

Constraints on new physics from long baseline neutrino oscillation experiments

ニュートリノで探る“non-standard” physics

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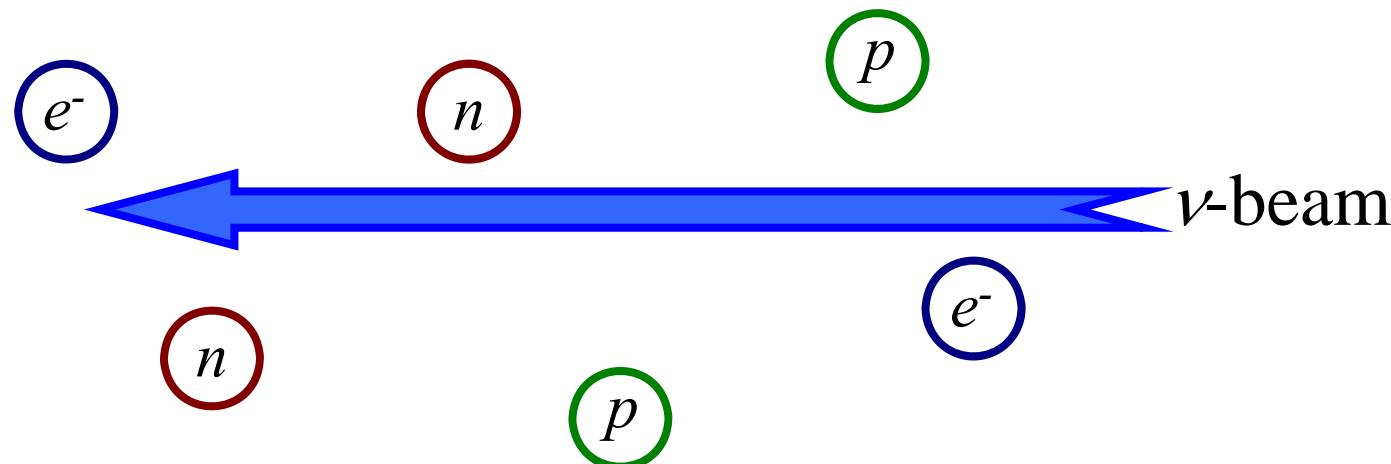
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第20回宇宙ニュートリノ研究会
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based on
[hep-ph/0602115](#)
[hep-ph/0603268](#)
[hep-ph/0702xxx](#)

Motivation

1. Can interactions between neutrinos and matter due to New Physics have any effect on neutrino oscillations ?

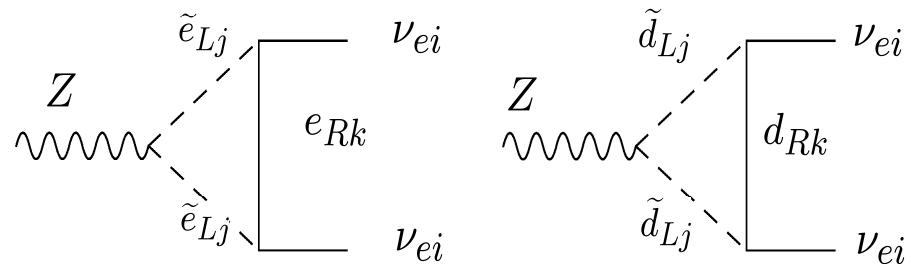


2. Can such effects be measured experimentally and be used to constrain New Physics ?

New Physics effect

- New Physics breaks the NC interaction

- radiative corrections
e.g. MSSM with
R-parity violation



- Direct exchange of the New Particle

- Gauged $B - \alpha L_e - \beta L_\mu - \gamma L_\tau$
 - Z' in topcolor assisted Technicolor
 - Leptoquarks that couple different generations
 - MSSM with R-Parity violating couplings
 - etc etc, Please consider the “fantastic model”.

Contents

- Motivation
- Can be measured ? (FNAL-to-HK)
- New Physics
 - Z' gauge boson
 - Leptoquarks
 - R -parity violation, SUSY
 - Extended Higgs Model
 -as you wish.....
- Summary

The effective Hamiltonian

U:MNS matrix

$$H_{\text{eff}}^2 = U \begin{bmatrix} 0 & 0 & 0 \\ 0 & \delta m_{21}^2 & 0 \\ 0 & 0 & \delta m_{31}^2 \end{bmatrix} U^\dagger + \begin{bmatrix} a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} b_e & 0 & 0 \\ 0 & b_\mu & 0 \\ 0 & 0 & b_\tau \end{bmatrix}$$

derived from universality violation
in NC and/or New Physics

ξ : universality violation in NC, New Physics

central value
of CHARM
 $\xi = 0.025$

$$= U \begin{bmatrix} 0 & 0 & 0 \\ 0 & \delta m_{21}^2 & 0 \\ 0 & 0 & \delta m_{31}^2 \end{bmatrix} U^\dagger.$$

$$= \tilde{U} \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \tilde{U}^\dagger$$

\tilde{U} : effective mixing matrix

$\lambda_1, \lambda_2, \lambda_3$: effective mass-squares

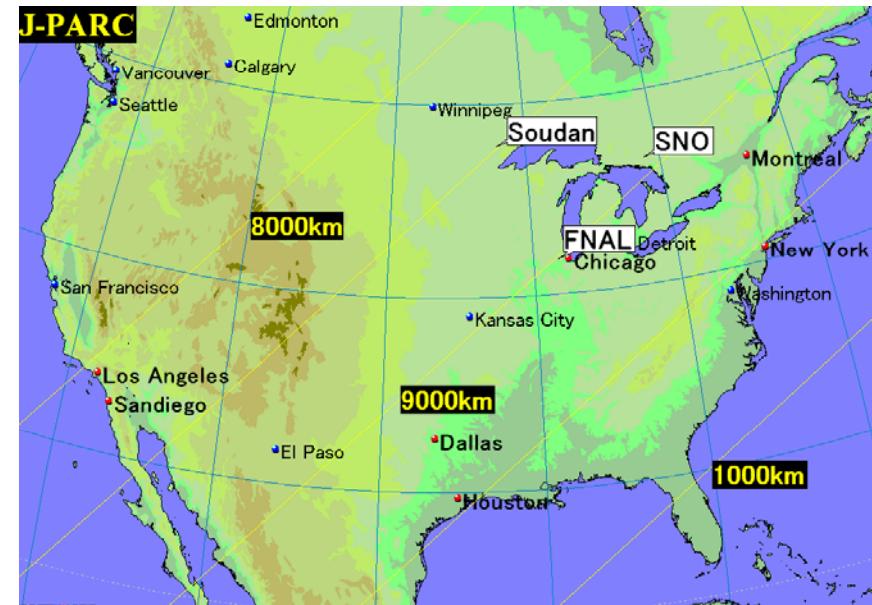
$$a \equiv 2EV_{CC} = 2\sqrt{2}G_F N_e E = 7.56 \times 10^{-5} (\text{eV})^2 \left(\frac{n_e}{g/\text{cm}^3} \right) \left(\frac{E}{g/\text{GeV}} \right)$$

$$b \equiv 2EV_{NC} = -\sqrt{2}G_F N_n E \approx -\frac{1}{2}a \quad (N_e = N_n = N_p)$$

loop correction

Can be measured ?

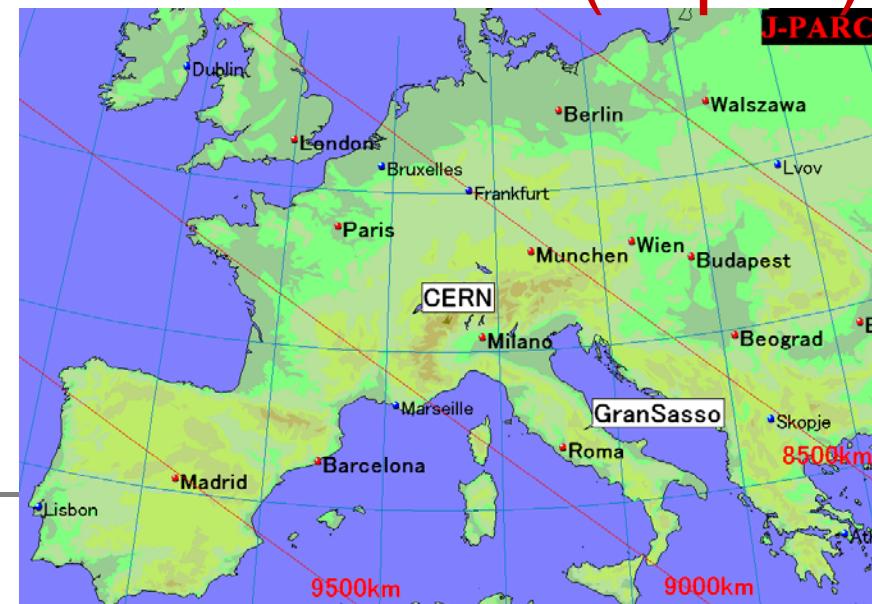
small value(ξ)
large matter effect
 \Leftrightarrow high energy
 \Leftrightarrow long base-line



