

地球の物質密度測定を使った MSW理論のテスト

“Testing MSW theory by Determination of Earth Matter Density”

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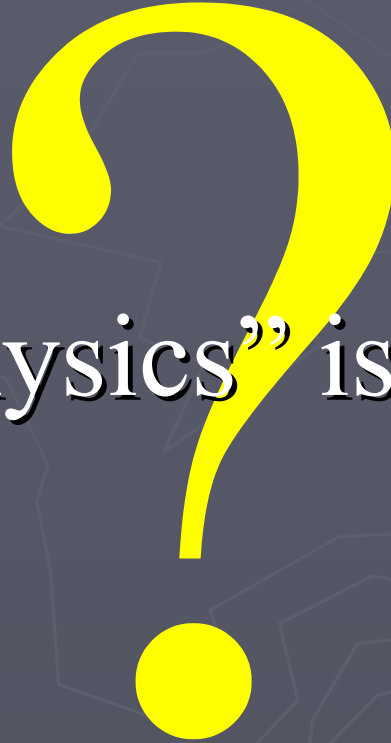
hep-ph/0612002 with Hisakazu Minakata

第20回「宇宙ニュートリノ」研究会
2007年2月20日 @ 宇宙線研究所

研究会テーマ

ニュートリノで探る

non-standard physics



“standard physics” is *standard* ?

neutrino oscillation

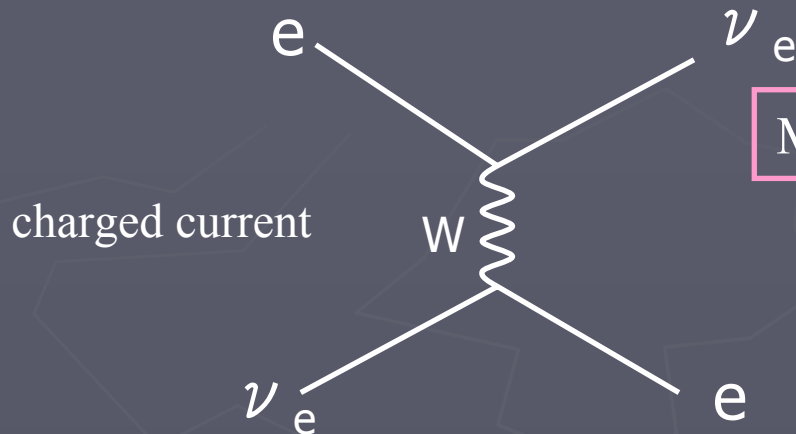
in vacuum

$$i \frac{\partial}{\partial t} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{\Delta m_{21}^2}{2E} & 0 \\ 0 & 0 & \frac{\Delta m_{31}^2}{2E} \end{pmatrix} U^{-1} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

$$\begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & e^{-i\delta}s_{13} \\ -c_{23}s_{12} - e^{i\delta}c_{12}s_{13}s_{23} & c_{12}c_{23} - e^{i\delta}s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{12}s_{23} - e^{i\delta}c_{12}c_{23}s_{13} & -e^{i\delta}c_{23}s_{12}s_{13} - c_{12}s_{23} & c_{13}c_{23} \end{pmatrix}$$

$$s_{ij} \equiv \sin \theta_{ij}, c_{ij} \equiv \cos \theta_{ij}$$

standard interaction theory



Mikheev-Smirnov-Wolfenstein

MSW effect

charged current interaction with electron

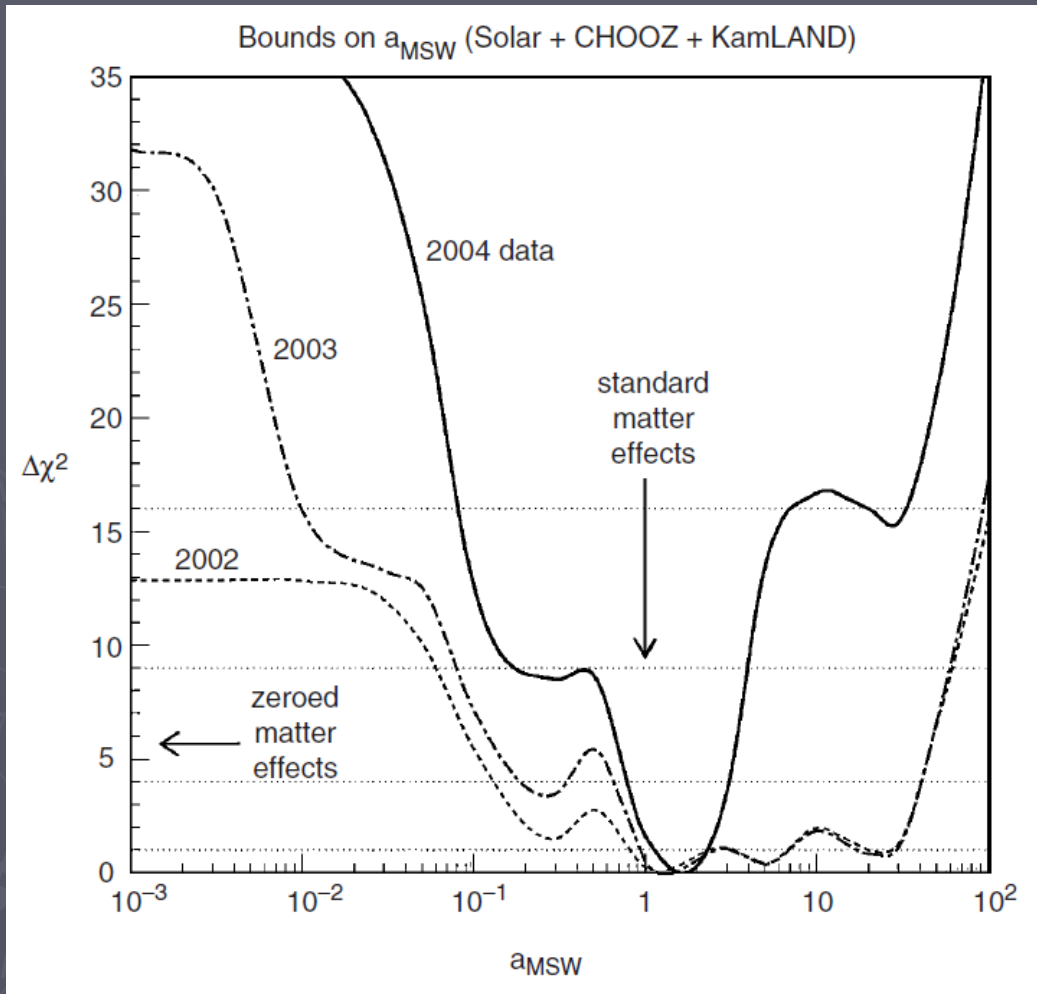
$$i \frac{\partial}{\partial t} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \left\{ U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{\Delta m_{21}^2}{2E} & 0 \\ 0 & 0 & \frac{\Delta m_{31}^2}{2E} \end{pmatrix} U^{-1} + \begin{pmatrix} \sqrt{2} G_F N_e & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right\} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

neutrino oscillations are modified

in matter

electron number density

Evidence for the MSW effect



Gianluigi Fogli, Eligio Lisi
 “Evidence for the MSW effect”
*New J.Phys.*6:139,2004.

$$V = a_{\text{MSW}} \times V_{\text{standard}}$$

$$= a_{\text{MSW}} \sqrt{2} G_F N_e$$

$a_{\text{MSW}} = 1$ for standard

a factor of ~ 2 uncertainty
 (at 2σ)

