Neutrino masses, dark matter
and baryon asymmetry

The νMSM

Takehiko Asaka
(Niigata University)

@ ICRR, Tokyo Univ.
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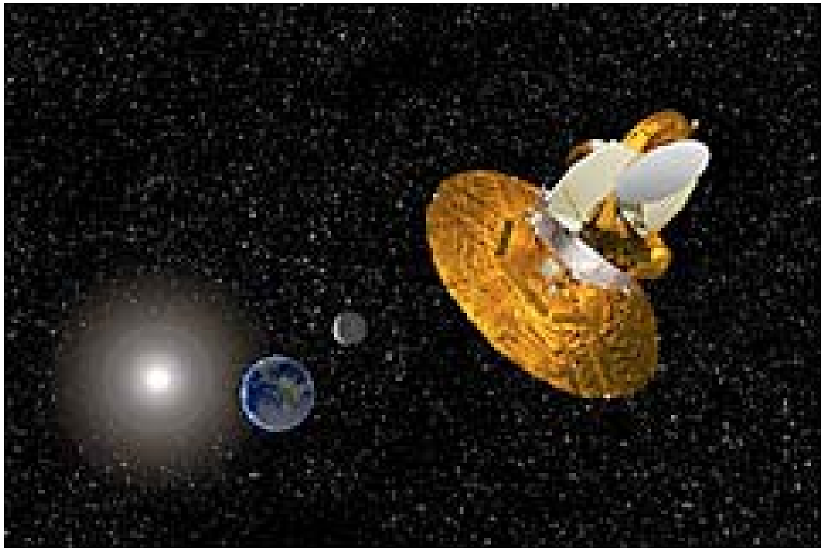
Prologue: Physics beyond the MSM

- About 10 years ago …,
  - There was no "convincing" evidence for physics beyond the minimal standard model (MSM)
  - People looked for physics beyond the MSM "mainly" based on theoretical arguments:
    - Hierarchy problem
    - Gravity, String, …
    - Strong CP problem
    - Why 3 generations?
    - Why anomalies cancel?
    - …

Neutrino Oscillations

[SuperK]

Cosmic Microwave Background (CMB)



[WMAP]

Physics beyond the MSM

- In the last decade(s), we have collected quite "convincing" evidences for physics beyond the MSM
  - Neutrino oscillations → non-zero neutrino masses
  - Baryon asymmetry
  - Dark matter
  - Dark energy
  - Scale-invariant density perturbations

Physics beyond the MSM

- In the last decade(s), we have collected quite "convincing" evidences for physics beyond the MSM

  - Neutrino oscillations → non-zero neutrino masses
  - Baryon asymmetry
  - Dark matter
  - ?? Dark energy
  - ?? Scale-invariant density perturbations

- Today, I would like to explain the νMSM, which can solve first three problems!

Outline

- The νMSM
  = the "*neutrino*" Minimal Standard Model
- Dark matter in the νMSM
- Baryogenesis in the νMSM
- Summary

Neutrino oscillations

- Evidence of neutrino oscillations
  → non-zero neutrino masses
  - Atmospheric $\Delta m^2_{\rm atm} \simeq 2.5 \times 10^{-3}\,{\rm eV}^2$
    - Atmospheric neutrino exps. (…, SuperK)
    - Long-baseline accelerator exps. (K2K, MINOS)
  - Solar $\Delta m^2_{\rm sol} \simeq 8.0 \times 10^{-5}\,{\rm eV}^2$
    - Solar neutrino exps. (…, SuperK, SNO)
    - Reactor exp. (KamLand)
- Need for
  physics beyond the minimal standard model (MSM)

The νMSM

- Adding three right-handed neutrinos N_I (I=1,2,3)

$$\mathcal{L}_{\nu\mathrm{MSM}} = \mathcal{L}_{\mathrm{MSM}} + i\overline{N}_I \partial\!\!\!/ N_I - F_{\alpha I}\overline{L}_\alpha \Phi N_I - \frac{M_I}{2}\overline{N^c_I} N_I + h.c.$$

- 18 new parameters
  - 3 Majorana masses
  - 15 parameters in Yukawa coupling matrix
    - 3 Yukawa couplings
    - 6 mixing angles
    - 6 phases

- Dirac and Majorana masses of neutrinos

$$M_D = F\langle\Phi\rangle \qquad\qquad M_M = M_I$$

Seesaw mechanism

- If Majorana masses >> Dirac masses,

$$\begin{pmatrix} 0 & M_D \\ M_D^T & M_M \end{pmatrix} \Rightarrow \begin{pmatrix} M_\nu & 0 \\ 0 & M_M \end{pmatrix}$$

$$M_\nu = -M_D^T \frac{1}{M_M} M_D$$

$$M_M = diag(M_1, M_2, M_3)$$

  - **active neutrinos**

    $$\nu_1,\ \nu_2,\ \nu_3\ (m_1 \le m_2 \le m_3)$$

    $$U^T M_\nu U = diag(m_1, m_2, m_3)$$

  - **sterile neutrinos**

    $$N_1,\ N_2,\ N_3\ (M_1 \le M_2 \le M_3)$$

- Mixing in CC current

$$\nu_\alpha = U_{\alpha a}\ \nu_a + \theta_{\alpha I}\ N_I^c$$

$(\alpha = e, \mu, \tau)$ $\quad$ $(a = 1, 2, 3)$

$$\theta_{\alpha I} = (M_D)_{\alpha I} / M_I \ll 1$$

active-sterile mixing

Scale for Majorana mass

- Neutrino oscillations are explained by flavor mixing between active neutrinos !
  - Masses of active neutrinos

$$M_\nu = -M_D^T \frac{1}{M_M} M_D$$

$$\Delta m^2_{\rm atm} \simeq 2.5 \times 10^{-3}\,{\rm eV}^2$$

$$\Delta m^2_{\rm sol} \simeq 8.0 \times 10^{-5}\,{\rm eV}^2$$

- Where is the scale of Majorana mass ??
  - Two "natural" options
    - Conventional seesaw
    - The νMSM

Conventstional seesaw scenario:

- Neutrino Yukawa couplings are comparable to those of quarks and charged leptons
  - $M_R >> 100\text{GeV}$

$$M_R \simeq 6 \times 10^{14}\,\text{GeV}\, f_\nu^2 \left(\frac{2.5 \times 10^{-3}\,\text{eV}^2}{m_\nu^2}\right)^{1/2} \qquad m_\nu \simeq \frac{M_D^2}{M_R}$$

  - Explain smallness of neutrino masses via seesaw [Yanagida; Gell-Mann, Ramond, Slansky]
  - Decays of RH neutrino(s) can account for baryon asymmetry through leptogenesis [Fukugita, Yanagida]
  - Physics of RH neutrino cannot be tested directly by experiments

The νMSM: [TA, Blanchet, Shaposhnikov; TA, Shposhnikov]

- No new mass scale is introduced

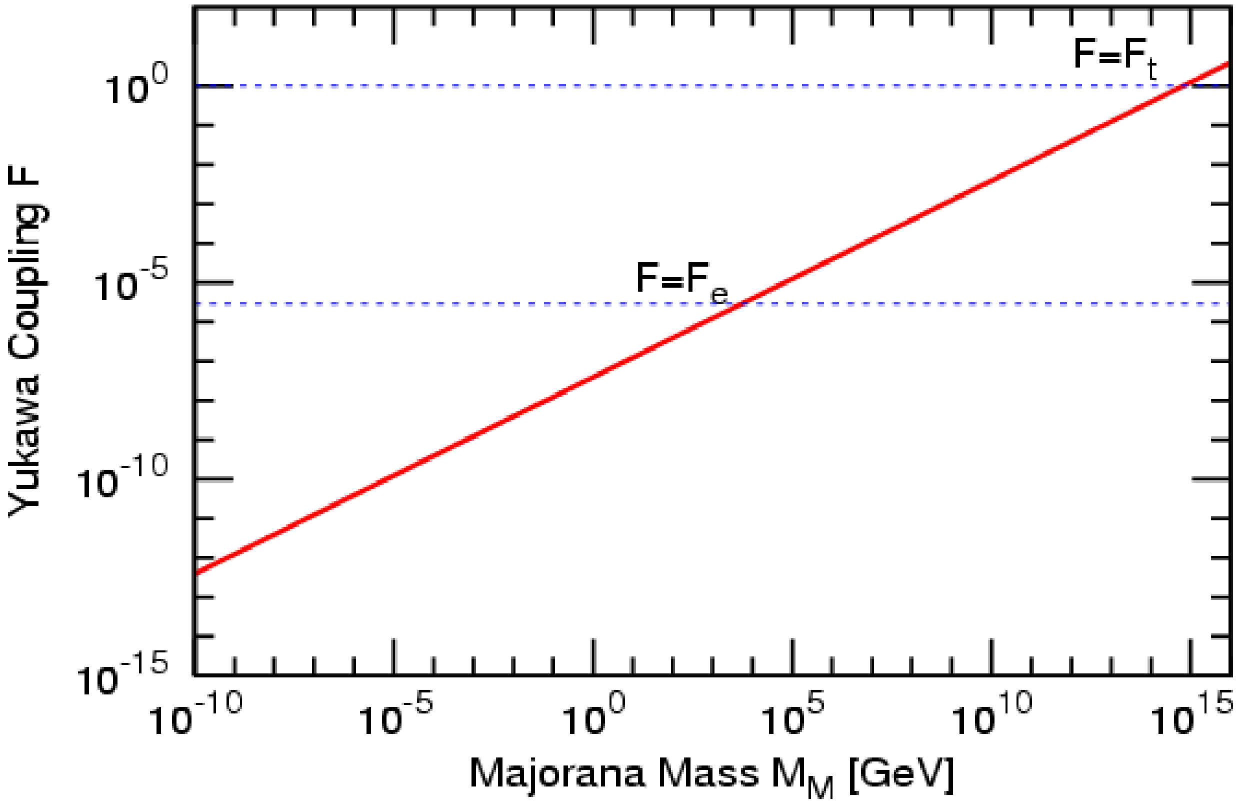
  - M_R ~< 100GeV

$$f_\nu \simeq 4 \times 10^{-7} \left(\frac{M_R}{100\mathrm{GeV}} \right)^{1/2} \left(\frac{m_\nu^2}{2.5 \times 10^{-3} \mathrm{eV}^2} \right)^{1/4} \qquad m_\nu \simeq \frac{M_D^2}{M_R}$$

    - Lightest RH neutrino (~keV) can be DM (?) [Dodelson, Widrow,…]
    - Oscillation of RH neutrinos can account for baryon asymmetry of the universe [Akhmedov, Rubakov, Smirnov]
    - Physics of RH neutrinos can potentially tested by experiments

Scale of Majorana mass

$$M_\nu = -M_D^T \frac{1}{M_M} M_D \quad \Rightarrow \quad F^2 = M_M M_\nu / \langle \Phi \rangle^2$$



$$M_\nu = \sqrt{\Delta m_{atm}^2}$$

Very small Yukawa couplings!

