Studies of Neutrino Oscillations *with* and *without* Mass using Super-K

Wei Wang, Boston University

Neutrino Workshop, Kashiwa, Feb 20, 2007

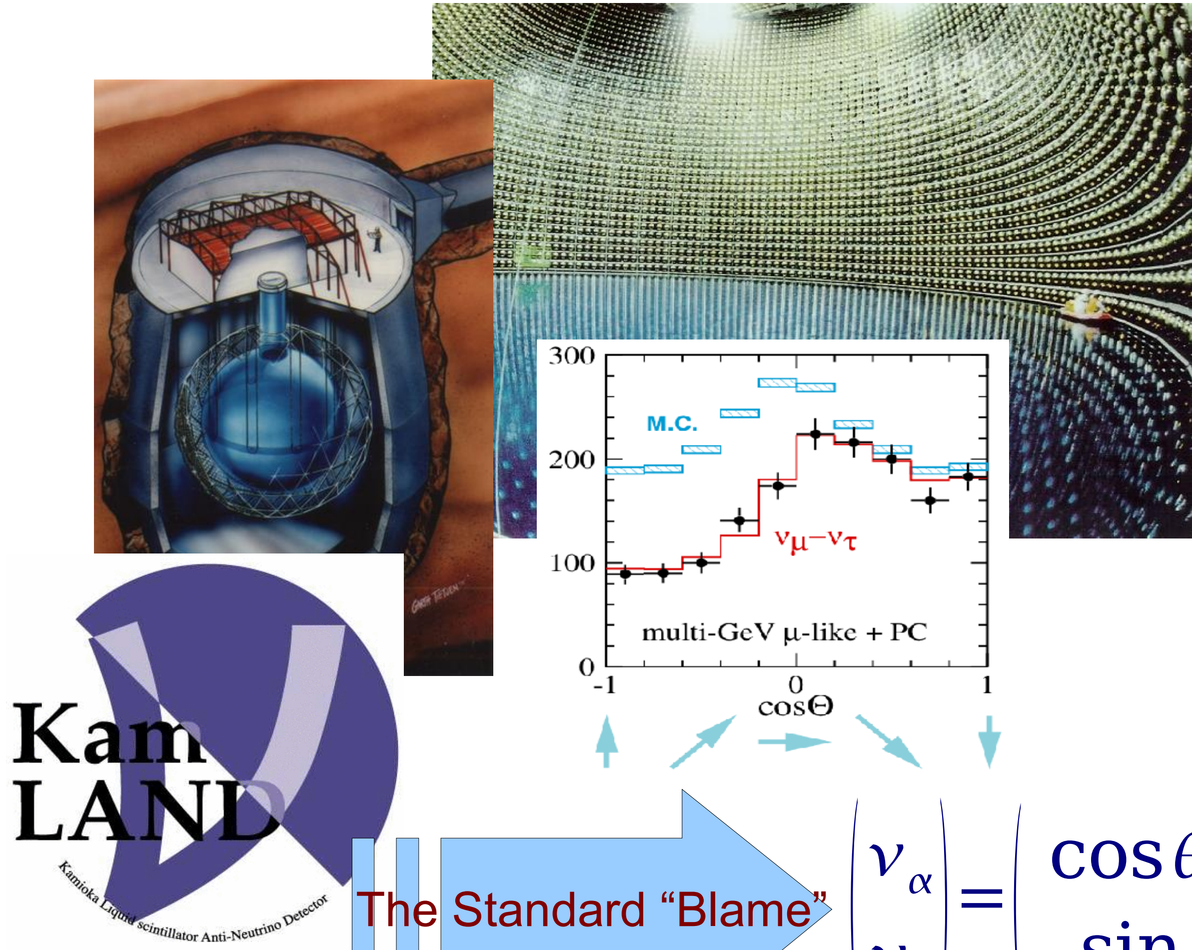
Preaching Buddhism to Buddha

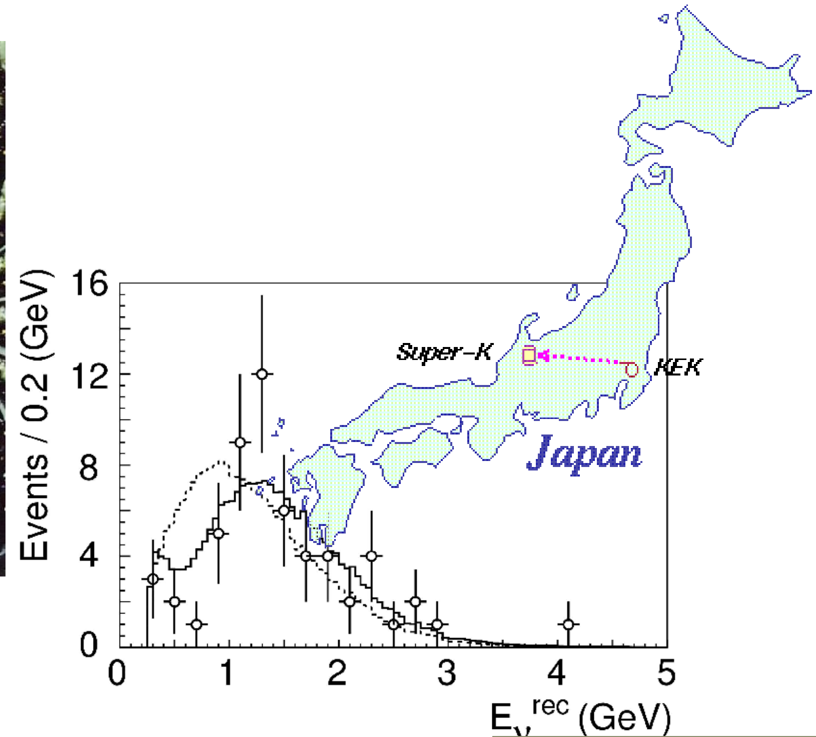
Syaka ni Seppou

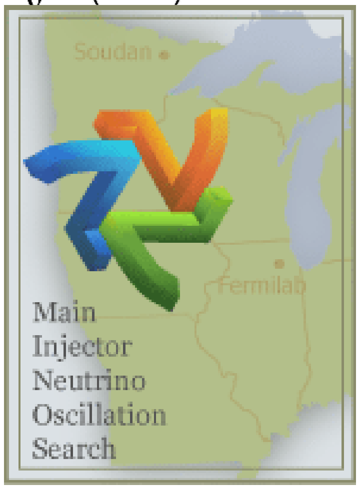
- Teaching Grandmas to Suck Eggs

- 班門弄斧
Ban-Men Nong-Fu

An Era of Discovery in Neutrino Physics







The Standard "Blame"

$$\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_{m_a} \\ \nu_{m_b} \end{pmatrix}$$

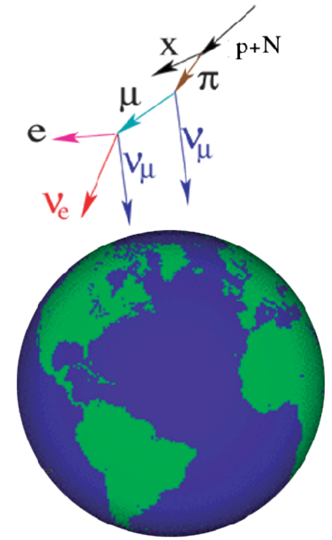
$$\rightarrow P_{\nu_\alpha \rightarrow \nu_\alpha} = 1 - \sin^2 2\theta \sin^2 \frac{\Delta m^2 L}{4E}$$