Testing MSW theory in neutrino factory

more precision

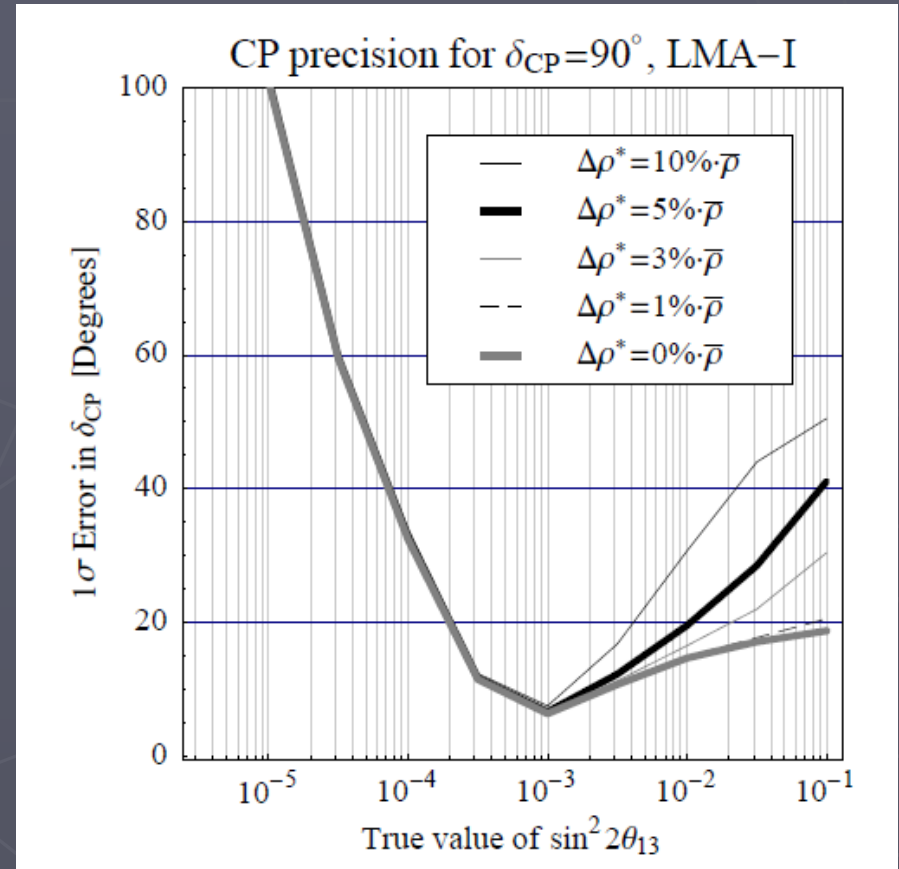
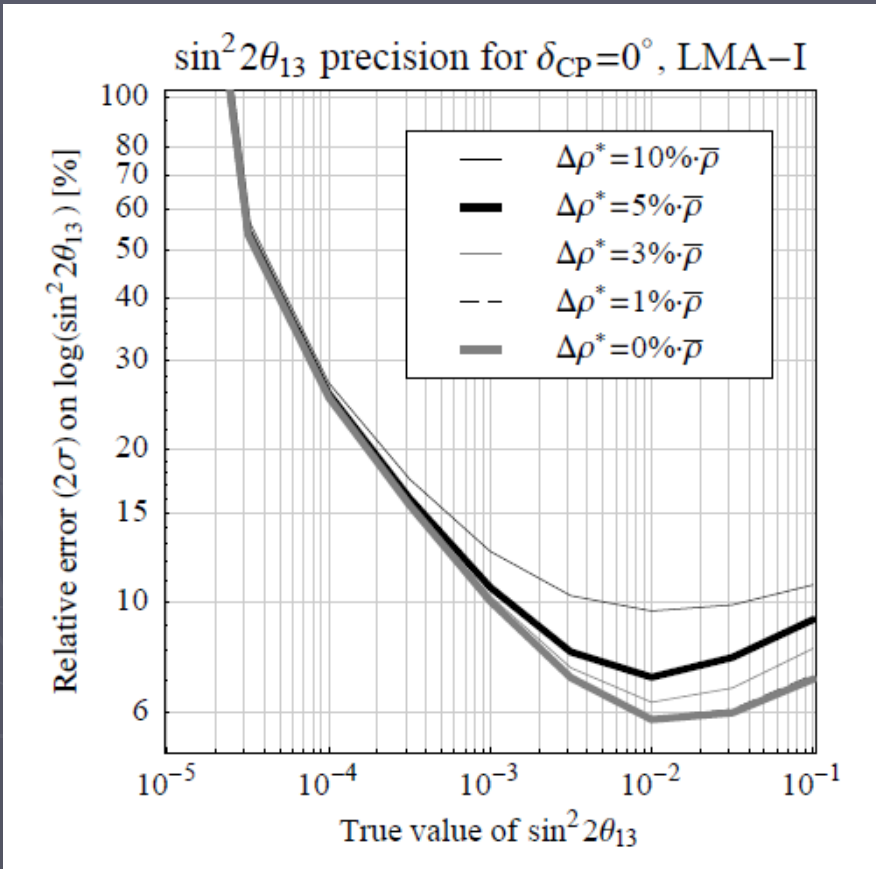
Original Motivation

