質量行列と世代間対称性

– tri-bimaximal 混合の世代構造と理論の現状 –

Tri-bimaximal Neutrino Mixing and Flavor Symmetry

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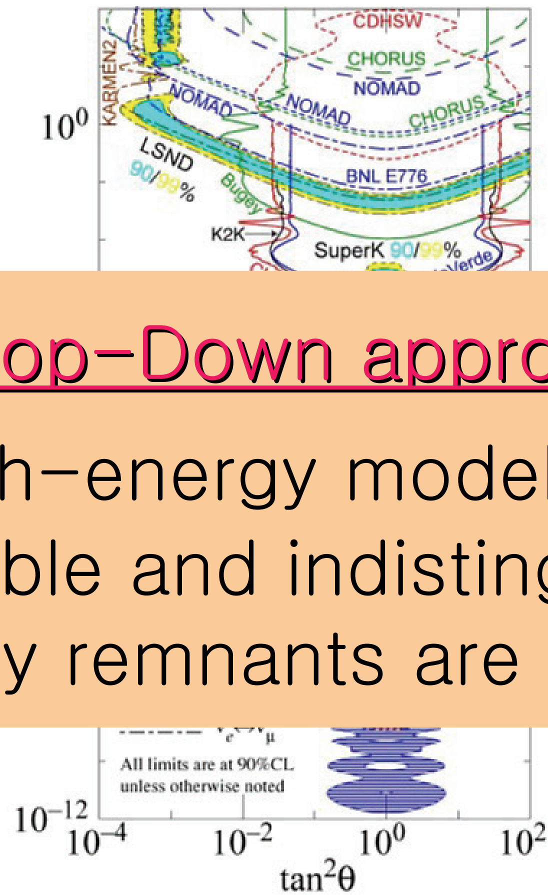
0. Introduction

Current exp results

model

model



Top-Down approach

high-energy models are undetectable and indistinguishable.....
(only remnants are found)

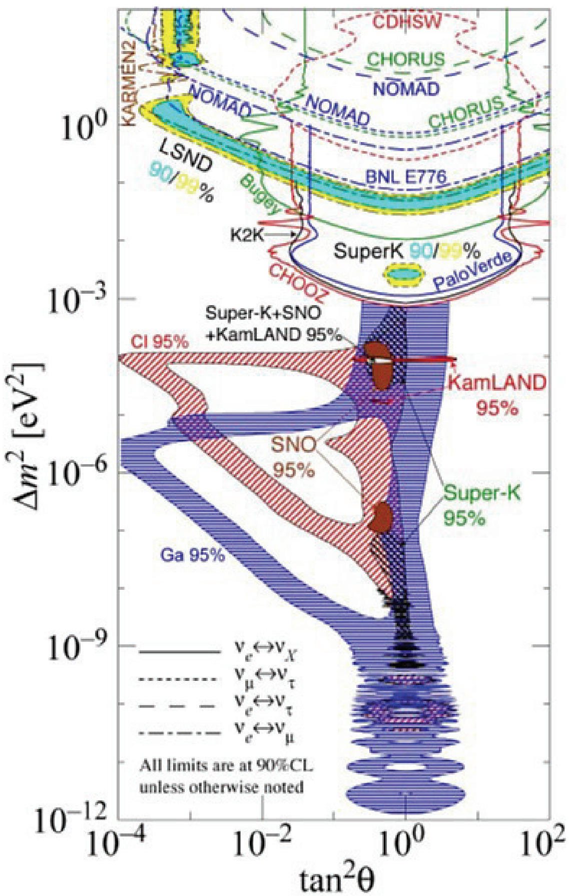
mode

model

model

H.Murayama (2006)

more "reasonable" approach (this talk) :



Extract and Identify the particle (neutrino) property (generation structure) in Nature (in fundamental theory)

Flavor Theory

Plan of talk

1. Lepton masses and mixing
2. Tri–bi (TB) maximal generation mixing
3. Flavor symmetry
4. TB mixing and Neutrino flavor structure
5. Summary