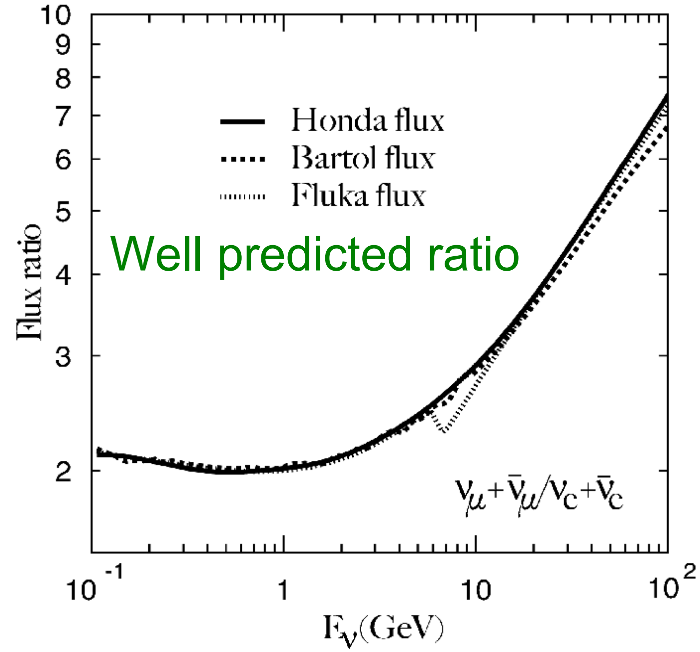
Outline

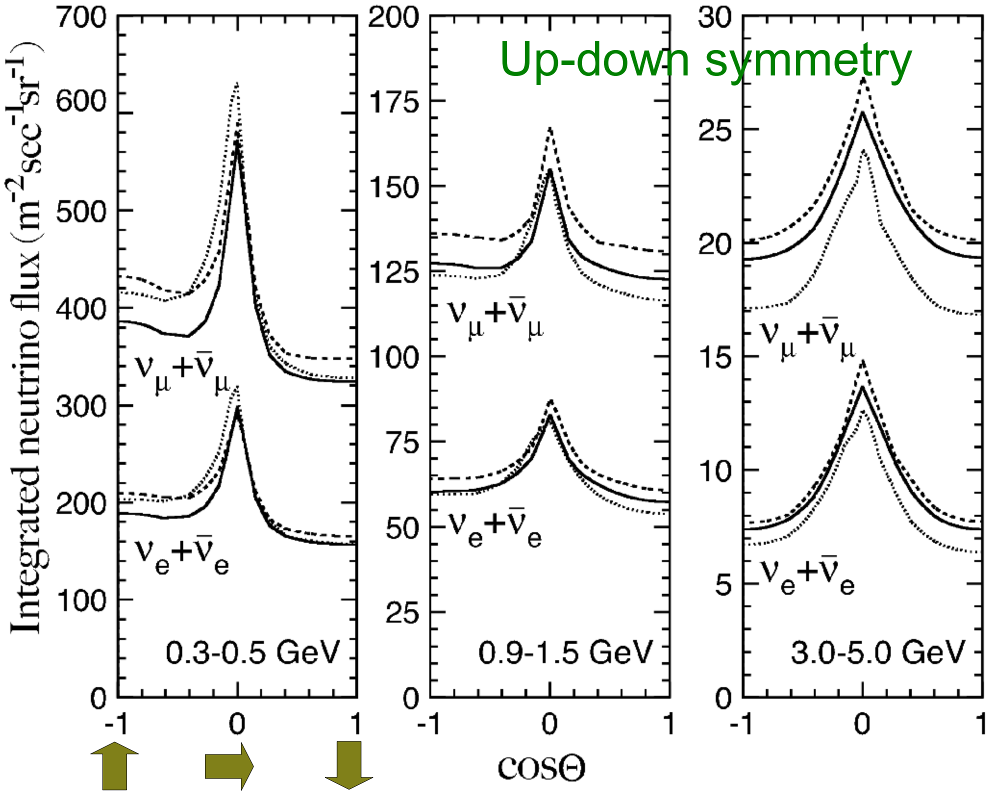
- Atmospheric neutrinos, Super-K experiment and events
- Standard ν_μ-ν_τ oscillation analysis using zenith distributions
- Sterile neutrino as a alternative to tau neutrino
  - ν_μ-ν_τ mixing vs ν_μ-ν_s mixing
  - An admixture analysis
- Neutrino oscillations induced by the violations of Lorentz (LIV) invariance and CPT (CPTV) symmetry
  - Fit LIV and CPTV induced oscillation against Super-K data
  - Allowed limits of LIV and CPTV
- Summary and conclusions

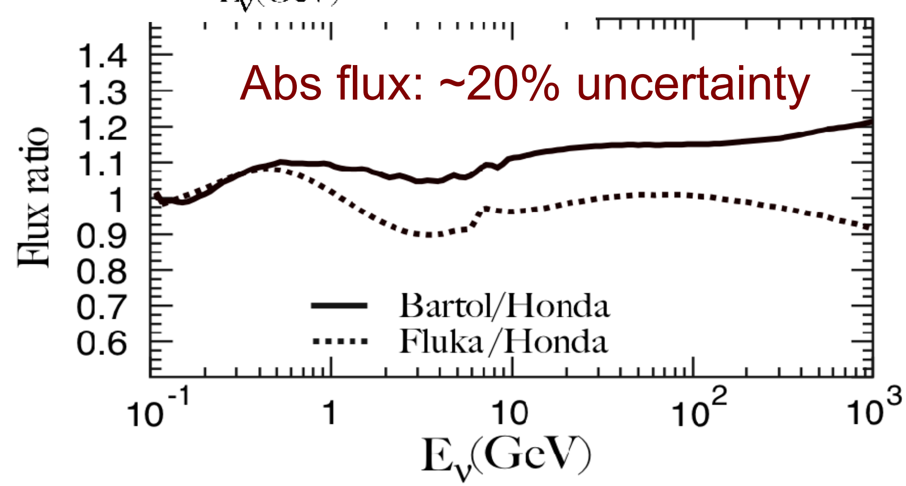
Atmospheric Neutrinos

- A large uncertainty on the absolute flux
- Good knowledge on flavor ratio
- Up-down symmetric



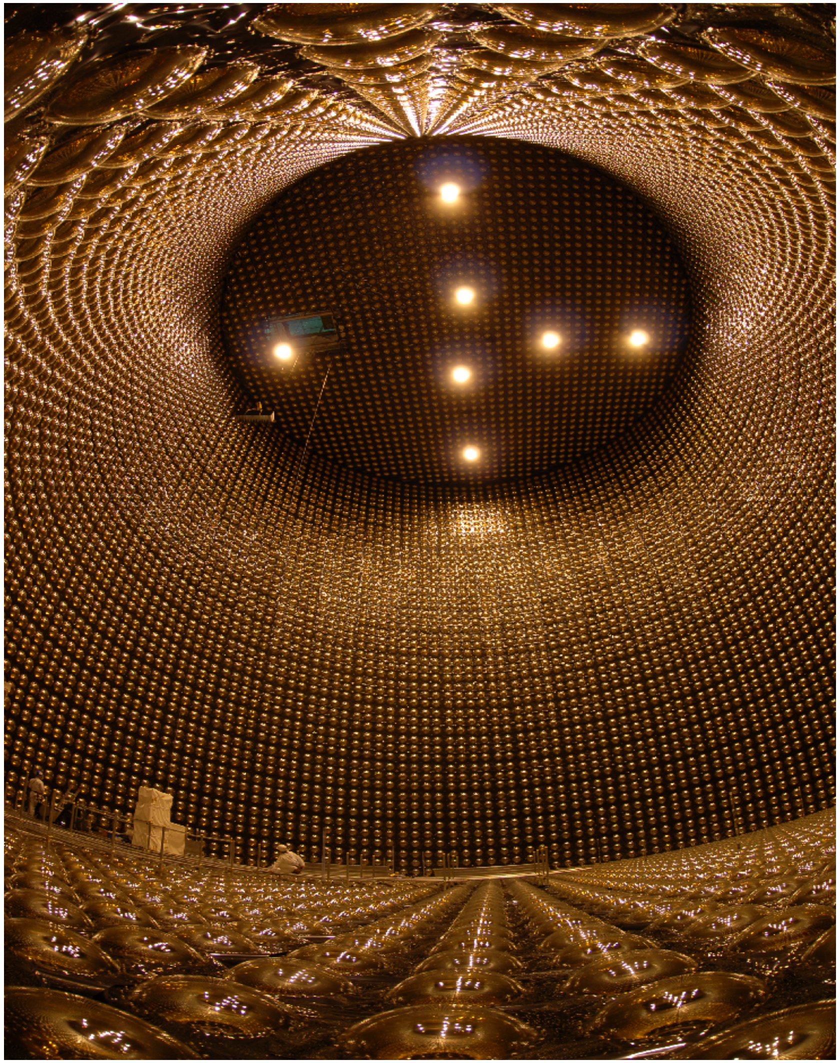






Super-Kamiokande Collaboration

- 140 collaborators from 35 institutes of 5 countries



S. Fukuda,[1] Y. Fukuda,[1] M. Ishitsuka,[1] Y. Itow,[1] T. Kajita,[1] J. Kameda,[1] K. Kaneyuki,[1] K. Kobayashi,[1] Y. Koshio,[1] M. Miura,[1] S. Moriyama,[1] M. Nakahata,[1] S. Nakayama,[1] Y. Obayashi,[1] A. Okada,[1] K. Okumura,[1] N. Sakurai,[1] M. Shiozawa,[1] Y. Suzuki,[1] H. Takeuchi,[1] Y. Takeuchi,[1] T. Toshito,[1] Y. Totsuka,[1] S. Yamada,[1] M. Earl,[2] A. Habig,[2,*] E. Kearns,[2] M. D. Messier,[2] K. Scholberg,[2] J. L. Stone,[2] L. R. Sulak,[2] C. W. Walter,[2] M. Goldhaber,[3] T. Barszczak,[4] D. Casper,[4] W. Gajewski,[4] W. R. Kropp,[4] S. Mine,[4] L. R. Price,[4] M. Smy,[4] H. W. Sobel,[4] M. R. Vagins,[4] K. S. Ganezer,[5] W. E. Keig,[5] R. W. Ellsworth,[6] S. Tasaka,[7] A. Kibayashi,[8] J. G. Learned,[8] S. Matsuno,[8] D. Takemori,[8] Y. Hayato,[9] T. Ishii,[9] T. Kobayashi,[9] K. Nakamura,[9] Y. Oyama,[9] A. Sakai,[9] M. Sakuda,[9] O. Sasaki,[9] M. Kohama,[10] A. T. Suzuki,[10] T. Inagaki,[11] K. Nishikawa,[11] T. J. Haines,[12,4] E. Blaufuss,[13] B. K. Kim,[13] R. Sanford,[13] R. Svoboda,[13] M. L. Chen,[14] J. A. Goodman,[14] G. Guillian,[14] G. W. Sullivan,[14] J. Hill,[15] C. K. Jung,[15] K. Martens,[15] M. Malek,[15] C. Mauger,[15] C. McGrew,[15] E. Sharkey,[15] B. Viren,[15] C. Yanagisawa,[15] M. Kirisawa,[16] S. Inaba,[16] C. Mitsuda,[16] K. Miyano,[16] H. Okazawa,[16] C. Saji,[16] M. Takahashi,[16] M. Takahata,[16] Y. Nagashima,[17] K. Nitta,[17] M. Takita,[17] M. Yoshida,[17] S. B. Kim,[18] T. Ishizuka,[19] M. Etoh,[20] Y. Gando,[20] T. Hasegawa,[20] K. Inoue,[20] K. Ishihara,[20] T. Maruyama,[20] J. Shirai,[20] A. Suzuki,[20] M. Koshiba,[21] Y. Hatakeyama,[22] Y. Ichikawa,[22] M. Koike,[22] K. Nishijima,[22] H. Fujiyasu,[23] H. Ishino,[23] M. Morii,[23] Y. Watanabe,[23] U. Golebiewska,[24] D. Kielczewska,[24,4] S. C. Boyd,[25] A. L. Stachyra,[25] R. J. Wilkes,[25] and K. K. Young[25,†]