We want to determine the Earth Matter Density
in Neutrino Factory

Effect of matter density uncertainties

on $\sin^2 2\theta_{13}$

on CP δ

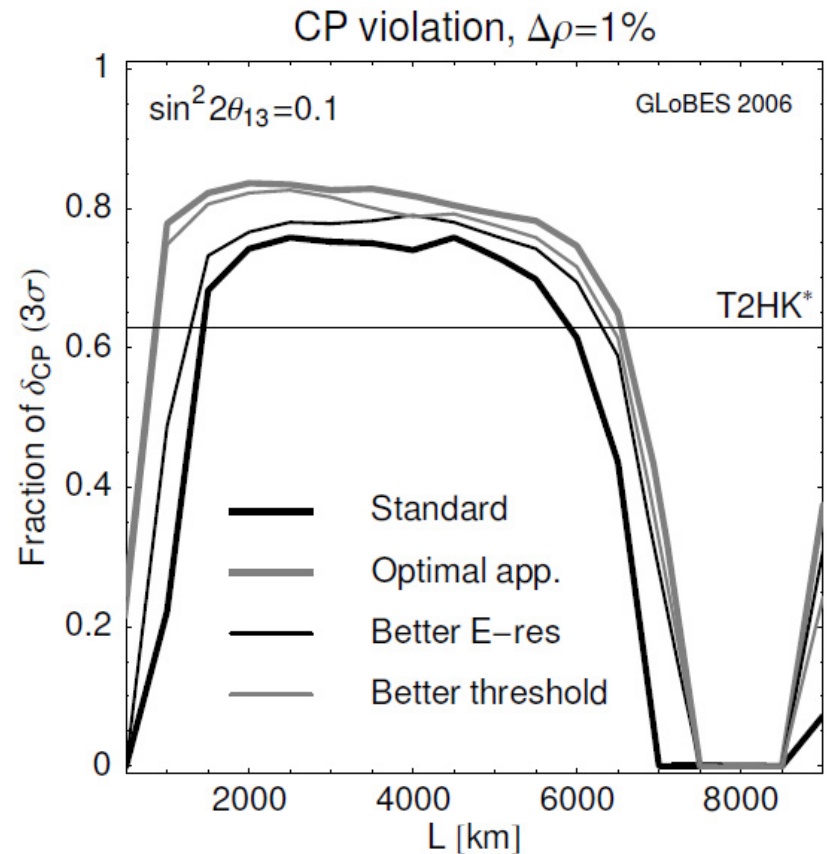
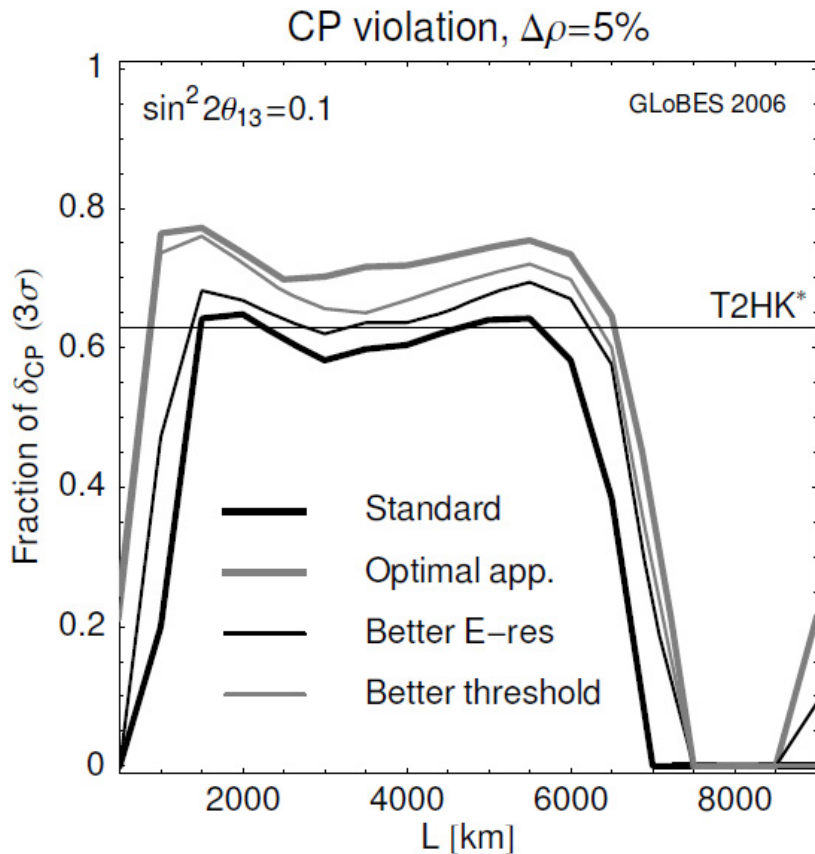


Tommy Ohlsson , Walter Winter

“The role of matter density uncertainties in the analysis of future neutrino factory experiments”

Phys.Rev.D68:073007,2003

Effect of matter density uncertainties on sensitivity to CP violation



P. Huber , M. Lindner , M. Rolinec , W. Winter

“Optimization of a neutrino factory oscillation experiment ”

Phys.Rev.D74:073003,2006

Determination of Earth Matter Density

(highly precise)



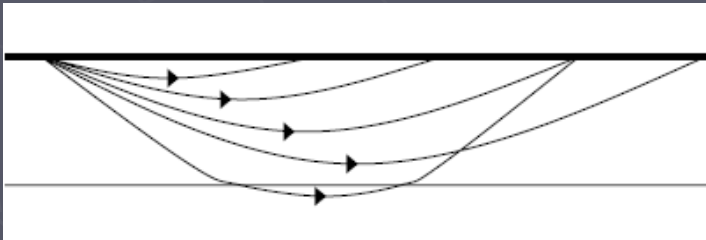
Test of MSW theory

(compare with geographic data)

