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非一様物質下での
ニュートリノ振動におけるパラメータ励振

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Plan

1. Introduction
2. Formulation
3. Parametric Resonance in Oscillation Probability
4. Summary & Outlook

Introduction

1

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Neutrino Oscillation

- Discovery, establishment of the oscillation
  - Solar — Homestake, Kamiokande...; Super-K, SNO, ...
  - Atmospheric — Kamiokande, ...; Super-K, ...
  - Reactor — Chooz, KamLAND,...; Double Chooz,...
  - Accelerator — K2K, MINOS; T2K,...

$$\delta m^2_{\rm atm} = 2.39\left(1^{+0.013}_{-0.084}\right) \times 10^{-3}{\rm eV}^2 \quad (2\sigma)$$

$$\delta m^2_{\rm sol} = 7.67\left(1^{+0.044}_{-0.047}\right) \times 10^{-5}{\rm eV}^2 \quad (2\sigma)$$

$$\delta_{\rm CP}$$

$$\sin^2\theta_{12} = 0.312\left(1^{+0.128}_{-0.109}\right) \quad (2\sigma)$$

$$\sin^2\theta_{23} = 0.466\left(1^{+0.292}_{-0.215}\right) \quad (2\sigma)$$

$$\sin^2\theta_{13} = 0.012 \pm 0.013 \quad (1\sigma)$$

Fogli et al. (2008)

Neutrino Oscillation

- Precision Measurement
  - Tiny unknown parameters: θ_{13}, δ_{CP}
  - Pin-down the parameter values
  - Lifting degeneracy
- Reactor
- Accelerator + Very-long-baseline
  - $L \gtrsim$ 1000km + 2 Detectors
  - $L \gtrsim$ 2000km
  - $L \gtrsim$ 7000km
  - etc. ⟶ Matter effect in control
- Others

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Matter effect

- Matter density on the baseline
  - Constant
  - Tabulation according to an Earth model
  - Analytic approximation

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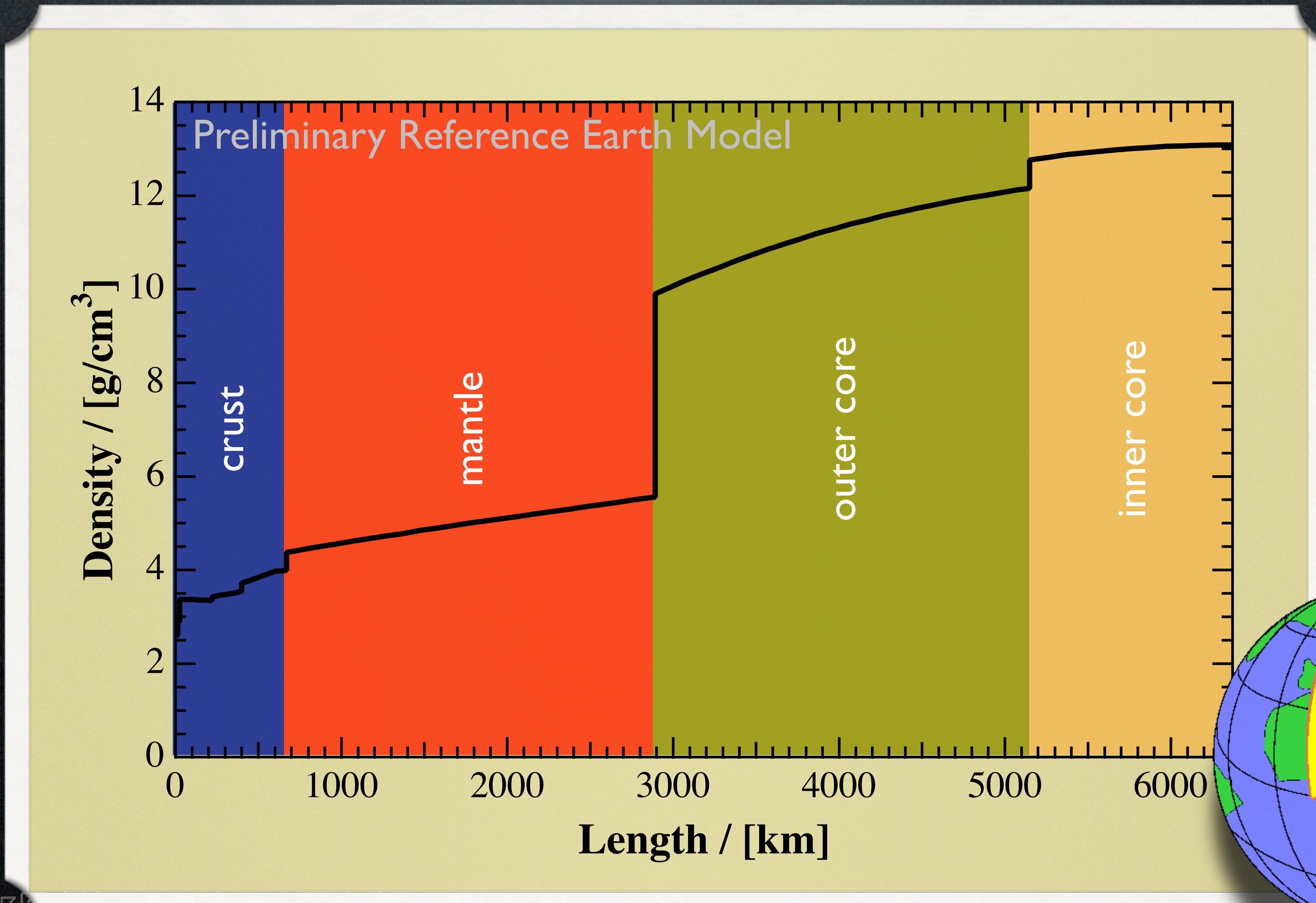
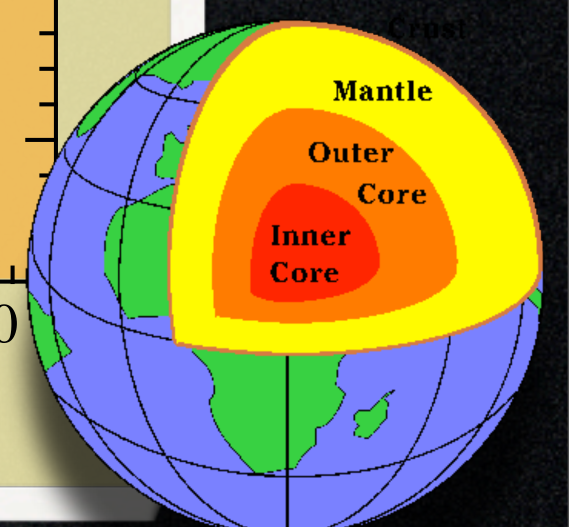
Earth Model, Example





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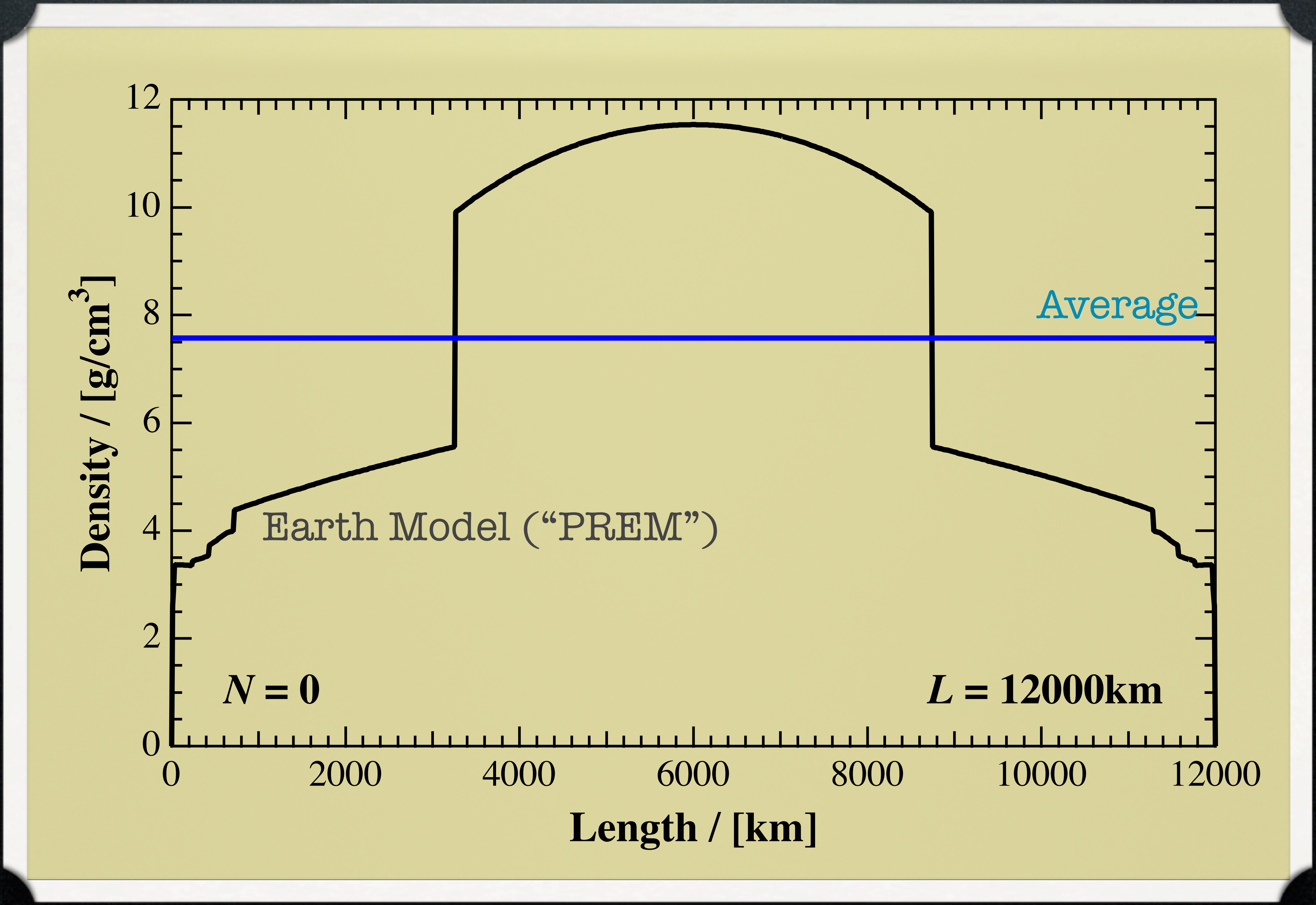
Matter Density on a Baseline