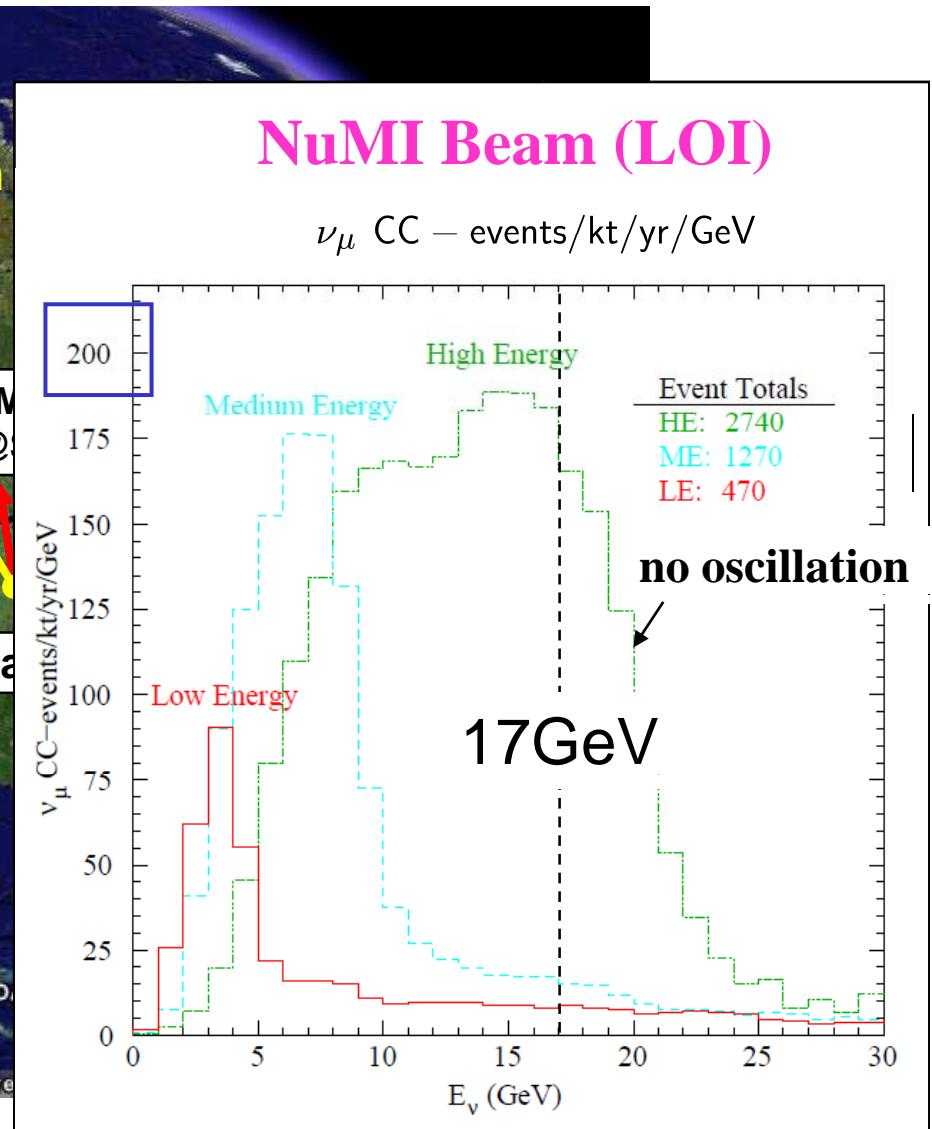
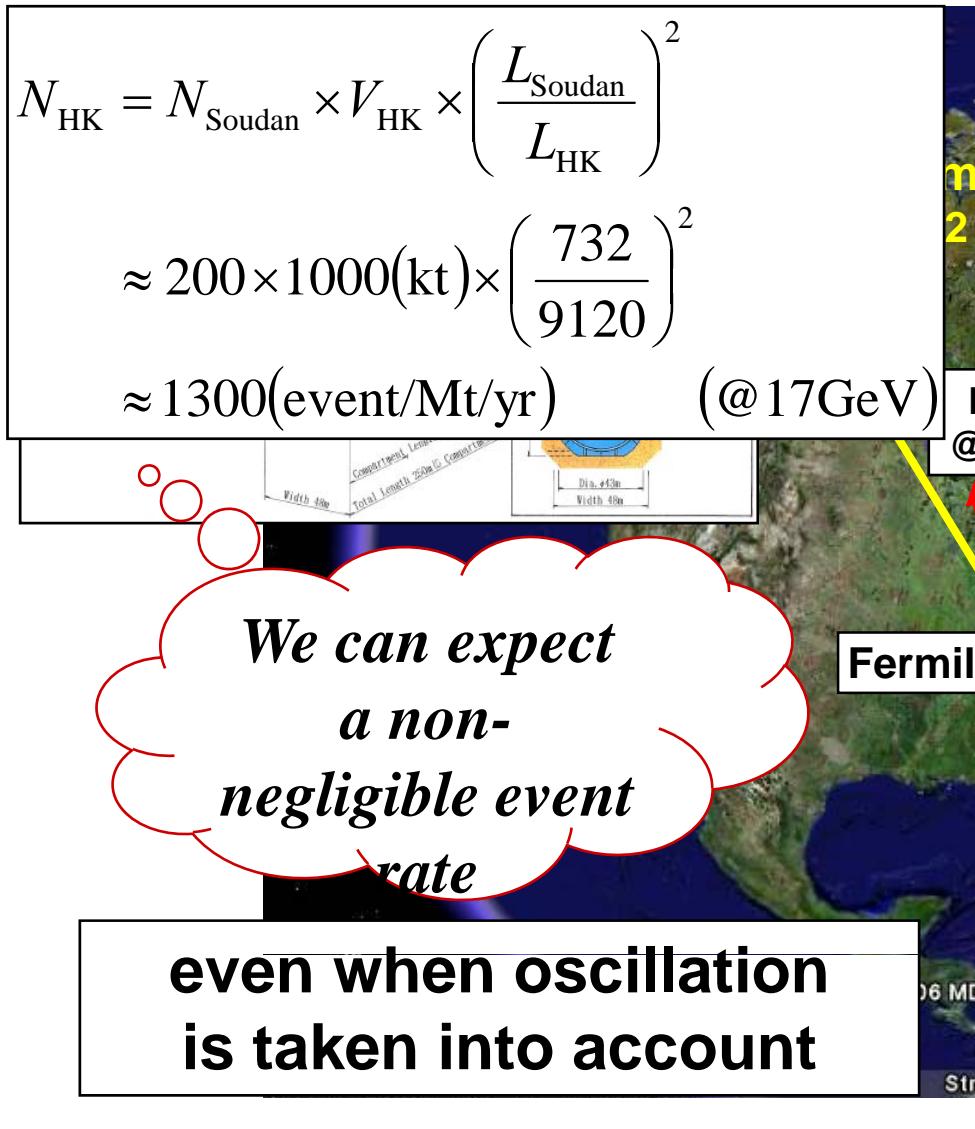
to Japan



from J-PARC (Japan)

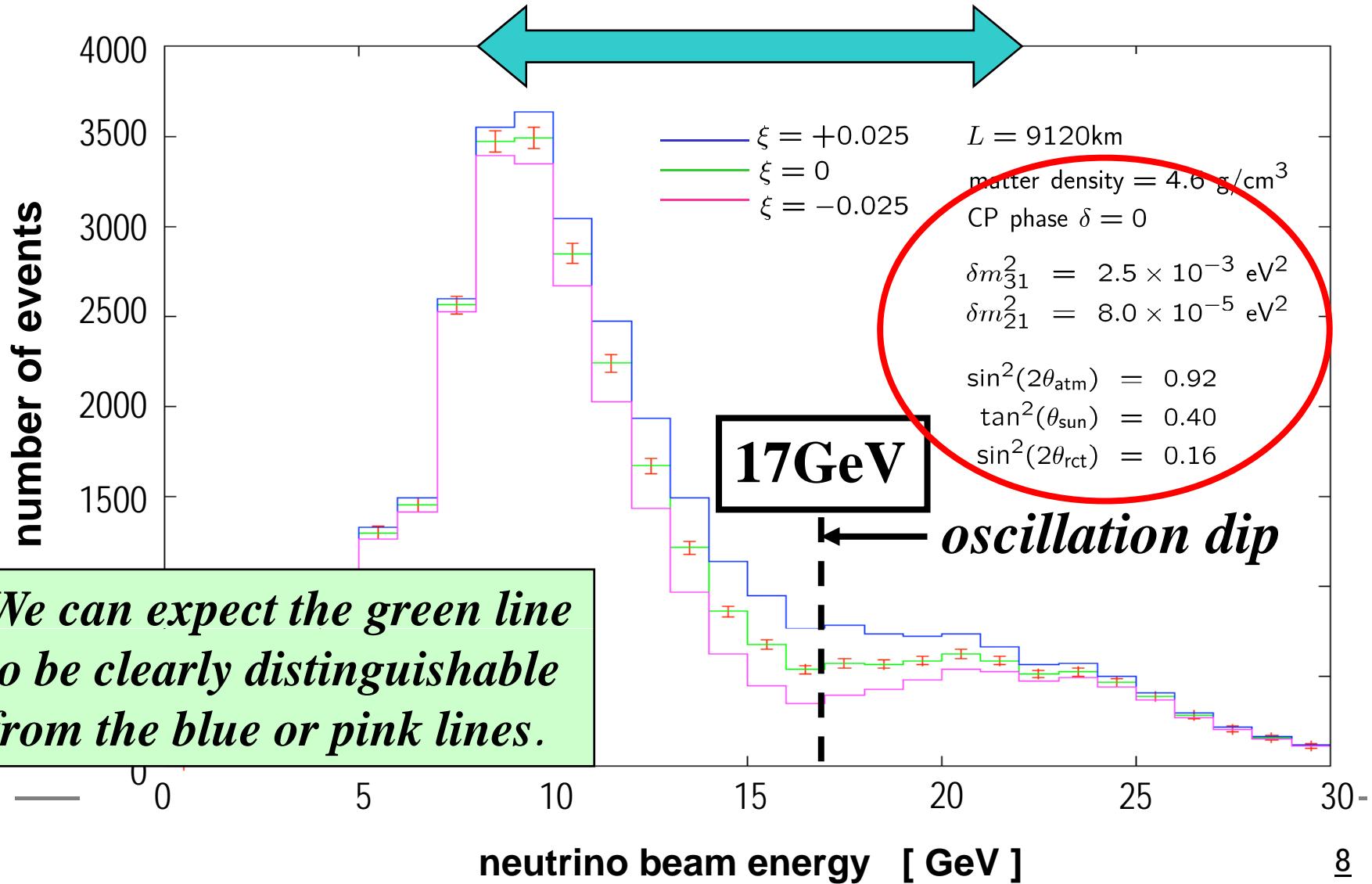


Fermilab $\xrightarrow{\nu_\mu}$ **HK [L~9,120 km]**

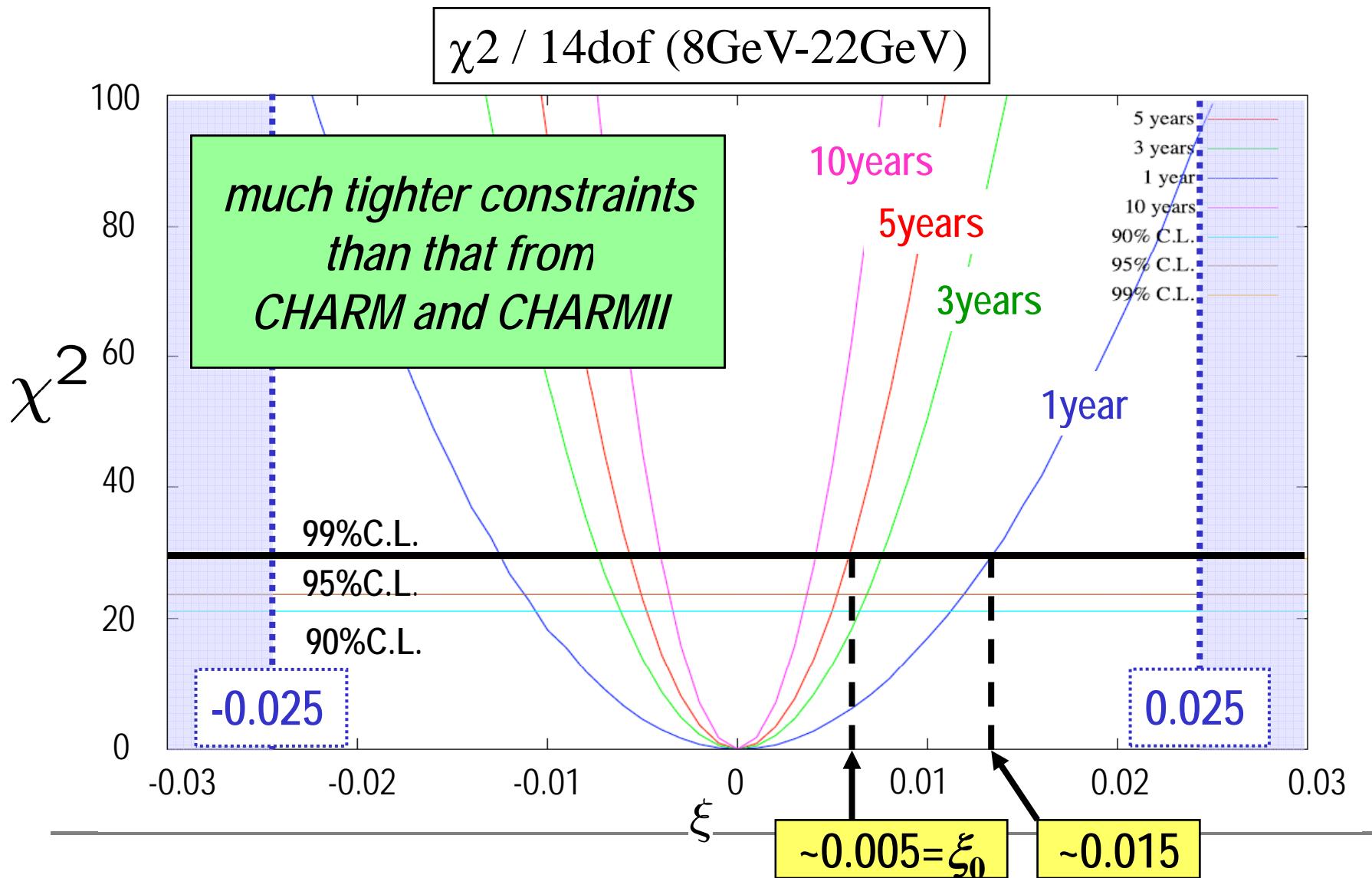


The expected number of $\nu_\mu \rightarrow \nu_\mu$ events@ HK

after 5 years of data taking (1Mton)



The constraints on universality violation ξ



New Physics

FNAL-to-HK

$$\xi > \xi_0 = 0.005$$

↓

constraints on new physics

Constraints on the New Physics

- Z' gauge boson
 - gauged $B-(\alpha L_e + \beta L_\mu + \gamma L_\tau)$ with $(\alpha + \beta + \gamma = 3)$
 - Topcolor Assisted Technicolor
- Leptoquarks
 - Scalar leptoquark
 - Vector leptoquark
- R -parity violation
- Extended Higgs Model