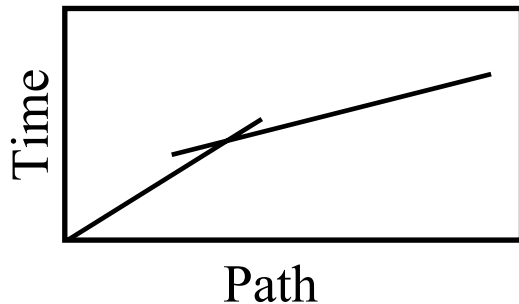
Earth Structure by Geophysics

Seismic waves tell us about Earth

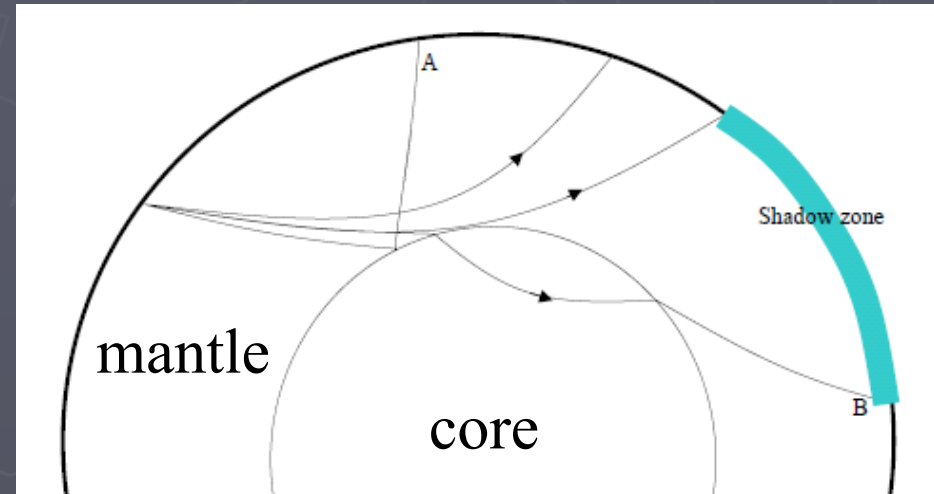
Mohorovičić discontinuity



travel-time curve

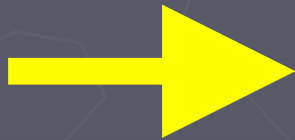


Core-Mantle boundary



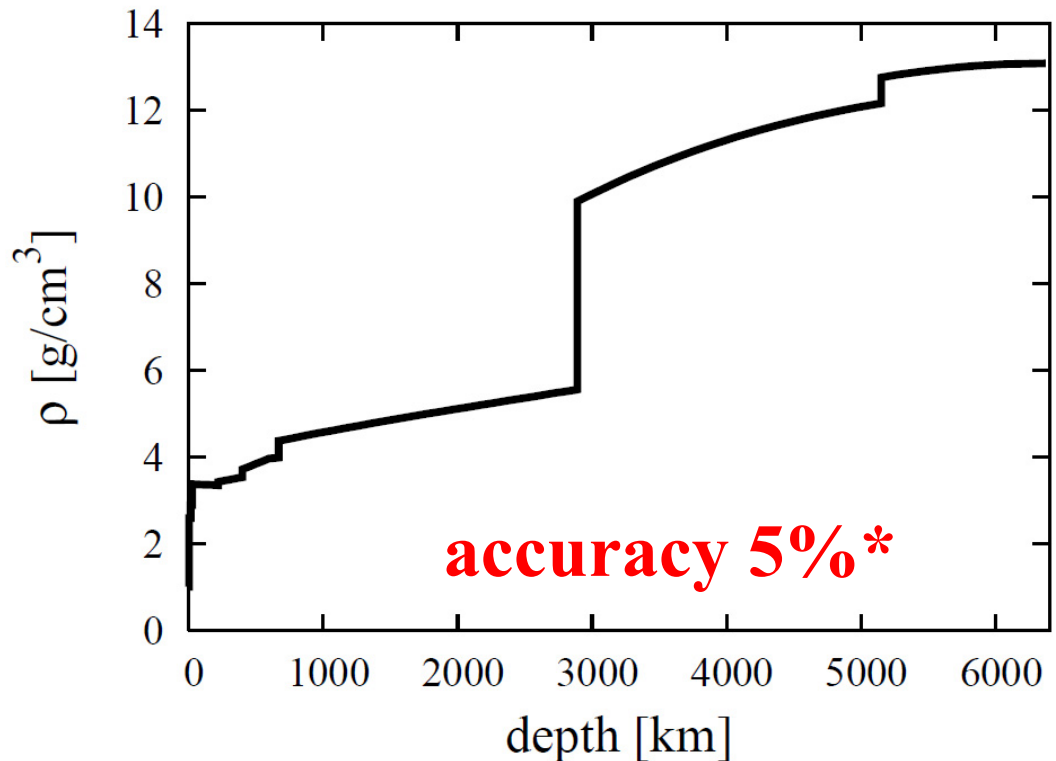
Density Profile

seismic +
free vibration by massive earthquake & inertia moment etc.



isotropic PREM

A. M. Dziewonski , D. L. Anderson
“Preliminary reference Earth model”
Phys.Earth. Planet. Inter. 25, 4, 297, (1981)



* Robert Geller , Tatsuhiko Hara
“Geophysical aspects of very long baseline neutrino experiments”
Nucl.Instrum.Meth.A503:187-191,2001

focus on

Neutrino Factory

NF have potential for determination of density

(very long baseline & $\nu_e \rightarrow \nu_\mu$ channel)

$\nu_e \rightarrow \nu_\mu$ (appearance) in matter

$$P(\nu_e \rightarrow \nu_\mu) = \underline{X_\pm} \sin^2 2\theta_{13} + \underline{Y_\pm} \sin 2\theta_{13} \cos(\pm\delta - \Delta_{31}) + \underline{P_{sol}}$$

$$X_\pm = s_{23}^2 \left(\frac{\Delta_{31} \sin(aL \mp \Delta_{31})}{aL \mp \Delta_{31}} \right)^2$$

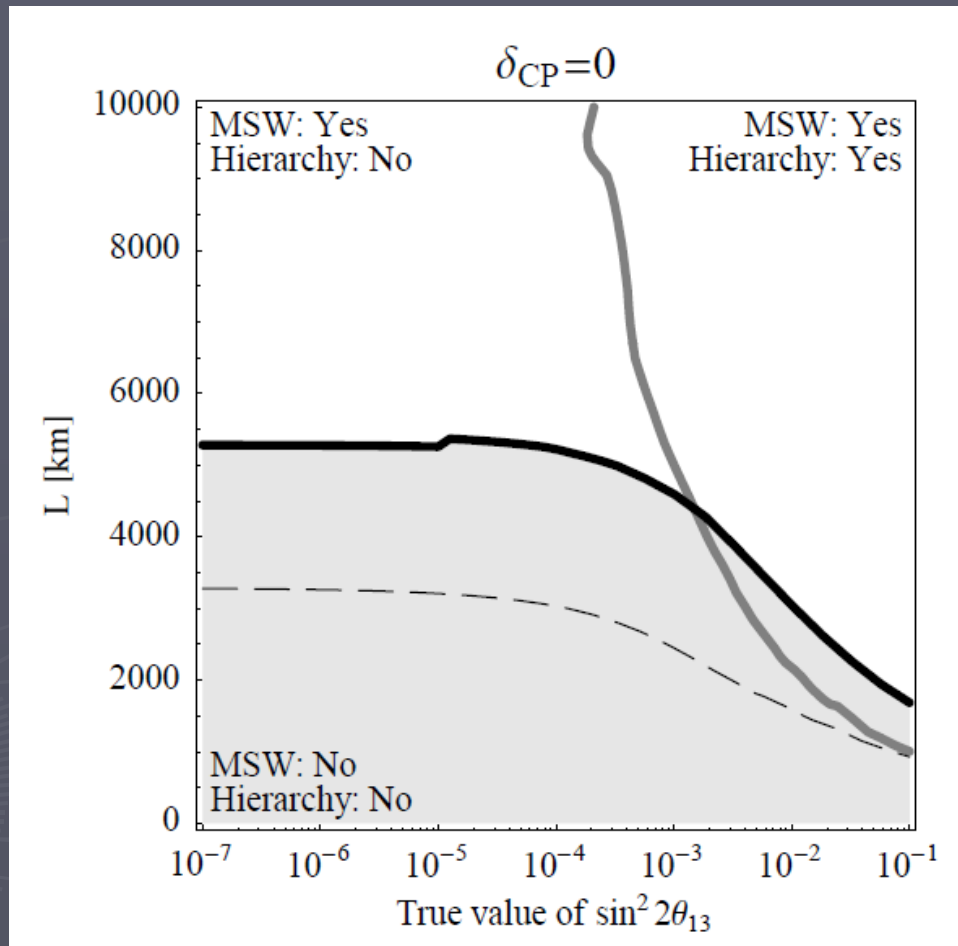
$$Y_\pm = \sin 2\theta_{12} \sin 2\theta_{23} \left(\frac{\Delta_{31} \sin(aL \mp \Delta_{31})}{aL \mp \Delta_{31}} \right) \left(\frac{\Delta_{21} \sin(aL)}{aL} \right)$$

$$P_{sol} = c_{23}^2 \sin^2 2\theta_{12} \left(\frac{\Delta_{21} \sin(aL)}{aL} \right)^2$$