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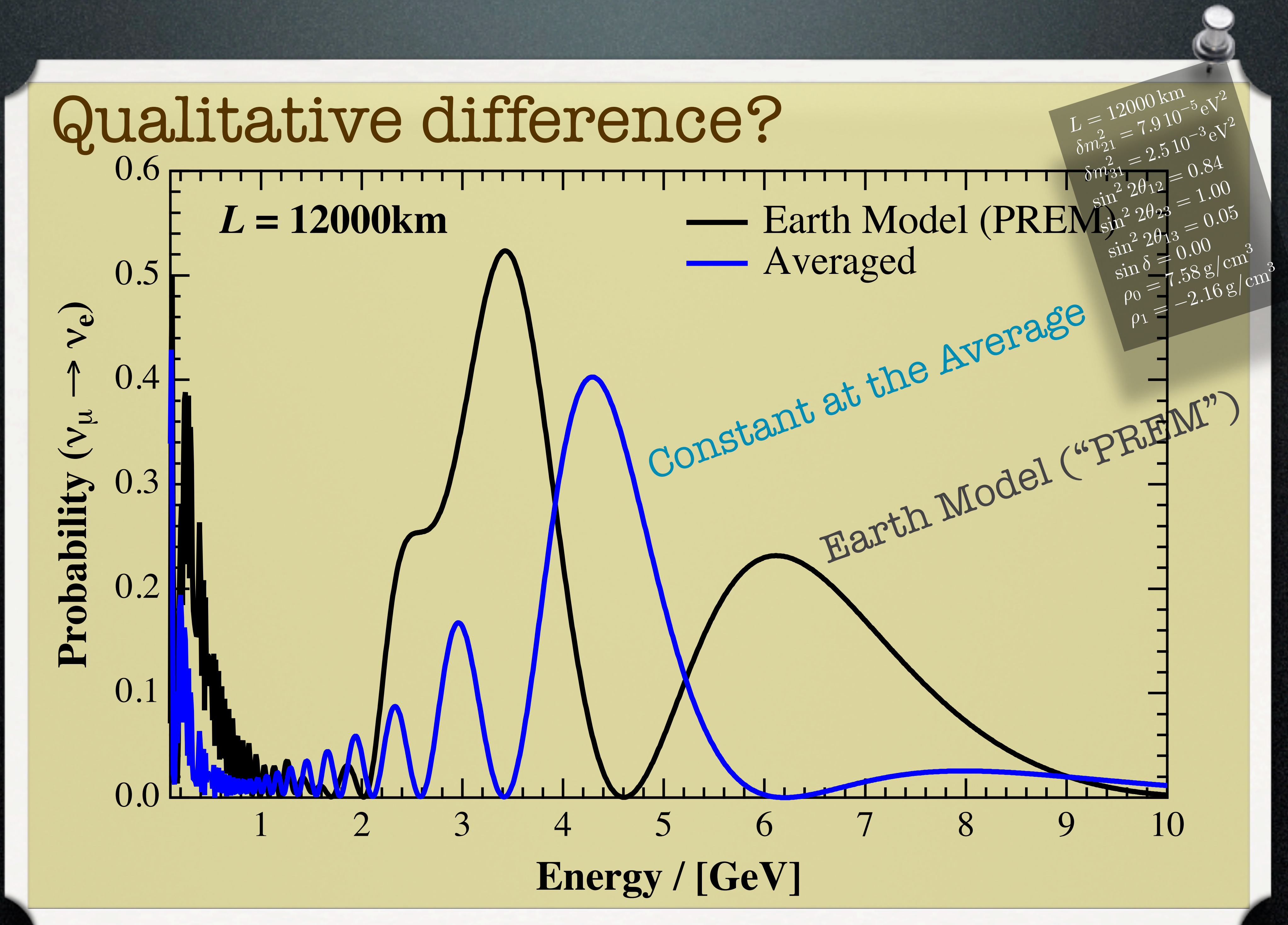
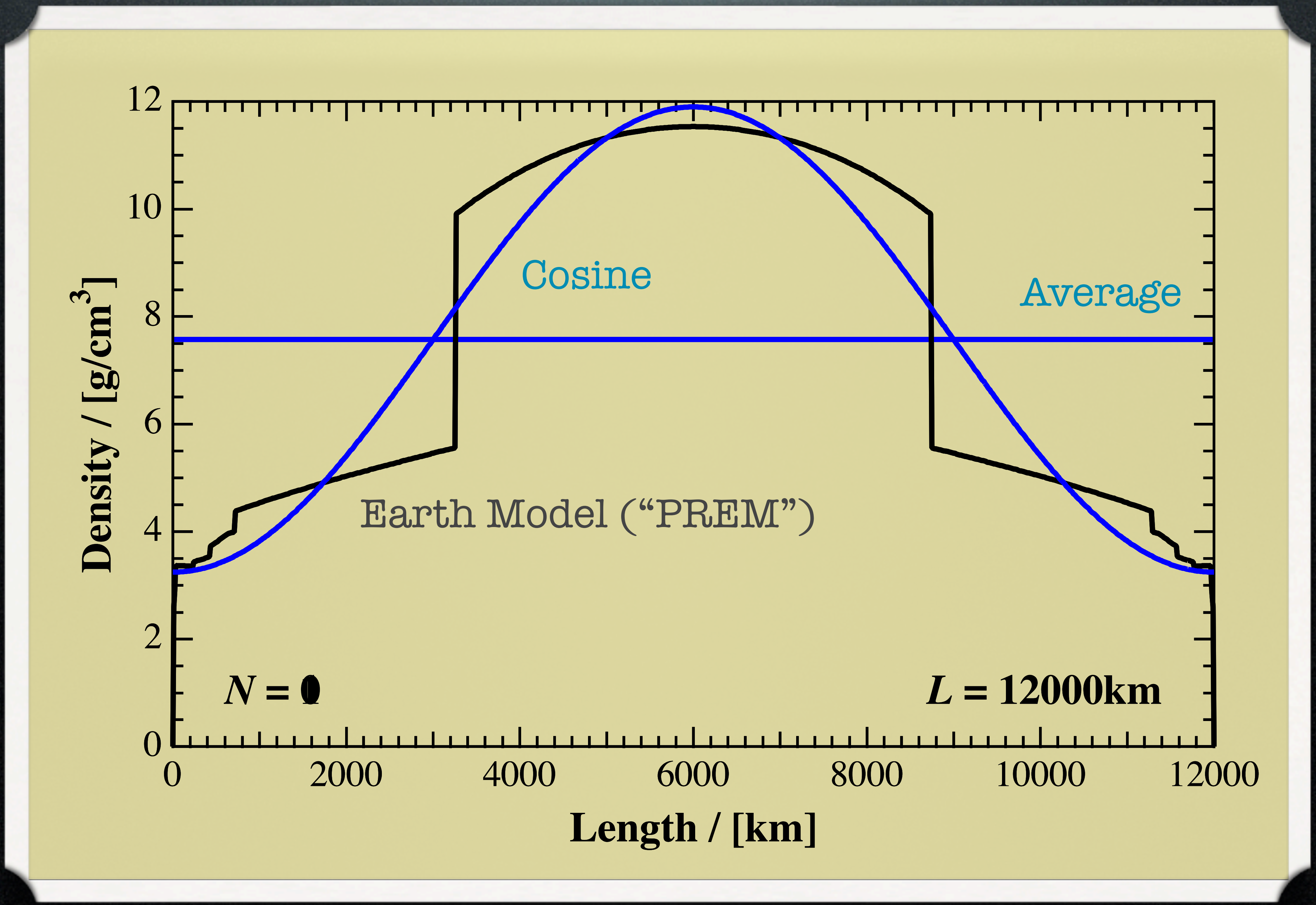
Constant vs. Earth Model



Matter Density Profile



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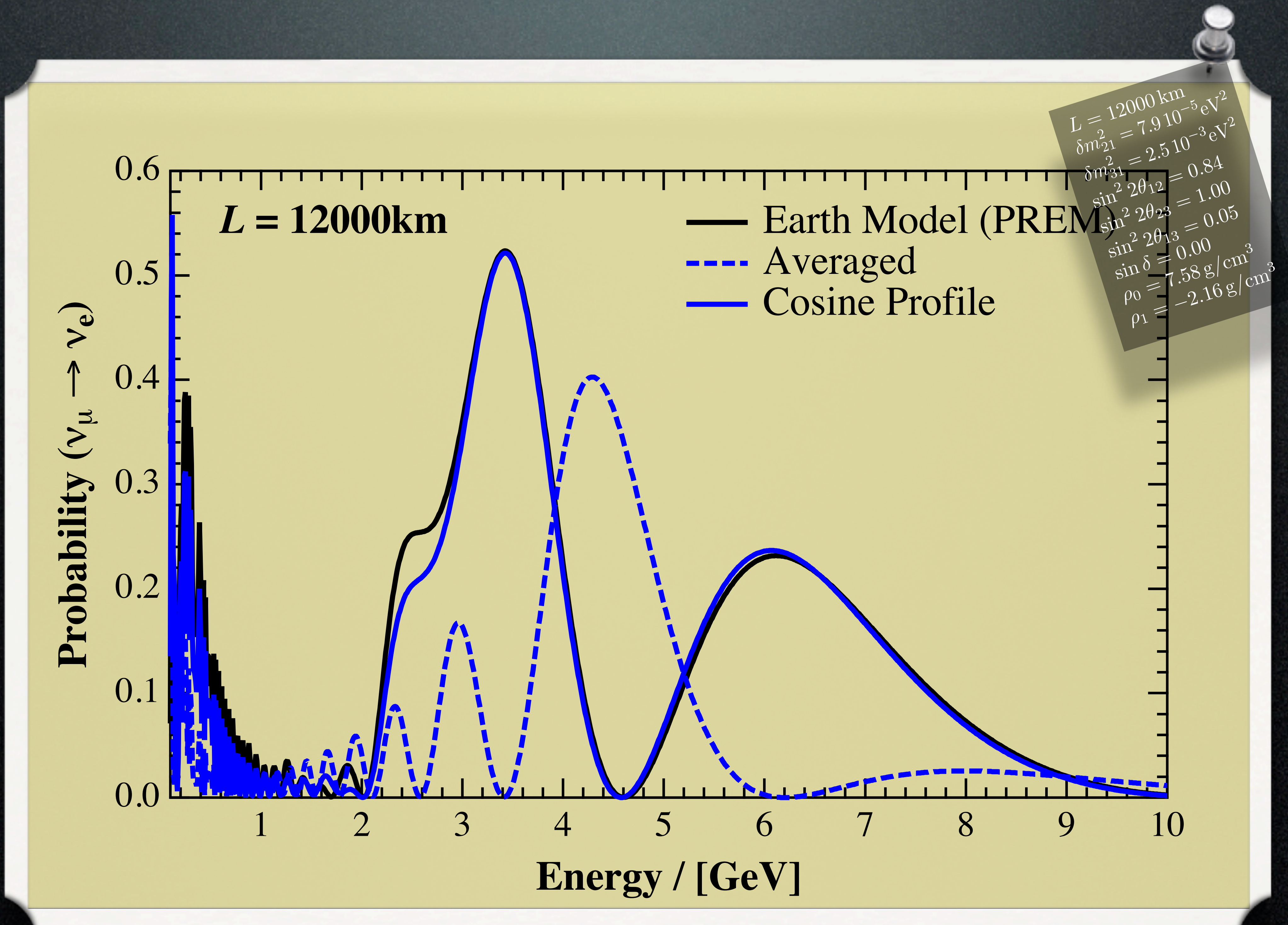
Constant vs. Earth Model



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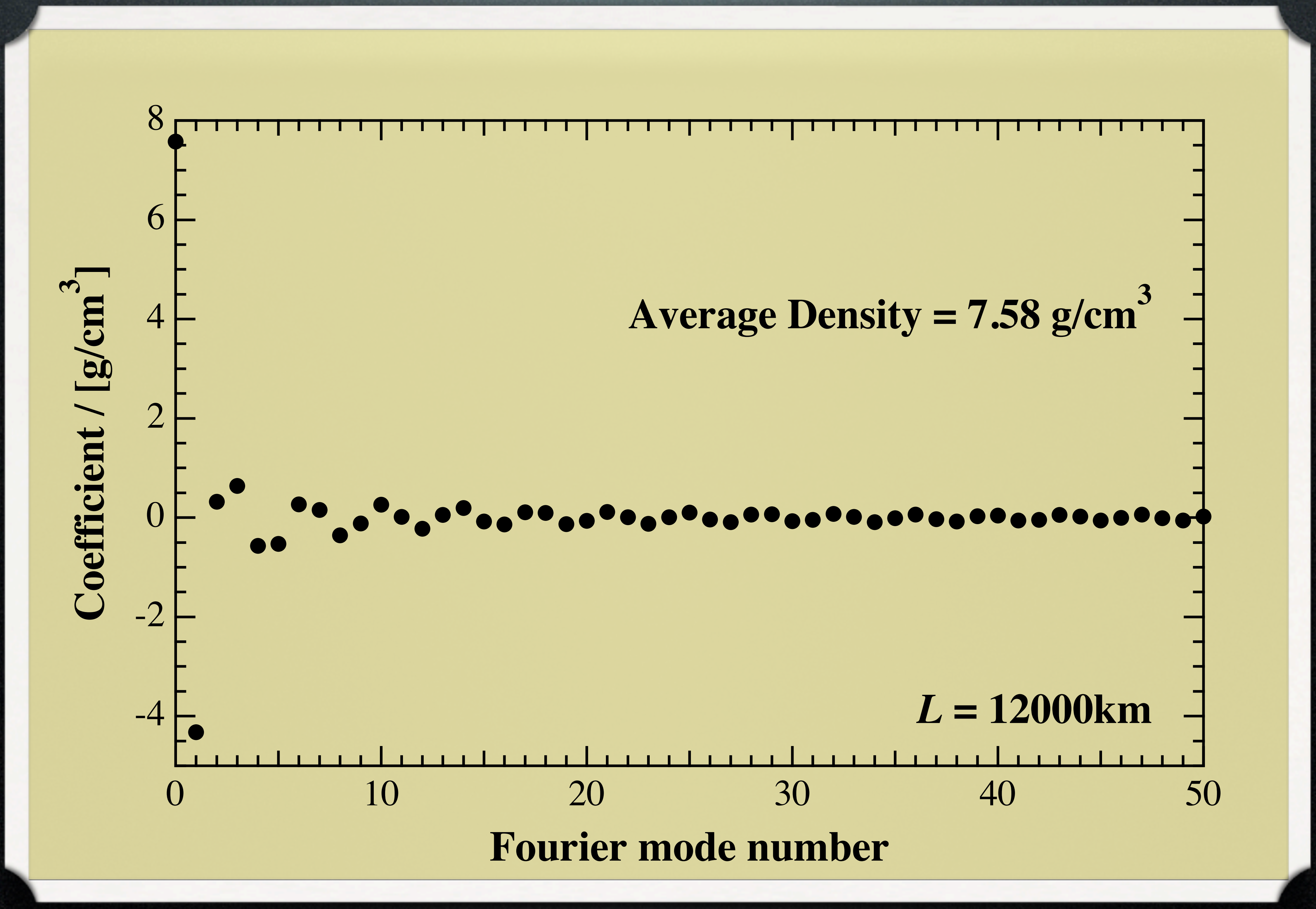
Fourier Coefficients