Dark matter in the νMSM

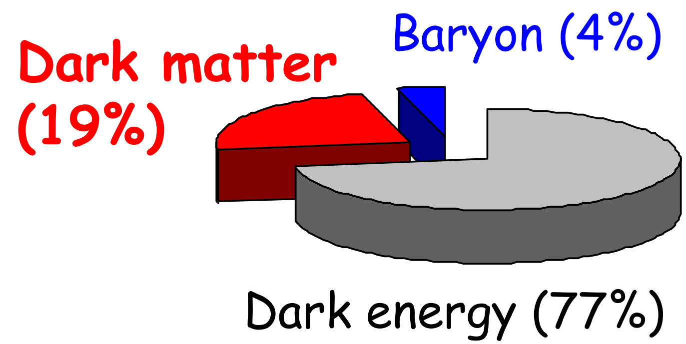
Dark matter

- Cosmological parameters are well determined now !
  - From CMBR anisotropy [WMAP '06]

$$\Omega_{\rm dm}h^2 = 0.105^{+0.007}_{-0.013}$$

$$\Omega_{\rm dm} = \rho^0_{\rm dm}/\rho_{\rm cr}$$

h : H_0 in units of 100km/sec/Mpc



- Particle physics candidate
  - "dark" (charge neutral)
  - stable within the age of the universe $\tau > t_U \sim 10^{17}$ sec
  - its abundance should be $\Omega_{\rm dm}h^2$
  - avoids cosmological constraints

→ No candidate in the MSM

Dark matter in the νMSM

- Unique candidate:
  lightest sterile neutrino N1 with ~ keV mass

Dodelson, Widrow / Shi, Fuller / Dolgov, Hansen / Abazajian, Fuller, Patel

Cf. Massive active neutrinos cannot be dark matter

- Active neutrinos are too "hot" (hot dark matter)

$$\sum m_\nu < 0.62\,\mathrm{eV}$$

[Hannestad, Raffelt]

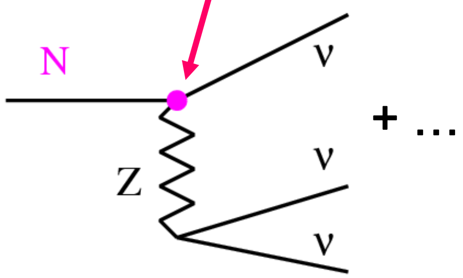
$$\Omega_\nu h^2 = \frac{\sum m_\nu}{93\,\mathrm{eV}} < 0.0067$$

But, $\Omega_{\mathrm{dm}} h^2 = 0.105^{+0.007}_{-0.013}$

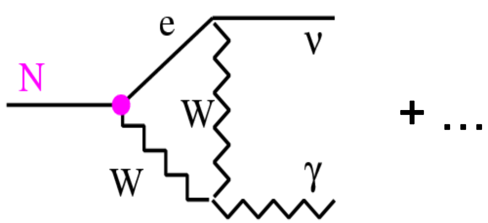
Decays of sterile neutrino

- N1 is not completely stable particle !
  - Dominant decay: $N_1 \to 3\nu$ for $M_1 \sim \text{keV}$
  - Lifetime can be very long
    - $\tau_{N_1} \simeq 5 \cdot 10^{26} \text{sec} \left(\frac{\text{keV}}{M_1}\right)^5 \left(\frac{10^{-8}}{\theta^2}\right)$

$\theta = M_D/M_1$



- N1 is not completely dark !
  - Subdominant decay: $N_1 \to \nu + \gamma$
  - Branching ratio is small
    - $\mathbf{Br} = 27\alpha_{\text{em}}/8\pi$
  - But, severely restricted from X-ray observations



Production of sterile neutrino

- To realize sterile neutrino DM, $\Omega_{N_1} h^2 = \Omega_{\rm dm} h^2 \simeq 0.1$
  - How are they produced???

    Cf. $\Omega_{N_1} = \rho_{N_1}^0 / \rho_{\rm cr}$

- In the early universe,
  - Interaction rate of N1 is very small:
    - Typically, $\Gamma_{\rm int} \sim f_\nu^2 T$

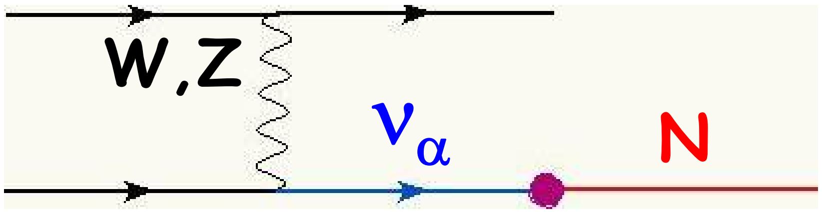
      $$\Gamma_{\rm int} > H \sim T^2/M_{\rm pl} \Rightarrow \quad f_\nu \gtrsim \sqrt{\frac{T}{M_{\rm pl}}} \sim 10^{-8} \left(\frac{T}{100\text{GeV}} \right)^{1/2}$$

  - We will be interested in $f_\nu = O(10^{-12})$

    → N1 is not thermalized !

Production of sterile neutrino

- Dodelson-Widrow scenario:
  - Production via active-sterile neutrino mixing θ



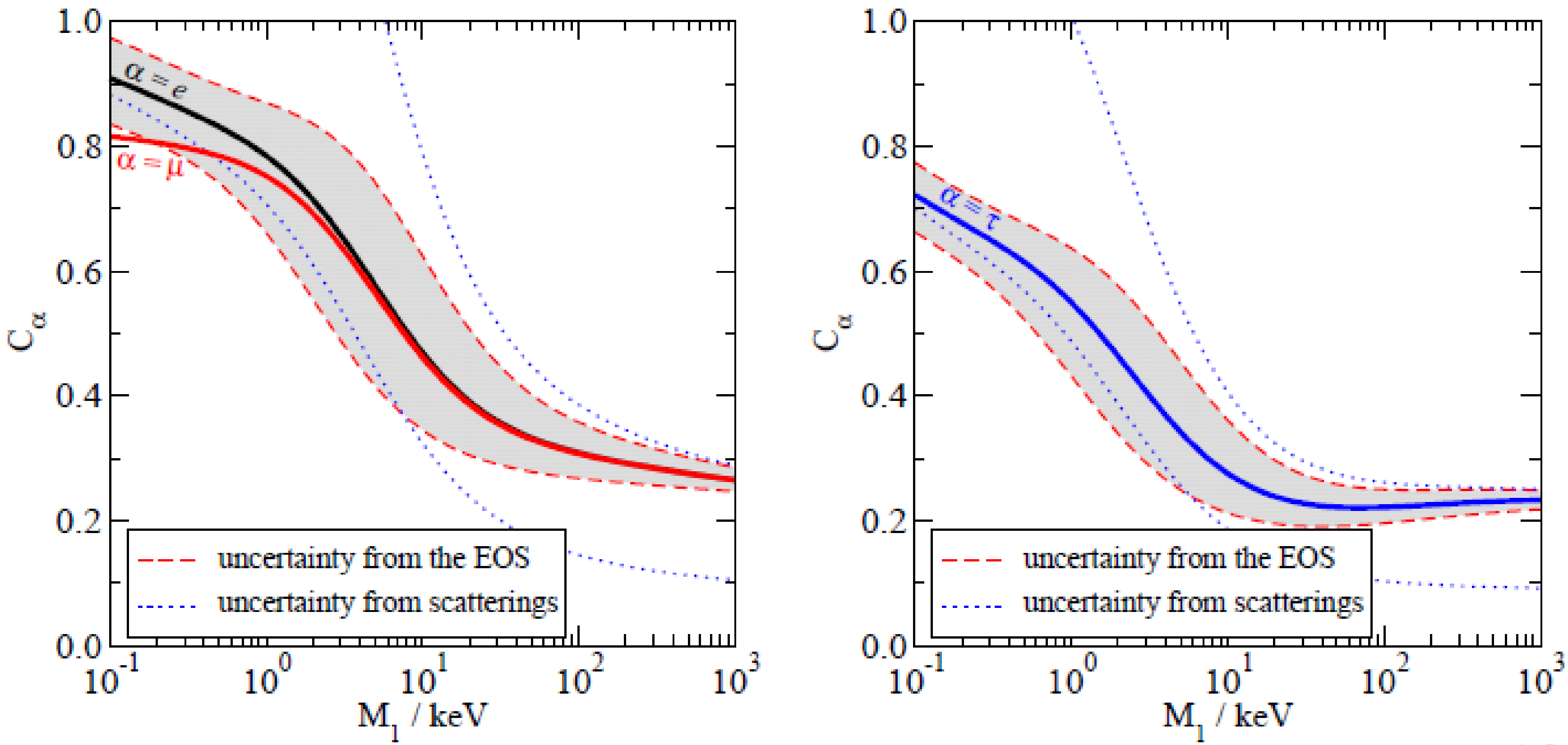
  - Dominant production at $T_* \simeq 100\text{MeV}\,(M_1/\text{keV})^{1/3}$

- Recently, we improve the estimate of the abundance
  - Use kinetic equation for density matrix
  - Study "hadronic uncertainties" in detail:
    - Hadronic contributions to production rate
    - QCD equation of state (time-temp. relation)

TA, Laine, Shaposhnikov JHEP06 ('06) 053 [hep-ph/0606209]
TA, Laine, Shaposhnikov [hep-ph/0612182]

Relic density of sterile neutrino

- $$\Omega_{N_1} h^2 = 0.11 \sum_{\alpha=e,\mu,\tau} C_\alpha(M_1) \left(\frac{|M_D|_{\alpha 1}}{0.1\text{eV}} \right)^2$$



$$\Omega_{N_1} = \Omega_{\rm dm} \Rightarrow |M_D|_{\alpha I} \sim 0.1 \text{ eV}, \quad |F_{\alpha I}| \sim 10^{-12}$$