$$\Delta_{ij} \equiv \frac{\Delta m_{ij}^2 L}{4E}$$

$$a = \frac{G_F Y_e}{\sqrt{2} m_N} \rho$$

Direct test of MSW in neutrino factory

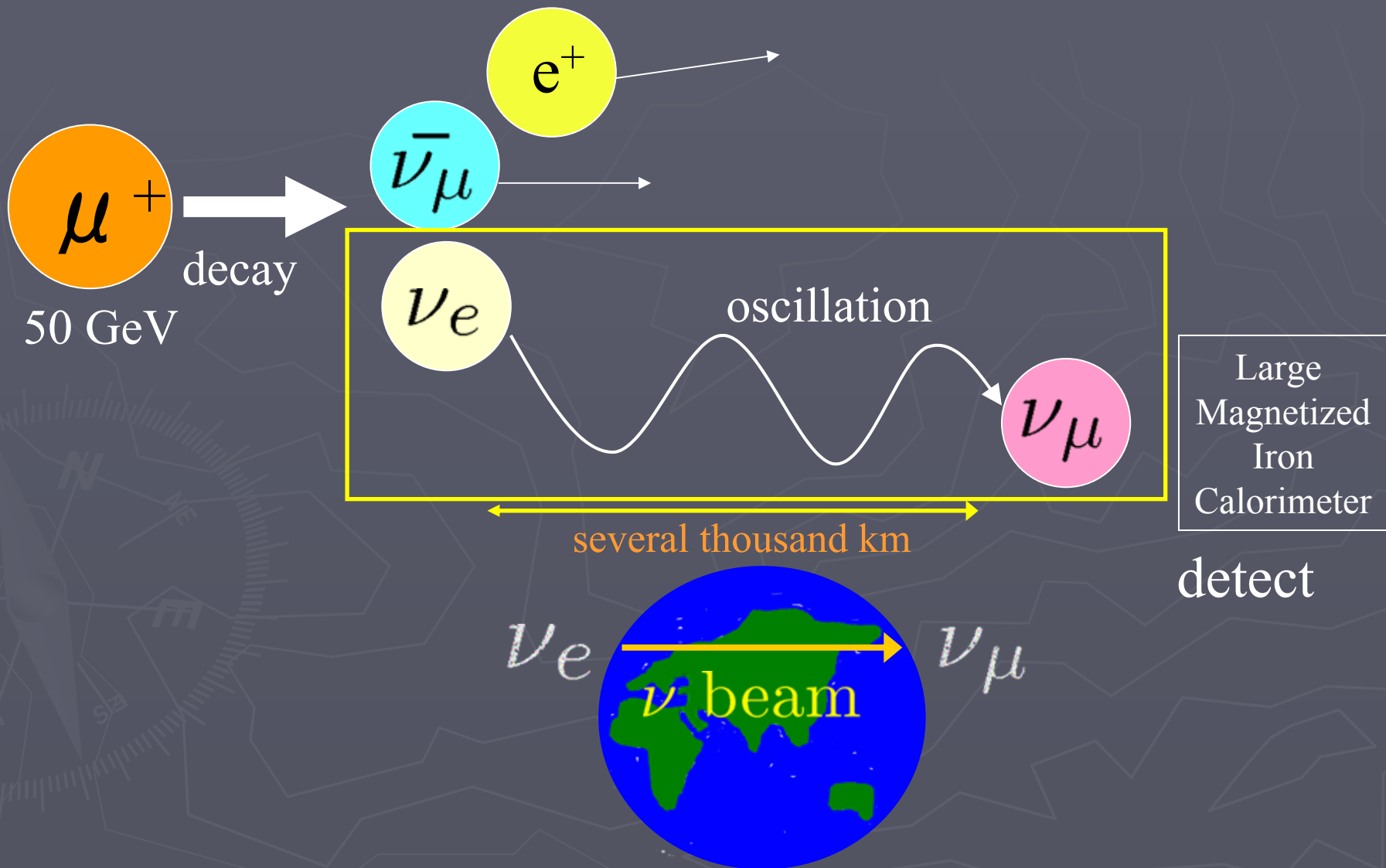


MSW effect $\neq 0$
(5σ)

Walter Winter

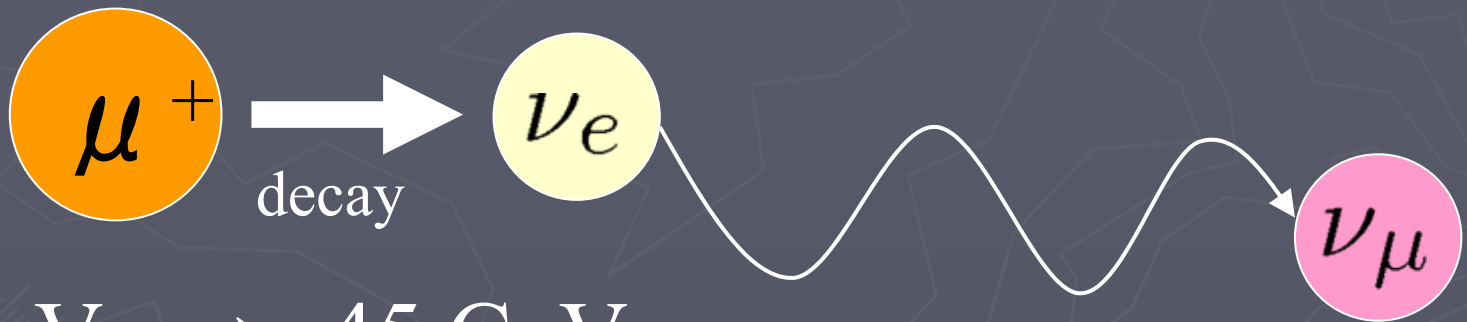
“Direct test of the MSW effect by the solar appearance term in beam experiments”
Phys.Lett.B613:67-73,2005.

neutrino factory



First Step (Which distance is better?)

tune a muon energy

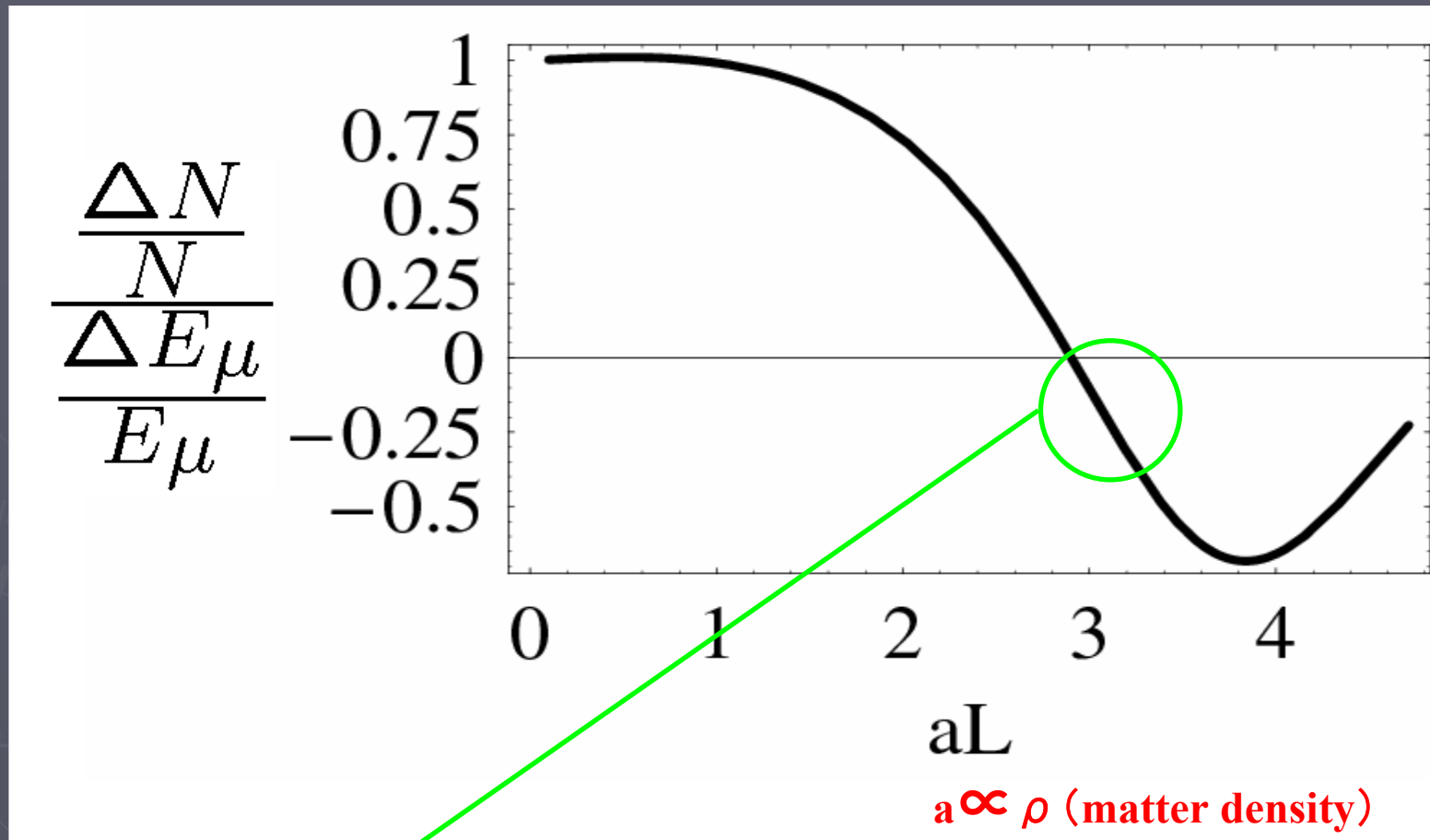


50 GeV \rightarrow 45 GeV
2 years 2 years

(energy scan)