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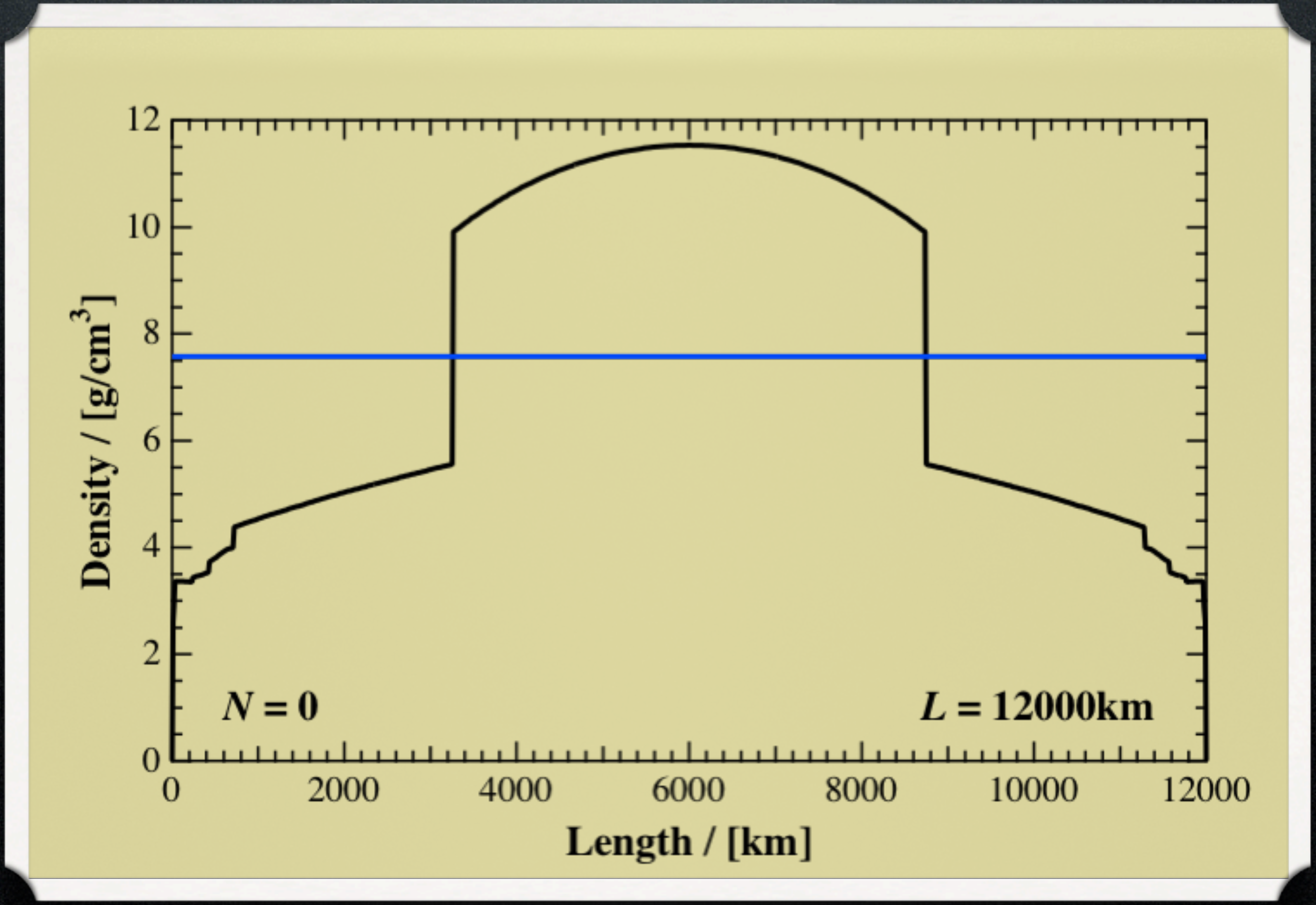
Matter Density Profile



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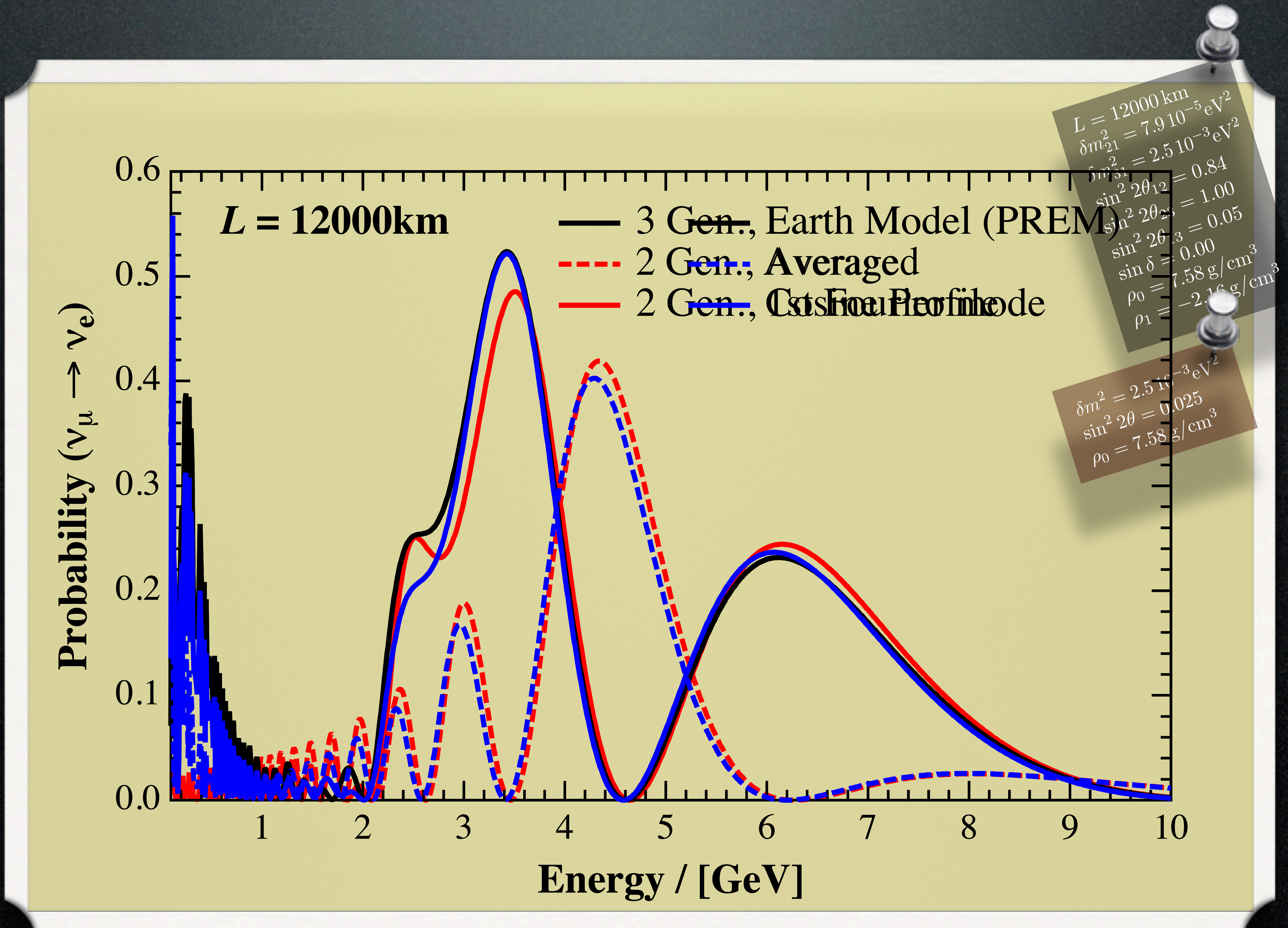
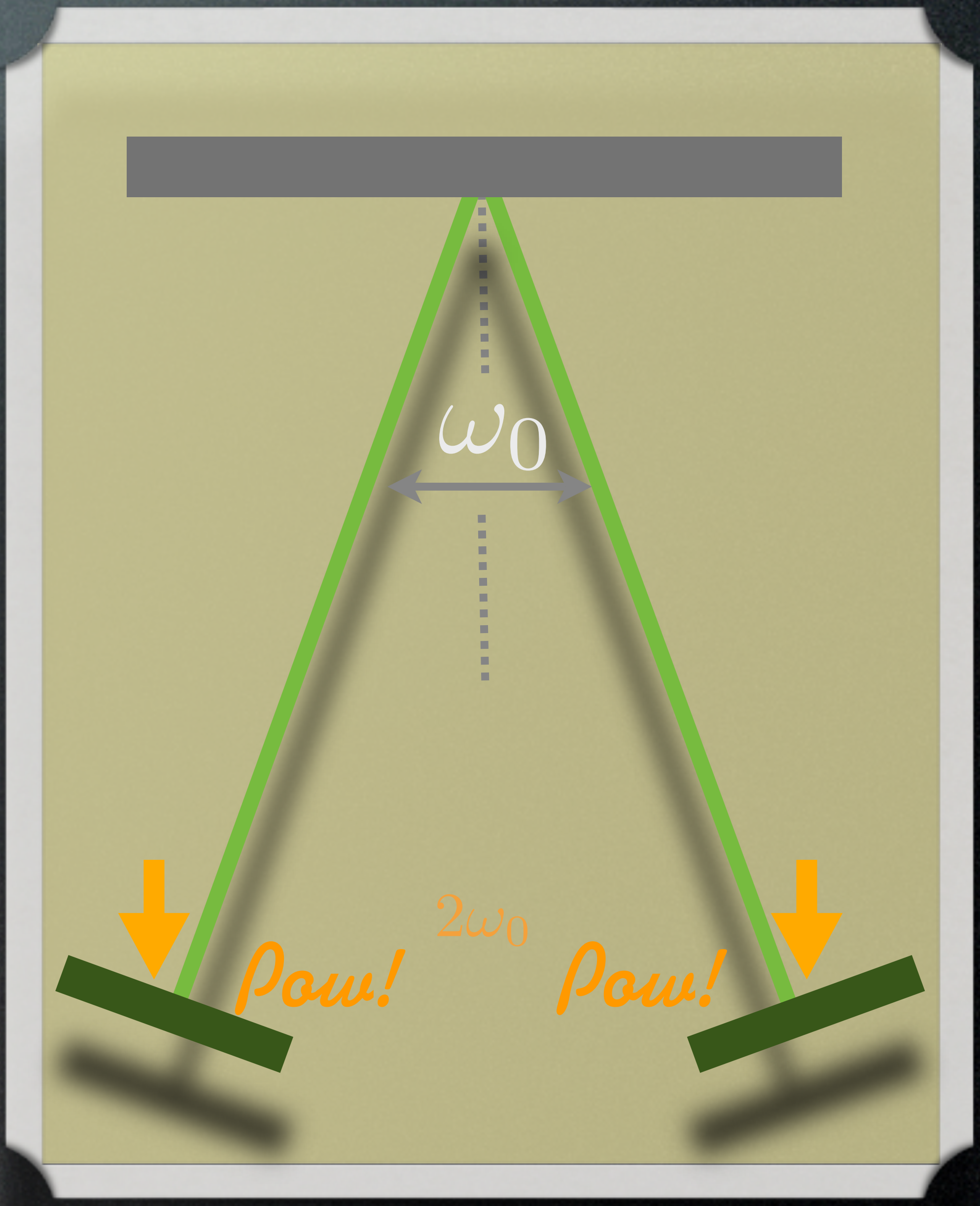
We’ll proceed with 2 flavors



Parametric Resonance

- Periodic perturbation
  - Twice in a period
  - Grows amplitude of oscillation
- Matter effect as a bunch of periodic perturbations



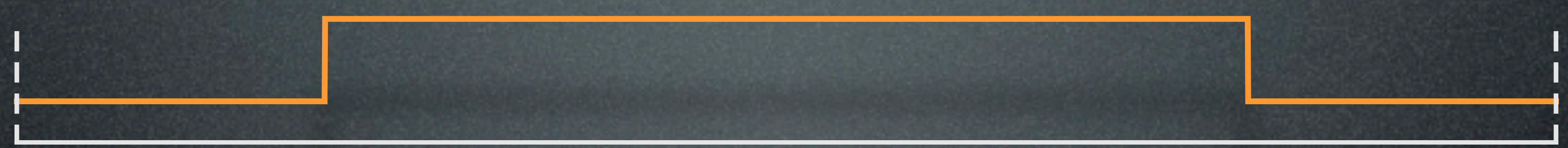
Ermilova et al. (1986), Akhmedov (1988), Krastev&Smirnov (1989), Krastev&Smirnov (1989), Liu&Smirnov (1998), Petcov (1998), Chizhov&Petcov (1998), ..., Akhmedov&Maltoni&Smirnov (2005), ...