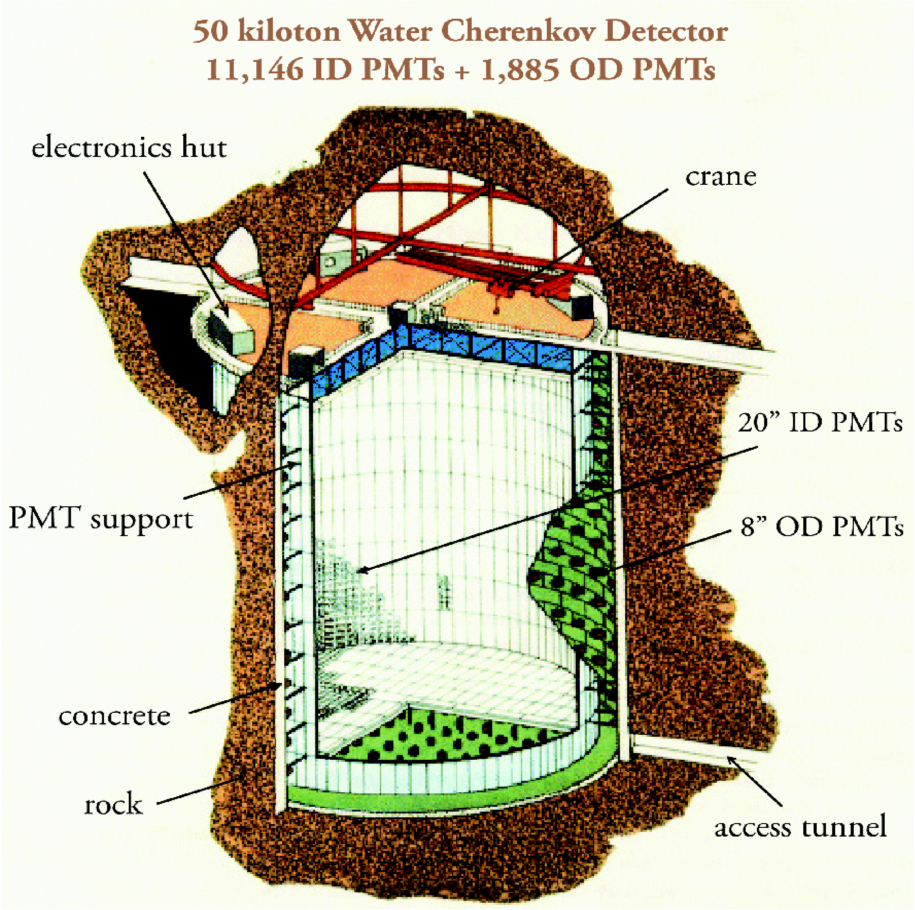
(Super-Kamiokande Collaboration)

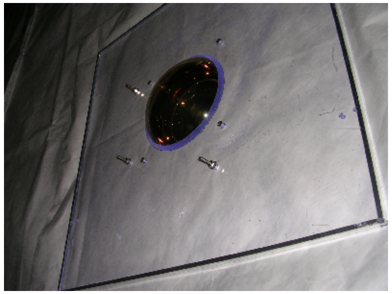
[1] *Institute for Cosmic Ray Research, University of Tokyo, Kashiwa, Chiba 277-8582, Japan*
[2] *Department of Physics, Boston University, Boston, Massachusetts 02215*
[3] *Physics Department, Brookhaven National Laboratory, Upton, New York 11973*
[4] *Department of Physics and Astronomy, University of California at Irvine, Irvine, California 92697-4575*
[5] *Department of Physics, California State University, Dominguez Hills, Carson, California 90747*
[6] *Department of Physics, George Mason University, Fairfax, Virginia 22030*
[7] *Department of Physics, Gifu University, Gifu, Gifu 501-1193, Japan*
[8] *Department of Physics and Astronomy, University of Hawaii, Honolulu, Hawaii 96822*
[9] *Institute of Particle and Nuclear Studies, High Energy Accelerator Research Organization (KEK), Tsukuba, Ibaraki 305-0801, Japan*
[10] *Department of Physics, Kobe University, Kobe, Hyogo 657-8501, Japan*
[11] *Department of Physics, Kyoto University, Kyoto 606-8502, Japan*
[12] *Physics Division, P-23, Los Alamos National Laboratory, Los Alamos, New Mexico 87544*
[13] *Department of Physics and Astronomy, Louisiana State University, Baton Rouge, Louisiana 70803*
[14] *Department of Physics, University of Maryland, College Park, Maryland 20742*
[15] *Department of Physics and Astronomy, State University of New York, Stony Brook, New York 11794-3800*
[16] *Department of Physics, Niigata University, Niigata, Niigata 950-2181, Japan*
[17] *Department of Physics, Osaka University, Toyonaka, Osaka 560-0043, Japan*
[18] *Department of Physics, Seoul National University, Seoul 151-742, Korea*
[19] *Department of Systems Engineering, Shizuoka University, Hamamatsu, Shizuoka 432-8561, Japan*
[20] *Research Center for Neutrino Science, Tohoku University, Sendai, Miyagi 980-8578, Japan*
[21] *The University of Tokyo, Tokyo 113-0033, Japan*
[22] *Department of Physics, Tokai University, Hiratsuka, Kanagawa 259-1292, Japan*
[23] *Department of Physics, Tokyo Institute for Technology, Meguro, Tokyo 152-8551, Japan*
[24] *Institute of Experimental Physics, Warsaw University, 00-681 Warsaw, Poland*
[25] *Department of Physics, University of Washington, Seattle, Washington 98195-1560*

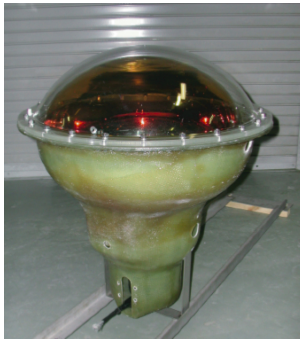
Super-Kamiokande Experiment

- A 50 kt water Cherenkov detector
  - Inner detector and outer detector optically separated
  - ID: 25in PMTs; gaps filled by black sheet
  - OD: 8in PMTs with wavelength shifters, wall covered by reflective Tyvek
- Operating periods
  - **SK-I: 1996 – 2001**
    - 1489 days livetime
    - ~40% ID coverage
  - **SK-II: 2003 – 2005**
    - 804 days livetime
    - Half ID tubes (acrylic&FRP) ~20% coverage
  - **SK-III: since Summer 2006**
    - ID tubes (acrylic&FRP) fully recovered

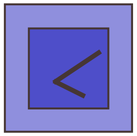
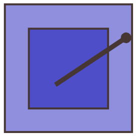
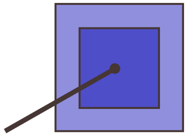
this analysis (SK-I, SK-II)