New Information
variation of event number

Energy Scan



$aL = \pi$ large gradient
→ strong response to density change

Good determination

$$aL = \pi$$

($a = \text{const} \times$ matter density)

special distance
(Magic Baseline)

$$\left(a = \frac{G_F Y_e}{\sqrt{2} m_N} \rho \right)$$

$$L = 7500 \text{ km}$$

Magic Baseline ($aL = \pi$)

$L = 7500$ km

$$P(\nu_e \rightarrow \nu_\mu) = X_\pm \sin^2 2\theta_{13} + \cancel{Y_\pm \sin 2\theta_{13} \cos(\pm\delta + \Delta_{31})} + P_{sol}$$

$$X_\pm = s_{23}^2 \left(\frac{\Delta_{31} \sin(aL \mp \Delta_{31})}{aL \mp \Delta_{31}} \right)^2$$

$$Y_\pm = \sin 2\theta_{12} \sin 2\theta_{23} \left(\frac{\Delta_{31} \sin(aL \mp \Delta_{31})}{aL \mp \Delta_{31}} \right) \left(\frac{\Delta_{21} \sin(aL)}{aL} \right)$$

$$P_{sol} = c_{23}^2 \sin^2 2\theta_{12} \left(\frac{\Delta_{21} \sin(aL)}{aL} \right)^2$$

vanish at $aL = \pi$

→ measure θ_{13} independently of CP δ
of course matter density!

analysis

at Magic BL

$$P(\nu_e \rightarrow \nu_\mu) = s_{23}^2 \left(\frac{\Delta_{31} \sin(aL \mp \Delta_{31})}{aL \mp \Delta_{31}} \right)^2 \sin^2 2\theta_{13}$$

event number scale depending on θ_{13}

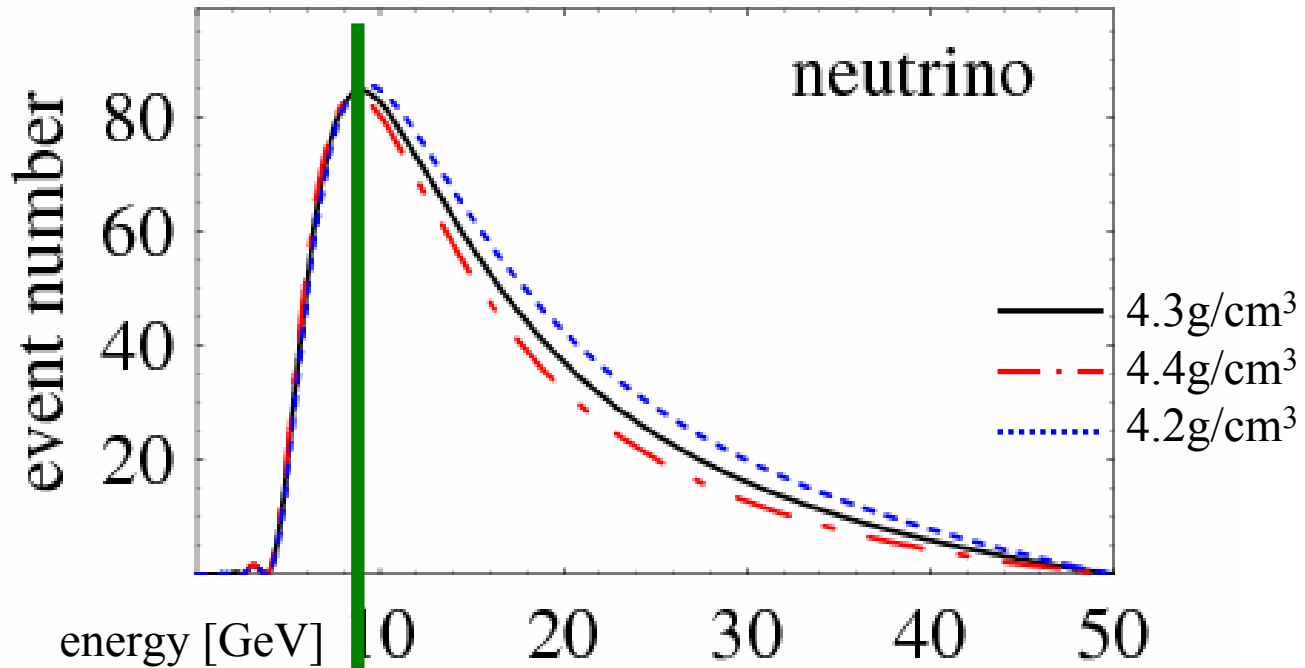


note energy spectra

analysis

high density → event up ↑

high density → event down ↓



low energy ↔ high energy

opposite response of density change

critical energy

response of density change

$$aL = \pi \rightarrow \pi + \varepsilon$$

$$\delta P = 2s_{23}^2 \sin^2 2\theta_{13} \left(\frac{\Delta_{31} \sin(\pi \mp \Delta_{31})}{\pi \mp \Delta_{31}} \right)^2 \times \left[\mp \cot(\Delta_{31}) - \frac{1}{\pi \mp \Delta_{31}} \right] \varepsilon$$