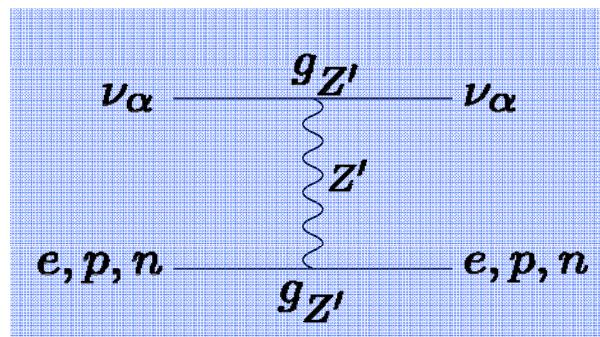
Z' boson

$SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$

Z' boson

1. gauged B -($\alpha L_e + \beta L_\mu + \gamma L_\tau$) with ($\alpha + \beta + \gamma = 3$)

$$L_{Z'} = g_{Z'} \left[\sum_q (\bar{q} \gamma^\mu q) - \alpha (\bar{l}_e \gamma^\mu l_e) - \beta (\bar{l}_\mu \gamma^\mu l_\mu) - \gamma (\bar{l}_\tau \gamma^\mu l_\tau) \right] Z'_\mu$$



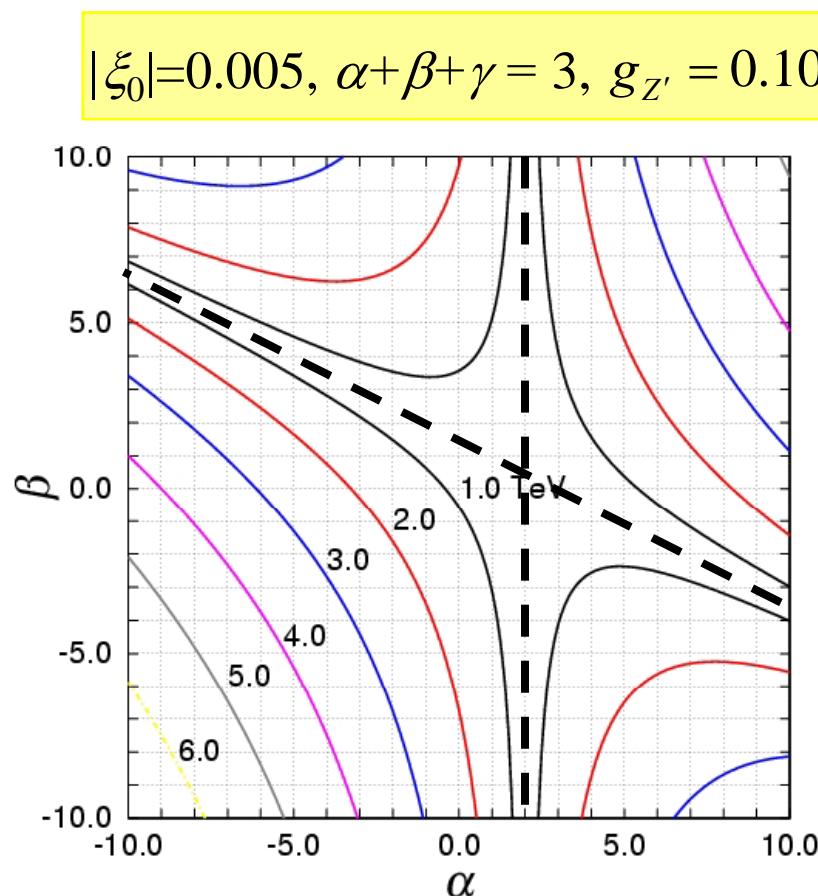
$$\begin{cases} V_{\nu_e} = \alpha [\alpha N_e - (N_p + N_n)] \frac{g_{Z'}^2}{M_{Z'}^2} \approx \alpha(\alpha-2)N \frac{g_{Z'}^2}{M_{Z'}^2} \\ V_{\nu_\mu} = \beta [\alpha N_e - (N_p + N_n)] \frac{g_{Z'}^2}{M_{Z'}^2} \approx \beta(\alpha-2)N \frac{g_{Z'}^2}{M_{Z'}^2} \\ V_{\nu_\tau} = \gamma [\alpha N_e - (N_p + N_n)] \frac{g_{Z'}^2}{M_{Z'}^2} \approx \gamma(\alpha-2)N \frac{g_{Z'}^2}{M_{Z'}^2} \end{cases}$$

$$\begin{aligned} \xi_{Z'} &= \frac{V_{\nu_\mu} - V_{\nu_\tau}}{2V_{NC}} \\ &= -4(\alpha-2)(\alpha+2\beta-3) \frac{(g_{Z'}/M_{Z'})^2}{(g/M_W)^2} \\ |\xi_{Z'}| &\leq \xi_0 = 0.005 \end{aligned}$$

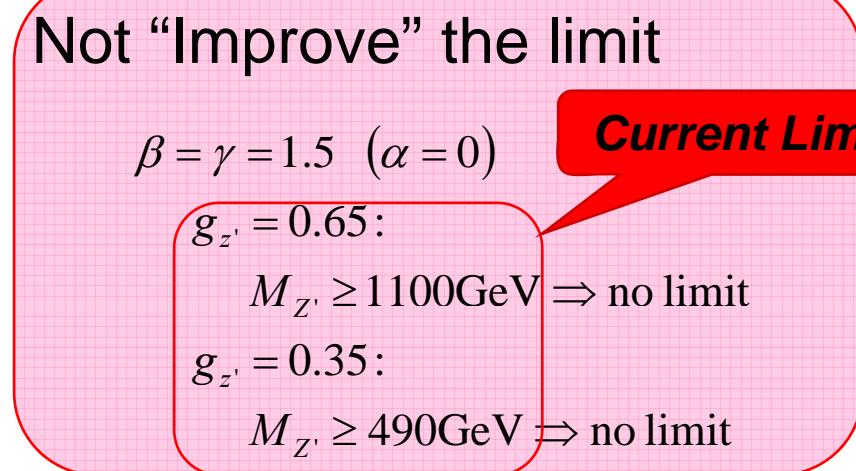
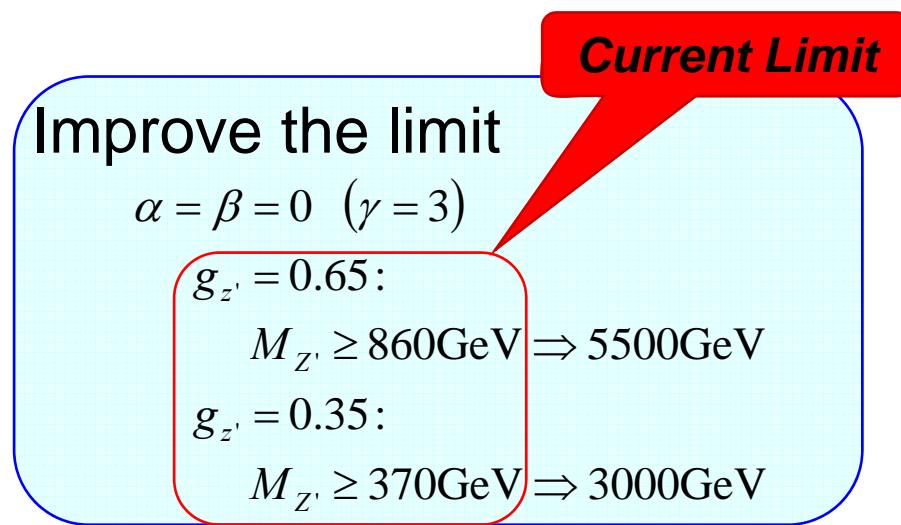
If ξ_0 is larger than 0.005,
the effect on the neutrino oscillation
gives the constraint on the mass of Z'

$$M_{Z'} \geq g_{Z'} \sqrt{\frac{|\alpha-2||\alpha+2\beta-3|}{\sqrt{2}G_F\xi_0}}$$

Constraint on $M_{Z'}$, (gauged)



$$M_{Z'} \geq g_{Z'} \sqrt{\frac{|\alpha - 2\beta| |\alpha + 2\beta - 3|}{\sqrt{2} G_F \xi_0}}$$



Z' boson

2. Topcolor Assisted Technicolor

C.T.Hill, PLB345,483(95)
 G.Buchalla,etal, PRD53,5185(96)
 W.Loinaz,etal,PRD60,015005(99)

$$L = g' (\cot \theta \cdot J_{1s}^\mu - \tan \theta \cdot J_{1w}^\mu) Z'_\mu + g' (J_{1s}^\mu + J_{1w}^\mu) B_\mu \quad \theta \ll 1$$

$$J_{1s}^\mu = \frac{1}{6} (\bar{t}_L \gamma^\mu t_L + \bar{b}_L \gamma^\mu b_L) + \frac{2}{3} \bar{t}_R \gamma^\mu t_R - \frac{1}{2} \bar{b}_R \gamma^\mu b_R - \frac{1}{2} (\bar{\tau}_L \gamma^\mu \tau_L + \bar{\nu}_\tau \gamma^\mu \nu_\tau) - \bar{\tau}_R \gamma^\mu \tau_R$$

J_{1w}^μ = [1st. and 2nd. generation]

Current limit
 $M_{Z'} \sim 200 \text{ GeV}$

$$V_{\nu_e} \approx 0, \quad V_{\nu_\mu} \approx 0, \quad V_{\nu_\tau} \approx -\frac{N}{8} \frac{g_{Z'}^2}{M_{Z'}^2}$$

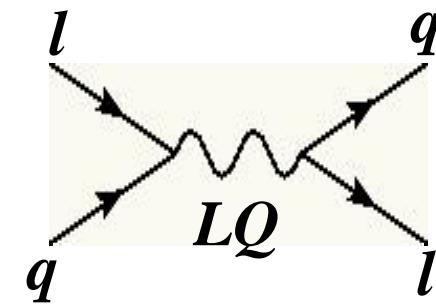
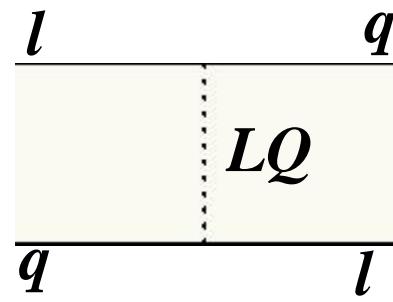
$$\xi_{TT} = -\frac{V_{\nu_\tau}}{2V_{NC}} = -\frac{1}{2} \frac{(g'/M_{Z'})^2}{(g/M_W)^2} = -\frac{1}{2} \sin^2 \theta_W \frac{M_Z^2}{M_{Z'}^2} \approx -0.1 \frac{M_Z^2}{M_{Z'}^2}$$

$$M_{Z'} \geq \sqrt{\frac{0.2311 \times (91.19)^2}{2 \times 0.005}} \approx 440 \text{ GeV} \left(\frac{0.005}{\xi_{TT}} \right)^{1/2}$$

If $M_{Z'} < 440 \text{ GeV} \Leftrightarrow$ The effect can be measured.