Upper bound on mixing angle

- $\Omega_{N_1} \propto \theta^2$

- **We find** $C_e > C_\mu > C_\tau$

  - **The case 1:**

    the largest abundance

    the strongest bound

    $|M_D|_{e1} \neq 0$

    $|M_D|_{\mu 1} = |M_D|_{\tau 1} = 0$

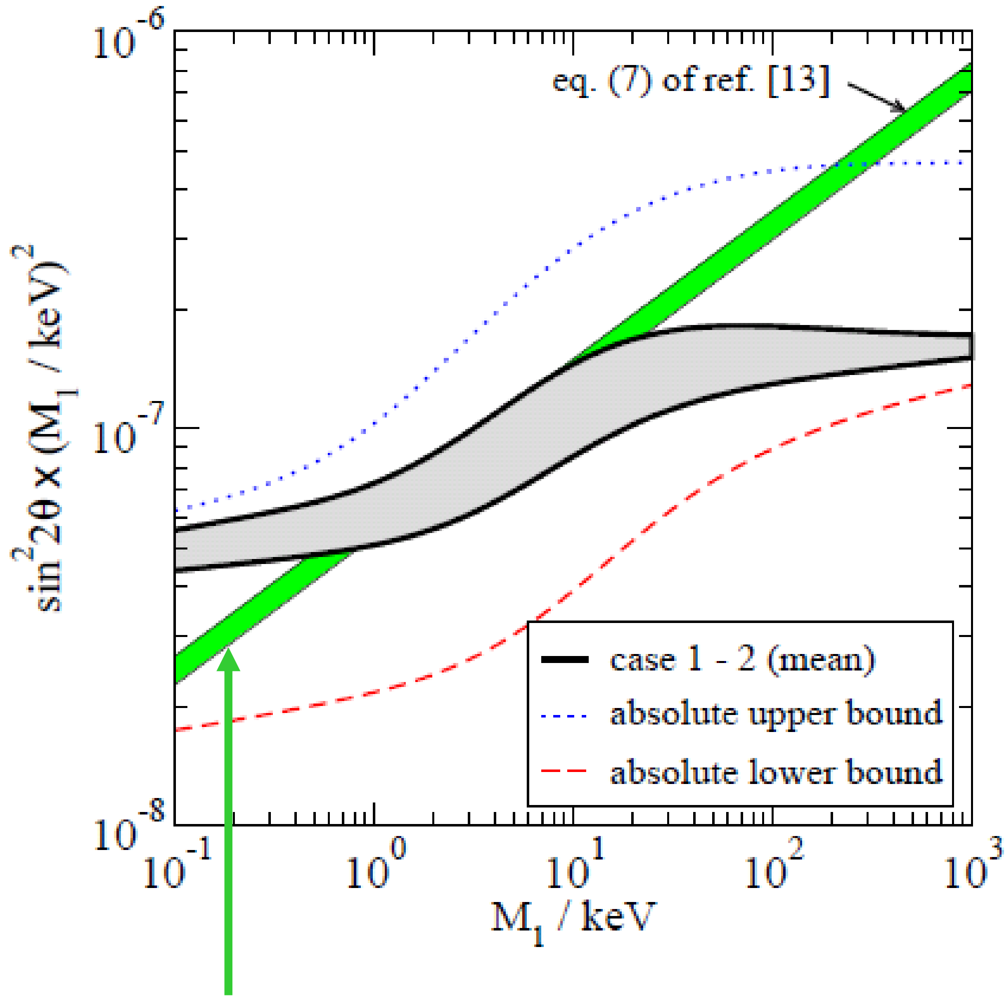
  - **The case 2:**

    the smallest abundance

    the weakest bound

    $|M_D|_{\tau 1} \neq 0$

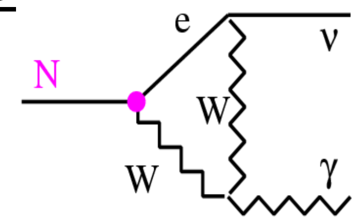
    $|M_D|_{e1} = |M_D|_{\mu 1} = 0$



Abazajian: Phys.Rev. D73 ('06) 063506

$\Omega_{\rm DM} = 0.2\,, \quad T_c = 150 - 200\mathrm{MeV}$

Constraints from X-rays



- Radiative decays of sterile neutrino DM
  - feature in X-ray background spectrum
  - line X-ray from clusters, galaxies, dwarf galaxies…
    - TEST for sterile neutrino DM!
    - No signal → Upper bound on mixing angle !

Dolgov, Hansen / Abazajian, Fuller, Tucker
Boyarsky, Neronov, Ruchayskiy, Shaposhnikov
Boyarsky, Neronov, Ruchayskiy, Shaposhnikov
Boyarsky, Neronov, Ruchayskiy, Shaposhnikov, Tkachev
Riemer-Sorensen, Hansen, Pedersen
Watson, Beacom, Yuksel, Walker
Boyarsky, Ruchayskiy, Markevitch
Riemer-Sorensen, Pedersen, Hansen, Dahle
Abazajian, Markevitch, Koushiappas, Hickox
Boyarsky, Herder, Neronov, Ruchayskiy

Constraints from structure formation

- **Light sterile neutrino = WDM** $\lambda_{\rm FS} \sim \text{Mpc}\left(\frac{\text{keV}}{M_1}\right)\frac{\langle|\mathbf{q}_{\rm s}|\rangle}{\langle|\mathbf{q}_{\rm a}|\rangle}$
  - **Erase structures on smaller scales**
  - **Lower bound on mass** $M_1 > \frac{\langle|\mathbf{q}_{\rm s}|\rangle}{\langle|\mathbf{q}_{\rm a}|\rangle} M_0$
- **From Ly-α forest observations**

$M_0 \simeq 14.4\text{keV}$ **Seljak, Makarov, McDonald, Trac '06**

$M_0 \simeq 10\text{keV}$ **Viel, Lesgourgues, Haehnelt, Matarresse, Riotto '06**

We find: $\frac{\langle|\mathbf{q}_{\rm s}|\rangle}{\langle|\mathbf{q}_{\rm a}|\rangle} \simeq 0.8$ for $M_1 \simeq 10\text{keV}$

➡ $M_1 \gtrsim 11.6\,\text{keV}$ (SMMT), $8\,\text{keV}$ (VLHMR)

Parameter space

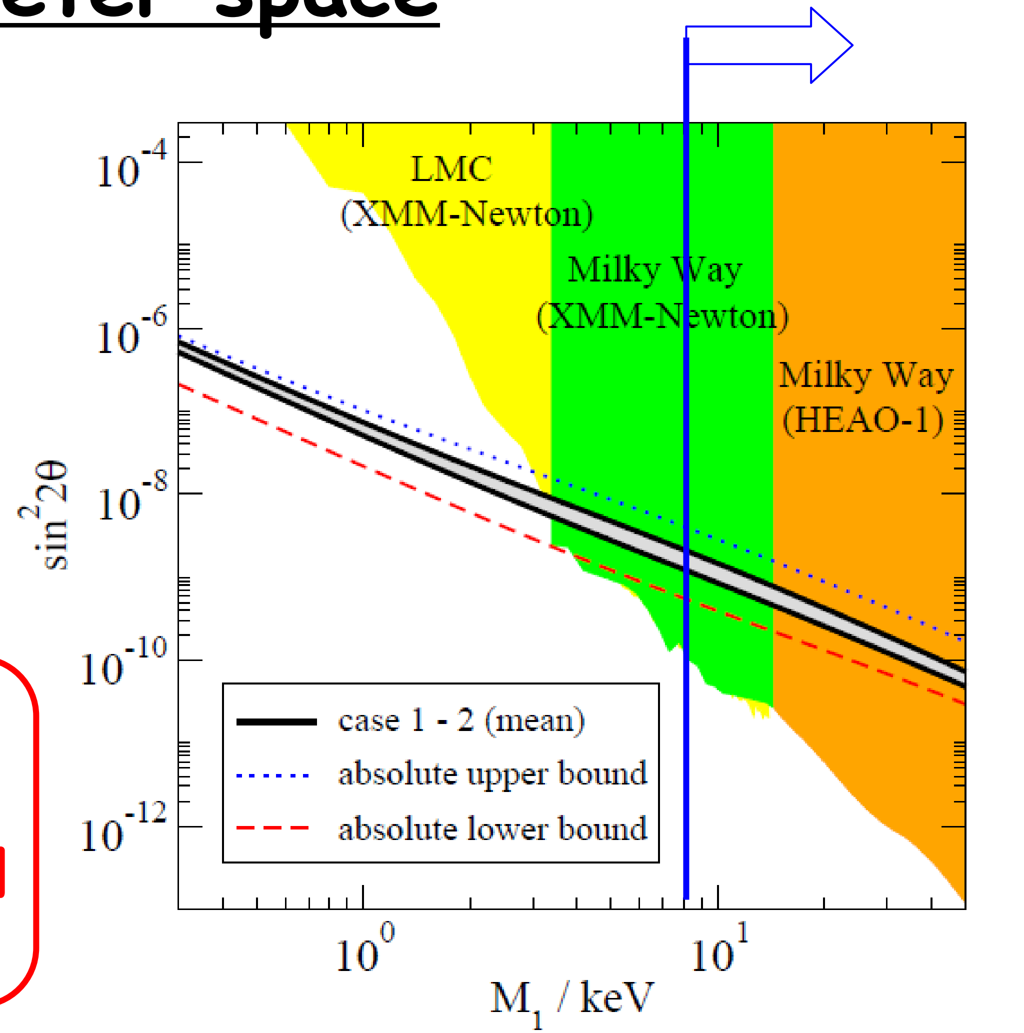
- **X-ray constraint:**

  $M_1 \lesssim (3.5)\ 6\ \text{keV}$

- **Ly-alpha constraint:**

  $M_1 \gtrsim 11.6\text{keV}$ (SMMT)

  $M_1 \gtrsim 8\text{keV}$ (VLHMR)



The Dodelson-Widrow scenario is ruled out in spite of all theoretical uncertainties !

X-ray constraints from

Boyarsky, Neronov, Ruchayskiy, Shaposhnikov, Tkachev (astro-ph/0603660)
Boyarsky, Nevalainen, Ruchayskiy (astro-ph/0610961)

Fate of sterile neutrino DM

- Dodelson-Widrow scenario assumes:
  - No initial abundance at T ~ 1GeV
  - No new interaction at E < 1GeV
  - Charge asymmetries smaller than baryon asymmetry
  - No low reheating, i.e. RD universe starts T>1GeV

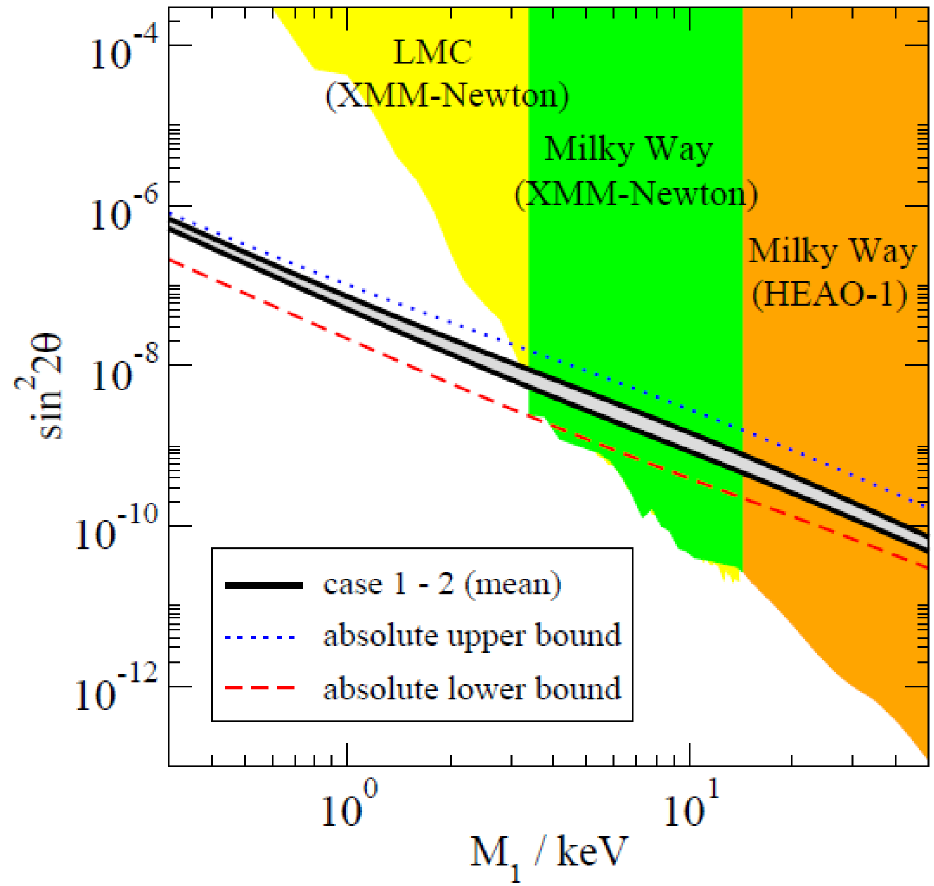
- Sterile neutrino DM is still possible, when
  - Large lepton asymmetry (Shi-Fuller)
    - resonant production via active-sterile oscillations
  - Production via decays of Inflaton/Scalar (Shaposhnikov, Tkachev / Kusenko)
  - . . .

Implications

- DM sterile neutrino N1 must have suppressed Yukawa interaction !

$$|F_{\alpha I}| \lesssim 10^{-12}$$

What this implies???