1. Lepton masses and mixing

M_e : charged lepton mass matrix

M_ν : neutrino mass matrix (Majorana or Dirac)

$$V_e^\dagger M_e V_{e_R} = M_e^{\rm diag}$$

$$V_\nu^{\rm T} M_\nu V_\nu = M_\nu^{\rm diag}$$

(similar exp for Dirac M_ν)

$$V_{\rm MNS} = V_e^\dagger V_\nu$$

$$= \begin{pmatrix} 1 & & \\ & \cos\theta_{23} & \sin\theta_{23} \\ & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \begin{pmatrix} \cos\theta_{13} & & \sin\theta_{13}e^{-i\delta} \\ & 1 & \\ -\sin\theta_{13}e^{i\delta} & & \cos\theta_{13} \end{pmatrix} \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & \\ -\sin\theta_{12} & \cos\theta_{12} & \\ & & 1 \end{pmatrix} V_{\rm phase}$$

Current experimental data

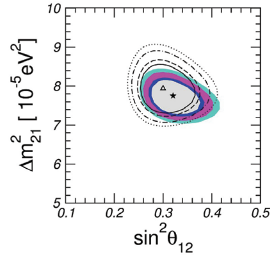
best fit

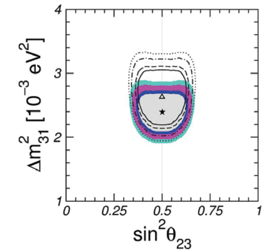
$$\sin^2\theta_{12} = 0.26 - \boxed{0.32} - 0.40 \quad \text{large}$$

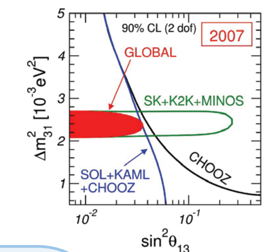
$$\sin^2\theta_{23} = 0.34 - \boxed{0.50} - 0.67 \quad \text{maximal}$$

$$\sin^2\theta_{13} \leq 0.050 \quad \text{small}$$

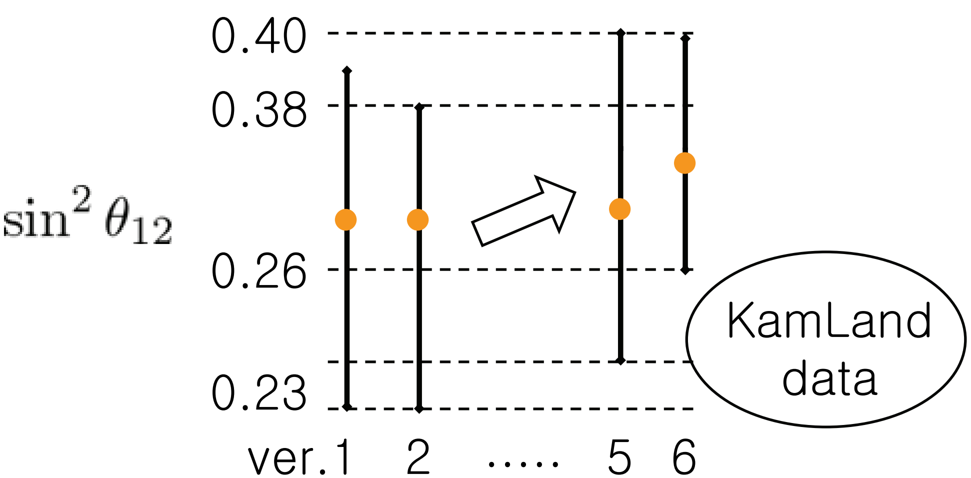
(3 sigma)