Formulation

2

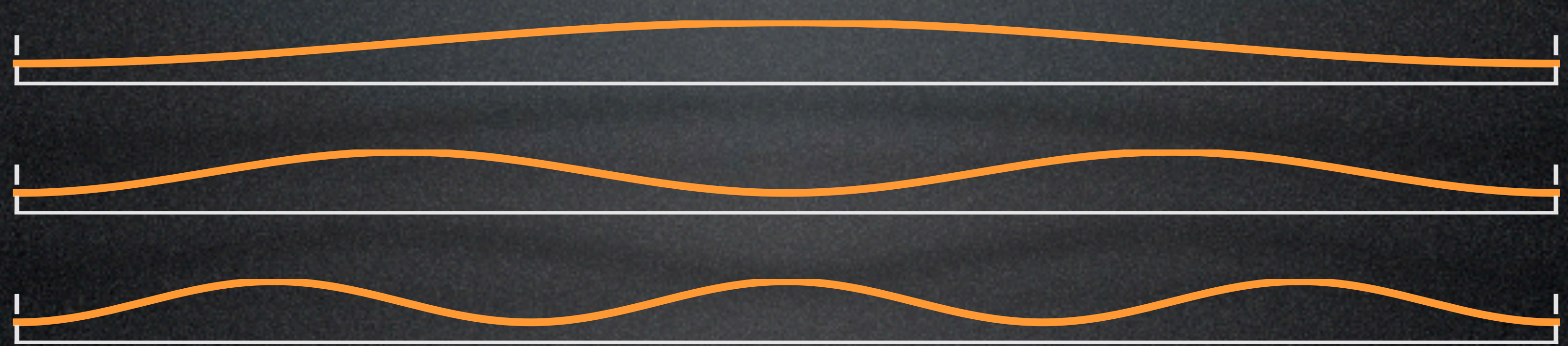
Modeling Density Profiles

- Step function



Akhmedov (1988), Krastev&Smirnov (1989), Krastev&Smirnov (1989), Liu&Smirnov (1998), Petcov (1998), Chizhov&Petcov (1998), ..., Akhmedov&Maltoni&Smirnov (2005), ...

- Fourier series



Koike&Sato (1998), Ota&Sato (2003), ...

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Two-Flavor Oscillation

- Evolution equation of the two-flavor neutrino

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}x}\begin{pmatrix}\nu_\mathrm{e}(x)\\ \nu_\mu(x)\end{pmatrix} = \frac{1}{2E}\left[\frac{\delta m^2}{2}\begin{pmatrix}-\cos 2\theta & \sin 2\theta\\ \sin 2\theta & \cos 2\theta\end{pmatrix} + \begin{pmatrix}a(x) & 0\\ 0 & 0\end{pmatrix}\right]\begin{pmatrix}\nu_\mathrm{e}(x)\\ \nu_\mu(x)\end{pmatrix}$$

  - Matter effect $a(x) = 2\sqrt{2}G_\mathrm{F}n_\mathrm{e}(x)E$
  - Dimensionless variables:

$$\xi \equiv \frac{x}{L} \qquad \Delta \equiv \frac{\delta m^2 L}{2E} \qquad \Delta_\mathrm{m}(\xi) \equiv \frac{a(\xi)L}{2E}$$

Distance — Reciprocal E — Matter Density

  - $z(\xi) = \nu_\mathrm{e}(\xi)\exp\left[\frac{\mathrm{i}}{2}\int_0^\xi \mathrm{d}s\,\Delta_\mathrm{m}(s)\right] \quad \cdots \quad \left|\nu_\mathrm{e}(\xi)\right|^2 = \left|z(\xi)\right|^2$
  - Initial conditions $\nu_\mathrm{e}(0) = 0, \nu_\mu(0) = 1 \longrightarrow z(0) = 0\,,\; z'(0) = -\mathrm{i}\frac{\Delta}{2}\sin 2\theta$

- Second-order equation in dimensionless variables

$$z''(\xi) + \frac{1}{4}\left[\left(\Delta_\mathrm{m}(\xi) - \Delta\cos 2\theta\right)^2 + \Delta^2\sin^2 2\theta + 2\mathrm{i}\Delta'_\mathrm{m}(\xi)\right]z(\xi) = 0$$

Constant-Density Matter

$$z''(\xi) + \frac{1}{4}\Big[\big(\Delta_{\mathrm{m}}(\xi) - \Delta\cos 2\theta\big)^2 + \Delta^2 \sin^2 2\theta + 2\mathrm{i}\Delta'_{\mathrm{m}}(\xi)\Big] z(\xi) = 0$$

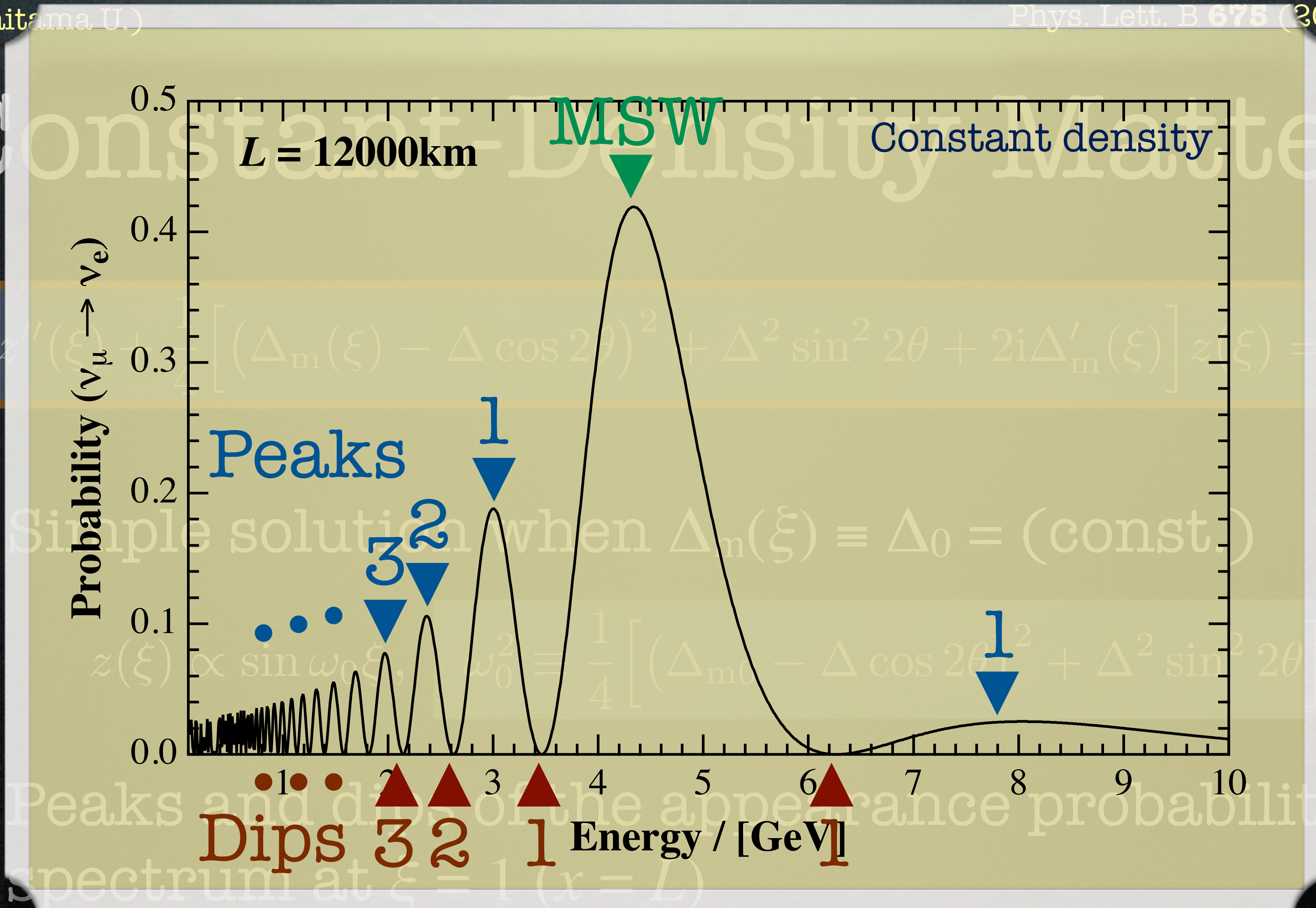
- Simple solution when $\Delta_{\mathrm{m}}(\xi) \equiv \Delta_0 = (\text{const.})$

$$z(\xi) \propto \sin\omega_0\xi\,, \quad \omega_0^2 \equiv \frac{1}{4}\Big[\big(\Delta_{\mathrm{m}0} - \Delta\cos 2\theta\big)^2 + \Delta^2\sin^2 2\theta\Big]$$

- Peaks and dips of the appearance probability spectrum at $\xi = 1$ $(x = L)$
  - $\omega_0 = \frac{1}{2}\Delta\sin 2\theta$ MSW resonance.
  - $\omega_0 = \left(n + \frac{1}{2}\right)\pi$ (n+1)-th peak.
  - $\omega_0 = n\pi$ n-th dip.