Leptoquarks



Leptoquarks

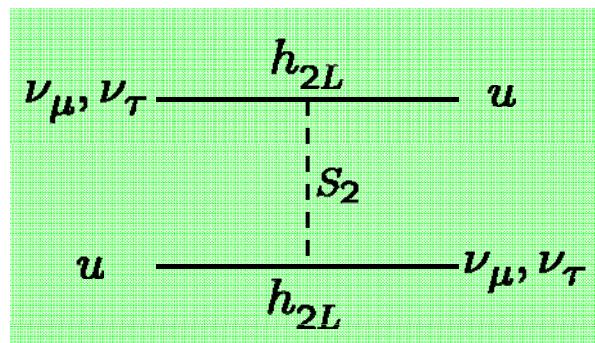
There are many leptoquarks (more than 10)

Fermion number : $\underline{F=3B+L}$, \underline{S} : scalar, \underline{V} : vector, subscript:dim.SU(2)

$$F = 2 : S_1, \tilde{S}_1, \vec{S}_3, V_{2\mu}, \tilde{V}_{2\mu} \quad F = 0 : S_2, \tilde{S}_2, V_{1\mu}, \tilde{V}_{1\mu}, \vec{V}_{3\mu}$$

$$\begin{aligned} L_{F=2} = & \left[g_{1L} \bar{q}_L^C i\tau_2 l_L + g_{1R} \bar{u}_R^C e_R \right] S_1 + \tilde{g}_{1R} \left[\bar{d}_R^C e_R \right] \tilde{S}_1 + g_{3L} \left[\bar{q}_L^C i\tau_2 \vec{\tau} l_L \right] \vec{S}_3 \\ & + \left[g_{2L} \bar{d}_R^C \gamma^\mu l_L + g_{2R} \bar{q}_R^C \gamma^\mu e_R \right] V_{2\mu} + \tilde{g}_{2L} \left[\bar{u}_R^C \gamma^\mu l_L \right] \tilde{V}_{2\mu} + h.c. \end{aligned}$$

$$\begin{aligned} L_{F=0} = & \left[h_{2L} \bar{u}_R l_L + h_{2R} \bar{q}_L i\tau_2 e_R \right] S_2 + \tilde{h}_{2L} \left[\bar{d}_R l_L \right] \tilde{S}_2 \\ & + \left[h_{1L} \bar{q}_L \gamma^\mu l_L + h_{1R} \bar{d}_R \gamma^\mu e_R \right] V_{1\mu} + \tilde{h}_{1R} \left[\bar{u}_R \gamma^\mu e_R \right] \tilde{V}_{1\mu} + h_{3L} \left[\bar{q}_L \vec{\tau} \gamma^\mu l_L \right] \vec{V}_{3\mu} + h.c. \end{aligned}$$

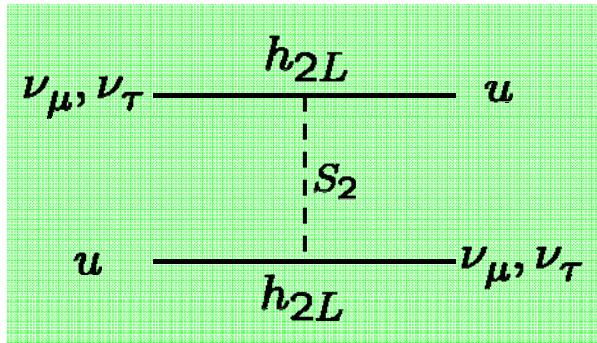


Only S_2 and V_3 cases are shown, today.
(The others are in our paper)
There is no special reason.

Leptoquarks (S_2)

$$L_{F=0} = \left[h_{2L}^{ijk} \bar{u}_{iR} l_{jL} + h_{2R}^{ijk} \bar{q}_{iL} i\tau_2 e_{jR} \right] S_{2k} + \tilde{h}_{2L}^{ijk} \bar{d}_{iR} l_{jL} \tilde{S}_{2k} + h.c. \quad (k=1 \cdots L)$$

$$= h_{2L}^{12k} (\bar{u}_R \nu_{\mu L}) S_{2k}^- + h_{2L}^{13k} (\bar{u}_R \nu_{\tau L}) S_{2k}^- + \tilde{h}_{2L}^{12k} (\bar{d}_R \nu_{\mu L}) \tilde{S}_{2k}^- + \tilde{h}_{2L}^{13k} (\bar{d}_R \nu_{\tau L}) \tilde{S}_{2k}^- + h.c.$$



$$V_{\nu_\mu} = \frac{3}{4} N \sum_{k=1}^L \frac{|h_{2L}^{12k}|^2}{M_{S_{2k}}}, \quad V_{\nu_\tau} = \frac{3}{4} N \sum_{k=1}^L \frac{|h_{2L}^{13k}|^2}{M_{S_{2k}}}$$

$$\xi_{S_2^-} = \frac{V_{\nu_\mu} - V_{\nu_\tau}}{2V_{NC}} = -3 \sum_{k=1}^L \frac{\left| h_{2L}^{12k} / M_{S_{2k}} \right|^2 - \left| h_{2L}^{13k} / M_{S_{2k}} \right|^2}{(g/M_W)^2}$$

For $M_{S_2^-} = 100\text{GeV}$ and $L = 1$

$$\left| |h_{2L}^{12}|^2 - |h_{2L}^{13}|^2 \right| < 1.1 \times 10^{-3}$$

$$\left| |\tilde{h}_{2L}^{12}|^2 - |\tilde{h}_{2L}^{13}|^2 \right| < 1.1 \times 10^{-3}$$

Current Limit

$$(h_{2L}^{12})^2 \leq 1 \quad (\mu N \rightarrow \mu X)$$

$$(\tilde{h}_{2L}^{12})^2 \leq 2 \quad (\mu N \rightarrow \mu X)$$

Leptoquarks (V_3)

$$\begin{aligned}
 L_{F=0} &= h_{3L}^{ijk} \left[\bar{q}_{iL} \vec{\tau} \gamma^\mu l_L \right] \vec{V}_{3k,\mu} \quad (k = 1 \dots L) \\
 &= h_{3L}^{12k} \left[(\bar{u}_L \gamma^\mu v_{\mu L}) V_{3k,\mu}^0 + \sqrt{2} (\bar{d}_L \gamma^\mu v_{\mu L}) V_{3k,\mu}^- \right] \\
 &\quad + h_{3L}^{13k} \left[(\bar{u}_L \gamma^\mu v_{\tau L}) V_{3k,\mu}^0 + \sqrt{2} (\bar{d}_L \gamma^\mu v_{\tau L}) V_{3k,\mu}^- \right] + h.c.
 \end{aligned}$$

$$V_{\nu_\mu} = 3N \sum_{k=1}^L \left| h_{3L}^{12k} \right|^2 \left[\frac{1}{2M_{V_{3k}^0}^2} + \frac{1}{M_{V_{3k}^-}^2} \right], \quad V_{\nu_\tau} = 3N \sum_{k=1}^L \left| h_{3L}^{13k} \right|^2 \left[\frac{1}{2M_{V_{3k}^0}^2} + \frac{1}{M_{V_{3k}^-}^2} \right]$$

$$\xi_{\vec{V}_3} = \frac{V_{\nu_\mu} - V_{\nu_\tau}}{2V_{NC}} = -\frac{6}{(g/M_W)^2} \sum_{k=1}^L \left(\left| h_{3L}^{12k} \right|^2 - \left| h_{3L}^{13k} \right|^2 \right) \left[\frac{1}{M_{V_{3k}^0}^2} + \frac{2}{M_{V_{3k}^-}^2} \right]$$

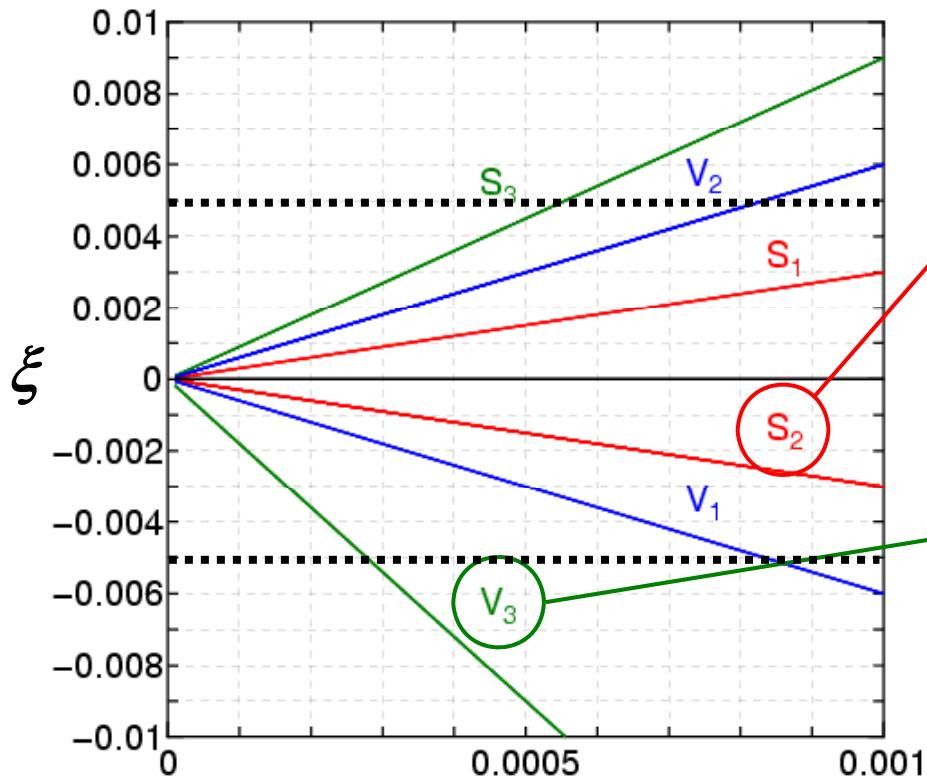
For $M_{S_2^-} = 100\text{GeV}$ and $L = 1$

Current Limit

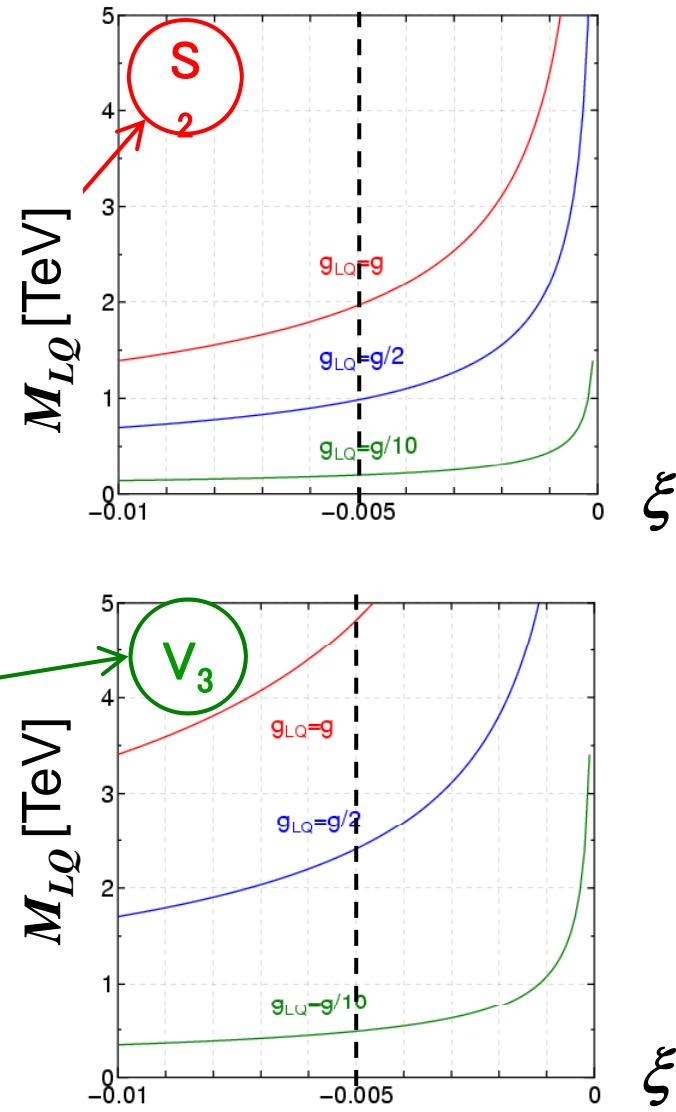
$$\left| \left| h_{3L}^{12} \right|^2 - \left| h_{3L}^{13} \right|^2 \right| < 1.8 \times 10^{-4} \quad \left(h_{3L}^{12} \right)^2 \leq 0.004 (R_\pi), \quad \left(h_{3L}^{13} \right)^2 \leq 0.1 (D \rightarrow \mu\nu)$$

Leptoquarks

S : scalar, V : vector,
subscript : dim of SU(2) rep.



$$\frac{(\Delta g_{LQ}/M_{LQ})^2}{(g/M_W)^2}$$



SUSY R-parity Violation

$$W_R = \frac{1}{2} \lambda_{ijk} \hat{L}_i \hat{L}_j \hat{E}_k + \lambda'_{ijk} \hat{L}_i \hat{Q}_j \hat{D}_k + \frac{1}{2} \lambda''_{ijk} \hat{U}_i \hat{D}_j \hat{D}_k$$

tightly
constrained
by lepton univ.