How many sterile neutrinos are needed?

- We need at least "two" sterile neutrinos to explain $\Delta m^2_{\rm atm} \simeq 2.5 \times 10^{-3}\,{\rm eV}^2, \quad \Delta m^2_{\rm sol} \simeq 8.0 \times 10^{-5}\,{\rm eV}^2$
  - In this case, the lightest active neutrino is massless

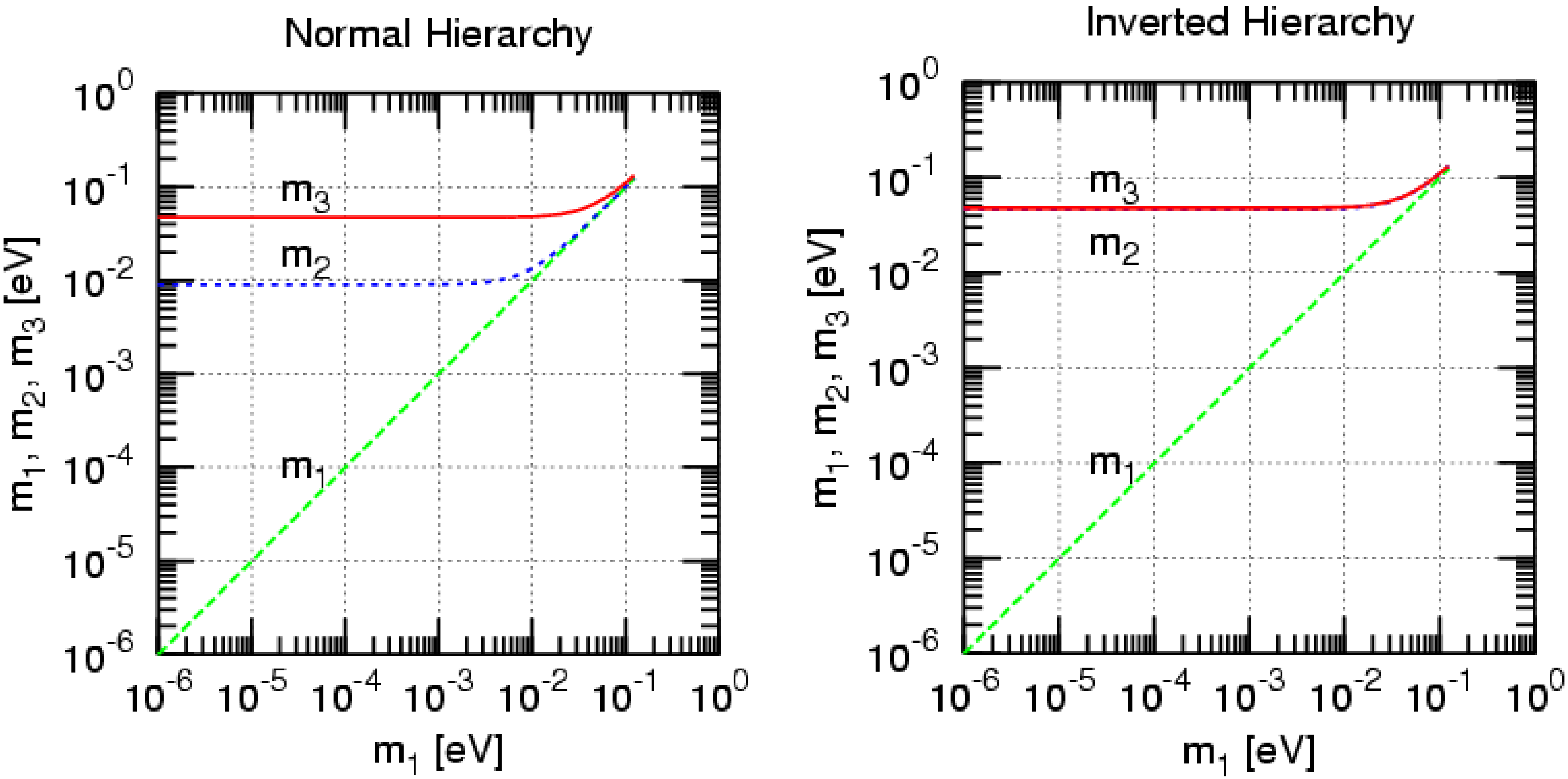
- DM sterile neutrino:

$$m_{\rm dm} = \sum_{\alpha=e,\mu,\tau} \frac{|M_D|^2_{\alpha 1}}{M_1} \simeq 2 \cdot 10^{-5}{\rm eV}\left(\frac{\rm keV}{M_1}\right) \ll \sqrt{\Delta m^2_{\rm sol}} \simeq 9 \times 10^{-3}{\rm eV}$$

  - DM sterile neutrino is irrelevant for explaining neutrino mass scales in oscillation experiments

- We need at least "three" sterile neutrinos !
  - In this case, $m_{\nu_1} \lesssim m_{\rm dm} \simeq 2 \cdot 10^{-5}{\rm eV}\left(\frac{\rm keV}{M_1}\right)$

Active neutrino masses



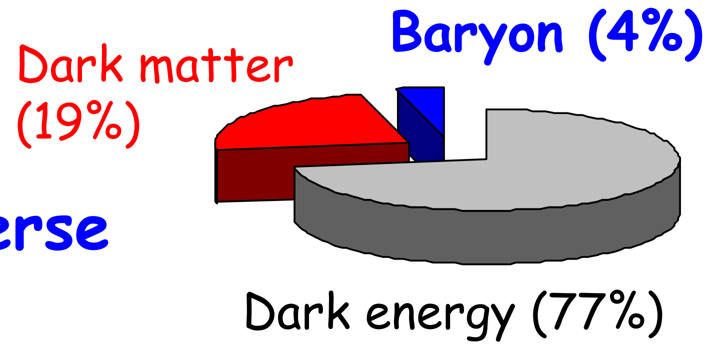
➡ **exclude the degenerate masses of active neutrinos**

Baryogenesis in the νMSM

Baryon asymmetry of the universe

- Observations:
  - Asymmetry of baryon and anti-baryon numbers in the universe

$$\frac{n_B}{s} = (8.4 - 8.9) \times 10^{-11}$$



- This non-zero asymmetry should be generated after, say, the reheating of inflation

  → Baryogenesis

- Three conditions for baryogenesis: Sakharov
  - Baryon number violation
  - C and CP violations
  - Out of equilibrium

Baryogenesis conditions in the MSM

- B and L violation
  - B and L violations in anomalous EW "sphaleron" which is in thermal equilibrium for T>100GeV
- CP violation
  - 1 CP phase in the quark-mixing (CKM) matrix

$$\mathrm{CPV} \propto J_{CP}(m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2)(m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2)/T_{EW}^{12} \sim 10^{-19}$$

→ too small

- Out of equilibrium
  - Strong 1st order phase transition if M_H<72GeV
    but $M_H > 114.4\mathrm{GeV}$ (exp.)

→ not satisfied

[Kajantie, Laine, Rummukainen, Shaposhnikov]

→ We have to go beyond the MSM !!

Baryogenesis in the νMSM

- B and L violations
  - EW sphaleron
  - L violation due to Majorana masses
    - Now we take Majorana masses < 100 GeV
      - Its violating effects can be neglected for high temperatures T>100 GeV

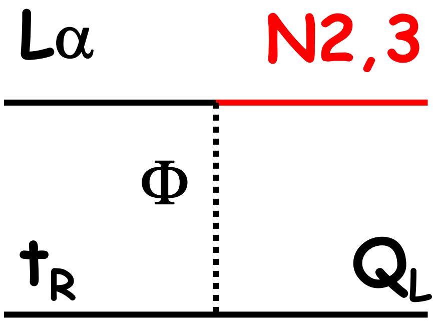
- C and CP violations
  - 1 CP phase in quark sector
  - 6 CP phases in lepton sector
    - Rich CP violation

Baryogenesis conditions in the νMSM

- Out of equilibrium
  - No 1st order EW phase transition as in the MSM
  - But, sterile neutrinos can be out of equilibrium if Yukawa couplings are small enough
    - To ensure this condition up to T~100GeV

➡ $f_{1,2,3} < 2\times 10^{-7}$

$[\mathrm{DM}: f_1 \simeq 6\times 10^{-13}]$



- To explain neutrino masses

➡ $M \leq 17\mathrm{GeV}$ (atm)

Baryogenesis in the νMSM

- The νMSM can potentially realize all three conditions for baryogenesis for T>100GeV
  - Masses of heavier sterile neutrinos N2 and N3

$$1\mathrm{GeV} \leq M_{2,3} \leq 17\mathrm{GeV}$$

BBN　　　　Out-of-equilibrium

- Is there a realistic scenario ???

Yes !

Baryogenesis via neutrino oscillations

Akhmedov, Rubakov, Smirnov '98

Idea: Sterile neutrino oscillation is a source of BAU

- Sterile neutrinos are created and oscillate with CPV
- The total lepton number is zero but is distributed between active and sterile neutrinos
- The asymmetry of active left-handed neutrinos is transferred into baryon asymmetry by sphaleron effects

Kinetic equation of the system

- **The system is described by 12x12 density matrix**

$$\rho = \begin{pmatrix} \rho_{LL} & \rho_{L\bar{L}} & \rho_{LN} & \rho_{L\bar{N}} \\ \rho_{\bar{L}L} & \rho_{\bar{L}\bar{L}} & \rho_{\bar{L}N} & \rho_{\bar{L}\bar{N}} \\ \rho_{NL} & \rho_{N\bar{L}} & \rho_{NN} & \rho_{N\bar{N}} \\ \rho_{\bar{N}L} & \rho_{\bar{N}\bar{L}} & \rho_{\bar{N}N} & \rho_{\bar{N}\bar{N}} \end{pmatrix} \qquad \rho_{IJ} = a_I^\dagger a_J \qquad \text{3x3 matrices}$$

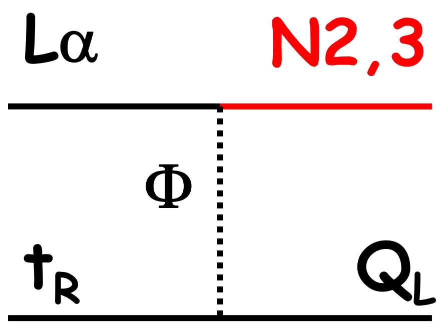
• **Diagonal terms give occupation numbers**
• **Off-diagonal terms contain correlations between states**

- **Kinetic equation**

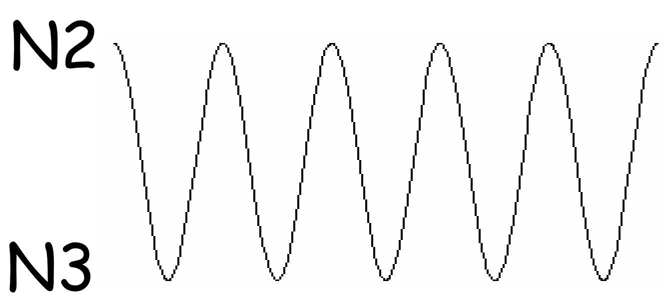
$$i\frac{d\rho}{dt} = [H,\rho] - \frac{i}{2}\left\{\Gamma^d,\rho\right\} + \frac{i}{2}\left\{\Gamma^p, 1-\rho\right\}$$

• **Effective Hamiltonian including medium effects** H
• **Destruction and production rates** Γ^d, Γ^p