Constant-Density Matter

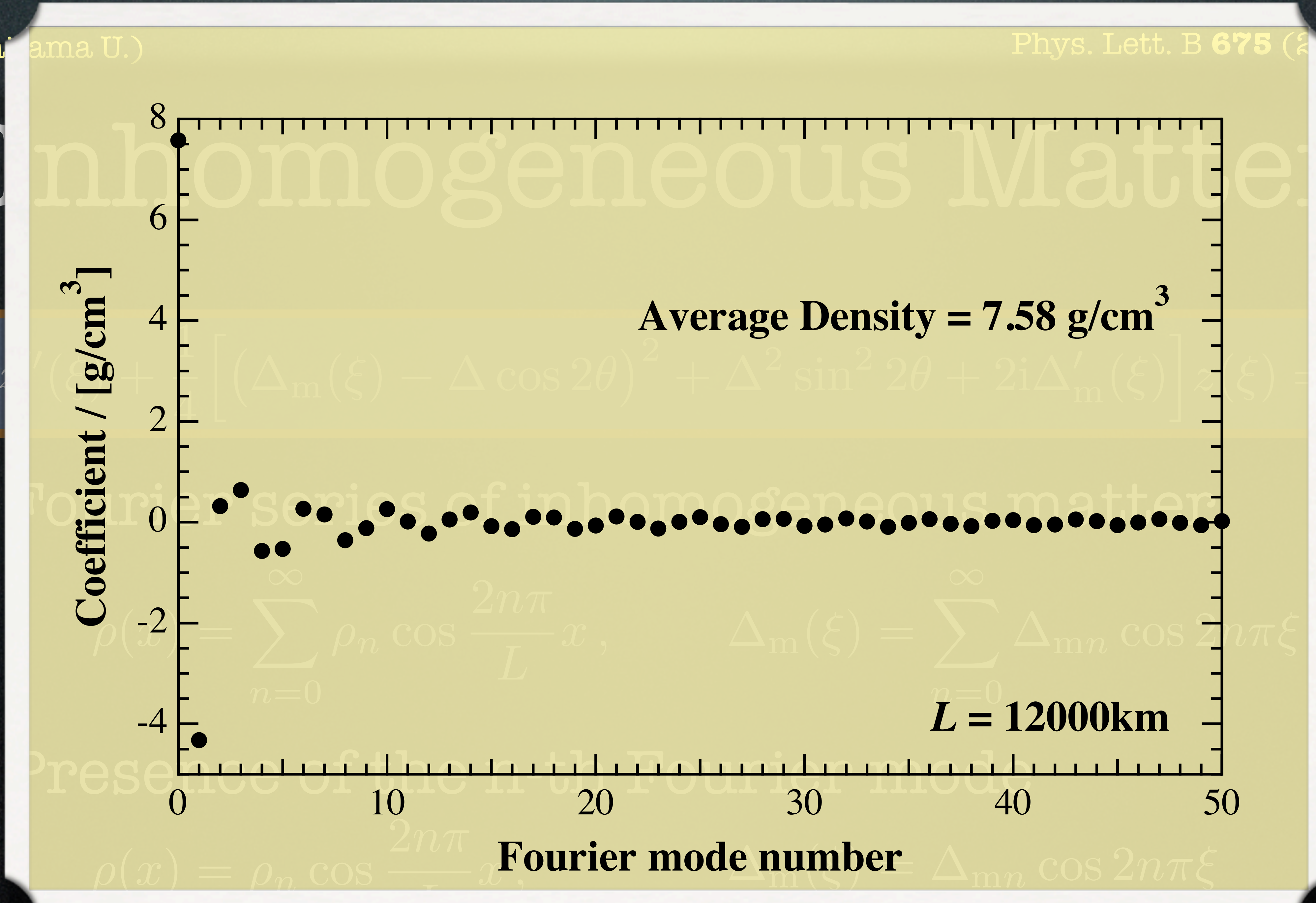
$$z''(\xi) + \frac{1}{4}\left[\left(\Delta_m(\xi) - \Delta\cos 2\theta\right)^2 + \Delta^2\sin^2 2\theta + 2i\Delta'_m(\xi)\right] z(\xi) = 0$$



- Simple solution when $\Delta_m(\xi) \equiv \Delta_0 = (\text{const.})$

$$z(\xi) \propto \sin\omega_0\xi, \quad \omega_0^2 = \frac{1}{4}\left[\left(\Delta_{m0} - \Delta\cos 2\theta\right)^2 + \Delta^2\sin^2 2\theta\right]$$

- Peaks and dips of the appearance probability spectrum at $\xi = 1$ ($x = L$)
  - $\omega_0 = \frac{1}{2}\Delta\sin 2\theta$ MSW resonance.
  - $\omega_0 = \left(n + \frac{1}{2}\right)\pi$ (n+1)-th peak.
  - $\omega_0 = n\pi$ n-th dip.


Average Density = 7.58 g/cm^3
L = 12000km
Coefficient / [g/cm^3]
Fourier mode number
8
6
4
2
0
-2
-4
0
10
20
30
40
50

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Inhomogeneous Matter

$$z''(\xi) + \frac{1}{4}\Big[\big(\Delta_{\mathrm{m}}(\xi) - \Delta\cos 2\theta\big)^2 + \Delta^2\sin^2 2\theta + 2\mathrm{i}\Delta'_{\mathrm{m}}(\xi)\Big]z(\xi) = 0$$

- Fourier series of inhomogeneous matter

$$\rho(x) = \sum_{n=0}^{\infty}\rho_n \cos\frac{2n\pi}{L}x\,, \qquad \Delta_{\mathrm{m}}(\xi) = \sum_{n=0}^{\infty}\Delta_{\mathrm{m}n}\cos 2n\pi\xi$$

- Presence of the n-th Fourier mode

$$\rho(x) = \rho_n \cos\frac{2n\pi}{L}x\,, \qquad \Delta_{\mathrm{m}}(\xi) = \Delta_{\mathrm{m}n}\cos 2n\pi\xi$$

Mathieu Equation

$$z''(t) + (\omega^2 - 2\varepsilon\cos t)z(t) = 0$$

$$z''(\xi) + \big(\omega_0^2 + \alpha_n\cos 2n\pi\xi - \mathrm{i}\beta_n\sin 2n\pi\xi + \gamma_n\cos 4n\pi\xi\big)z(\xi) = 0$$

$$\omega_0^2 = \frac{1}{4}(\Delta_{\mathrm{m}0} - \Delta\cos 2\theta)^2 + \frac{1}{4}\Delta^2\sin^2 2\theta + \frac{1}{8}\Delta_{\mathrm{m}n}^2\,,$$

$$\alpha_n = \frac{1}{2}(\Delta_{\mathrm{m}0} - \Delta\cos 2\theta)\Delta_{\mathrm{m}n}\,, \quad \beta_n = n\pi\Delta_{\mathrm{m}n}\,, \quad \gamma_n = \frac{1}{8}\Delta_{\mathrm{m}n}^2$$

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Parametric Resonance in Appearance Probability

3

Condition of Resonance

$$z''(\xi) + \left(\omega_0^2 + \alpha_n \cos 2n\pi\xi - \mathrm{i}\beta_n \sin 2n\pi\xi + \gamma_n \cos 4n\pi\xi\right) z(\xi) = 0$$

n-th mode

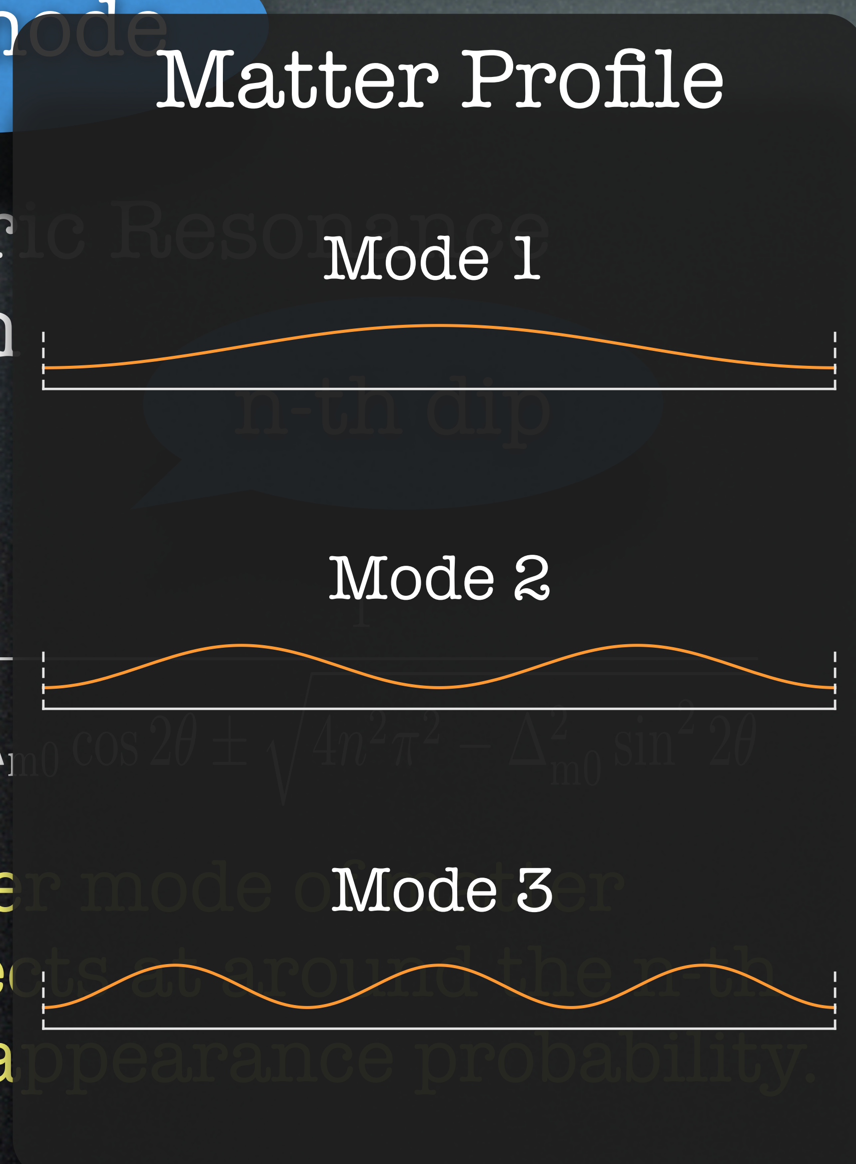
- Parametric Resonance Condition

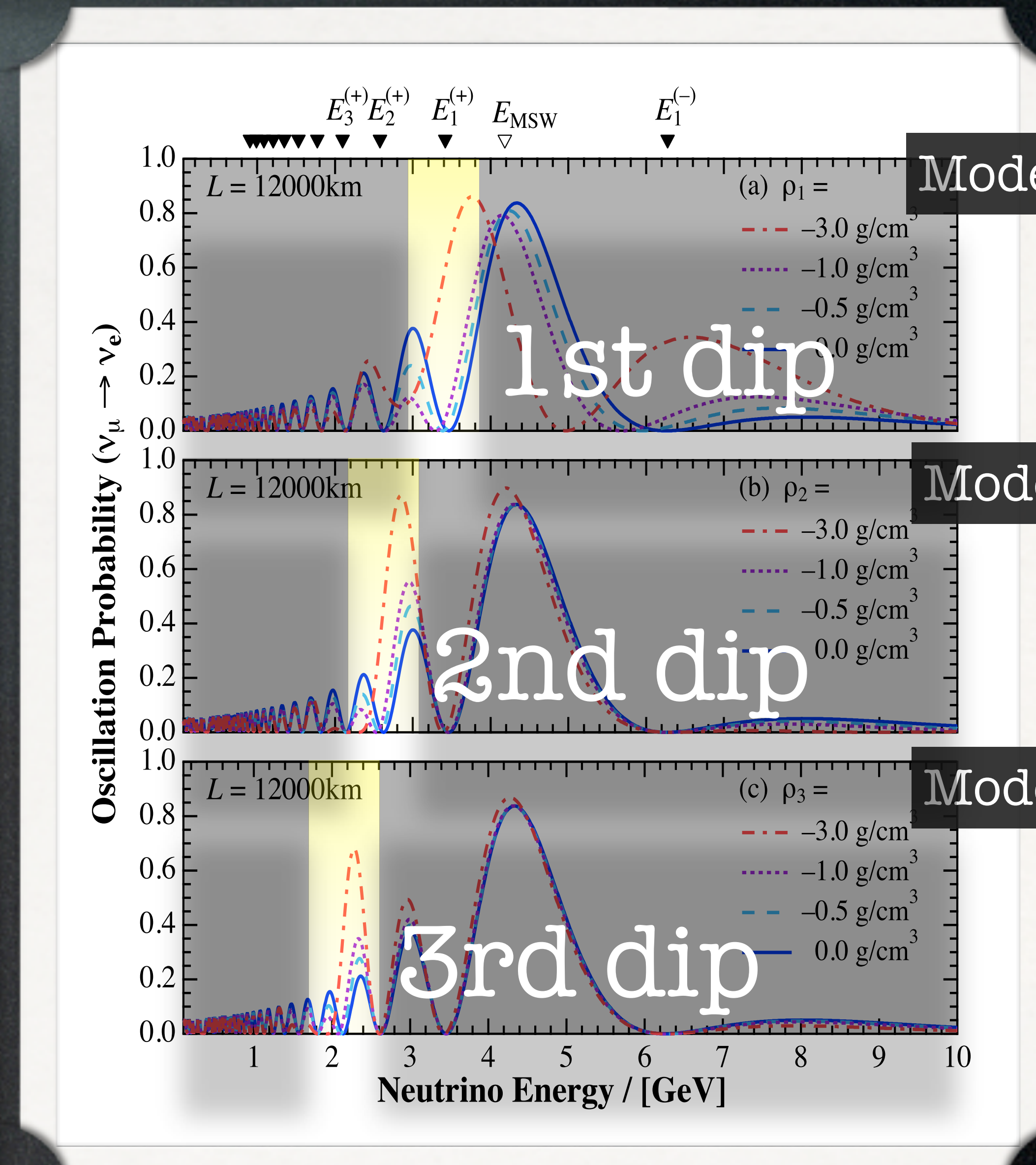
$$\omega_0 = n\pi$$

$$E = E_n^{(\pm)} \equiv \frac{\delta m^2 L}{2} \frac{\quad}{\Delta_{m0} \cos 2\theta \pm \sqrt{4n^2\pi^2 - \Delta_{m0}^2 \sin^2 2\theta}}$$