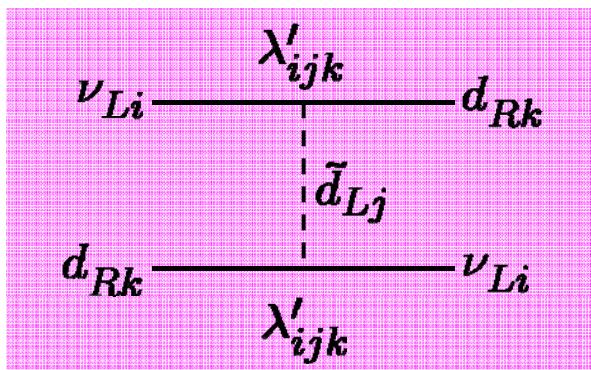
less stringent

irrelevant

SUSY

R-parity violation

$$W_R = \frac{1}{2} \lambda_{ijk} \hat{L}_i \hat{L}_j \hat{E}_k + \underline{\lambda'_{ijk} \hat{L}_i \hat{Q}_j \hat{D}_k} + \frac{1}{2} \lambda''_{ijk} \hat{U}_i \hat{D}_j \hat{D}_k$$



Similar to S_1 and \tilde{S}_2^- type leptoquarks

$$V_{\nu_\mu} = \sum_{j=1}^3 \frac{3N}{4M_{\tilde{d}_j}^2} \left(|\lambda'_{2j1}|^2 - |\lambda'_{21j}|^2 \right)$$

$$V_{\nu_\tau} = \sum_{j=1}^3 \frac{3N}{4M_{\tilde{d}_j}^2} \left(|\lambda'_{3j1}|^2 - |\lambda'_{31j}|^2 \right)$$

$$\xi_{\tilde{d}} = \frac{V_{\nu_\mu} - V_{\nu_\tau}}{2V_{NC}} = -3 \sum_{j=1}^3 \frac{\left(|\lambda'_{2j1}|^2 - |\lambda'_{21j}|^2 \right) - \left(|\lambda'_{3j1}|^2 - |\lambda'_{31j}|^2 \right)}{(g/M_W)^2} \frac{1}{M_{\tilde{d}_j}^2} = -\frac{3\sqrt{2}}{8G_F} \frac{\Delta \lambda_{\tilde{d}}^2}{M_{\tilde{d}}^2}$$

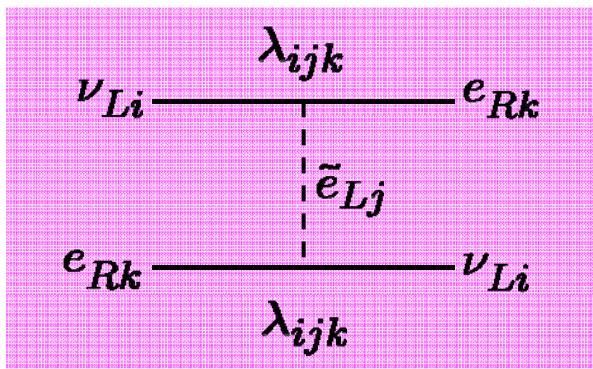
$$M_{\tilde{d}} \geq \sqrt{|\Delta \lambda_{\tilde{d}}^2|} \frac{\sqrt{3}}{2} \sqrt{\frac{1}{\sqrt{2}G_F \xi_{\tilde{d}}}} \approx 3.0[\text{TeV}] \sqrt{|\Delta \lambda_{\tilde{d}}^2|} \left(\frac{0.005}{\xi_{\tilde{d}}} \right)^{1/2}$$

Current Limit
 $M_d > 300\text{GeV}$

SUSY

R-parity violation

$$W_R = \frac{1}{2} \lambda_{ijk} \hat{L}_i \hat{L}_j \hat{E}_k + \lambda'_{ijk} \hat{L}_i \hat{Q}_j \hat{D}_k + \frac{1}{2} \lambda''_{ijk} \hat{U}_i \hat{D}_j \hat{D}_k$$



Similar S_1 , S_2^- and \tilde{S}_2^- type leptoquarks

$$V_{\nu_\mu} = \frac{N}{4} \left(\sum_{i=1,3} \frac{|\lambda'_{i21}|^2}{M_{\tilde{e}_i}^2} - \sum_{i=1}^3 \frac{|\lambda'_{21i}|^2}{M_{\tilde{e}_i}^2} \right)$$

$$V_{\nu_\tau} = \frac{N}{4} \left(\sum_{i=1,2} \frac{|\lambda'_{i31}|^2}{M_{\tilde{e}_i}^2} - \sum_{i=1}^3 \frac{|\lambda'_{31i}|^2}{M_{\tilde{e}_i}^2} \right)$$

$$\xi_{\tilde{e}} = \frac{V_{\nu_\mu} - V_{\nu_\tau}}{2V_{NC}} = - \frac{\sum_{i=1,3} |\lambda_{i21}|^2 / M_{\tilde{e}_i}^2 + \sum_{i=1,2} |\lambda_{i31}|^2 / M_{\tilde{e}_i}^2 - \sum_{i=1}^3 (|\lambda_{21i}|^2 + |\lambda_{31i}|^2) / M_{\tilde{e}_i}^2}{(g/M_W)^2}$$

$$= -\frac{\sqrt{2}}{8G_F} \frac{\Delta \lambda_{\tilde{e}}^2}{M_{\tilde{e}}^2},$$

$$M_{\tilde{e}} \geq 1.7[\text{TeV}] \sqrt{|\Delta \lambda_{\tilde{e}}^2|} \left(\frac{0.005}{\xi_{\tilde{e}}} \right)^{1/2}$$

Current Limit
 $\lambda \sim O(10^{-2})$

Extended Higgs

triplet Higgs for neutrino mass

Extended Higgs Models

$$L = f_{ij} \left(\bar{l}_{iL}^C i\tau_2 l_{jL} \right) h^+ + h.c.$$

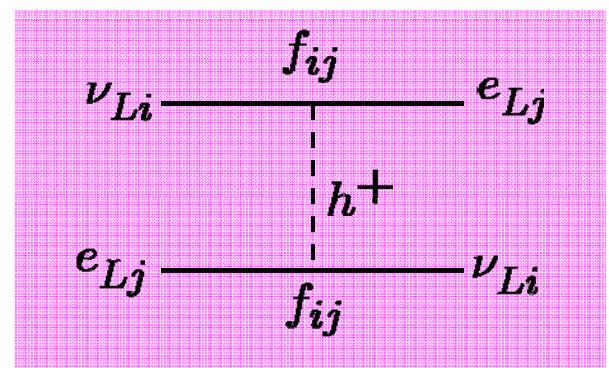
$$= -2 \left(f_{e\mu} \bar{\nu}_\mu^C e_L + f_{e\tau} \bar{\nu}_\tau^C e_L \right) h^+ + h.c. + \dots$$

f_{ij} : Yukawa couplings (anti-symmetric)
 h^+ : Higgs particle with $Y=1$

$$V_{\nu_\mu} = -N \frac{|f_{e\mu}|^2}{M_{h^+}^2}, \quad V_{\nu_\tau} = -N \frac{|f_{e\tau}|^2}{M_{h^+}^2}$$

$$\xi_{h^+} = \frac{V_{\nu_\mu} - V_{\nu_\tau}}{2V_{NC}} = 4 \frac{\left(|f_{e\mu}|^2 - |f_{e\tau}|^2 \right) / M_{h^+}^2}{\left(g/M_W \right)^2} = \frac{4\sqrt{2}}{8G_F} \frac{\Delta\lambda_{h^+}^2}{M_{h^+}^2}$$

K.S.Babu, PLB203, 132 (88)
A.Zee, PLB93, 389 (80)
E.Accomande etal,
hep-ph/0608079, p497



$$\left| \frac{|f_{e\mu}|^2 - |f_{e\tau}|^2}{M_{h^+}^2} \right| = \sqrt{2} G_F \xi_0 \approx 8.2 \times 10^{-8}$$

↔ Current Limit

$$3.4 \times 10^{-8} (\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)$$



Summary

- New Physics effects on neutrino oscillation
- FNAL -to- HK
 - $\xi_0 = (V_{\nu\mu} - V_{\nu\tau})/V_{NC} \sim 0.005$
 - (coupling / mass)²
- Loop correction to the NC
- Direct interaction
 - Z' gauge boson
 - Leptoquarks
 - R-parity violation
 - Extended Higgs Model

*Thank you very much
for your attention*



topcolor assisted technicolor

topcolor

top quark condensate gives rise to a triplet of NG bosons $\Leftrightarrow W, Z$.

$$f_\pi^2 = m_t^2 \left(\frac{N_C}{16\pi^2} \ln \frac{\Lambda^2}{\mu^2} \right), \quad O(\mu) \sim O(m_t)$$

top-pions

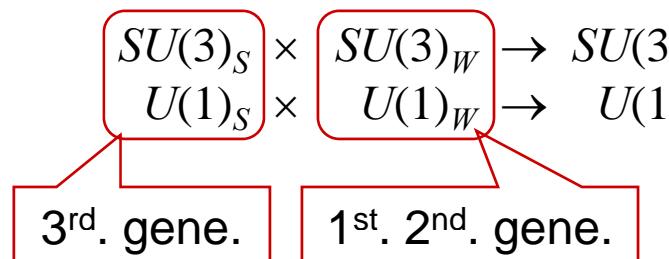
$f_\pi = 174 \text{ GeV} \Leftrightarrow \Lambda \sim 10^{13-14} \text{ GeV} \Leftrightarrow$ large hierarchy \Leftrightarrow fine tuning of the c.c.

T.A.T.

$$\begin{aligned} SU(3)_S \times SU(3)_W \times U(1)_S \times U(1)_W \times SU(2)_L &\xrightarrow{\text{TechniC}} SU(3)_C \times U(1)_Y \times SU(2)_L \\ &\xrightarrow{\text{TopC}} SU(3)_C \times U(1)_{em} \end{aligned}$$

$$F_\pi \approx 167 \text{ GeV}, \text{ TechniC} \quad f_\pi \approx 50 \text{ GeV}, \quad \Lambda_{\text{TopC}} \sim \text{TeV}$$

The majority of the W and Z masses come from the technifermion condensate.