$$\Delta_{31} = \frac{\Delta m_{31}^2 L}{4E}$$

critical energy E_C

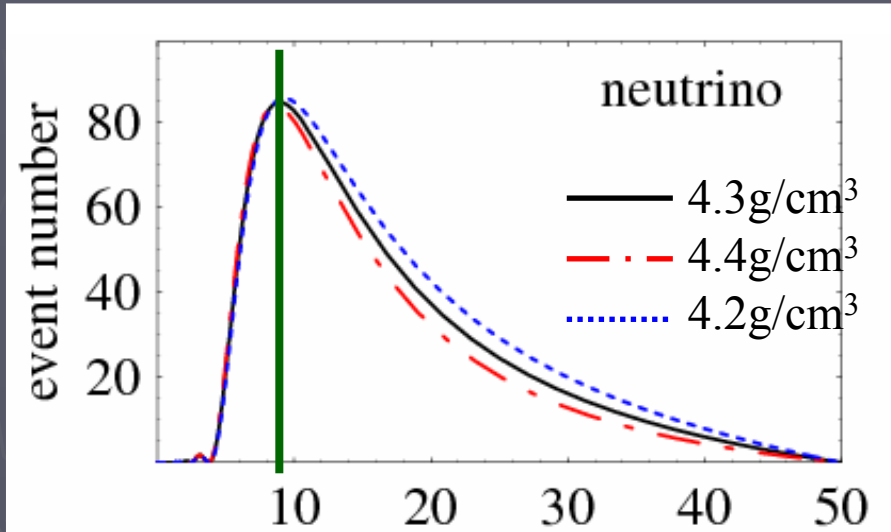
normal hierarchy
neutrino channel

$$\delta P > 0 \quad (E < E_C)$$

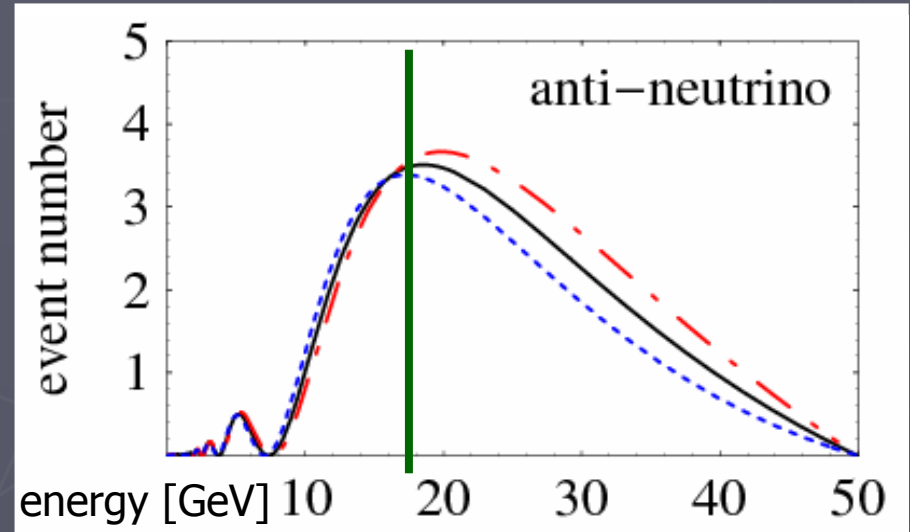
$$\delta P < 0 \quad (E > E_C)$$

response of density change

neutrino



anti-neutrino



(low E : event few & high E : event large)

→ low density

(low E : event large & high E : event few)

→ high density

(low E : event large & high E : event few)

→ high density

(low E : event few & high E : event large)

→ low density

low-high energy 2 bin analysis

analysis

chi-square test

$$\Delta\chi^2 \equiv \min_{\alpha\text{'s}} \sum_{a=\nu, \bar{\nu}} \left[\sum_{i=1,2} \left\{ \frac{(N_{ai}^{obs} - (1 + \alpha_i + \alpha_a + \alpha)N_{ai}^{exp})^2}{N_{ai}^{exp} + \sigma_{ai}^2(N_{ai}^{exp})^2} + \frac{\alpha_i^2}{\sigma_i^2} \right\} + \frac{\alpha_a^2}{\sigma_a^2} \right] + \frac{\alpha^2}{\sigma_{corr}^2}$$

N_1 : event number of low energy bin

N_2 : event number of high energy bin

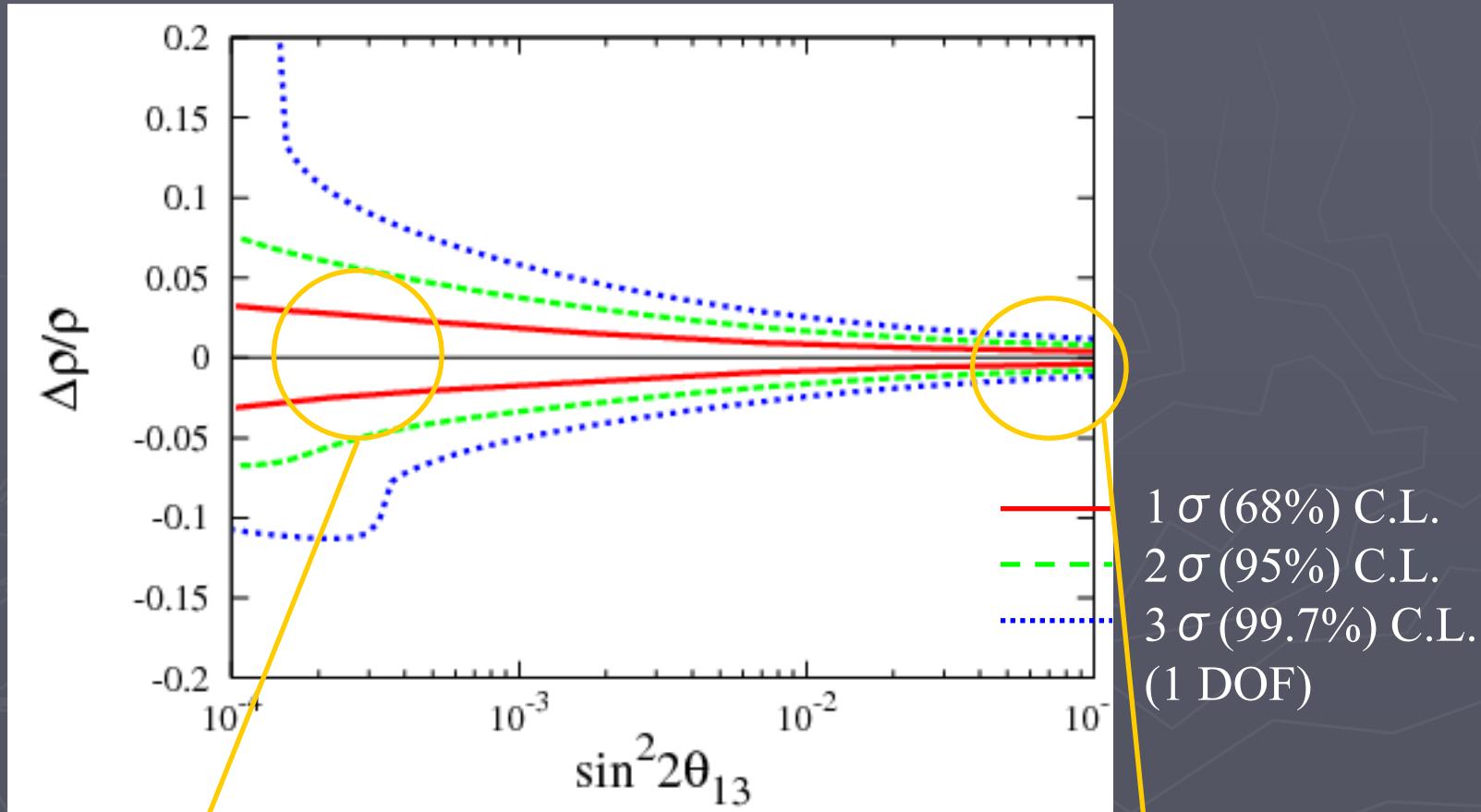
σ : systematic error

$$\sigma_{corr} = \sigma_a = \sigma_i = 2\% , \sigma_{ai} = 1\%$$

fit θ_{13} and ρ

result

accuracy of matter density $\Delta \rho / \rho$



a few (<5) % (1 σ)

1% ! (3 σ)

Magic Baseline ($aL = \pi$)