n-th dip

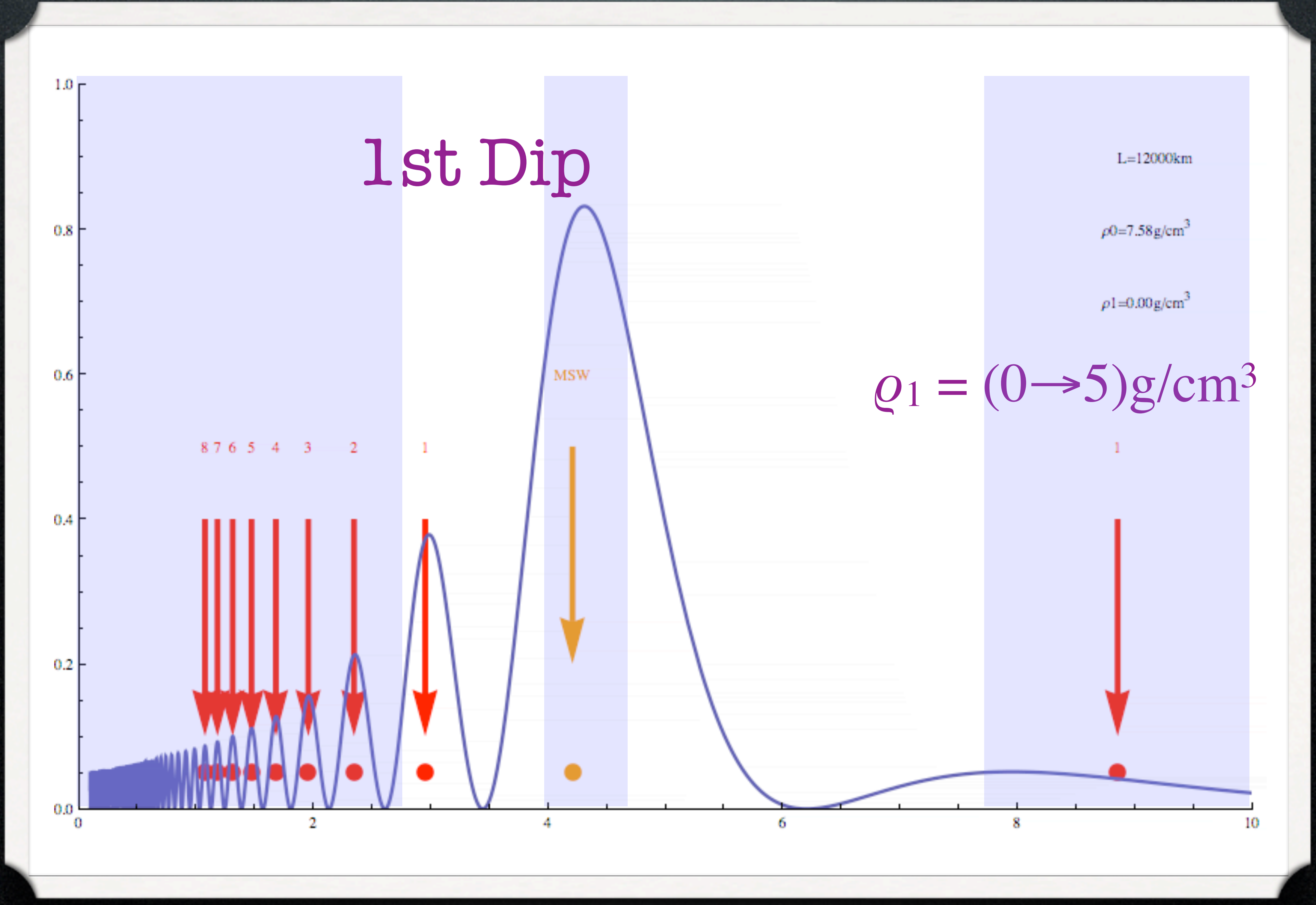
- n-th Fourier mode of matter profile affects at around the n-th dip of the appearance probability.



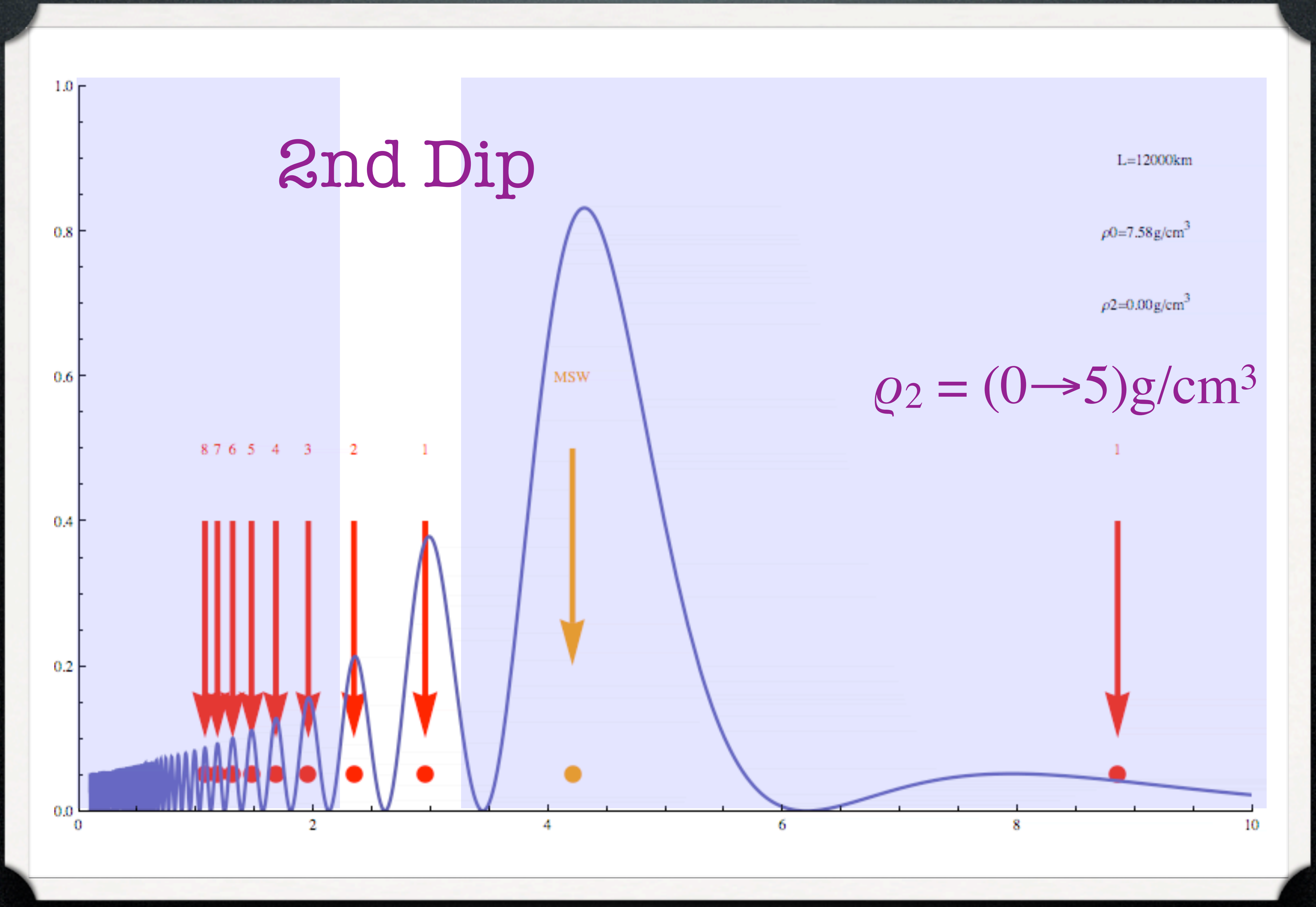


The values of other parameters used in this plot found in arXiv:0902.1597 [hep-ph].

Effect of the Mode 1



Effect of the Mode 2



Beyond a single swing, $\xi > 1$

$$z''(\xi) + \left(\omega_0^2 + \alpha_n \cos 2n\pi\xi - \mathrm{i}\beta_n \sin 2n\pi\xi + \gamma_n \cos 4n\pi\xi\right) z(\xi) = 0$$

- What if the matter profile is repetitively applied?

Approximate solution at $\omega_0 = n\pi$:

$$z(\xi) = \frac{\mathrm{i}\Delta \sin 2\theta}{2\Omega(4n^2\pi^2 - \alpha_n - 2\Omega^2)} \left[-\mathrm{i}\beta_n \sin \Omega\xi \sin n\pi\xi + (\alpha_n + 2\Omega^2) \sin \Omega\xi \cos n\pi\xi - 4n\pi\Omega \cos \Omega\xi \sin n\pi\xi \right],$$

where $\Omega = \pm\sqrt{2} n\pi \left[1 - \sqrt{1 - \frac{\beta_n^2 - \alpha_n^2}{(2n\pi)^4}} \right]^{1/2}$

$$|z(\xi)|^2 = \frac{\Delta^2 \sin^2 2\theta}{4\Omega^2(4n^2\pi^2 - b - 2\Omega^2)^2} \times \Big[(b + 2\Omega^2)^2 \sin^2 \Omega\xi \cos^2 n\pi\xi + \left(16n^2\pi^2\Omega^2 \cos^2 \Omega\xi + c^2 \sin^2 \Omega\xi\right) \sin^2 n\pi\xi - 8n\pi\Omega(b + 2\Omega^2) \cos \Omega\xi \sin \Omega\xi \cos n\pi\xi \sin n\pi\xi \Big]$$

Beyond a single swing, $\xi > 1$

$$z''(\xi) + \left(\omega_0^2 + \alpha_n \cos 2n\pi\xi - i\beta_n \sin 2n\pi\xi + \gamma_n \cos 4n\pi\xi\right) z(\xi) = 0$$

$$|z(\xi)|^2 = \frac{\Delta^2 \sin^2 2\theta}{4\Omega^2(4n^2\pi^2 - b - 2\Omega^2)^2} \times \Big[(b+2\Omega^2)^2 \sin^2 \Omega\xi \cos^2 n\pi\xi + (16n^2\pi^2\Omega^2 \cos^2 \Omega\xi + c^2 \sin^2 \Omega\xi) \sin^2 n\pi\xi$$
$$-8n\pi\Omega(b+2\Omega^2) \cos\Omega\xi \sin\Omega\xi \cos n\pi\xi \sin n\pi\xi\Big]$$

$$\Omega = \pm\sqrt{2}n\pi\left[1 - \sqrt{1 - \frac{\beta_n^2 - \alpha_n^2}{(2n\pi)^4}} \ldots\right.$$

- Resonant growth of the appearance probability confirmed
- Resonance turns to an oscillation
- Unitarity ensured