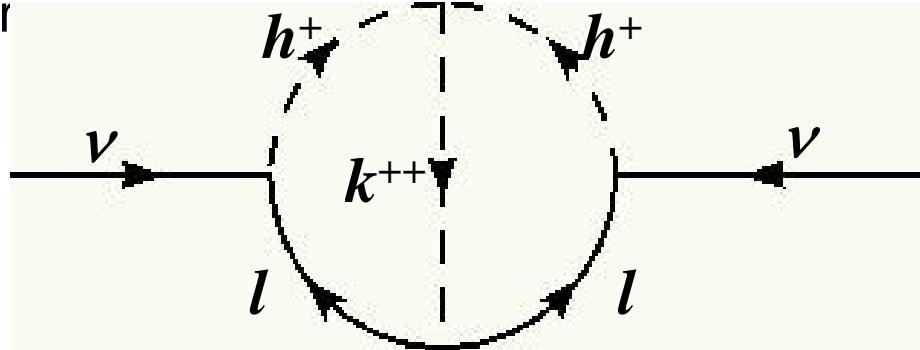
Extended Higgs Models

$$L = f_{ij} \left(\bar{l}_{iL}^C i\tau_2 l_{jL} \right) h^+ + h_{ij} \left(\bar{l}_{iR}^C l_{jR} \right) k^{++} - \mu h^+ h^+ k^{--} + h.c.$$

f_{ij} : anti-symmetric, h_{ij} : symmetric

h^+ : Higgs $\#L = -2$

k^{++} : Higgs $\#L = -2$



$$(M_\nu)_{ij} = 8\mu f_{ia} m_a h_{ab}^* m_b f_{bj} \cdot I_{ab}$$

$$I_{ab} \approx \frac{1}{(16\pi^2)^2} \frac{1}{m_h^2} \tilde{I}\left(\frac{m_k^2}{m_h^2}\right)$$

$$\tilde{I}(r) = - \int_0^1 dx \int_0^{1-x} dy \frac{1-y}{x + (r-1)y + y^2} \log \frac{y(1-y)}{x + ry}$$



“ぺんぎんだいあぐらむ”って
知ってる？

こんな格好
でしょ？



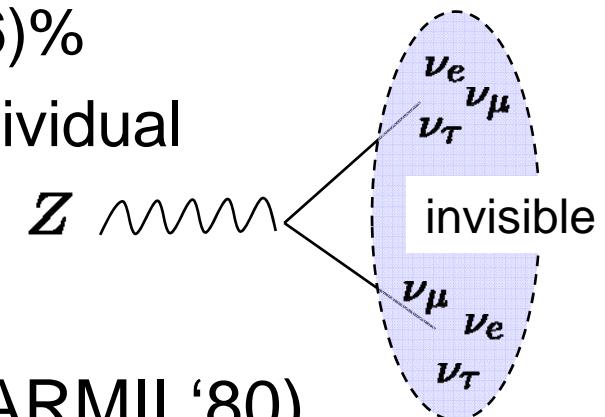
Experimental Constraints (NC)

- Invisible decay width (LEP/SLC)
 - Invisible Z decay width, $(20.00 \pm 0.06)\%$
 - No information is available on the individual decay widths into each flavor.
- Individual decay width (CHARM/CHARMII '80)
 - Z couplings to neutrinos [PLB180,303('86), PLB320 203('94)]

$$g^{\nu_e} : 0.528 \pm 0.085$$

$$g^{\nu_e} : 0.502 \pm 0.017$$

$$g^{\nu_e}/g^{\nu_\mu} = 1.05^{+0.15}_{-0.18} \rightarrow \boxed{0.9-1.2}$$



- The theoretical possibility of universality violation in many models
- The weakness of the current experimental bounds

Z DECAY MODES	Fraction (Γ_i/Γ)
$e^+ e^-$	(3.363 \pm 0.004) %
$\mu^+ \mu^-$	(3.366 \pm 0.007) %
$\tau^+ \tau^-$	(3.370 \pm 0.008) %
invisible	(20.00 \pm 0.06) %
hadrons	(69.91 \pm 0.06) %

CHARM,CHARM II

g^{ν_e} **0.528 ± 0.085**

g^{ν_μ} **0.502 ± 0.017**

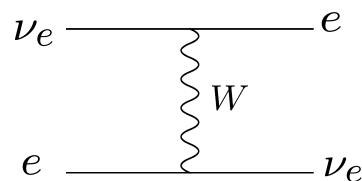
g^{ν_e}/g^{ν_μ} **$1.05^{+0.15}_{-0.18}$**

*It would be interesting
to analyze the effect of
such a violation on neutrino oscillations.*



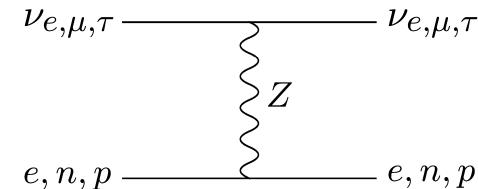
The effective potentials

- charged current interaction



$$\begin{aligned} \mathcal{M}_{CC} &= -\frac{G_F}{\sqrt{2}} [\bar{e}\gamma^\mu(1-\gamma_5)\nu_e] [\bar{\nu}_e\gamma_\mu(1-\gamma_5)e] \\ &\quad \downarrow \text{Fierz tran.} \\ &= +\frac{G_F}{\sqrt{2}} \underbrace{[\bar{e}\gamma^\mu(1-\gamma_5)e]}_{N_e} \underbrace{[\bar{\nu}_e\gamma_\mu(1-\gamma_5)\nu_e]}_2 \end{aligned}$$

- neutral current interaction



$$\begin{aligned} \mathcal{M}_{NC} &= \rho \frac{G_F}{\sqrt{2}} \underbrace{[\bar{\nu}_\alpha\gamma^\mu(1-\gamma_5)\nu_\alpha]}_2 \\ &\quad \times \left[g_L^f \underbrace{\bar{\psi}_f\gamma_\mu(1-\gamma_5)\psi_f}_{N_f} + g_R^f \underbrace{\bar{\psi}_f\gamma_\mu(1+\gamma_5)\psi_f}_{N_f} \right] \\ I_L^f &= I_3^f - Q_f \sin^2 \theta_W, \quad g_L^f = -Q_f \sin^2 \theta_W \quad (f = e, n, p) \end{aligned}$$

→ $V_{CC} = \sqrt{2} G_F N_e$

N_e : electron number density
in matter

$$g_L^f + g_R^f = I_3^f - 2Q_f \sin^2 \theta_W$$

$$f = e^- : (-1/2) - 2(-1) \sin^2 \theta_W$$

$$p : (+1/2) - 2(+1) \sin^2 \theta_W$$

$$\pi : (-1/2) - 2(0) \sin^2 \theta_W \sim \frac{1}{2}$$

$$\begin{aligned} V_{NC} &= \rho \sqrt{2} G_F N_f (g_L^f + g_R^f) \\ &= \rho \sqrt{2} G_F N_n (-1/2) \end{aligned}$$

$V_{NC} \approx (-\sqrt{2}/2) G_F N_n$
 $\approx (-1/2) V_{CC}$

N_n : neutron number density

effective mixing angles (normal)

$\delta = 0$ case, expanded in powers of $\varepsilon \equiv \sqrt{\frac{\delta m_{21}^2}{|\delta m_{31}^2|}} = 0.15 \sim 0.24$

in the region of $\frac{a}{|\delta m_{31}^2|} \sim O(\varepsilon^{-1})$: ($E(\text{GeV}) \sim O(10)$), $\xi \sim O(\varepsilon^2)$

neutrino

$$\tilde{\theta}_{13} = \theta'_{13} \approx \frac{\pi}{2} - \left(\frac{\delta m_{31}^2}{a} \right) \theta_{13}$$

$$\tilde{\theta}_{12} = \frac{\pi}{2} - \frac{c_{13}}{c'_{13}} \left(\frac{\delta m_{21}^2}{2a} \right) \sin 2\theta_{12} - \xi \frac{a}{2\delta m_{31}^2}$$

$$\tilde{\theta}_{23} = \theta_{23} + \frac{s_\phi}{c'_{13}} \left(\frac{\delta m_{21}^2}{2a} \right) \sin 2\theta_{12}$$

$$\tilde{\delta} = 0$$

$$\left(\phi = \frac{\pi}{2} - \theta_{13} - \left(\frac{\delta m_{31}^2}{a} \right) \theta_{13} \right)$$

anti - neutrino

$$\tilde{\theta}_{13} = \theta'_{13} \approx \left(\frac{\delta m_{31}^2}{a} \right) \theta_{13}$$

$$\tilde{\theta}_{12} = \left(\frac{\delta m_{21}^2}{2a} \right) \sin 2\theta_{12}$$

$$\tilde{\theta}_{23} = \theta_{23} + \xi \frac{a}{2\delta m_{31}^2}$$

$$\tilde{\delta} = 0$$

effective mixing angles (inverted)

$\delta = 0$ case, expanded in powers of $\varepsilon \equiv \sqrt{\frac{\delta m_{21}^2}{|\delta m_{31}^2|}} = 0.15 \sim 0.24$

in the region of $\frac{a}{|\delta m_{31}^2|} \sim O(\varepsilon^{-1}) : (E(\text{GeV}) \sim O(10)), \xi \sim O(\varepsilon^2)$

neutrino

$$\tilde{\theta}_{13} = \theta'_{13} \approx -\left(\frac{\delta m_{31}^2}{a}\right)\theta_{13}$$

$$\tilde{\theta}_{12} = \frac{\pi}{2} - \left(\frac{\delta m_{21}^2}{2a}\right) \sin 2\theta_{12}$$

$$\tilde{\theta}_{23} = \theta_{23} + \xi \frac{a}{2|\delta m_{31}^2|}$$

$$\tilde{\delta} = 0$$

anti - neutrino

$$\tilde{\theta}_{13} = \theta'_{13} \approx \frac{\pi}{2} + \left(\frac{\delta m_{31}^2}{a}\right)\theta_{13}$$

$$\tilde{\theta}_{12} = \frac{c_{13}}{c'_{13}} \left(\frac{\delta m_{21}^2}{2a}\right) \sin 2\theta_{12} - \xi \frac{a}{2|\delta m_{31}^2|}$$

$$\tilde{\theta}_{23} = \theta_{23} - \frac{s_\phi}{c'_{13}} \left(\frac{\delta m_{21}^2}{2a}\right) \sin 2\theta_{12}$$

$$\tilde{\delta} = 0$$

$$\left(\phi = \frac{\pi}{2} - \theta_{13} - \left(\frac{|\delta m_{31}^2|}{a}\right)\theta_{13} \right)$$

shift of effective mixing by ξ

	<i>neutrino</i>	<i>anti-neutrino</i>
<i>normal hierarchy</i>	$\tilde{\theta}_{12}$ is shifted by $-\frac{a}{2 \delta m_{31}^2 }\xi$	$\tilde{\theta}_{23}$ is shifted by $+\frac{a}{2 \delta m_{31}^2 }\xi$
<i>inverted hierarchy</i>	$\tilde{\theta}_{23}$ is shifted by $+\frac{a}{2 \delta m_{31}^2 }\xi$	$\tilde{\theta}_{12}$ is shifted by $-\frac{a}{2 \delta m_{31}^2 }\xi$

in the region of

$$\frac{a}{|\delta m_{31}^2|} \approx O(\varepsilon^{-1}): (E(\text{GeV}) \sim O(10)), \quad \xi \approx O(\varepsilon^2), \quad \left(\varepsilon \equiv \sqrt{\frac{\delta m_{21}^2}{|\delta m_{31}^2|}} = 0.15 \sim 0.24 \right)$$

Survival Probability

$$\begin{aligned}
 \tilde{P}(\nu_\mu \rightarrow \nu_\mu) &= 1 - 4|\tilde{U}_{\mu 2}|^2 \left(1 - |\tilde{U}_{\mu 2}|^2\right) \sin^2 \frac{\tilde{\Delta}_{21}}{2} - 4|\tilde{U}_{\mu 3}|^2 \left(1 - |\tilde{U}_{\mu 3}|^2\right) \sin^2 \frac{\tilde{\Delta}_{31}}{2} \\
 &\quad + 2|\tilde{U}_{\mu 2}|^2 |\tilde{U}_{\mu 3}|^2 \left(4 \sin^2 \frac{\tilde{\Delta}_{21}}{2} \sin^2 \frac{\tilde{\Delta}_{31}}{2} + \sin \tilde{\Delta}_{21} \sin \tilde{\Delta}_{31}\right) \\
 \tilde{U}_{\mu 2} &= \tilde{c}_{12} \tilde{c}_{23} - \tilde{s}_{12} \tilde{s}_{13} \tilde{s}_{23}, \quad \tilde{U}_{\mu 3} = \tilde{c}_{13} \tilde{s}_{23}
 \end{aligned}$$

neutrino (normal hierarchy)

$$\tilde{U}_{\mu 2} \approx \cos(\tilde{\theta}_{12} + \tilde{\theta}_{23}), \quad \tilde{U}_{\mu 3} \approx 0$$

$$\tilde{P}(\nu_\mu \rightarrow \nu_\mu) \approx 1 - \sin^2 \left[2 \left(\theta_{23} - \frac{a}{2|\delta m_{31}^2|} \xi \right) \right] \sin^2 \frac{\tilde{\Delta}_{21}}{2}$$

$$\tilde{\Delta}_{21} \approx \delta m_{31}^2 c_{13}^2 - \delta m_{21}^2 c_{12}^2$$

neutrino (inverted hierarchy)

$$\tilde{U}_{\mu 2} \approx 0, \quad \tilde{U}_{\mu 3} \approx \tilde{s}_{23}$$

$$\tilde{P}(\nu_\mu \rightarrow \nu_\mu) \approx 1 - \sin^2 \left[2 \left(\theta_{23} + \frac{a}{2|\delta m_{31}^2|} \xi \right) \right] \sin^2 \frac{\tilde{\Delta}_{31}}{2}$$

$$\tilde{\Delta}_{31} \approx \delta m_{31}^2 c_{13}^2 - \delta m_{21}^2 c_{12}^2$$

anti - neutrino (normal hierarchy)

$$\tilde{U}_{\mu 2} \approx \tilde{c}_{23}, \quad \tilde{U}_{\mu 3} \approx \tilde{s}_{23}$$

$$\tilde{P}(\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu) \approx 1 - \sin^2 \left[2 \left(\theta_{23} + \frac{a}{2|\delta m_{31}^2|} \xi \right) \right] \sin^2 \frac{\tilde{\Delta}_{32}}{2}$$

$$\tilde{\Delta}_{32} \approx \delta m_{31}^2 c_{13}^2 - \delta m_{21}^2 c_{12}^2$$

anti - neutrino (inverted hierarchy)