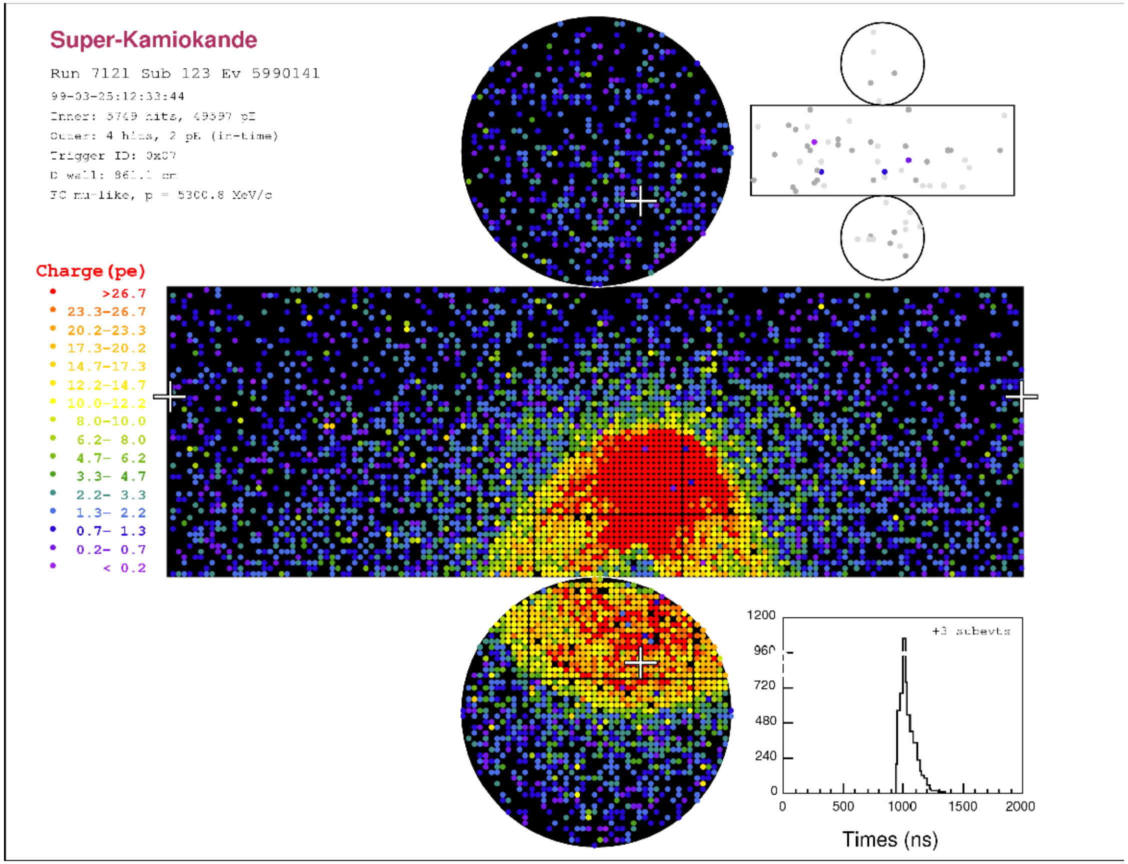






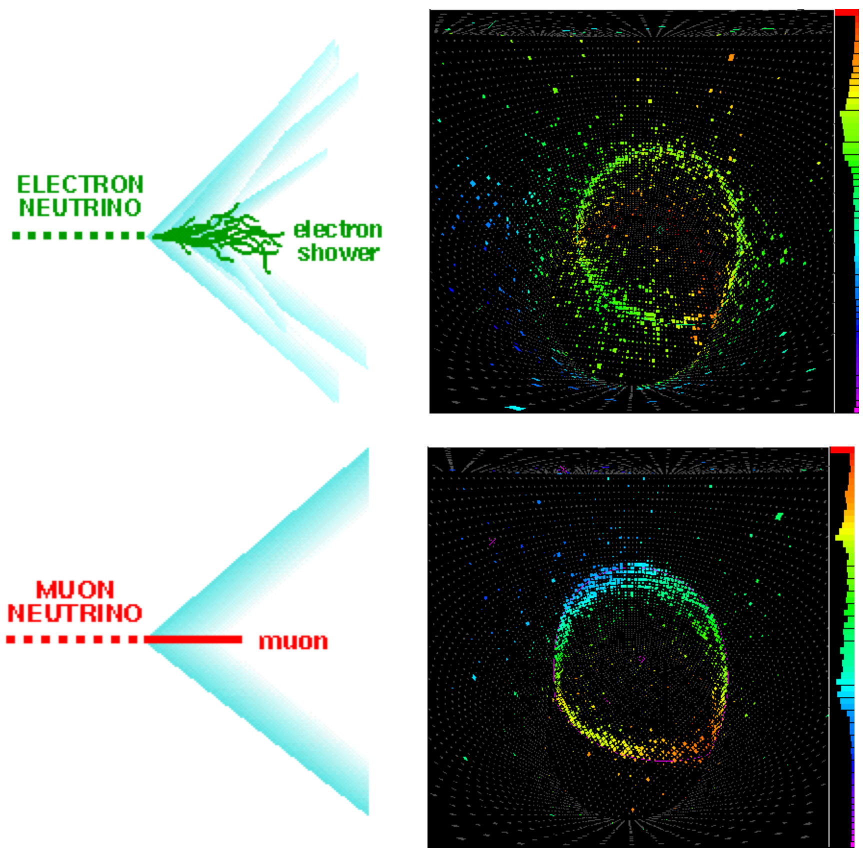
Super-K Neutrino Events

- Neutrino interaction
  → charged particles
  → Cherenkov radiation
  → recorded by PMTs
- Neutrino event categories
  - Fully contained 
  - Partially contained 
  - Upward going μ 

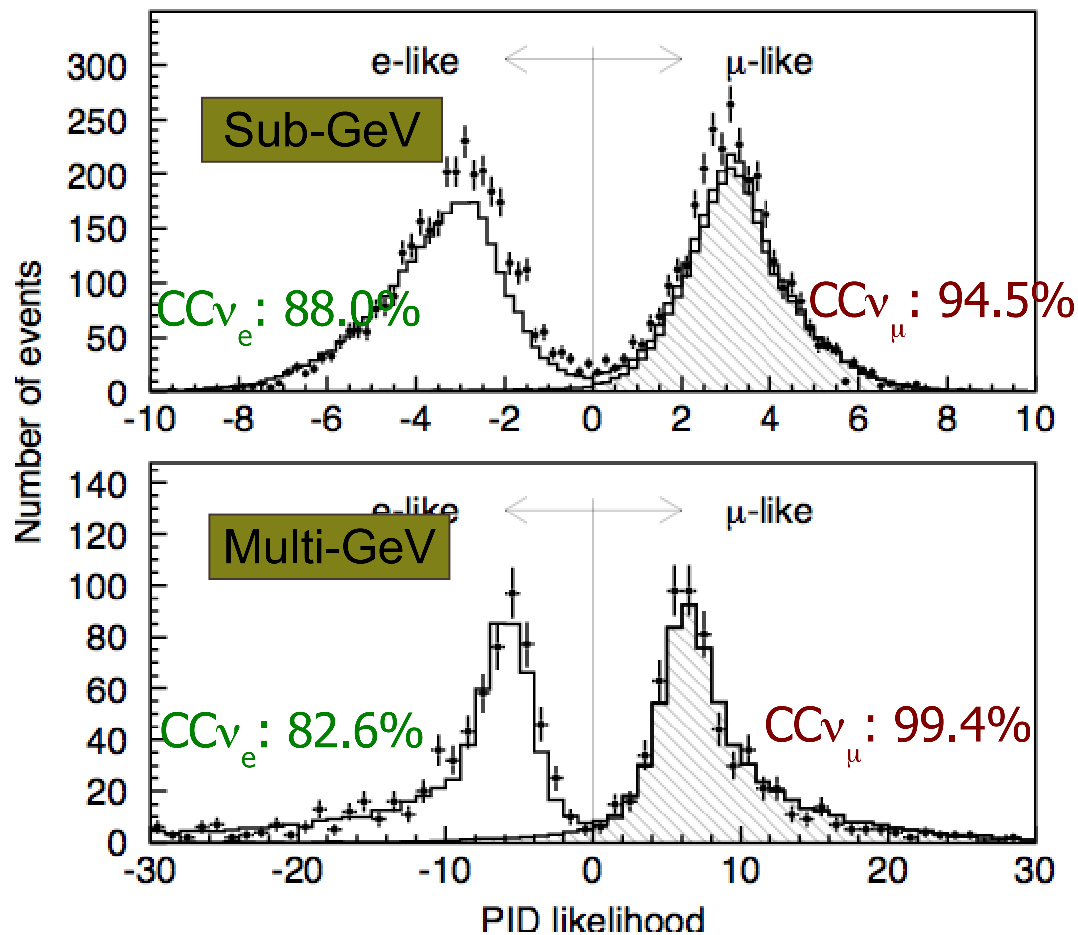


Event Reconstruction

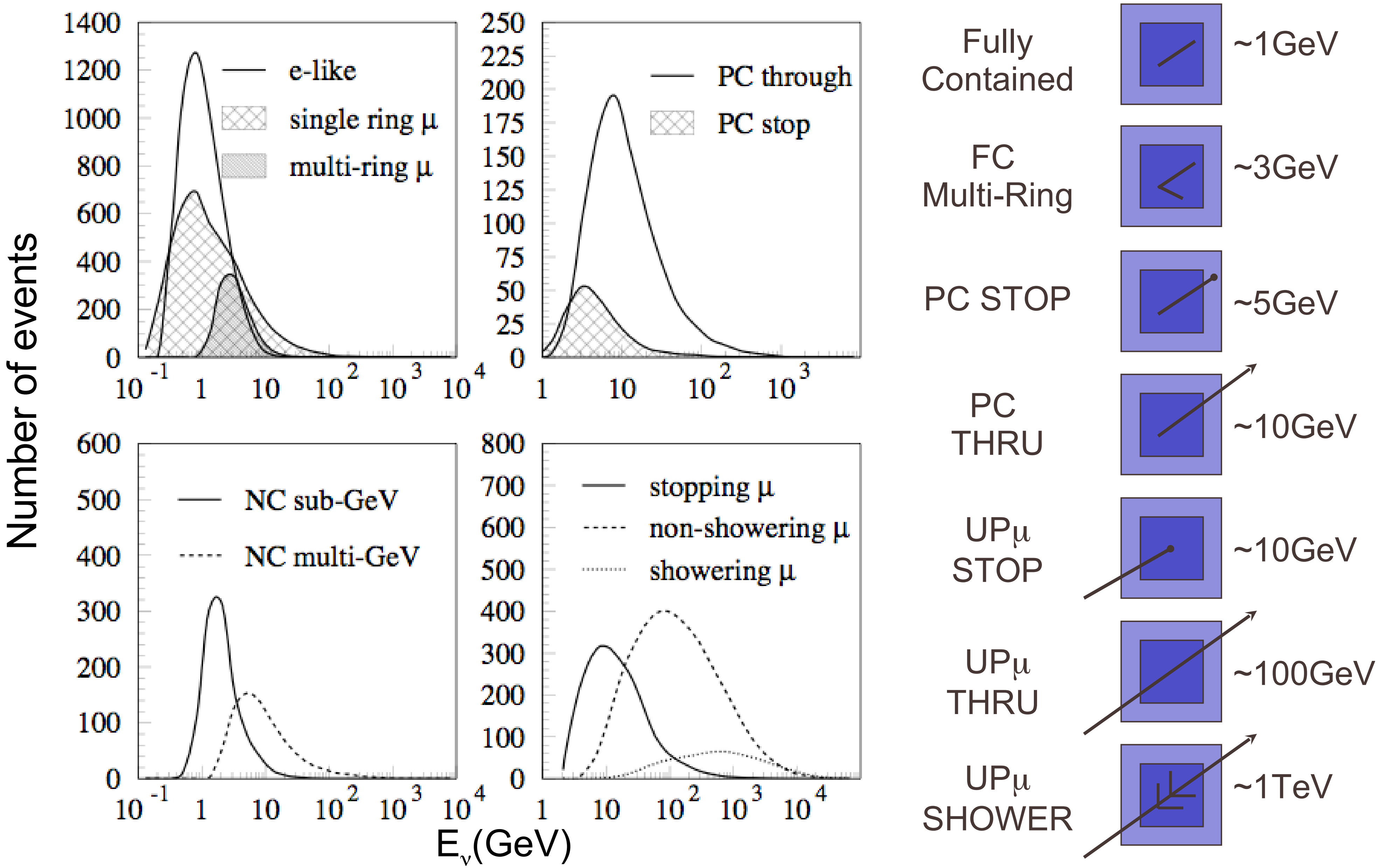
- Vertex finding
- Ring recognition
- PID (e-/μ-like)
- Momentum reconstruction