Mode 1

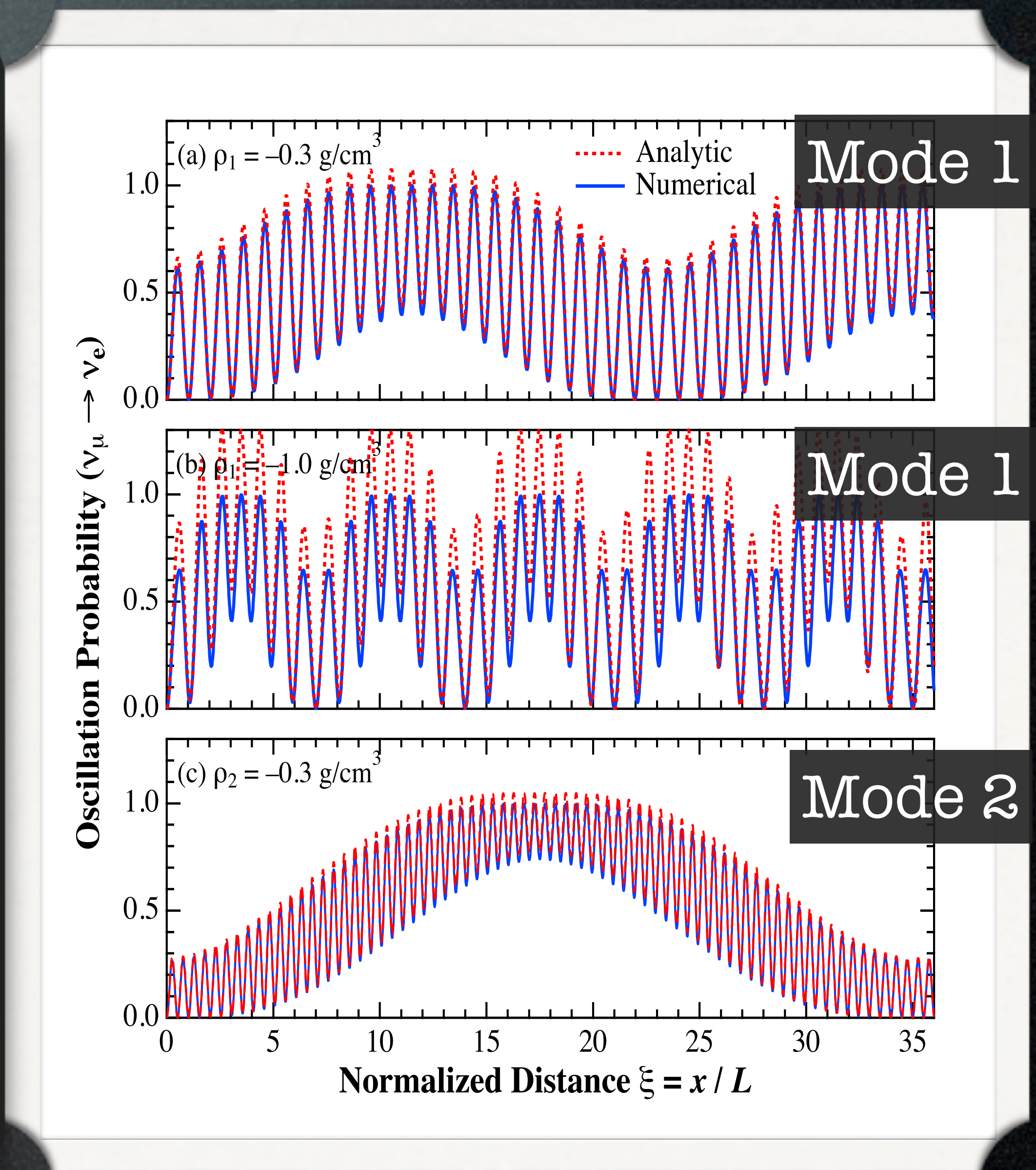
$\rho_1 = -0.3\,\mathrm{g/cm^3}$

Mode 1

$\rho_1 = -1.0\,\mathrm{g/cm^3}$

Mode 2

$\rho_2 = -0.3\,\mathrm{g/cm^3}$



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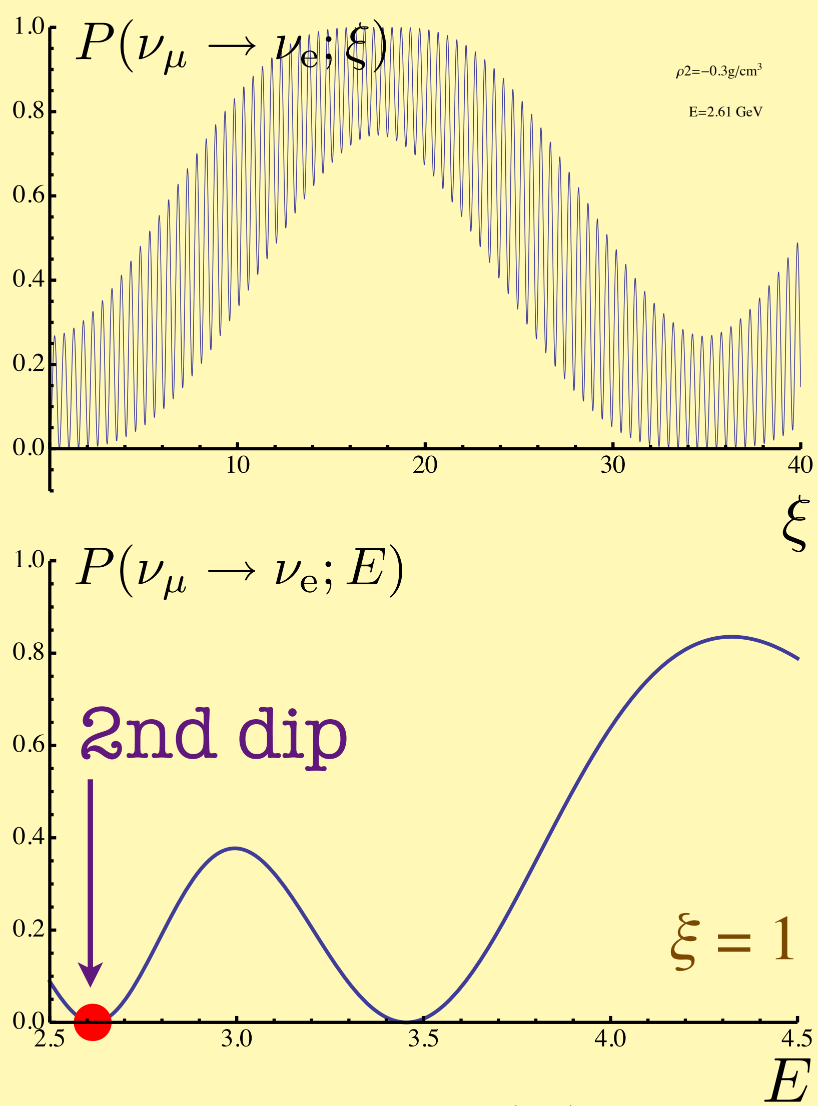
Off the Resonance Energy



$$E = E_2^{(+)}$$

Off the Resonance

- Off-the-resonance ($\omega_0 \neq n\pi$, but $\omega_0 \simeq n\pi$)

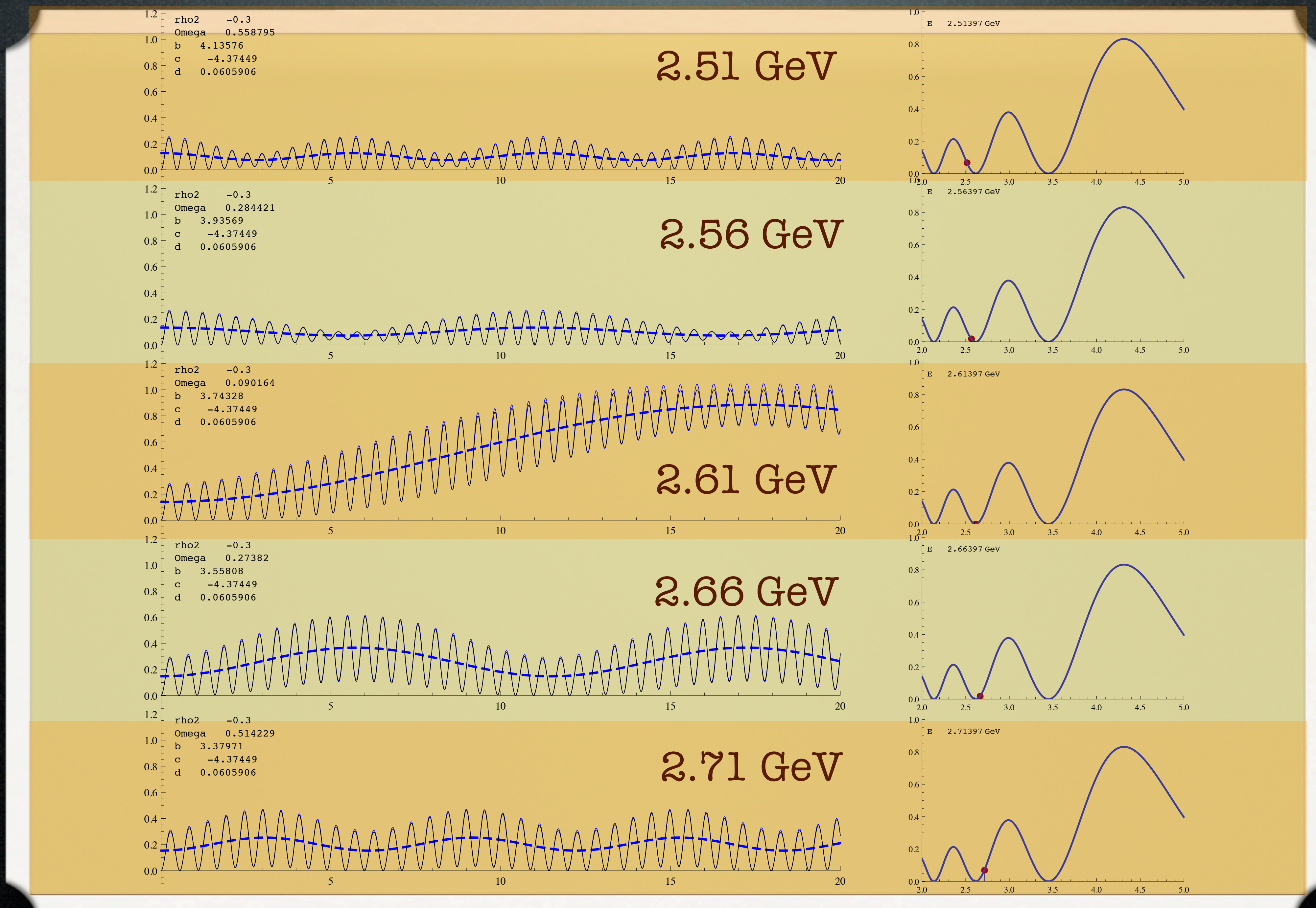
$$z(\xi) = \frac{\Delta \sin 2\theta}{2\Omega[4n^2\pi^2 - \alpha_n - 2\Omega^2 + 2(\omega_0^2 - n^2\pi^2)]} \times \Big\{ i[\alpha_n + 2\Omega^2 - 2(\omega_0^2 - n^2\pi^2)] \sin \Omega\xi \cos n\pi\xi + \beta_n \sin \Omega\xi \sin n\pi\xi - i4n\pi\Omega \cos \Omega\xi \sin n\pi\xi \Big\}$$

where

$$\Omega \equiv \sqrt{2} n\pi \left[1 + \frac{\omega_0^2 - n^2\pi^2}{2n^2\pi^2} - \sqrt{1 - \frac{\beta_n^2 - \alpha_n^2}{16n^4\pi^4} + \frac{\omega_0^2 - n^2\pi^2}{n^2\pi^2}} \right]^{1/2} \simeq \frac{\sqrt{\beta_n^2 - \alpha_n^2}}{4n\pi} \left[1 + \frac{2(\omega_0^2 - n^2\pi^2)^2}{\beta_n^2 - \alpha_n^2} - \frac{\omega_0^2 - n^2\pi^2}{4n^2\pi^2} + \frac{\beta_n^2 - \alpha_n^2}{128 n^4 \pi^4} \right]^{1/2}$$

$$|z(\xi)|^2 = \frac{\Delta^2 \sin^2 2\theta}{4\Omega^2 \left[4n^2\pi^2 - \alpha_n - 2\Omega^2 + 2(\omega_0^2 - n^2\pi^2)\right]^2} \times \Big\{ \left[\alpha_n + 2\Omega^2 - 2(\omega_0^2 - n^2\pi^2)\right]^2 \sin^2 \Omega\xi \cos^2 n\pi\xi + \left(16n^2\pi^2\Omega^2 \cos^2 \Omega\xi + \beta_n^2 \sin^2 \Omega\xi\right) \sin^2 n\pi\xi - 8n\pi\Omega \left[\alpha_n + 2\Omega^2 - 2(\omega_0^2 - n^2\pi^2)\right] \cos \Omega\xi \sin \Omega\xi \cos n\pi\xi \sin n\pi\xi \Big\}$$