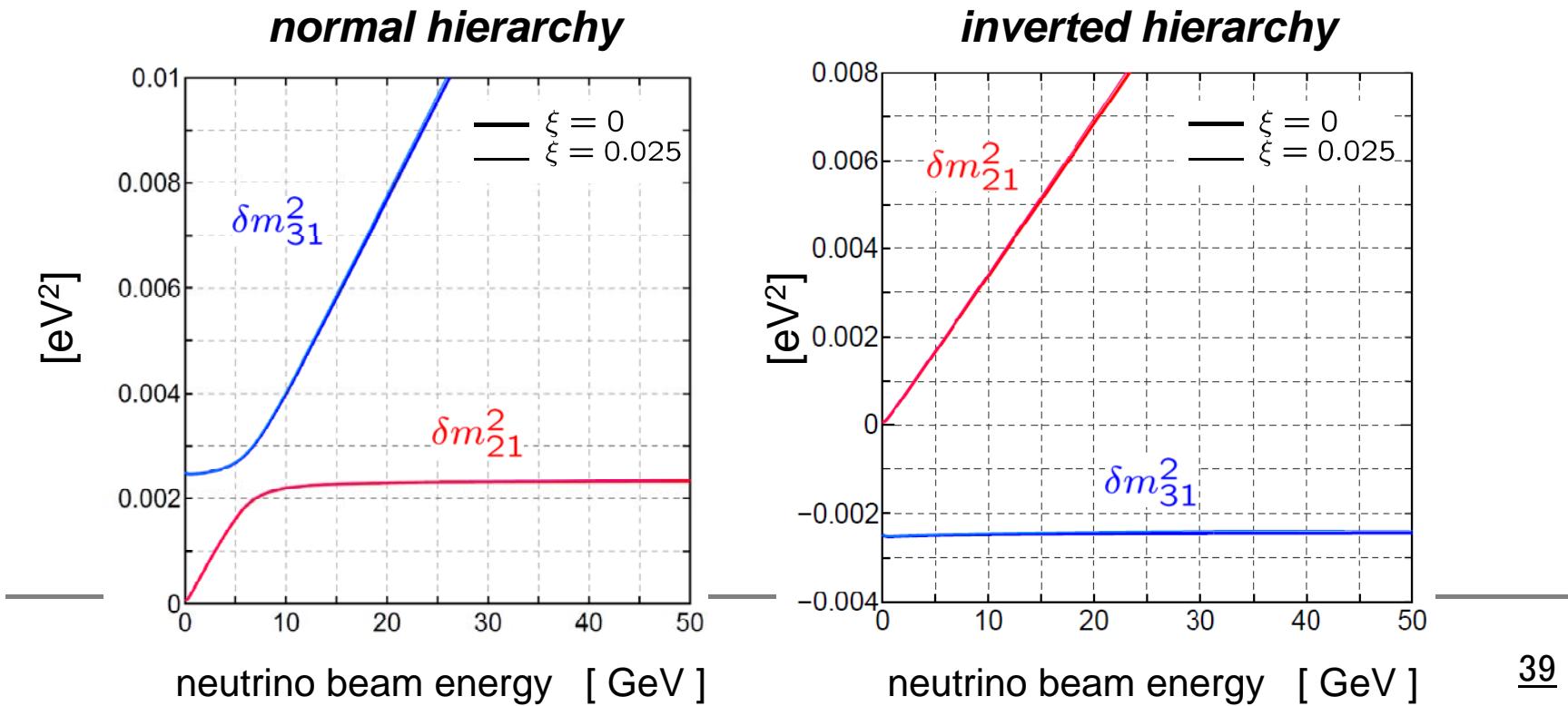
$$\tilde{U}_{\mu 2} \approx \cos(\tilde{\theta}_{12} + \tilde{\theta}_{23}), \quad \tilde{U}_{\mu 3} \approx 0$$

$$\tilde{P}(\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu) \approx 1 - \sin^2 \left[2 \left(\theta_{23} - \frac{a}{2|\delta m_{31}^2|} \xi \right) \right] \sin^2 \frac{\tilde{\Delta}_{21}}{2}$$

$$\tilde{\Delta}_{21} \approx -\delta m_{31}^2 c_{13}^2 + \delta m_{21}^2 c_{12}^2$$

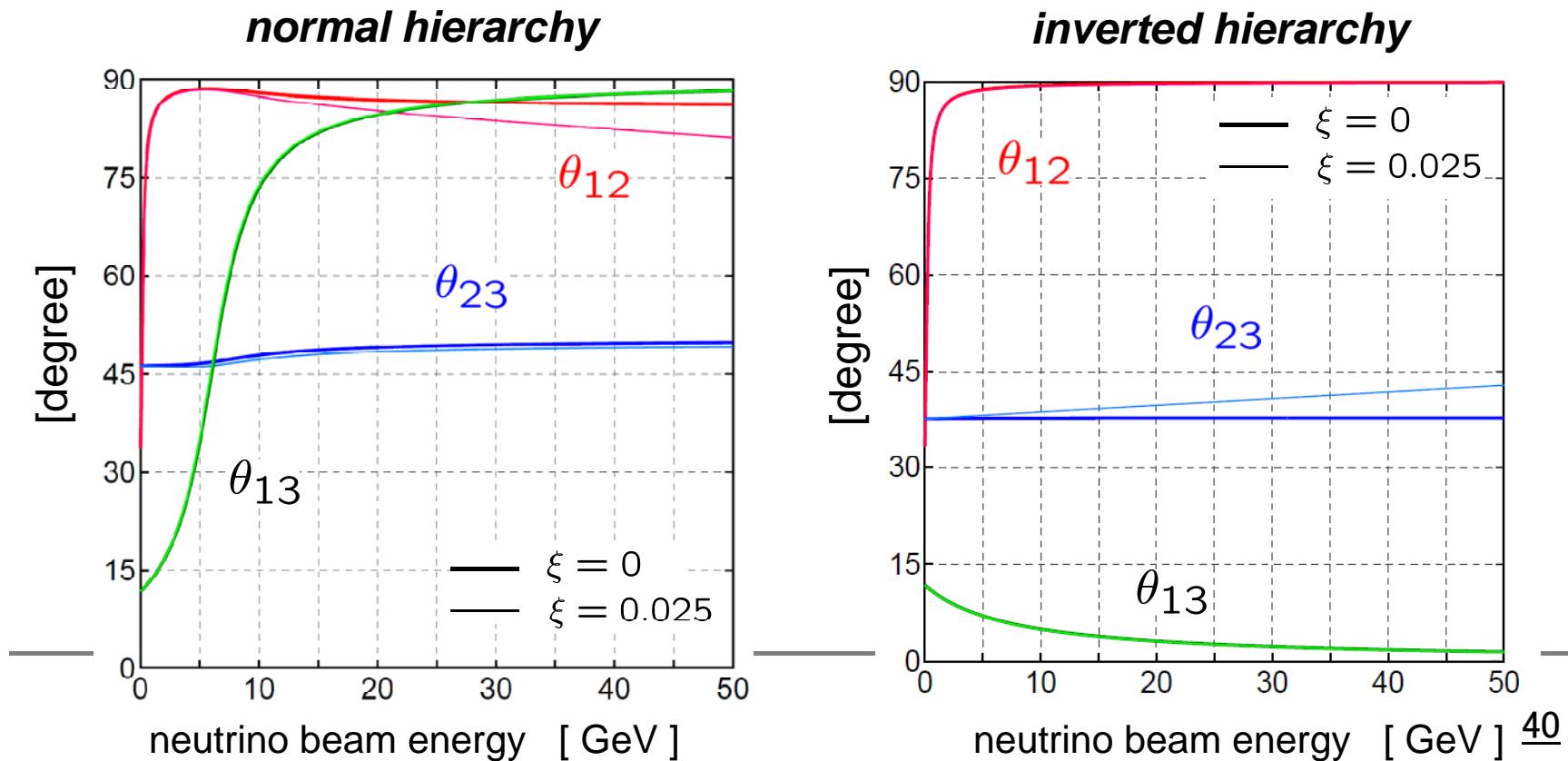
The energy dependence of the effective mass squared differences

matter density = 4.6 g/cm ³	$\sin^2(2\theta_{\text{atm}}) = 1.0$	$\delta m_{31}^2 = 2.5 \times 10^{-3} \text{ eV}^2$
CP phase $\delta = 0$	$\tan^2(\theta_{\text{sun}}) = 0.40$	$\delta m_{21}^2 = 8.0 \times 10^{-5} \text{ eV}^2$
	$\sin^2(2\theta_{\text{rd}}) = 0.16$	



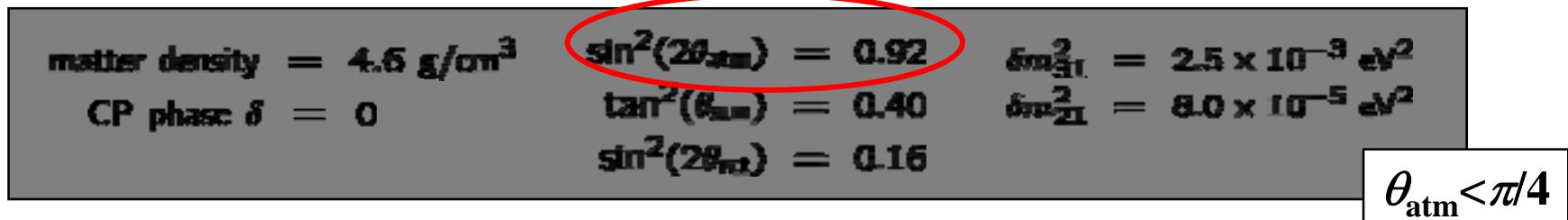
The energy dependence of the effective mixing angles

matter density = 4.6 g/cm ³	$\sin^2(2\theta_{\text{eff}}) = 1.0$	$\delta m_{31}^2 = 2.5 \times 10^{-3} \text{ eV}^2$
CP phase $\delta = 0$	$\tan^2(\theta_{\text{eff}}) = 0.40$	$\delta m_{21}^2 = 8.0 \times 10^{-5} \text{ eV}^2$
	$\sin^2(2\theta_{\text{wt}}) = 0.16$	