Maltoni et al [hep-ph/0405172 ver.6 (Sep 2007)]



$$\sin^2\theta_{12} \simeq \frac{1}{3}$$

$$\sin^2\theta_{23} \simeq \frac{1}{2}$$

2. Tri-bi maximal generation mixing

Harrison–Perkins–Scott (2002)

$$V_{\text{tri-bi}} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\left.\begin{aligned} \nu_2 &= \text{tri-maximal mixture of } \nu_e, \nu_\mu, \nu_\tau \\ \nu_3 &= \text{bi-maximal mixture of } \nu_\mu, \nu_\tau \end{aligned}\right\}$$

unique

$\Rightarrow$

- $\sin^2\theta_{12} = \frac{1}{3}, \quad \sin^2\theta_{23} = \frac{1}{2}, \quad \sin^2\theta_{13} = 0$

  close to the best fit values ! $\left(V_{\text{tri-bi}} \simeq V_{\text{MNS}}^{\text{exp}} \right)$

- no Dirac CP phase

Comparison to the democratic mixing

$$V_{\rm MNS} = V_e^\dagger = V_{\rm demo}$$: democratic mixing

$$V_{\rm demo} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{-1}{\sqrt{6}} & \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}$$

Harari–Haut–Weyers
Koide
Fukugita–Tanimoto–Yanagida
:

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \xrightarrow{V_{\rm demo}} \begin{pmatrix} & & \\ & & \\ & & 3 \end{pmatrix}$$

2 mass eigen leptons are

tri–maximal mixture of e, μ, τ and
bi–maximal mixture of e, μ

$V_{\rm demo}$ is a TB mixing matrix in the charged–lepton sector

[but experimentally disfavored (by the solar angle)]

Theoretical perspective

$$\begin{pmatrix} 0.785 & 0.620 & 0.023 \\ -0.396 & 0.530 & -0.750 \\ -0.477 & 0.580 & 0.661 \end{pmatrix}$$

~~"parameter fitting"~~

$$\begin{pmatrix} 0.805 & 0.585 & 0.101 \\ -0.479 & 0.542 & 0.690 \\ 0.349 & -0.603 & 0.717 \end{pmatrix} \longrightarrow V_{\mathrm{MNS}}^{\mathrm{exp}} \Longrightarrow \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

TB mixing

- good theoretical motivation
- a key to looking for "hidden" flavor structure
- flavor symmetry plays important role

(cf.) S_3 group $\leftrightarrow$ V_{demo}

3. Flavor symmetry

Standard Model $=$ gauge sector $+$ Yukawa sector

gauge sym　　flavor sym

- abelian or non−abelian ?

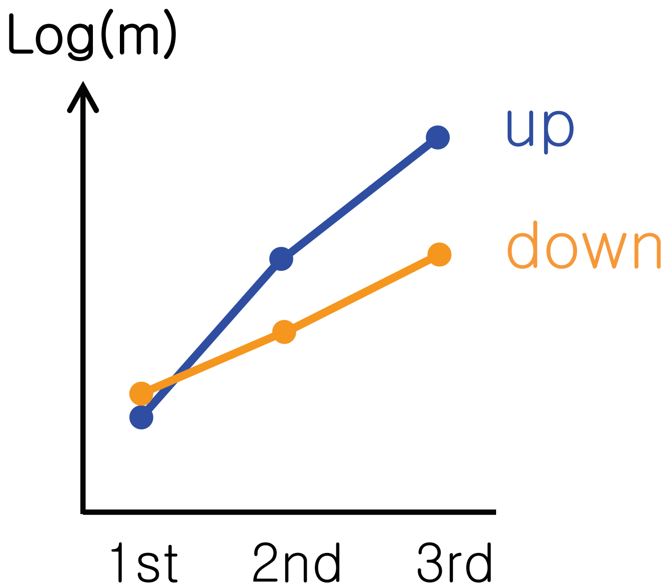
abelian : discriminate between generations
non−abelian : connect different generations

- continuous or discrete ?

continuous : free rotation between generations
discrete : definite meaning of generations

Non−Abelian Discrete flavor symmetry is appropriate for (lepton) flavor physics

The quark sector



For example :

$$M_{\rm up} \sim \begin{pmatrix} \epsilon^8 & \epsilon^6 & \epsilon^4 \\ \epsilon^6 & \epsilon^4 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & \epsilon^0 \end{pmatrix}$$

$$M_{\rm down} \sim \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^3 \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon^2 & \epsilon^1 & \epsilon^0 \end{pmatrix}$$

- large mass hierarchy
- small mixing

i.e. "separate" generations

$\cancel{\Leftrightarrow}$ non-abelian flavor sym ....