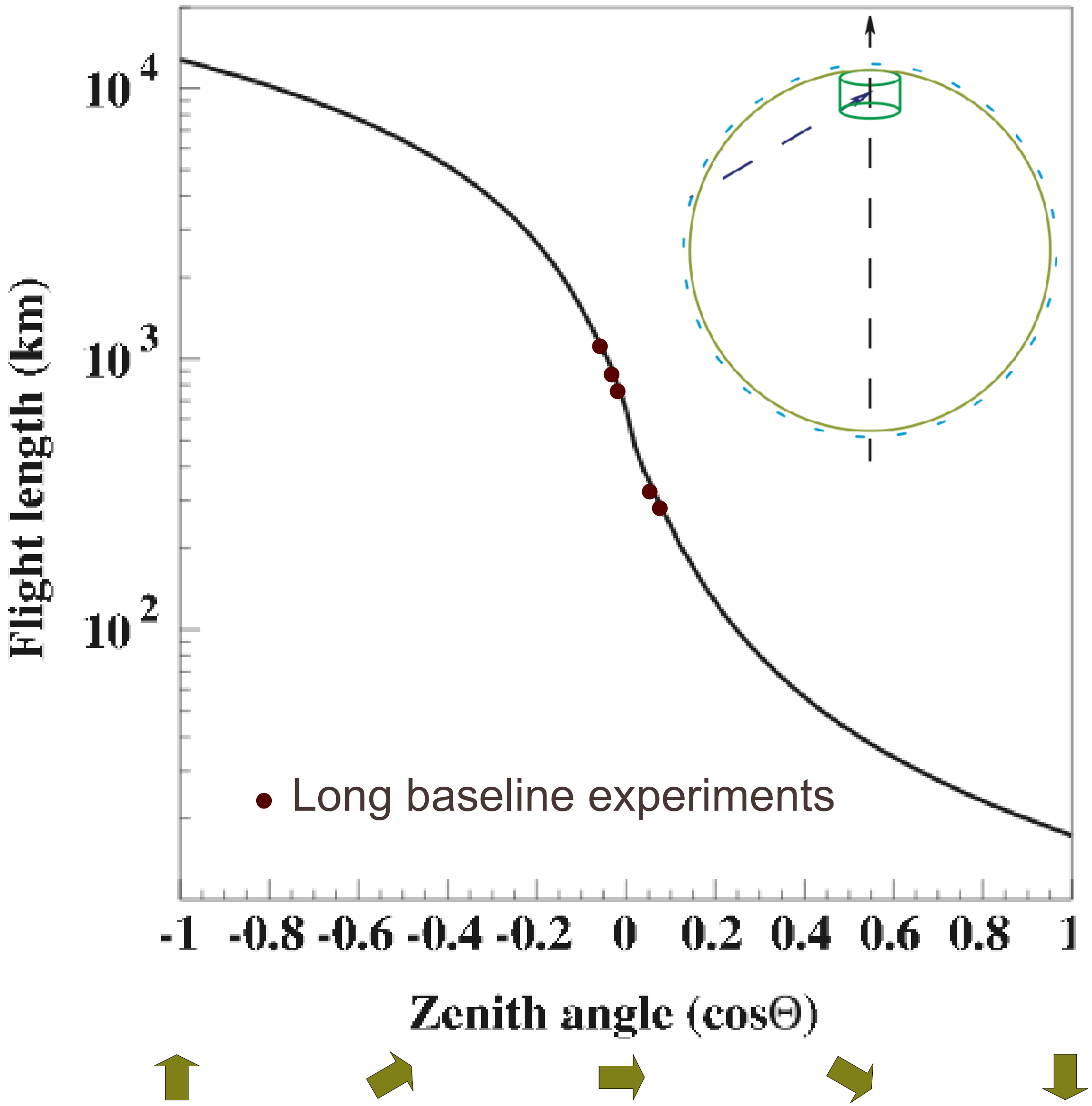
FC Single Ring Events

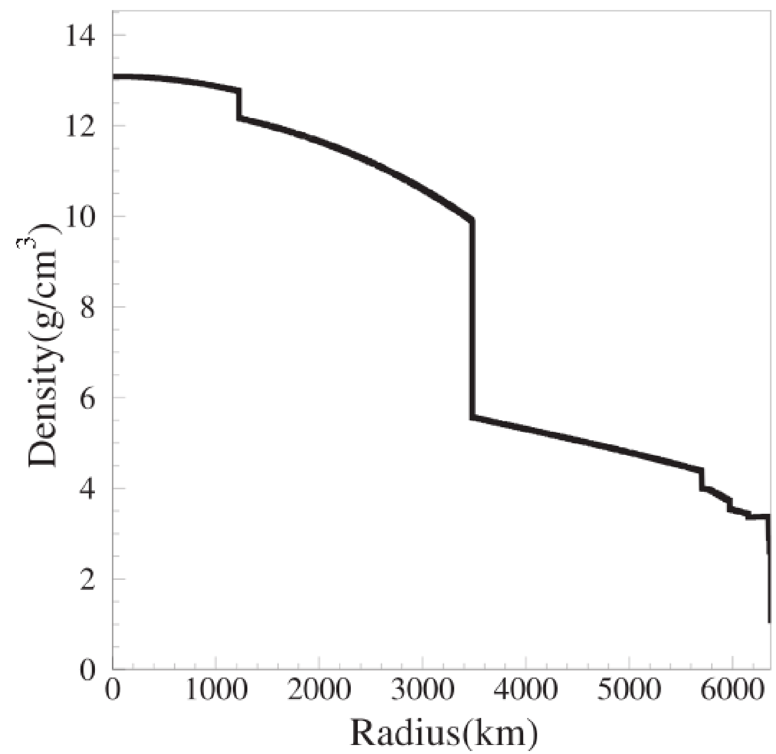


Five Decades of Energy



Four Decades of Pathlengths

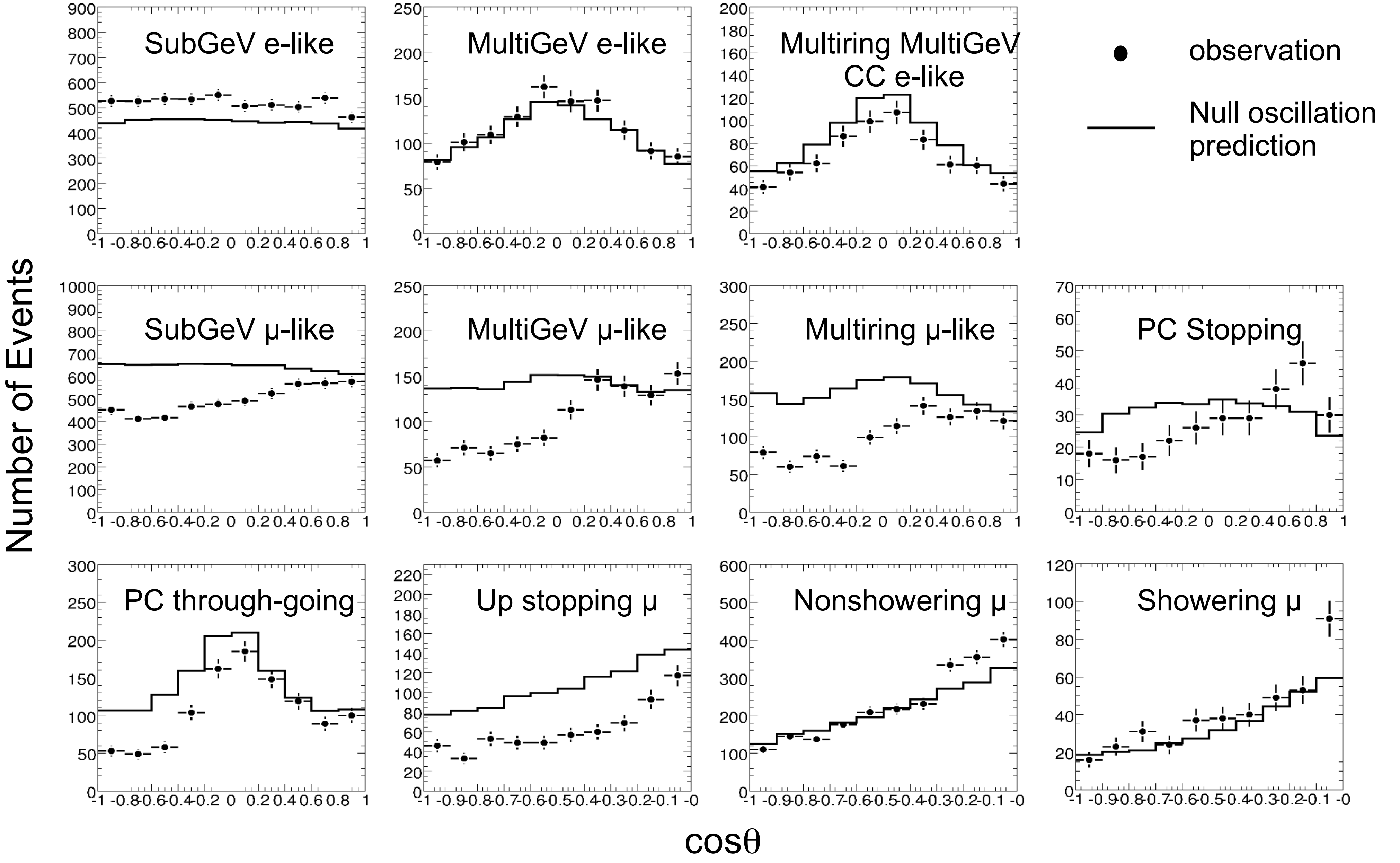




- →Large ranges of *L* and *E*
- →Various matter densities

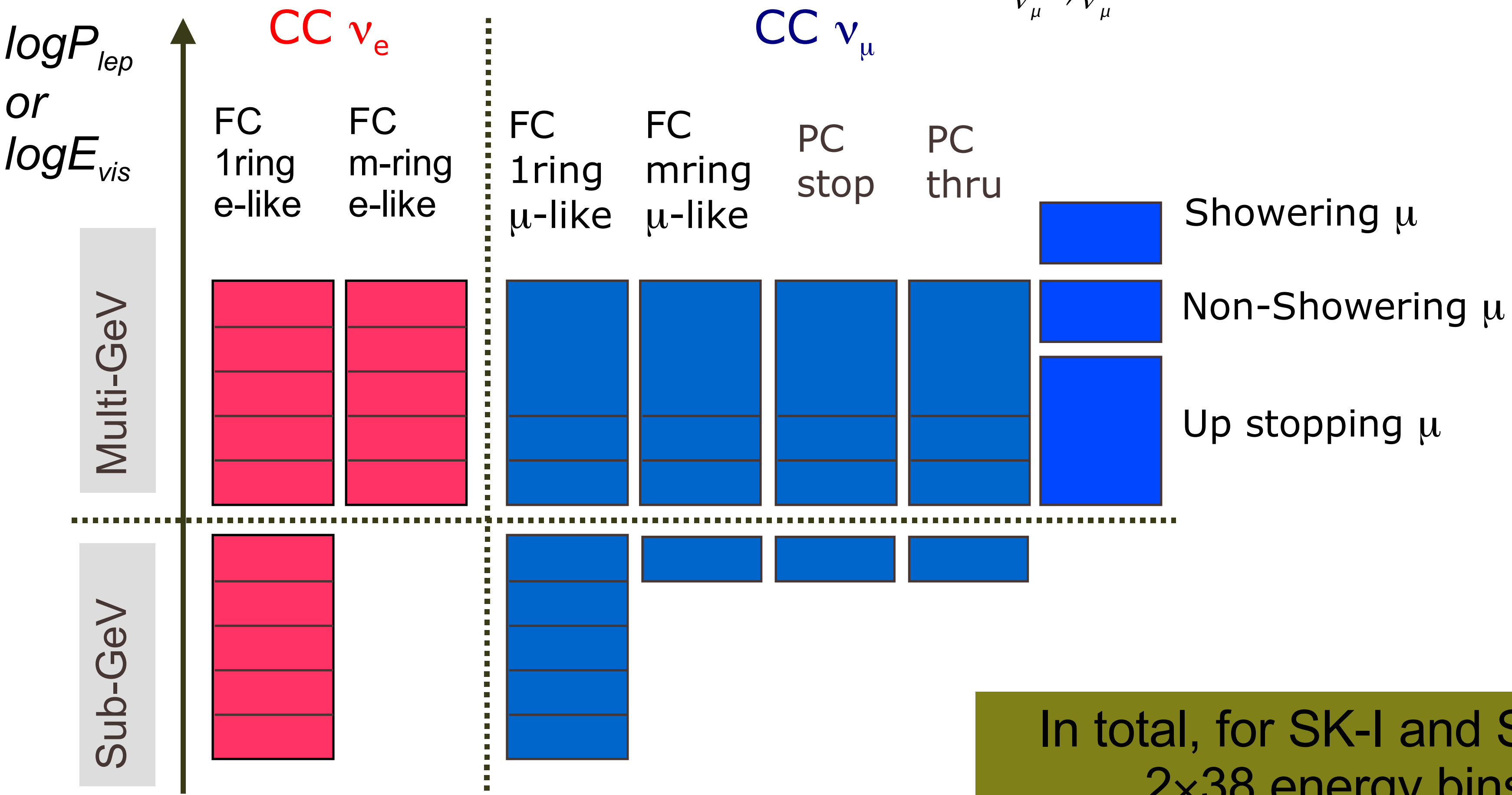
⇒ great advantages for studying exotic phenomena

Atmospheric Neutrino Observations



Data Analysis: Binning

$$P_{\nu_\mu \to \nu_\mu} = 1 - \sin^2 2\theta \sin^2 \frac{\Delta m^2 L}{4E}$$



e-like events for flux normalization

μ-like events for oscillation signal

In total, for SK-I and SK-II, 2×38 energy bins ×10 zenith bins = 760 bins

Data Analysis: Pull Method

$$\chi^2 = \underbrace{\sum_{i=1}^{N} 2\left(N_i^{\text{exp}} - N_i^{\text{obs}} - N_i^{\text{obs}} \ln \frac{N_i^{\text{obs}}}{N_i^{\text{exp}}}\right)}_{\text{Data