- First step: at F^2 order
  - N2 and N3 are produced (N1 production is suppressed)



  - N2 and N3 are oscillate



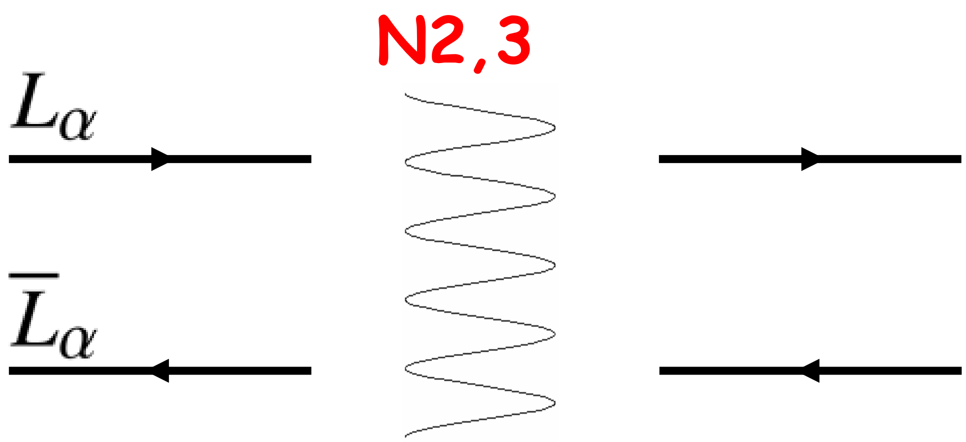
CPV in these processes are suppressed in the νMSM

- **Second step: at F⁴ order** [TA, Shaposhnikov]
  - **Active flavor asymmetries are generated**

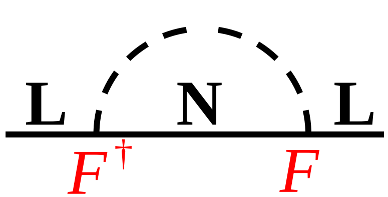
$$\Delta L_e \neq 0 \quad \Delta L_\mu \neq 0 \quad \Delta L_\tau \neq 0$$

but $\Delta L_{\rm tot} = \Delta L_e + \Delta L_\mu + \Delta L_\tau = 0$

Cf. $\Delta N_{\rm tot} = 0 \quad \Delta N_I = 0$

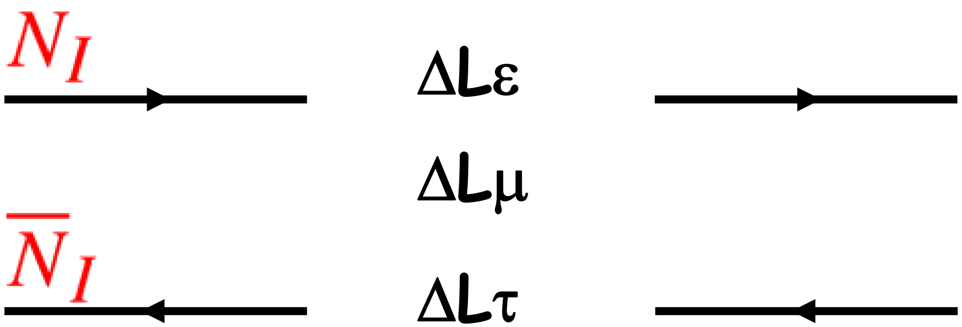


Evolution rates of L_α and $\overline{L}_\alpha$ are different due to CPV in

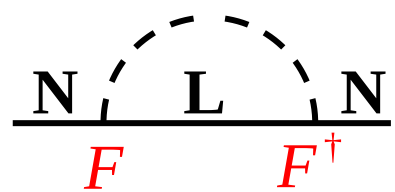


- **Final step: at F^6 order**
  - **Total asymmetries in active and sterile sectors are generated.**

$$\Delta L_{\rm tot} \neq 0 \quad \Delta N_{\rm tot} \neq 0 \quad \textbf{but} \quad \Delta L_{\rm tot} + \Delta N_{\rm tot} = 0$$

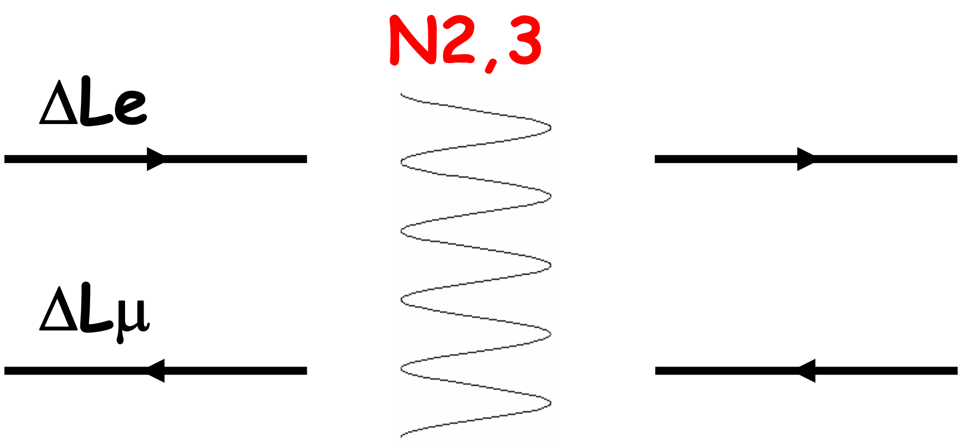


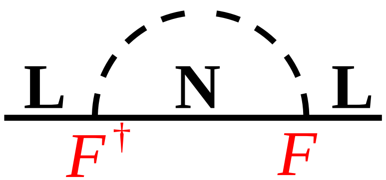
Evolution rates of N_I and $\overline{N}_I$ are different due to $\Delta L\alpha$ and CPV in



- Final step: at F^6 order (2)
  - Total asymmetries in active and sterile sectors are generated.

$$\Delta L_{\rm tot} \neq 0 \;\; \Delta N_{\rm tot} \neq 0 \quad \textbf{but} \quad \Delta L_{\rm tot} + \Delta N_{\rm tot} = 0$$

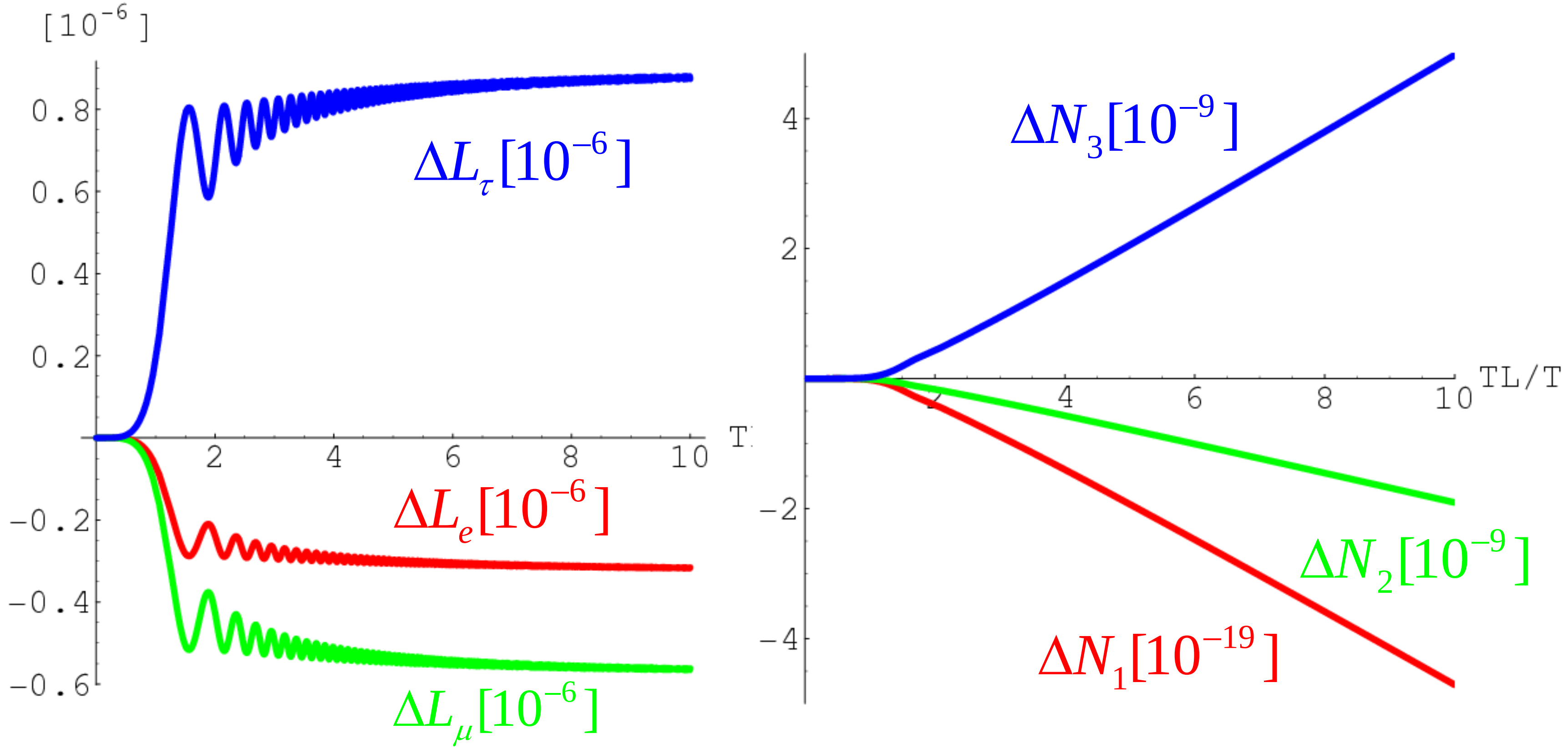




Evolution rates of ΔLe $\Delta L\mu$ $\Delta L\mu$
are different due to different Yukawa couplings

Evolution of asymmetries

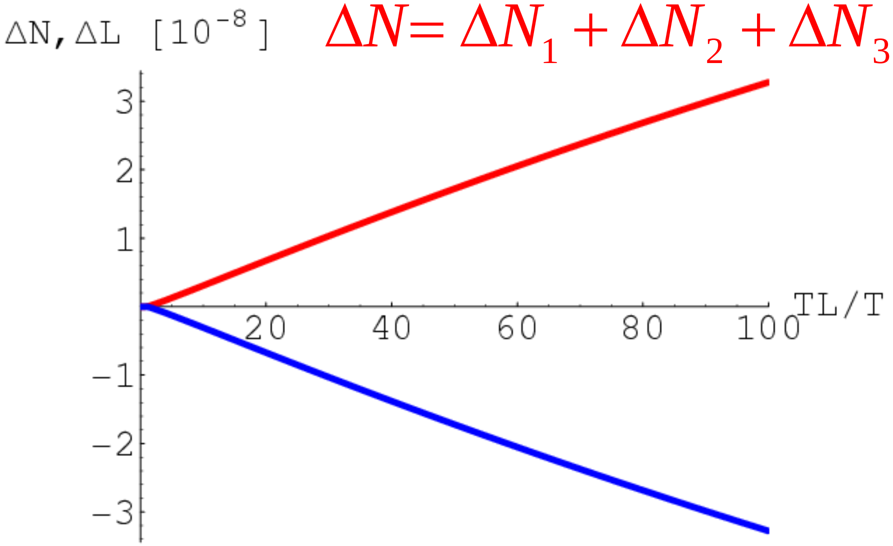
Active sector **Sterile sector**



$$T_L \sim 10^4\,\mathrm{GeV}$$

Total asymmetries in active and sterile sectors

$\Delta N = \Delta N_1 + \Delta N_2 + \Delta N_3$



$\Delta L = \Delta L_e + \Delta L_\mu + \Delta L_\tau$

- **We can generate**

  $\Delta N \neq 0, \ \Delta L \neq 0$

  but $\Delta N + \Delta L = 0$

- **Production starts**

  $T_L \sim 10^4 \, \mathrm{GeV}$

Shaleron converts ΔL partially into baryon asymmetry

$$\Delta B = -\frac{28}{79} \Delta L \neq 0$$

Kuzmin, Rubakov, Shaposhnikov

Baryon asymmetry of the universe

$$\frac{n_B}{s} \simeq 2\times10^{-10}\ \delta_{CP}\left(\frac{10^{-5}}{\Delta M_{32}^2/M_3^2}\right)^{2/3}\left(\frac{M_3}{10\mathrm{GeV}}\right)^{5/3}$$

in NH $m_3 \approx \sqrt{\Delta m_{atm}^2},\ m_2 \approx \sqrt{\Delta m_{sol}^2}$