U(1) Froggatt-Nielsen mechanism ○?

4. TB mixing and Neutrino flavor structure

What is the form of M_ν for TB mixing $(V_{\rm MNS}^{\rm exp})$?

$$M_\nu^{\rm exp} \simeq V_{\rm tri-bi}^* \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix} V_{\rm tri-bi}^\dagger$$

$$= \boxed{\frac{m_1+m_3}{2}\begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} + \frac{m_2-m_1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{m_1-m_3}{2}\begin{pmatrix} 1 & & \\ & & 1 \\ & 1 & \end{pmatrix}}$$

- integer (inter-family related) matrix elements

  $\Longleftrightarrow$ non-abelian discrete flavor sym

- the exp data ($V_{\rm tri-bi}$) implies the neutrino mixing is determined independently of mass eigenvalues

$\left(\theta_{ij} \cancel{\sim} \sqrt{\frac{m_i}{m_j}}\right)$

TB mixing indicates that Flavor structure

S_3 or A_4

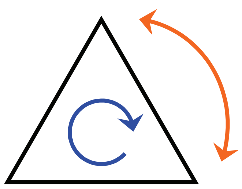
is hidden in the lepton sector

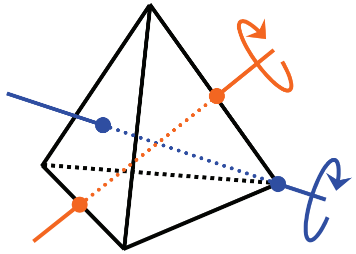
S_3 : permutation of 3 objects
(3 generations)

A_4 : even permutation of 4 objects

$$\begin{pmatrix} \text{3-dim representation x 1} \\ \text{1-dim representation x 3} \end{pmatrix}$$

suitable for 3 generations





TB mixing and S_3

$$M_\nu^{\rm exp} = \frac{m_1+m_3}{2}\begin{pmatrix}1&&\\&1&\\&&1\end{pmatrix} + \frac{m_2-m_1}{3}\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix} + \frac{m_1-m_3}{2}\begin{pmatrix}1&&\\&&1\\&1&\end{pmatrix}$$

- The 1st and 2nd terms are S_3 symmetric
  (the most general S_3–invariant forms)
- The 3rd is not (SS_3 flavor sym breaking)

KEY for $V_{\text{tri–bi}}$

(cf.) gauge sym breaking
in the SM gauge sector

But...

- fine tuning required experimentally

  the 3rd term $\begin{pmatrix} \gamma & & \\ & & \delta \\ & \delta & \end{pmatrix} \Longrightarrow \left|1 - \frac{\gamma}{\delta}\right| \lesssim 0.12 \left(\frac{\Delta m_{21}^2}{\Delta m_{31}^2}\right)^{\frac{1}{2}\sim 1} \lesssim O(0.01)$

- any subgroup of S_3 does not work .....

$\Longrightarrow$ Extra property of neutrinos :

- $m_1 \simeq m_3$ Chen–Wolfenstein
  - * the 3rd term negligible
  - * degenerate neutrino masses
- Magic matrix $\sum_i M_{\nu_{ij}} = \sum_j M_{\nu_{ij}}$ Lam
- Twisted flavors Haba–Watanabe–KY
- Extra higgs contributions Mohapatra–Nasri–Yu

  ⋮

TB mixing and A4

$$M_\nu^{\exp} = \frac{m_1+m_3}{2}\begin{pmatrix}1 & & \\ & 1 & \\ & & 1\end{pmatrix} + \frac{m_2-m_1}{3}\begin{pmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix} + \frac{m_1-m_3}{2}\begin{pmatrix}1 & & \\ & & 1\\ & 1 & \end{pmatrix}$$

