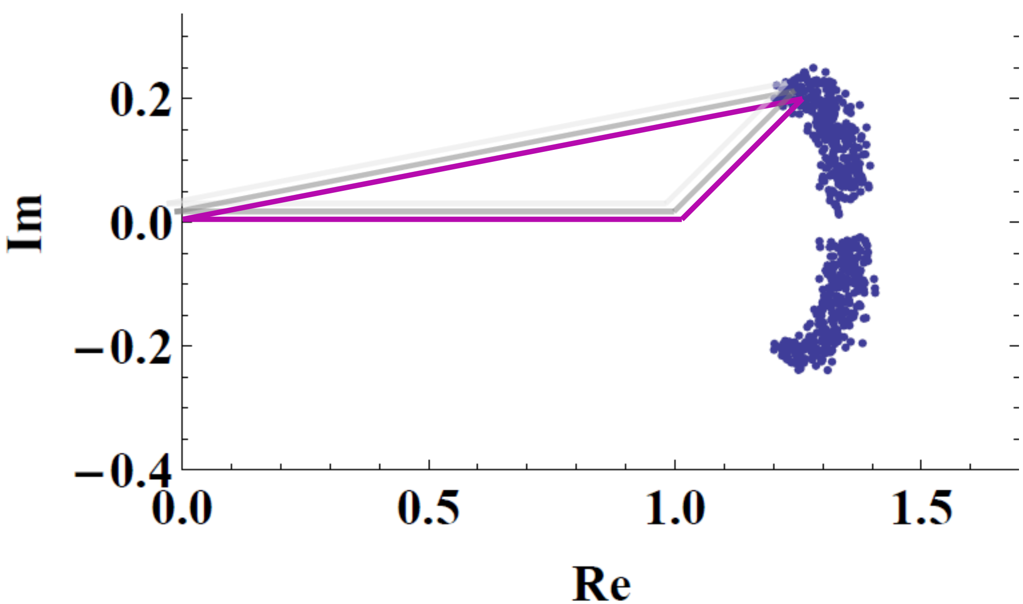


4 - 5.5 meV



CP Violation

Non-Zero Jcp !



Unitarity Triangle

$$U_{e1}U_{\mu 1}^{*}+U_{e2}U_{\mu 2}^{*}+U_{e3}U_{\mu 3}^{*}=0$$

5 Summary

Large $\theta_{13} \fallingdotseq 0.15$ suggests

☆**breaking with tri-bimaximal mixing.**

θ_{23} and θ_{12} may be independent neutrino masses.

θ_{13} probably depends on neutrino masses !

More Precise determination of θ_{13} is required in order to clarify the mass dependence.

Specific textures will be tested clearly.