bins: likelihood ratio}} + \underbrace{\sum_{j=1}^{M} \left(\frac{\epsilon_j}{\sigma_j}\right)^2}_{\text{Systematic uncertainties: Gaussian}}$$

Data bins: likelihood ratio

Systematic uncertainties: Gaussian

Predicted events based on ν flux

$$\begin{cases} N_i^{\text{exp0}} = P_{\text{survival}}(\text{model x with parameters}\,\vec{x})\, N_i^{\text{nosc}} \\ N_i^{\text{exp}} = \left(1 + \sum_{j=1}^{M} f_i^j \epsilon_j\right) N_i^{\text{exp0}} \end{cases}$$

N_i^{exp0}: Expected number of events without considering systematics

N_i^{exp}: Expected number of events

Plug in different models and find the minimum chi-squares

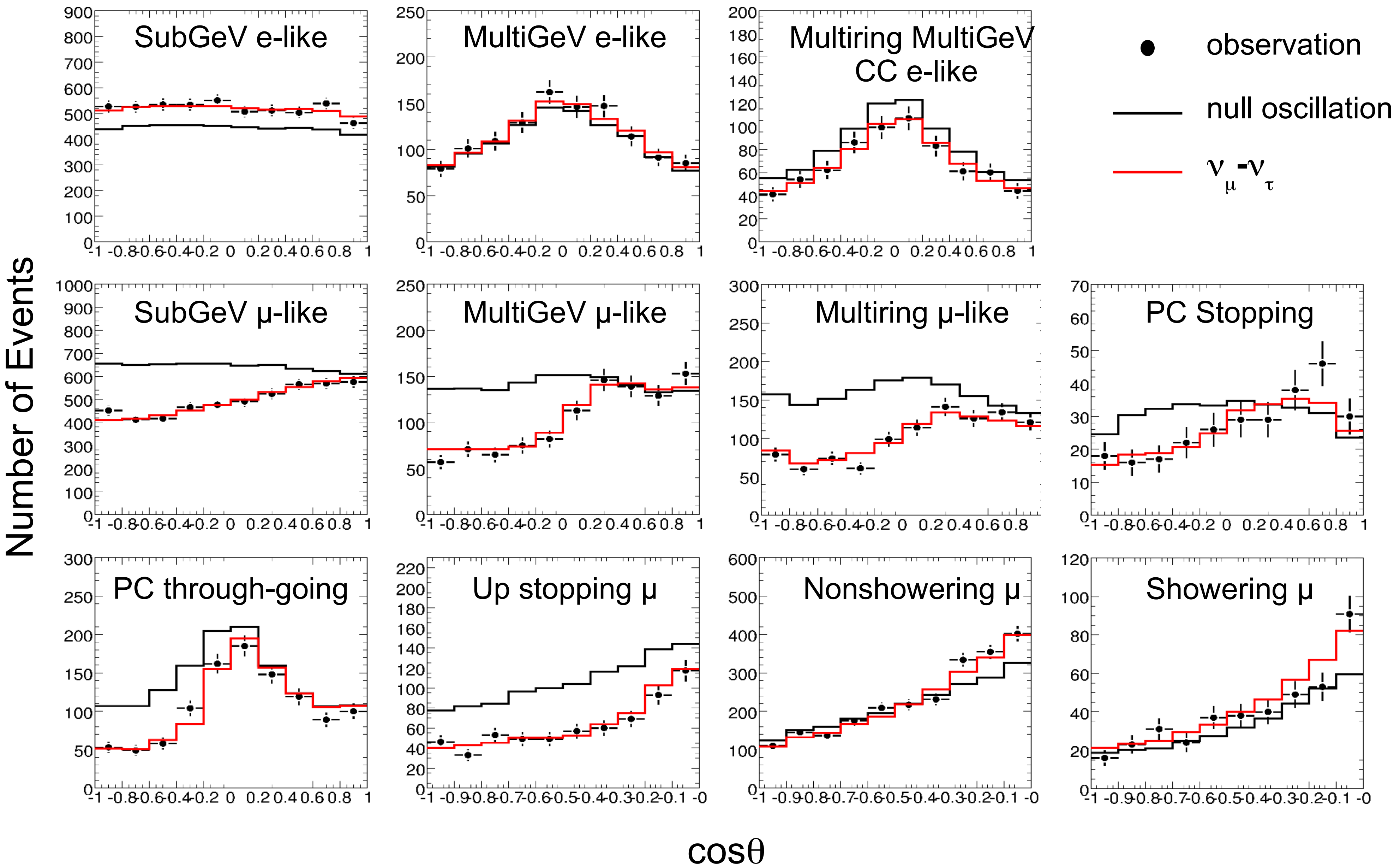
1. Minimize wrt systemic terms
   - Solving a linear equation set
2. Minimize wrt model parameters
   - searching on a grid in parameter space)