- **The effective CP violation parameter**

$$\delta_{CP} = 4 s_{R23} c_{R23}\Big[s_{L12} s_{L13} c_{L13}\left(\left(c_{L23}^4 + s_{L23}^4\right)c_{L13}^2 - s_{L13}^2\right)\cdot\sin\left(\delta_L + \alpha_2\right)$$
$$+ c_{L13} s_{L13}^2 s_{L23} c_{L23}\left(c_{L23}^2 - s_{L23}^2\right)\cdot\sin\alpha_2\Big]$$

$\delta_{CP} \sim 1$ may be possible

- **Heavier sterile neutrinos should be degenerate in mass**

$M_2, M_3 \sim 10\mathrm{GeV}$ $\qquad$ $M_3 - M_2 \sim \mathrm{keV} \sim M_1$

$$\left(\frac{n_B}{s}\right)_{\mathrm{OBS}} = (8.4-8.9)\times10^{-11}$$

III. Summary

Summary

- The MSM + three right-handed neutrinos (νMSM)
  - Lightest sterile neutrino ~10keV can be dark matter
    - The simplest Dodelson-Widrow scenario conflicts with X-ray and Ly-alpha constraints
    - Some other production mechanism is needed
  - Heavier sterile neutrinos can be responsible to baryon asymmetry of the universe
    - Baryogenesis via neutrino oscillations

Three sterile neutrinos

- We may call as "bright", "clear" and "dark"

  - Clear and Bright, N_C and N_B: Heavier ones
    - Neutrino oscillations
    - Baryon asymmetry
    - $M \sim 10$GeV, $F \sim 10^{-7}$, $\theta \sim 10^{-6}$

  - Dark, N_D: Lightest one
    - Dark matter (production?)
    - $M \sim$keV, $F < 10^{-12}$, $\theta < 10^{-4}$

Neutrino Yukawa coupling constants

$$F = K_L \cdot P_\alpha \cdot F_d \cdot K_R^\dagger \cdot P_\beta$$

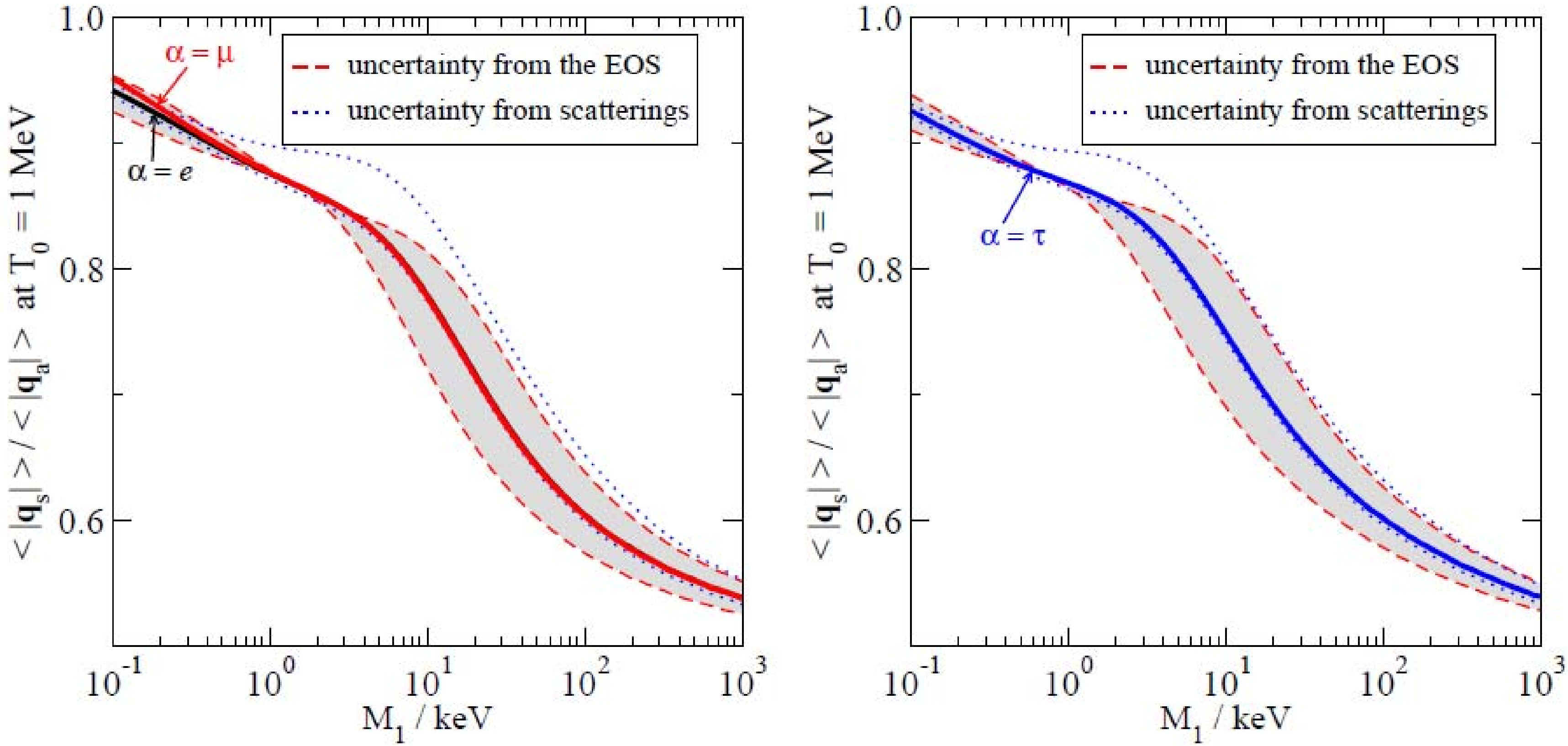
$$F_d = \mathrm{diag}(f_1, f_{2,} f_3)$$

$$P_\alpha = \mathrm{diag}(e^{i\alpha_1}, e^{i\alpha_2}, 1),\ P_\beta = \mathrm{diag}(e^{i\beta_1}, e^{i\beta_2}, 1)$$

$$K_L = \begin{pmatrix} 1 & & \\ & c_{L23} & s_{L23} \\ & -s_{L23} & c_{L23} \end{pmatrix} \begin{pmatrix} c_{L13} & & s_{L13}e^{-i\delta_L} \\ & 1 & \\ -s_{L13}e^{i\delta_L} & & c_{L13} \end{pmatrix} \begin{pmatrix} c_{L12} & s_{L12} & \\ -s_{L12} & c_{L12} & \\ & & 1 \end{pmatrix}$$

$$K_R = \begin{pmatrix} 1 & & \\ & c_{R23} & s_{R23} \\ & -s_{R23} & c_{R23} \end{pmatrix} \begin{pmatrix} c_{R13} & & s_{R13}e^{-i\delta_R} \\ & 1 & \\ -s_{R13}e^{i\delta_R} & & c_{R13} \end{pmatrix} \begin{pmatrix} c_{R12} & s_{R12} & \\ -s_{R12} & c_{R12} & \\ & & 1 \end{pmatrix}$$

Averaged momentum



We find: $\dfrac{\langle|\mathbf{q}_s|\rangle}{\langle|\mathbf{q}_a|\rangle} \simeq 0.8$ for $M_1 \simeq 10\text{keV}$

$M_1 \gtrsim 11.6\text{keV}$ (SMMT)

$M_1 \gtrsim 8\text{keV}$ (VLHMR)

<u>Estimation of sterile neutrino abundance</u>

General formalism

TA, Laine, Shaposhnikov

- **Using density matrix:** $i\frac{d\hat{\rho}}{dt} = \left[\hat{H}\,,\,\hat{\rho}\right]$
  - Initial condition $\hat{\rho}(0) = \hat{\rho}_{\mathrm{MSM}} \otimes |0\rangle\langle 0|$
  - Distribution function: $n_I(t,\mathbf{q}) = \frac{1}{V}\mathrm{Tr}\left[\sum_{s=\pm 1} \hat{a}_s^\dagger(\mathbf{q})\hat{a}_s(\mathbf{q})\,\hat{\rho}(t)\right]$

$$\left(\frac{\partial}{\partial t} - Hq_i\frac{\partial}{\partial q_i}\right) n_I(t,\mathbf{q}) = R(t,\mathbf{q})$$

  - $\hat{H} = \hat{H}_{\mathrm{MSM}} + \hat{H}_{\mathrm{S}} + \hat{H}_{\mathrm{int}}$ ⟵ Neutrino Yukawa interaction

$|F_{\alpha 1}| \lesssim 10^{-11}$

Cf. $|F_{\alpha 2}|\,,\ |F_{\alpha 3}| \lesssim 10^{-7}$

For the production, the leading term is $\mathcal{O}(F^2)$

$$R(t,\mathbf{q}) = -\frac{1}{V}\mathrm{Tr}\left\{\sum_{s=\pm} \hat{a}_s^\dagger(\mathbf{q})\hat{a}_s(\mathbf{q}) \int_0^t dt'[\hat{H}_I(t),[\hat{H}_I(t'),\hat{\rho}(0)]]\right\}$$

Kinetic equation

$$\left(\frac{\partial}{\partial t} - H q_i \frac{\partial}{\partial q_i}\right) n_I(t,\mathbf{q}) = R(t,\mathbf{q})$$

$$R(t,\mathbf{q}) = \frac{4 n_F(q^0)}{(2\pi)^3 2q^0} \sum_{\alpha=1}^{3} \frac{|M_D|^2_{\alpha I}}{\left\{[Q+\mathrm{Re}\Sigma]^2 - [\mathrm{Im}\Sigma]^2\right\}^2 + 4\{[Q+\mathrm{Re}\Sigma]\,\mathrm{Im}\Sigma\}^2}$$
$$\times \mathrm{Tr}\{\not{Q}\, a_L (2[Q+\mathrm{Re}\,\Sigma]\cdot \mathrm{Im}\,\Sigma\ [\not{Q} + \mathrm{Re}\,\not{\Sigma}\,]$$
$$- \{[Q+\mathrm{Re}\,\Sigma]^2 - [\mathrm{Im}\,\Sigma]^2\}\ \mathrm{Im}\,\not{\Sigma}\,) a_R\}$$

1. Dirac neutrino masses: $[M_D]_{\alpha 1}$
2. Majorana mass: M_1
3. Self-energy of active neutrinos $\Sigma_{\alpha\alpha}$