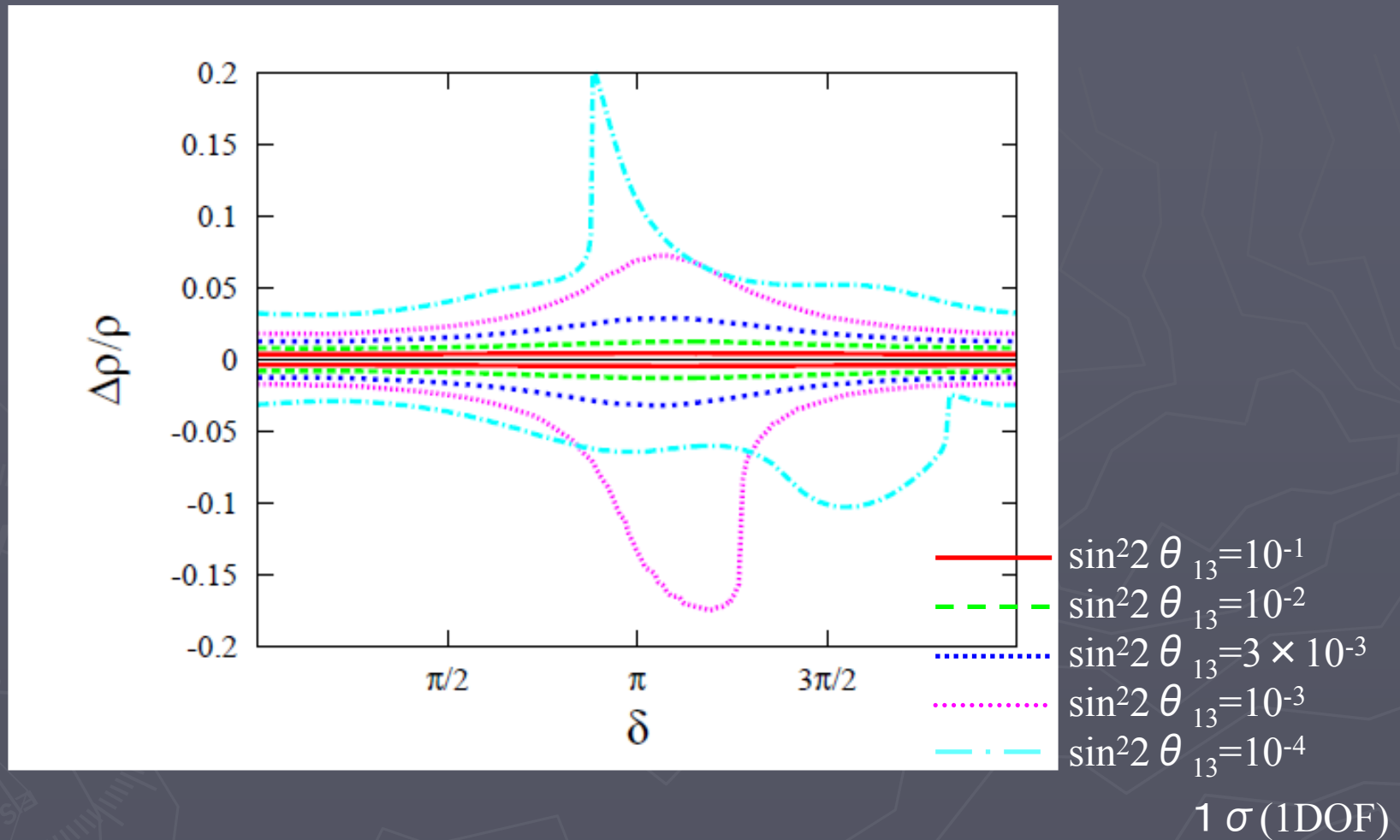
$L = 7500$ km

$$P(\nu_e \rightarrow \nu_\mu) = X_{\pm} \sin^2 2\theta_{13} + \cancel{Y_{\pm} \sin 2\theta_{13} \cos(\pm\delta - \Delta_{31})} + P_{sol}$$

→ It was independently of CP δ

But...

δ dependence

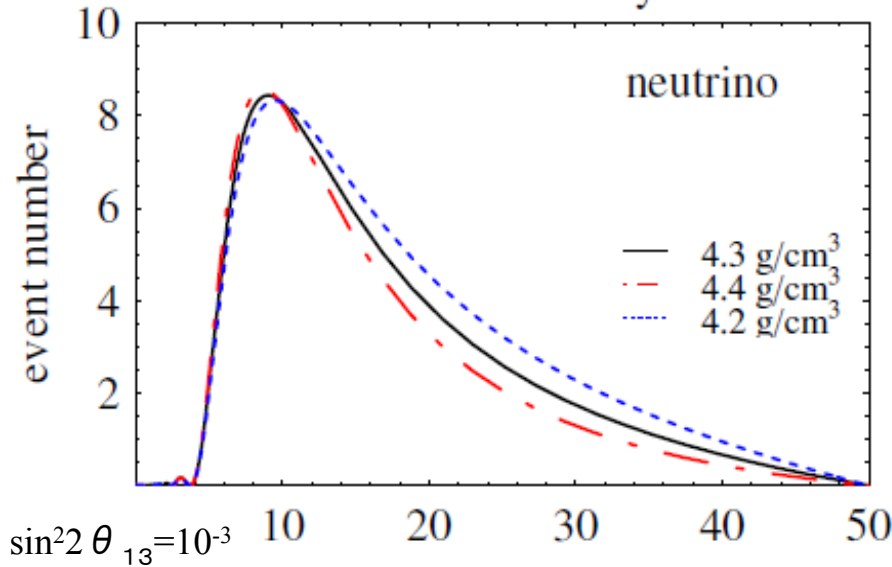


if θ_{13} is small ($\sin^2 2\theta_{13} < 10^{-3}$),
accuracy of matter density get worse depending on δ

δ dependence

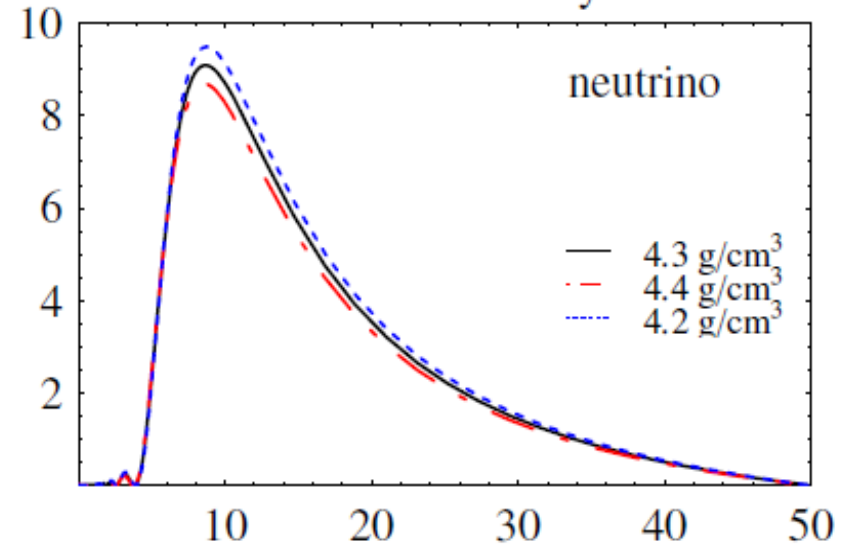
(small θ_{13})

normal hierarchy



$$\delta = 0$$

normal hierarchy



$$\delta = 7\pi/6$$

oscillation probability change is small

δ dependence

response of density change

$$aL = \pi \rightarrow \pi + \varepsilon$$

$$\delta P(\nu_e \rightarrow \nu_\mu) = -\epsilon A [\pm \cos(\delta \mp \Delta_{31}) + B \sin 2\theta_{13}]$$

If θ_{13} is small
canceled out!

poor response to density change



enlarged error in $\Delta \rho / \rho$

summary

- ▶ very precise determination of ρ at Magic BL
- ▶ large $\theta_{13} \rightarrow 1\% (3\sigma)$
- ▶ small $\theta_{13} \rightarrow <5\% (1\sigma)$

MSW theory 200% (2σ) accuracy \rightarrow 10% ?

in future...

- ▶ Analysis with density profile
- ▶ New Physics search !