A4 structure is hidden :

- The 3rd term is A4 symmetric
- A 3-dim higgs gives the general A4−symmetric Majorana mass term:

$$M_\nu^{A_4} = \begin{pmatrix}a & & \\ & b & \\ & & c\end{pmatrix} - \frac{1}{3}\begin{pmatrix}a & c & b\\ c & b & a\\ b & a & c\end{pmatrix} + x\begin{pmatrix}1 & & \\ & & 1\\ & 1 & \end{pmatrix}$$

$$a = b = c \iff V_{\text{tri}-\text{bi}}$$

A4 flavor sym breaking $\rightarrow$ Z2 : $\frac{1}{3}\begin{pmatrix}-1 & 2 & 2\\ 2 & -1 & 2\\ 2 & 2 & -1\end{pmatrix}$

Other approaches

* different flavor structure

(tri−bi maximal) = (tri maximal) − (maximal)

$$V_{\rm MNS}^{\rm exp} \simeq V_{\rm tri-bi} = V_{\rm tri}^{\dagger} V_{\rm max}$$

from V_e

from V_ν

cubic roots of unity

$$V_{\rm tri} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & e^{\frac{2\pi}{3}i} & e^{\frac{4\pi}{3}i} \\ 1 & e^{\frac{4\pi}{3}i} & e^{\frac{2\pi}{3}i} \end{pmatrix} \qquad V_{\rm max} = \begin{pmatrix} \frac{1}{\sqrt{2}} & & \frac{-1}{\sqrt{2}} \\ & 1 & \\ \frac{1}{\sqrt{2}} & & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$V_{\text{tri−bi}}$ is divided by 2 pieces
both of which may be theoretically justified
(e.g. almost all A_4 work adopt this flavor structure)

* different flavor symmetry (discrete, non-abelian)

of group elements

6	S_3
8	D_4 Q_4
10	D_5
12	D_6 Q_6 A_4
14	D_7
16	D_8 Q_8 $Z_2 \times D_4$ $Z_2 \times Q_4$
18	D_9 $Z_3 \times D_3$
20	D_{10} Q_{10}
22	D_{11}
24	D_{12} Q_{12} $Z_2 \times D_6$ $Z_2 \times Q_6$ $Z_2 \times A_4$
	$Z_3 \times D_4$ $Z_3 \times Q$ $Z_4 \times D_3$ S_4
26	D_{13}
28	D_{14} Q_{14}
30	D_{15} $D_5 \times Z_3$ $D_3 \times Z_5$

= many people

= some people

= curious people

- models become complex
- little advantage

Nature does not like these complex structures...?

Deviations from exact TB mixing

- TB mixing is a (good) approximation
- TB mixing is exact only in flavor–symmetric theory

  (flavor sym breaking is important)
- Uncertainty in exp data

a clue to deeper understanding

⇨ Deviations from TB mixing

⇨
- decreasing θ_{12} (really needed...?)
- nonzero θ_{13}
- (Dirac phase) CP violation

Future neutrino experiments will give further information

5. Summary

The tri-bimaximal generation mixing $\left(\sin^2\theta_{12} = \frac{1}{3} \quad \sin^2\theta_{23} = \frac{1}{2} \quad \sin^2\theta_{13} = 0\right)$

- good approximation of $V_{\mathrm{MNS}}^{\mathrm{exp}}$ (close to the best-fit values)
- theoretically well motivated
- indicates definite symmetric structure of Yukawa sector
    - non-abelian discrete flavor symmetry
    - S_3 or A_4
- independent of mass eigenvalues
- incompatible with exact flavor symmetry
- important to understand flavor symmetry breaking
    - deviation from exact $V_{\mathrm{tri-bi}}$
    - nonzero θ_{13}
    - CP violation in future neutrino exp.

will provide deeper understanding of flavor physics