Combining SK-I and SK-II

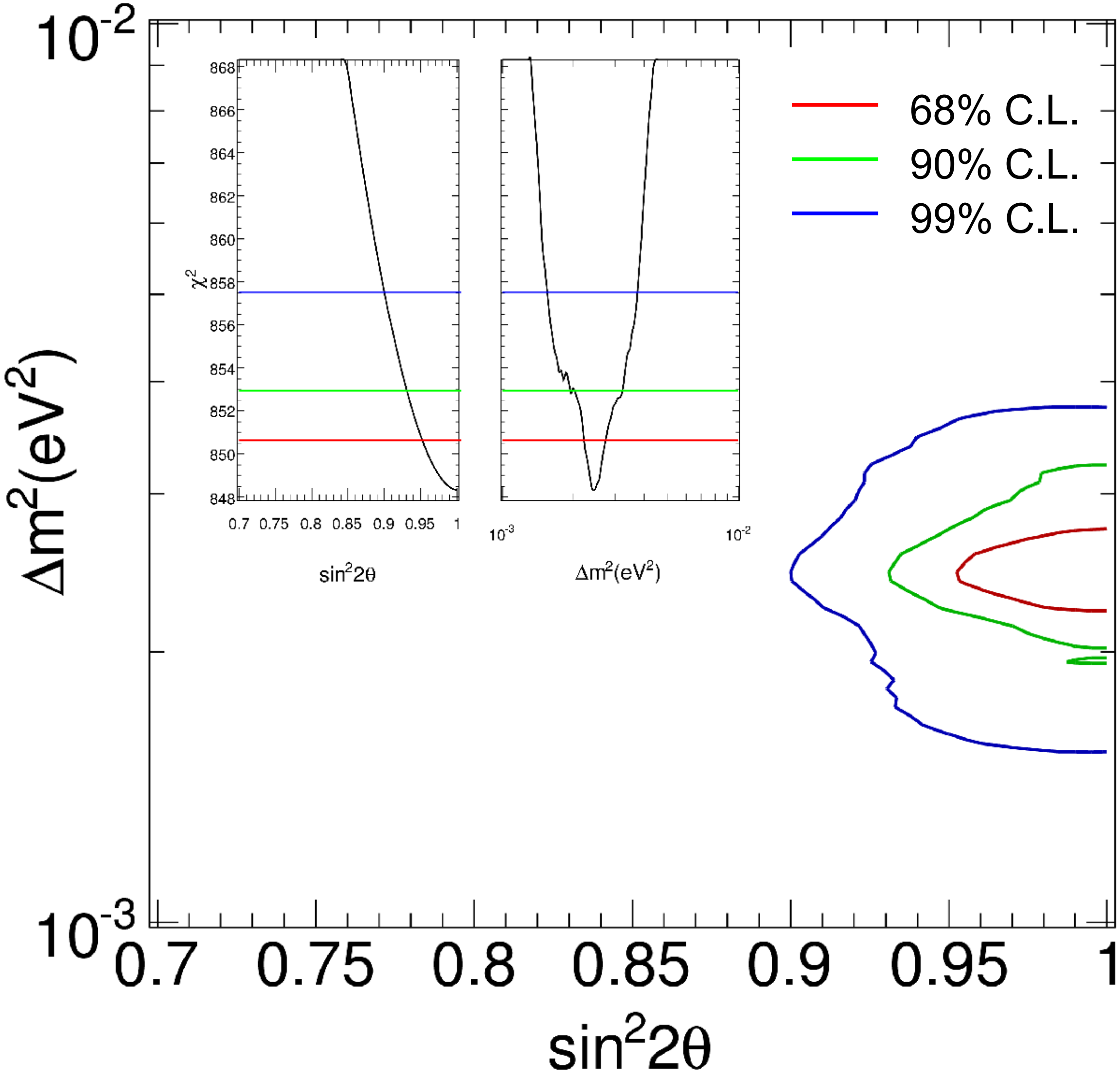
- Data bins are considered as independent observations
- Systematic uncertainties
  - → Identical for SK-I and SK-II
    - Atm neutrino flux (14)
    - Neutrino interaction (12)
  - → Independent for SK-I and SK-II
    - Solar activity (1)
    - Data selection and event reconstruction (21)

→ In total, 70 systematic uncertainties in SK-I and SK-II combined analysis

Zenith Distributions of ν_μ-ν_τ Oscillation



Standard Mixing Parameters



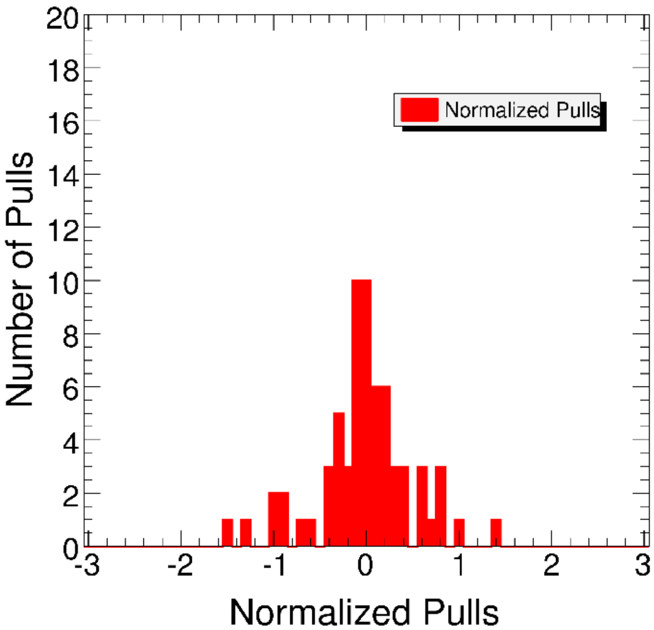
$\nu_\mu \rightarrow \nu_\tau$

$\sin^2 2\theta = 1$

$\Delta m^2 = 2.5 \times 10^{-3}\, eV^2$

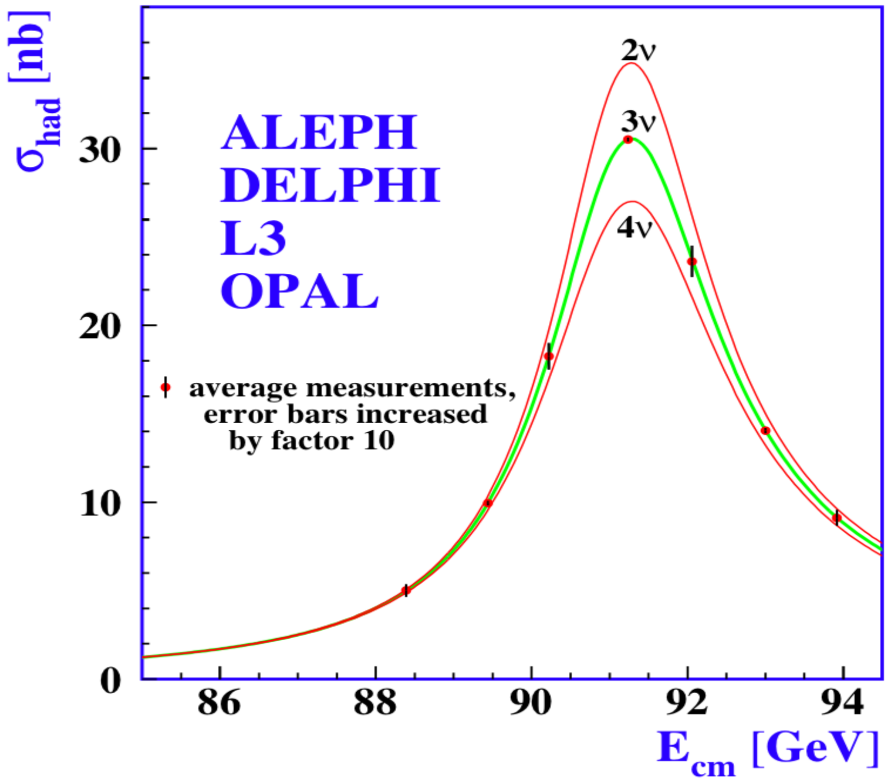
$\chi^2 / dof = 839.7/755$

$p - value = 18\,\%$



Must It Be Tau Neutrino?

- LEP experiments:
  Z decay cross section indicates there are only three neutrino flavors:
  N_ν=2.992±0.020



- If only three flavors of neutrinos, it must be tau neutrino
  - ν_μ-ν_e oscillation does not explain the Super-K observation
  - Chooz and Palo Verde experiments
    $\Rightarrow$ NO $\bar{\nu}_e \rightarrow \bar{\nu}_x$ oscillation at the scale of $\Delta m^2 \sim 10^{-3}$eV^2

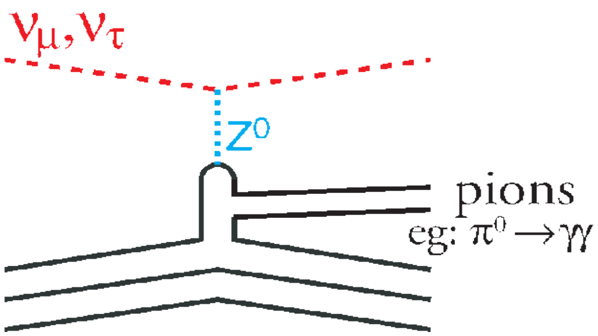
Sterile Neutrinos Are Possible

- Sterile neutrino (ν_s: no electric, strong or weak charge) is not charged under Standard Model → a potential candidate of atmospheric neutrino oscillation
  - Some theoretical models do predict the existence of sterile neutrinos
    - e.g. right-handed neutrino to explain neutrino mass
  - Some observation are in favor of the existence of sterile neutrinos
    - Sterile neutrino helps to solve the LSND anomaly
    - Sterile neutrino helps to solve the nuclear synthesis problem during the supernova R-process