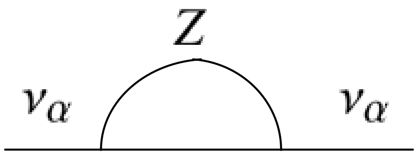
$$Q^2 = q_0^2 - |\mathbf{q}|^2 = M_1^2$$

calculated within the MSM at finite temperatures

Active neutrino self-energy $\Sigma_{\alpha\alpha}$

- **Real part:**

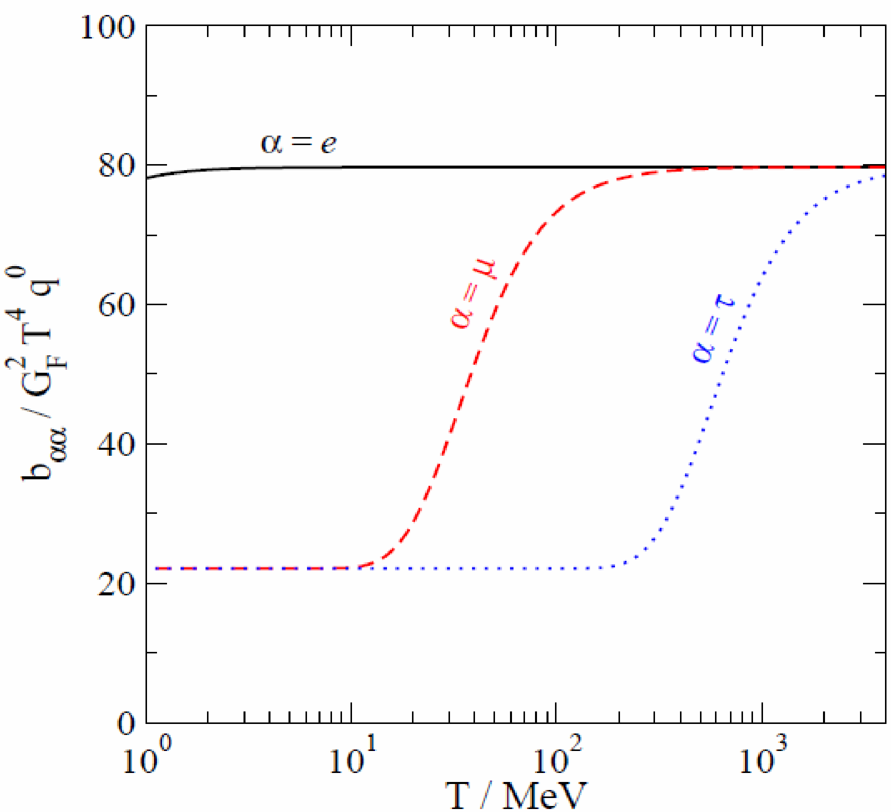
$$\mathrm{Re}\, \not{\Sigma}_{\alpha\alpha}(Q) = \not{Q}\, a_{\alpha\alpha}(Q) + \not{u}\, b_{\alpha\alpha}(Q)\ ,$$



+…

- Generated at 1-loop diagrams of W and Z
- Hadronic contributions are suppressed by α_W ➡ negligible
- $a_{\alpha\alpha}$ is negligibly small compared with 1
- $b_{\alpha\alpha}$ plays a role of effective potential

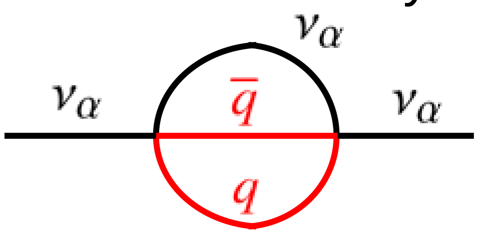
Notzold, Raffelt
Enquvist, Kainulainen, Maalampi



Active neutrino self-energy (2)

■ Imaginary part:

- 1-loop diagrams of W and Z are suppressed by $\sim \exp(m_W/T)$
- Dominant contributions are 2-loop graphs in the Fermi theory



$+\ldots$

Hadronic contributions:

- We can express these contributions by two point functions of mesonic currents $A^0_\mu = \bar{q}\gamma_\mu\gamma_5 q\,,\ \ V^0_\mu = \bar{q}\gamma_\mu q\,,\ \ V^a_\mu = \bar{q}\gamma_\mu T^a q$

$$\langle A^0_\mu(x) A^0_\nu(0)\rangle \quad \langle V^0_\mu(x) V^0_\nu(0)\rangle \quad \langle V^a_\mu(x) V^b_\nu(0)\rangle$$

- These quantities at T ~ Tc are poorly understood

hadronic uncertainty

Hadronic contributions

- **In general,**
  - We may estimate by free quarks for $T \gg T_c$
  - We may estimate by the chiral perturbation theory for $T \ll T_c$
    - ✓ Dominant pion contributions <1% of leptonic ones

- Phenomenological approach
  - We expect the hadronic contributions are proportional to the hadronic degrees of freedom, say $h_{\rm eff}^{\rm QCD}(T)$
  - We estimate by massive free quarks:
    - *"strict"* upper bound: $N_C = 3$
    - *"strict"* lower bound: $N_C = 0$
    - *"mean"* value: $N_C = 3 \times h_{\rm eff}^{\rm QCD}(T)/58$

58=d.o.f for
four quark flavors

QCD equation of state

- **Time-temperature relation:**

$$\rho = \frac{\pi^2 T^4}{30} g_{\text{eff}}$$

$$s = \frac{2\pi^2 T^3}{45} h_{\text{eff}}$$

$$\frac{dT}{dt} = -\left(\frac{90}{\pi^2 g_{\text{eff}}(T)}\right)^{\frac{1}{2}} \frac{M_P}{T^3}\left[1 + \frac{T}{3 h_{\text{eff}}(T)} \frac{d h_{\text{eff}}(T)}{dT}\right]$$

hadronic uncertainty

- **Phenomenological approach**

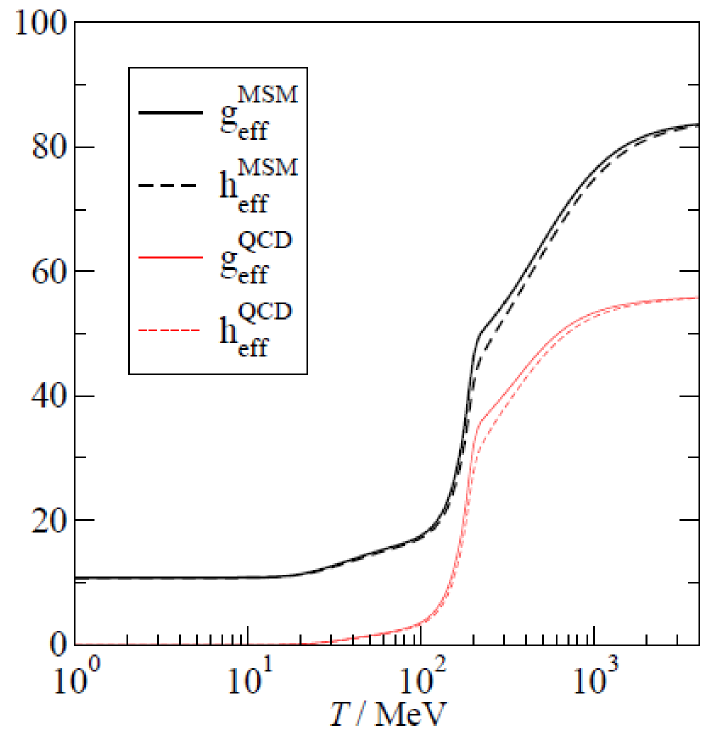
Laine, Schroder

- For high temperatures
  perturbative results (resummed 4-loop)
  Kajantie, Laine, Rummukainen, Schroder

- For low temperatures
  a dilute gas of resonances

- Interpolate two regions smoothly

$$\tilde{T}_c = 200^{+40}_{-40}\,\text{MeV}$$



Entropy dilution

- **Decays of heavier sterile neutrinos can produce entropy by $S \lesssim 30$ within the νMSM**

TA, Kusenko, Shaposhnikov

- Bound from overclosure: $\sin^2(2\theta) < f(M_1) \cdot S$
- Bound from X-rays: does not change
- Bound from Ly-α: $M_1 > \frac{\langle|\mathbf{q}_s|\rangle}{\langle|\mathbf{q}_a|\rangle} M_0 \cdot S^{-1/3}$

- Allowed region opens

$$S \gtrsim 155\,, \quad (M_1 \lesssim 1.5\text{keV}, \ \sin^2(2\theta) < 1.9 \cdot 10^{-6})$$

$$\text{for} \ \ M_1 \gtrsim 8\text{keV} \cdot S^{-1/3} \quad \text{(VLHMR)}$$

- Such rate cannot be obtained within the νMSM
- Sterile neutrino dark-matter cannot be realized within the νMSM, if we apply the current Ly-α constraints !!

Upper bound on mixing angle

- **We find**

$$C_e > C_\mu > C_\tau$$

- The case 1:
  the largest abundance
  the strongest bound

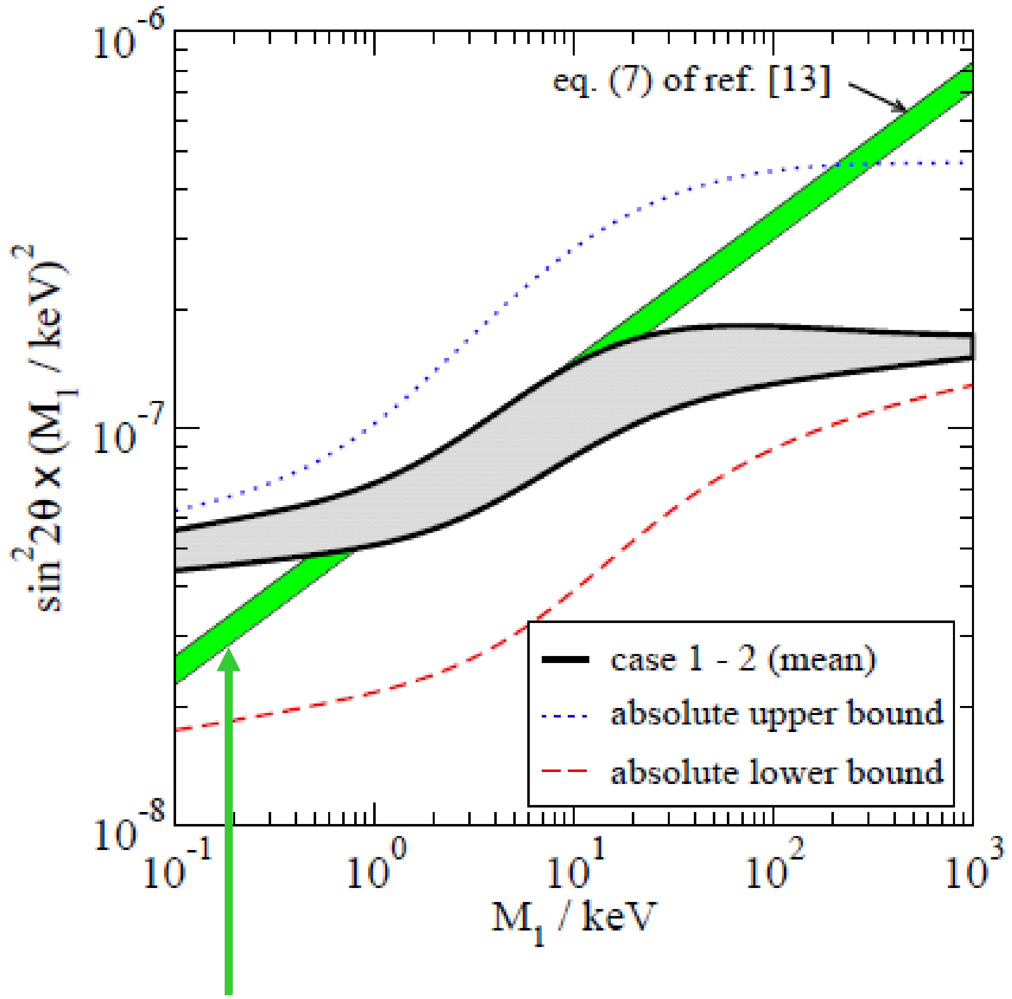
$$|M_D|_{e1} \neq 0$$

$$|M_D|_{\mu 1} = |M_D|_{\tau 1} = 0$$

- The case 2:
  the smallest abundance
  the weakest bound

$$|M_D|_{\tau 1} \neq 0$$

$$|M_D|_{e1} = |M_D|_{\mu 1} = 0$$



Abazajian: Phys.Rev. D73 ('06) 063506

$$\Omega_{\mathrm{DM}} = 0.2\,, \quad T_c = 150 - 200\mathrm{MeV}$$

Kinetic equation of the system (2)