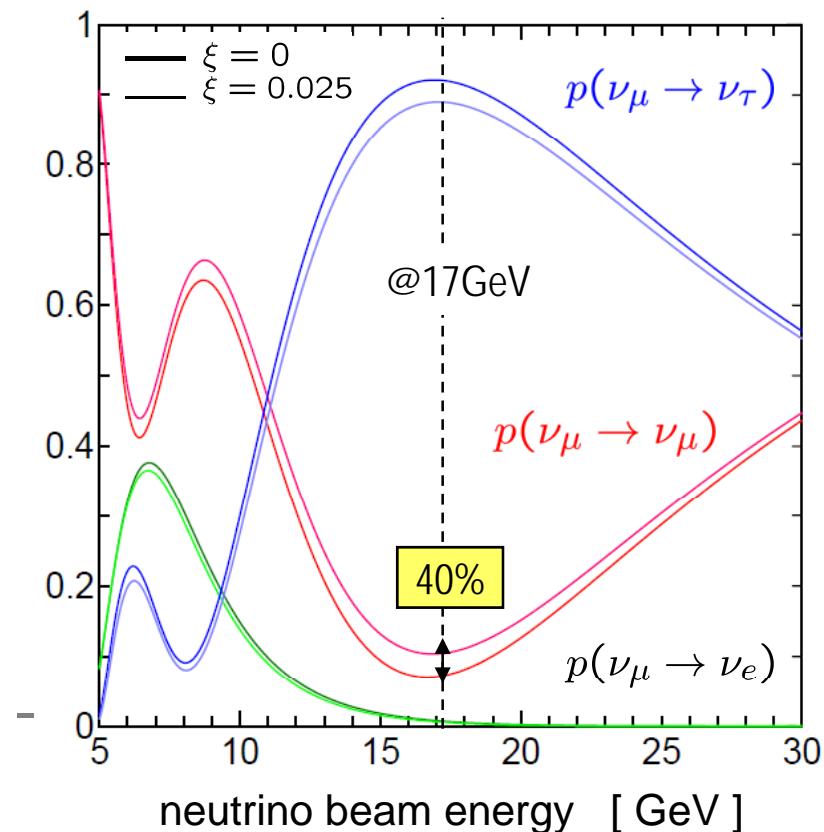


The oscillation probabilities (L=9,120km)

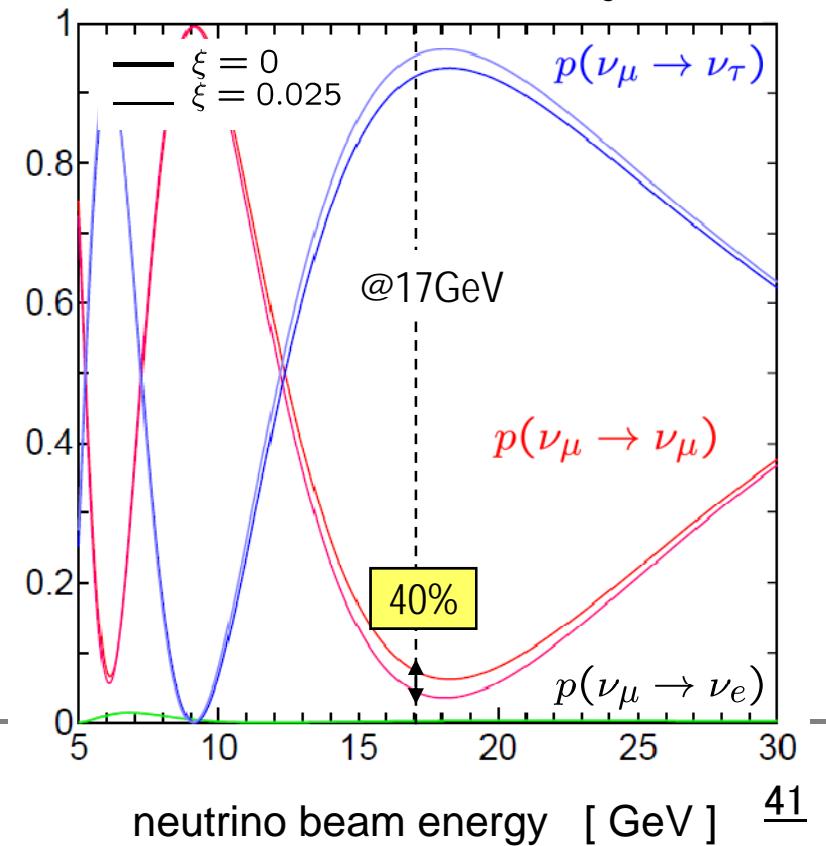
$$\sin(2\theta_{\text{atm}}) = \sin \theta_{23} \cos \theta_{13} \approx \sin \theta_{23}$$



normal hierarchy



inverted hierarchy

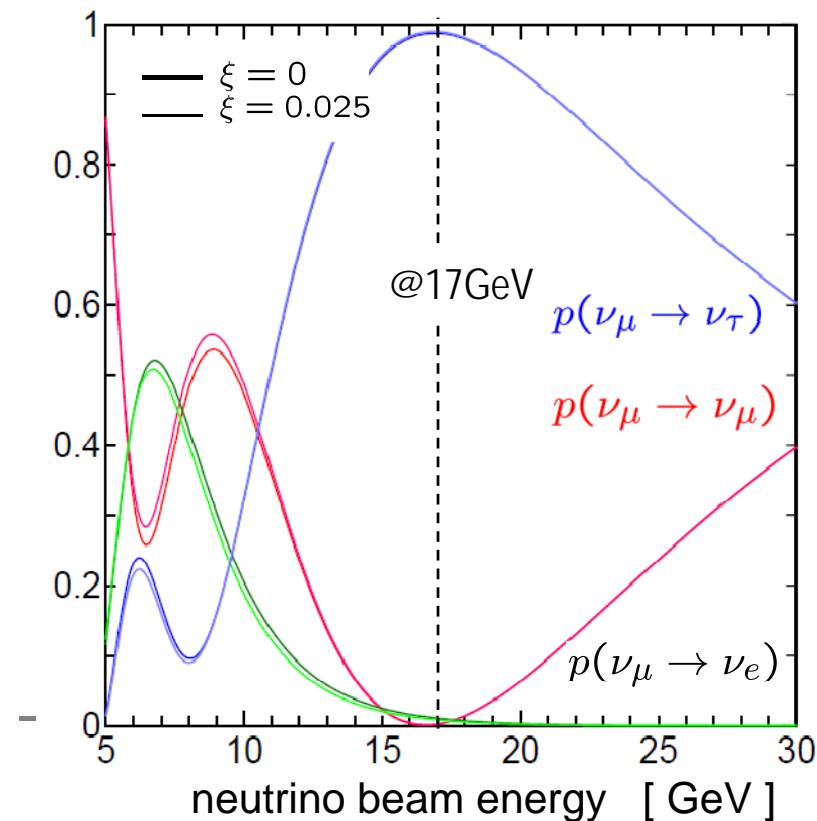


The oscillation probabilities (L=9,120km)

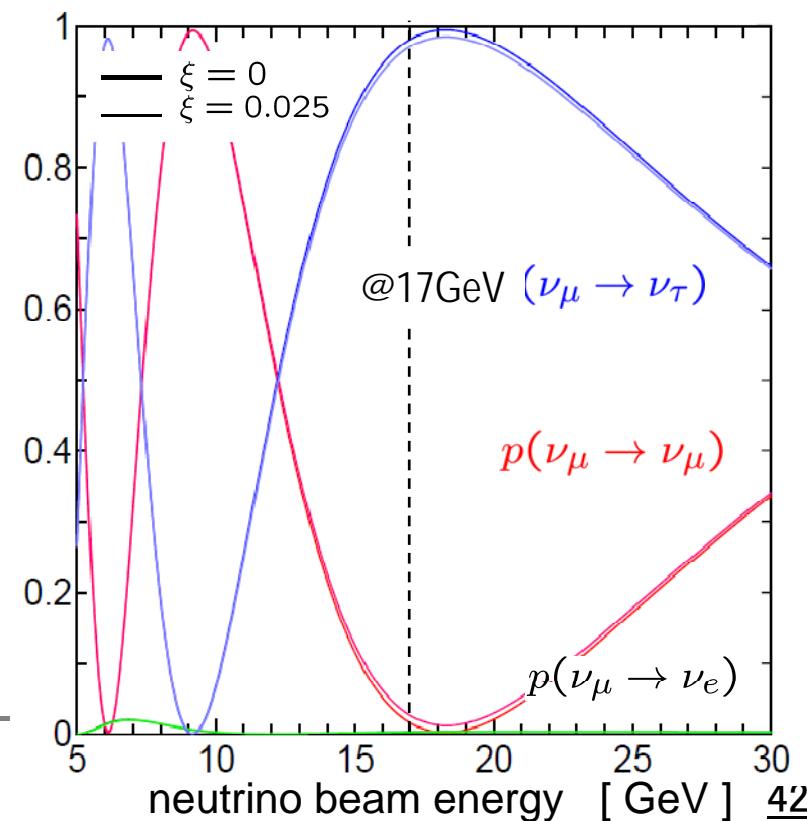
$$\sin(2\theta_{\text{atm}}) = \sin \theta_{23} \cos \theta_{13} \approx \sin \theta_{23}$$

matter density = 4.6 g/cm ³	$\sin^2(2\theta_{\text{atm}}) = 1$	$\delta m_{31}^2 = 2.5 \times 10^{-3} \text{ eV}^2$
CP phase $\delta = 0$	$\tan^2(\theta_{\text{atm}}) = 0.40$	$\delta m_{21}^2 = 8.0 \times 10^{-5} \text{ eV}^2$
	$\sin^2(2\theta_{\text{atm}}) = 0.16$	

normal hierarchy

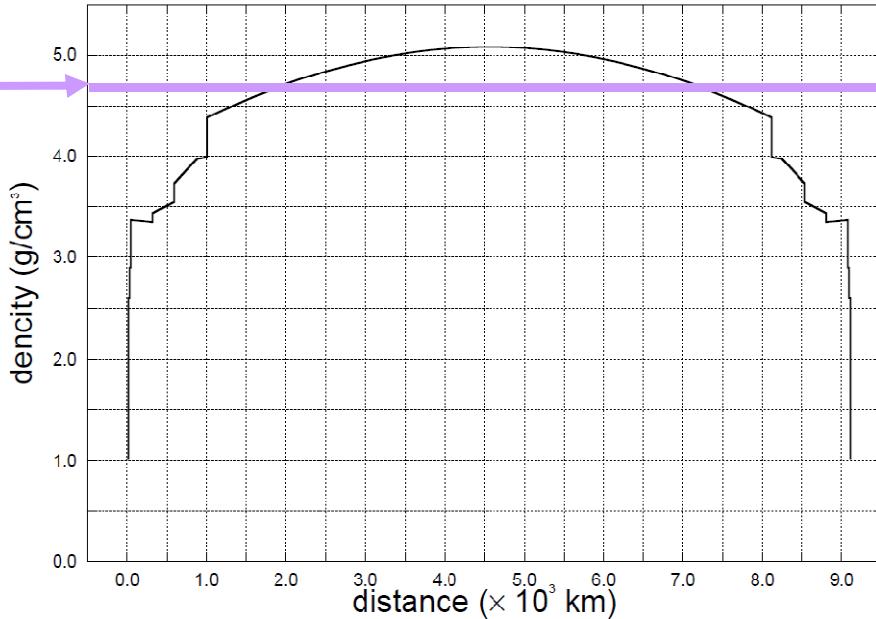


inverted hierarchy



Matter Profile

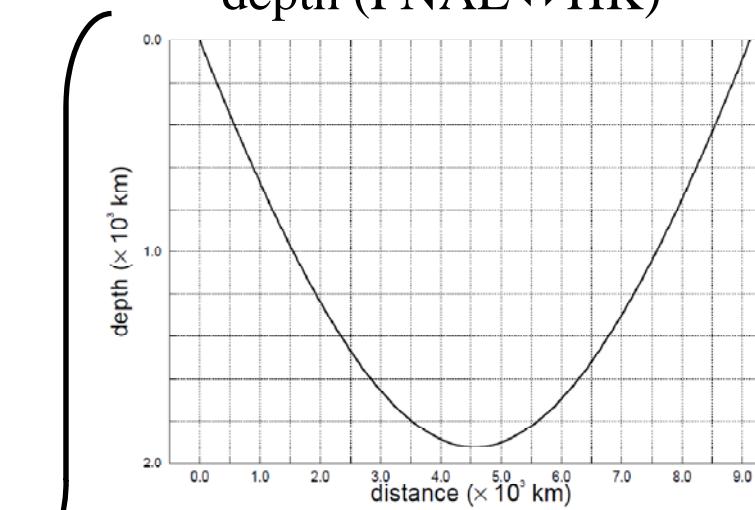
density (FNAL \leftrightarrow HK)



average matter density

$$\rho = 4.6 \text{ g/cm}^3$$

depth (FNAL \leftrightarrow HK)



+
density profile