Off the Resonance



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Summary & Outlook

- Fourier analysis is shown to be powerful to account for the parametric resonance in neutrino oscillation.
    - Neutrino oscillation under inhomogeneous matter is analyzed, analytically and numerically.
    - n-th Fourier mode of the matter profile affects at around the n-th dip of the appearance probability.
    - Inhomogeneity effect leads to parametric resonance, resulting in a slow oscillation.
    - Resonance effect deminishes as the energy goes off the resonance point.
- Search for observable effects of interst (at $\xi = 1$)
    - Beyond a small potential: Large effect in a single swing?
- Further mathematical investigation

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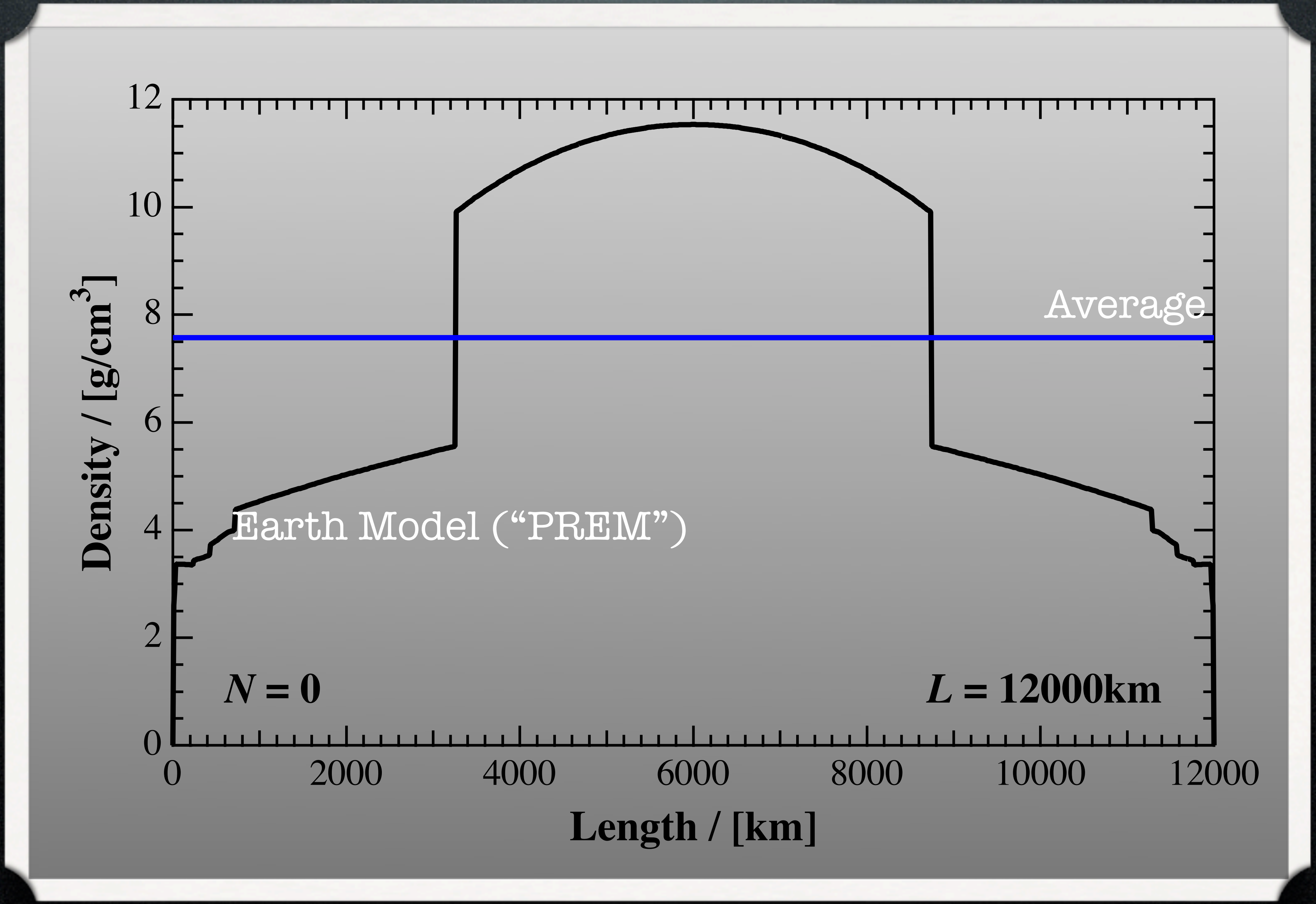
Matter density on a baseline



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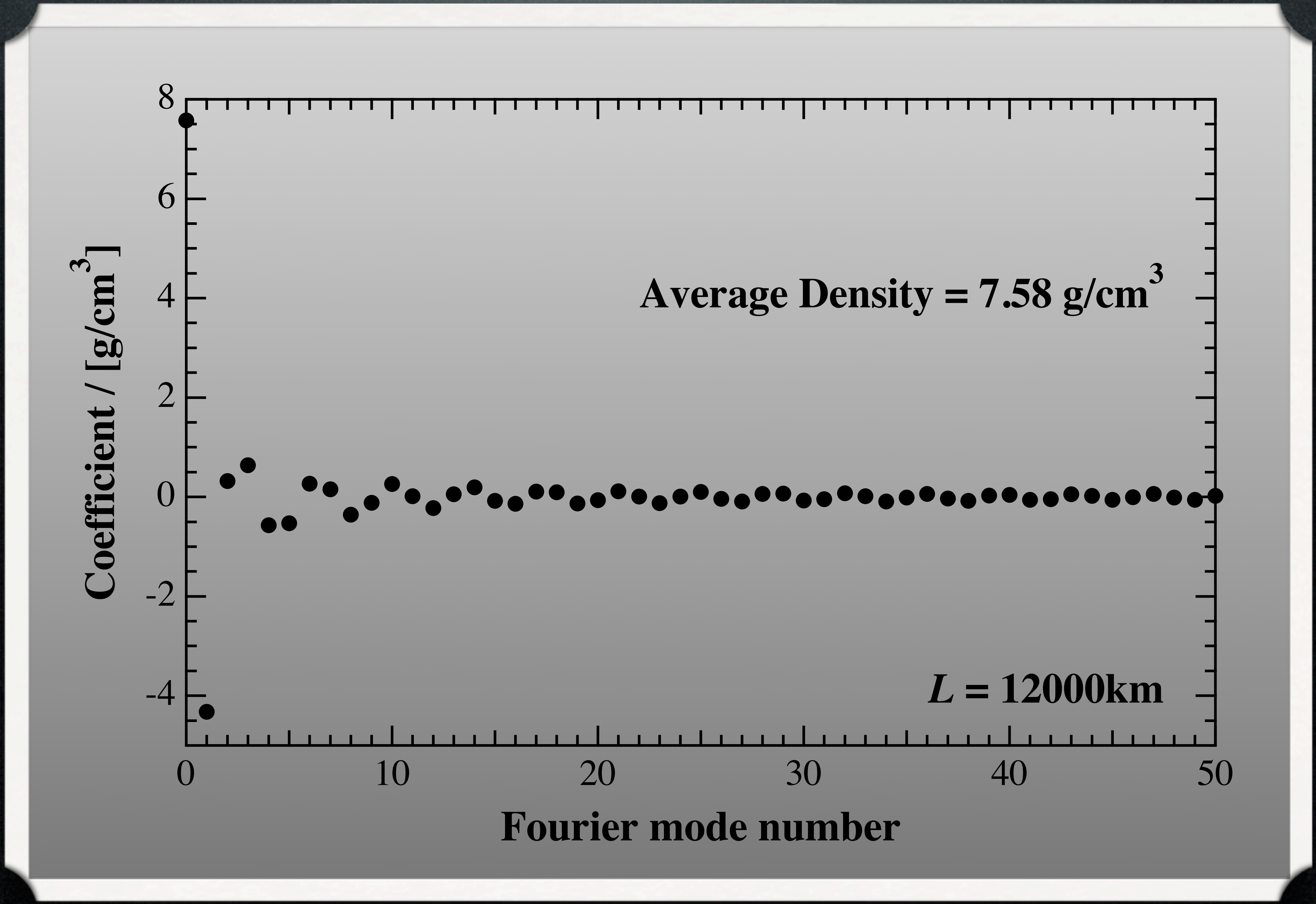
Fourier coefficients



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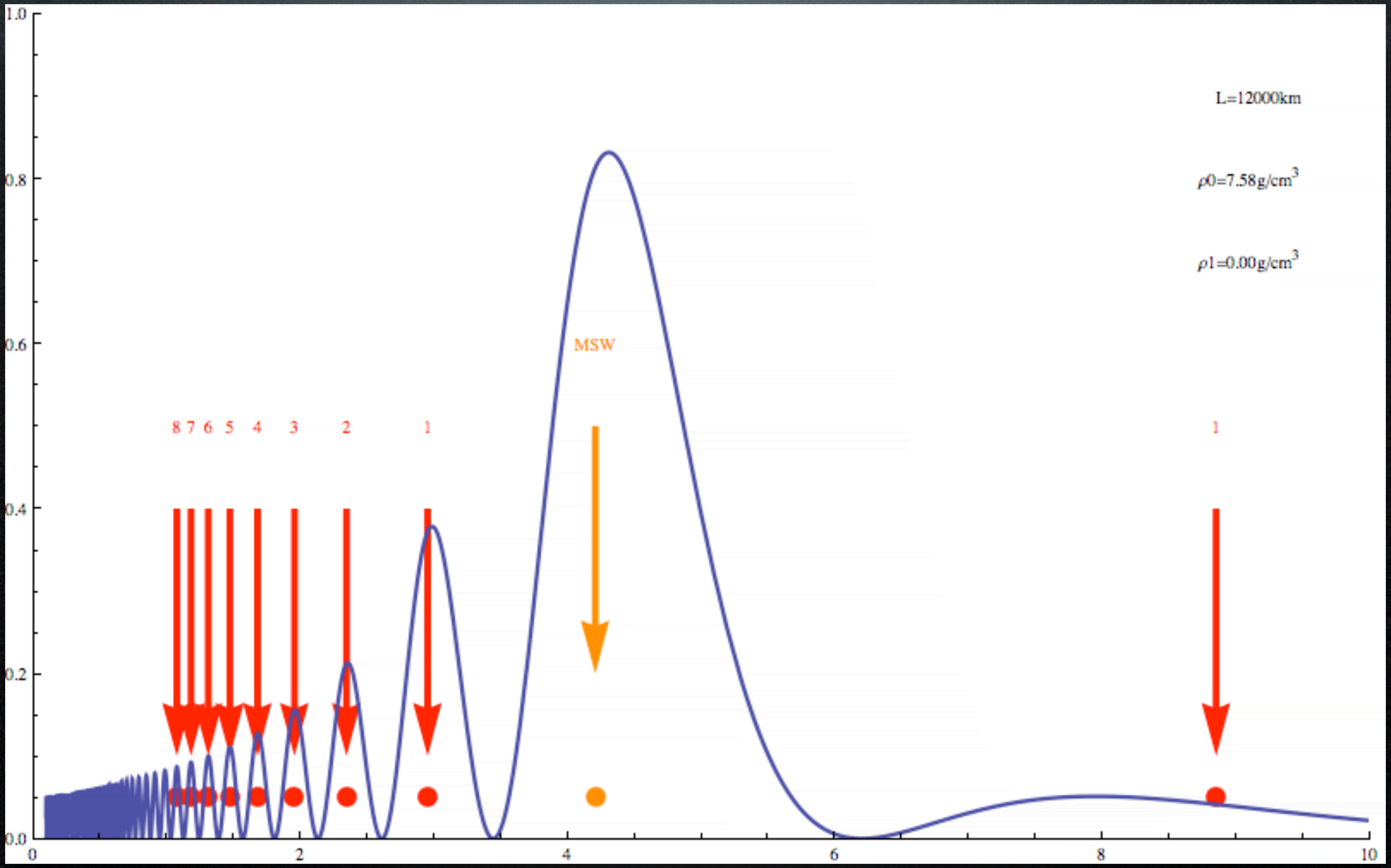
First-mode effect



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Second-mode effect