- **We apply the Bolzmann approximation**

$$i\frac{d\rho}{dt}=[H,\rho]-\frac{i}{2}\left\{\Gamma^{d},\rho\right\}+\frac{i}{2}\left\{\Gamma^{p},1-\rho\right\}$$

$$\Rightarrow i\frac{d\rho}{dt}=[H,\rho]-\frac{i}{2}\left\{\Gamma^{d},\rho\right\}+i\Gamma^{p}$$

- **If the system is in thermal equilibrium,**

$$\Rightarrow i\frac{d\rho}{dt}=[H,\rho]-\frac{i}{2}\left\{\Gamma^{d},\rho-\rho^{eq}\right\}$$

$$\Gamma^{p}=\frac{1}{2}\left\{\Gamma^{d},\rho^{eq}\right\}$$

Kinetic equation of the system (3)

- **Effective Hamiltonian**

$$H = k + H^0 + H^{\mathrm{int}}$$

H^{int} : Contributions from Dirac Yukawa F

- **If Dirac Yukawa F=0, the Hamiltonian becomes diagonal**

$$H^0 = diag(H^0_{LL}, H^0_{\bar{L}\bar{L}}, H^0_{NN}, H^0_{\bar{N}\bar{N}})$$

Medium effects

$$H^0_{LL} = H^0_{\bar{L}\bar{L}} = \frac{T^2}{k}\left[\frac{3g_W^2 + g_Y^2}{32} diag(1,1,1) + \frac{1}{8} diag(h_e^2, h_\mu^2, h_\tau^2)\right]$$

$$H^0_{NN} = H^0_{\bar{N}\bar{N}} = \frac{1}{2k} diag(M_1^2, M_2^2, M_3^2)$$

Majorana masses

These structures simply the discussion

Kinetic equation of the system (4)

- **Let us go to the interaction picture and remove the trivial time dependence due to H^0**

$$\rho = U \cdot \tilde{\rho} \cdot U^{\dagger} \qquad U(t) = \exp\left(-i\int_0^t dt' H^0(t')\right)$$

Very roughly speaking,

$$\rho_{IJ} \sim \exp\left[-i\left(H_I^0 - H_J^0\right)t\right] \cdot \tilde{\rho}_{IJ}$$

- **For** $t \gg 1/(H_I^0 - H_J^0)$
  - Off-diagonal elements of density matrix undergo rapid oscillations and can be put zero due to averaging effects
  - Diagonal elements must be kept

Kinetic equation of the system (5)

- **For our interesting time range,** $T < (\Delta M^2 \cdot M_P)^{1/3}$

$$\rho = \begin{pmatrix} \rho_{LL} & \rho_{L\bar{L}} & \rho_{LN} & \rho_{L\bar{N}} \\ \rho_{\bar{L}L} & \rho_{\bar{L}\bar{L}} & \rho_{\bar{L}N} & \rho_{\bar{L}\bar{N}} \\ \rho_{NL} & \rho_{N\bar{L}} & \rho_{NN} & \rho_{N\bar{N}} \\ \rho_{\bar{N}L} & \rho_{\bar{N}\bar{L}} & \rho_{\bar{N}N} & \rho_{\bar{N}\bar{N}} \end{pmatrix} \Rightarrow \begin{pmatrix} \rho_{LL}^{diag} & \rho_{L\bar{L}}^{diag} & & \\ \rho_{\bar{L}L}^{diag} & \rho_{\bar{L}\bar{L}}^{diag} & & \\ & & \rho_{NN} & \rho_{N\bar{N}} \\ & & \rho_{\bar{N}N} & \rho_{\bar{N}\bar{N}} \end{pmatrix}$$

- When the effects of sterile neutrino oscillations become important, most of elements are irrelevant.
- Active neutrinos and charged leptons receive flavor-dependent effective masses due to $h_{e,\mu,\tau}$

Kinetic equation of the system (6)

- **Since lepton number violation can be neglected,**

$$\rho \Rightarrow \begin{pmatrix} \rho_{LL}^{diag} & \rho_{L\overline{L}}^{diag} & & \\ \rho_{\overline{L}L}^{diag} & \rho_{\overline{L}\,\overline{L}}^{diag} & & \\ & & \rho_{NN} & \rho_{N\overline{N}} \\ & & \rho_{\overline{N}N} & \rho_{\overline{N}\,\overline{N}} \end{pmatrix} \Rightarrow \begin{pmatrix} \rho_{LL}^{diag} & & & \\ & \rho_{\overline{L}\,\overline{L}}^{diag} & & \\ & & \rho_{NN} & \\ & & & \rho_{\overline{N}\,\overline{N}} \end{pmatrix}$$

  - Only block-diagonal parts are left !

-

Finally, we reach the following equations

Kinetic equations of the system (7)

Sterile neutrinos:

$$i\frac{d\rho_{NN}}{dt} = \left[H_{NN}, \rho_{NN}\right] - \frac{i}{2}\left\{\Gamma^{d}_{NN}, \rho_{NN} - \rho^{eq}_{NN}\right\} + \frac{i\sin\phi}{4} T \cdot F^{\dagger}(\rho_{LL} - \rho^{eq}_{LL})F$$

Active neutrinos:

$$i\frac{d\rho^{diag}_{LL}}{dt} = [H_{LL}, \rho^{diag}_{LL}] - \frac{i}{2}\left\{\Gamma^{d}_{LL}, \rho^{diag}_{LL} - \rho^{eq}_{LL}\right\} + \frac{i\sin\phi}{4} T \cdot F(\rho_{NN} - \rho^{eq}_{NN})F^{\dagger}$$

- **ARS discussed only the terms colored by black**
- **We include the evolution of both sterile and active neutrinos**

Let us see the meaning of the each term

Kinetic equations (2)

Sterile neutrinos:

$$i\frac{d\rho_{NN}}{dt} = [H_{NN}, \rho_{NN}] - \frac{i}{2}\{\Gamma^d_{NN}, \rho_{NN} - \rho^{eq}_{NN}\} + \frac{i\sin\phi}{4} T \cdot F^\dagger(\rho_{LL} - \rho^{eq}_{LL})F$$

Oscillation

Production and destruction

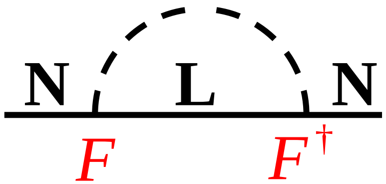
$$H_{NN} = H^0_{NN} + H^{\text{int}}_{NN}$$

$$\Gamma^d_{NN} = 2\sin\phi \cdot H^{\text{int}}_{NN} \quad [\sin\phi \simeq 2\times 10^{-2}]$$

$$H^0_{NN} = \frac{1}{2k} diag(M_1^2, M_2^2, M_3^2)$$

$$H^{\text{int}}_{NN} = \frac{T^2}{8k} F^\dagger F$$

Medium effects



Kinetic equations (3)

couple two sectors

$$i\frac{d\rho_{NN}}{dt} = [H_{NN}, \rho_{NN}] - \frac{i}{2}\left\{\Gamma^d_{NN}, \rho_{NN} - \rho^{eq}_{NN}\right\} + \frac{i\sin\phi}{4} T \cdot F^\dagger(\rho_{LL} - \rho^{eq}_{LL})F$$

$$i\frac{d\rho^{diag}_{LL}}{dt} = [H_{LL}, \rho^{diag}_{LL}] - \frac{i}{2}\left\{\Gamma^d_{LL}, \rho^{diag}_{LL} - \rho^{eq}_{LL}\right\} + \frac{i\sin\phi}{4} T \cdot F(\rho_{NN} - \rho^{eq}_{NN})F^\dagger$$

- **Total lepton number conservation** $\frac{d}{dt}Tr[\rho_{NN} + \rho^{diag}_{LL}] = 0$
- **If the system gets thermalized** $(\Gamma^d > H)$

$$\rho_{NN}, \rho_{\overline{N}\,\overline{N}} \to \rho^{eq}_{NN}$$
$$\rho_{LL}, \rho_{\overline{L}\,\overline{L}} \to \rho^{eq}_{LL}$$

No asymmetry is generated !

$$Tr[\rho_{NN} + \rho^{diag}_{LL}] = 0$$

Kinetic equations (3)

couple two sectors

$$i\frac{d\rho_{NN}}{dt}=[H_{NN},\rho_{NN}]-\frac{i}{2}\left\{\Gamma^{d}_{NN},\rho_{NN}-\rho^{eq}_{NN}\right\}+\frac{i\sin\phi}{4}T\cdot F^{\dagger}(\rho_{LL}-\rho^{eq}_{LL})F$$

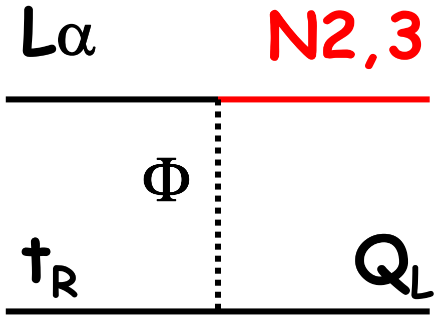
$$i\frac{d\rho^{diag}_{LL}}{dt}=[H_{LL},\rho^{diag}_{LL}]-\frac{i}{2}\left\{\Gamma^{d}_{LL},\rho^{diag}_{LL}-\rho^{eq}_{LL}\right\}+\frac{i\sin\phi}{4}T\cdot F(\rho_{NN}-\rho^{eq}_{NN})F^{\dagger}$$

- **Total lepton number conservation** $\frac{d}{dt}Tr[\rho_{NN}+\rho^{diag}_{LL}]=0$
- **If the system gets thermalized** $(\Gamma^{d}>H)$

$$\rho_{NN},\rho_{\overline{N}\,\overline{N}}\rightarrow\rho^{eq}_{NN}$$
$$\rho_{LL},\rho_{\overline{L}\,\overline{L}}\rightarrow\rho^{eq}_{LL}$$

No asymmetry is generated !

$$Tr[\rho_{NN}+\rho^{diag}_{LL}]=0$$


Lα
N2,3
Φ
t_R
Q_L