➔ Compare ν_μ-ν_τ oscillation and ν_μ-ν_s oscillation

Signatures of Sterile Neutrinos

Based on the definition of sterile neutrino:



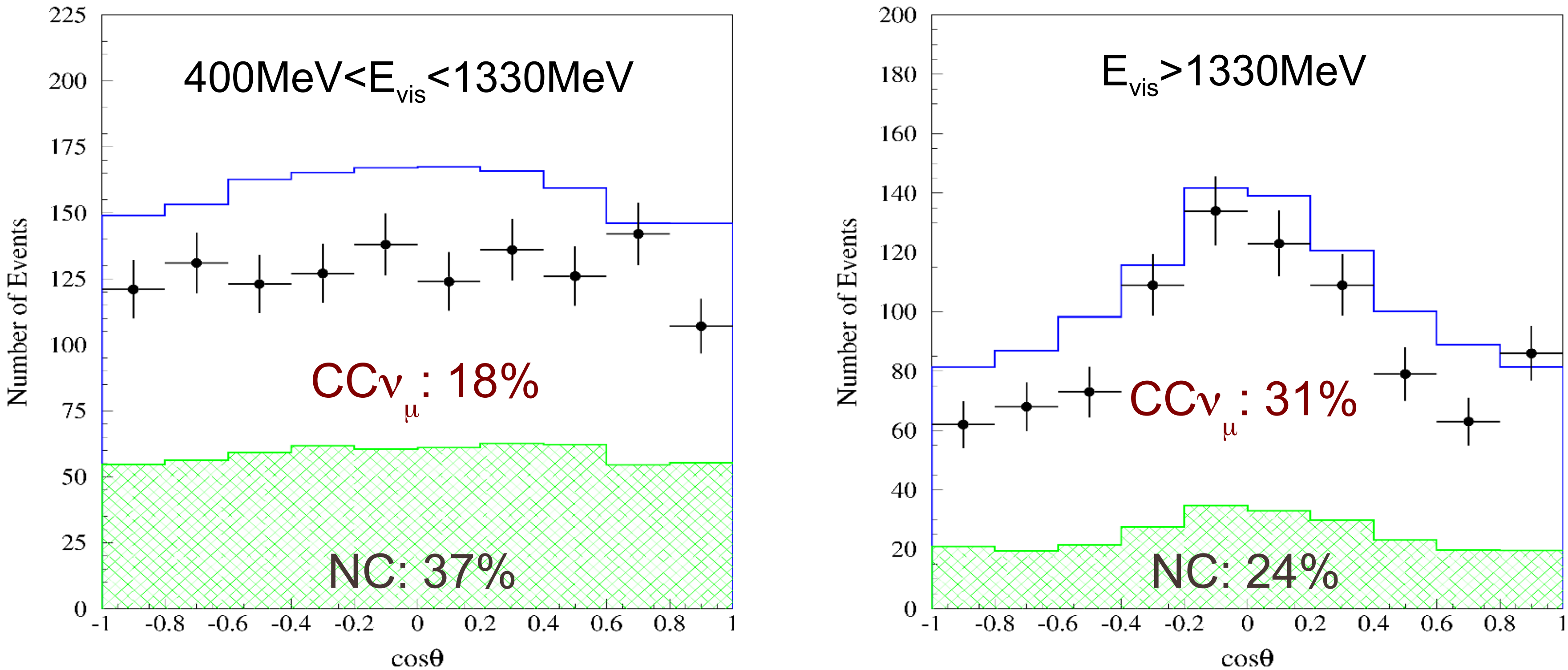
ν_s: no interaction

Difference between ν_μ-ν_τ oscillation and ν_μ-ν_s oscillation:

1. Inside the detector: less neutral current events
2. During the propagation: Matter Effect

1. NC Events at Super-K

- Multi-ring events: neutral pions are the NC signature at SK
- Brightest ring e-like: to remove CC ν_μ events
- E_{vis}>400MeV: low energy events do not point well



2. Matter Effect

- If two neutrino flavors interact differently in matter

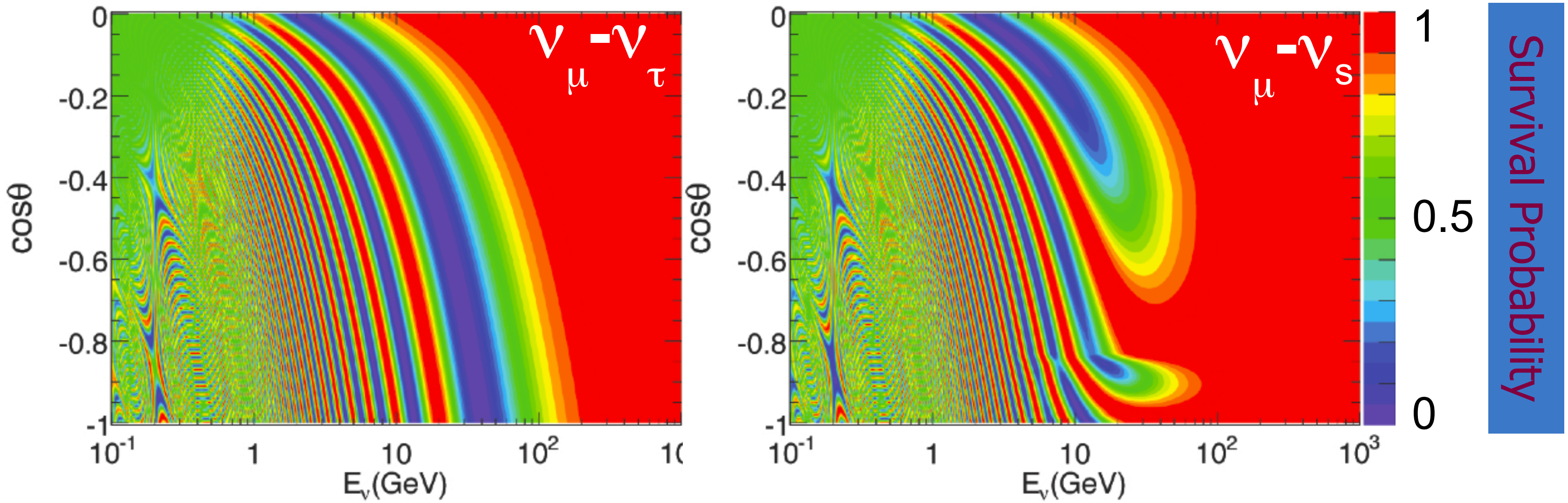
$$P_{osc} = \sin^2 2\theta_M \sin^2 \frac{\Delta m_M^2 L}{4E}$$

$$\begin{cases} \sin^2 2\theta_M = \dfrac{\sin^2 2\theta}{(2E\Delta V/\Delta m^2 - \cos 2\theta)^2 + \sin^2 2\theta} \\ \Delta m_M^2 = \Delta m^2 \sqrt{(2E\Delta V/\Delta m^2 - \cos 2\theta)^2 + \sin^2 2\theta} \end{cases}$$

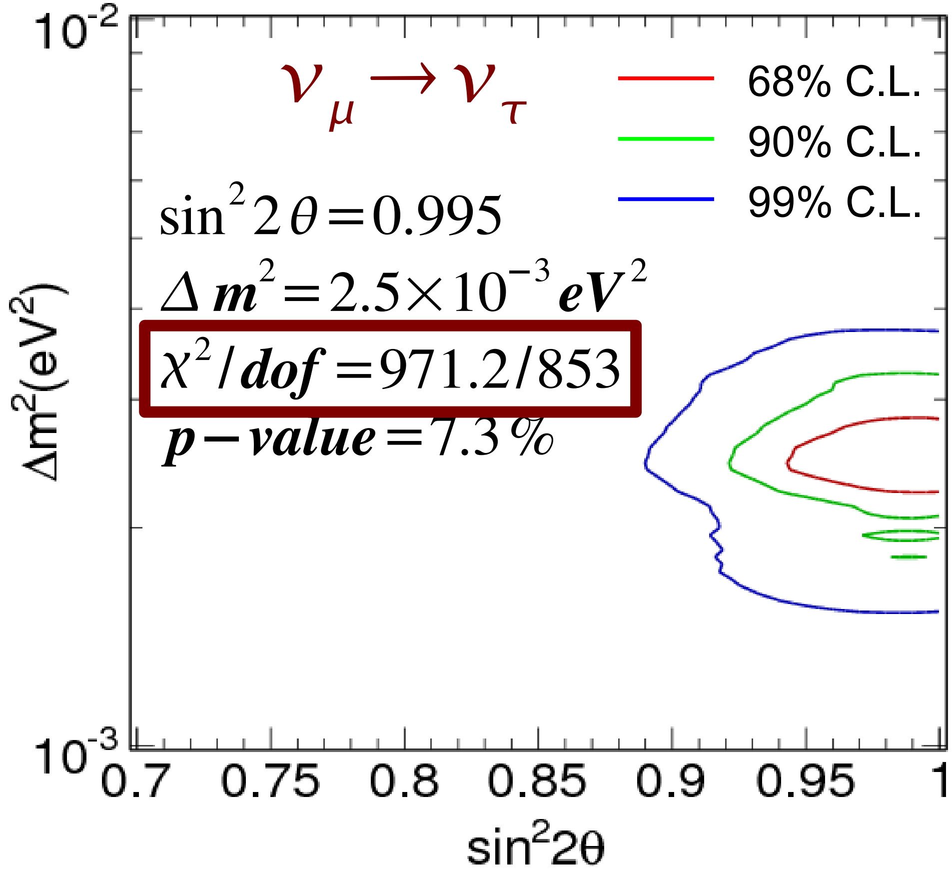
- ν_μ-ν_s: ν_μ and ν_s interact with matter differently $\quad \Delta V = \mp\sqrt{2}\, G_F \frac{\rho_n}{2}$

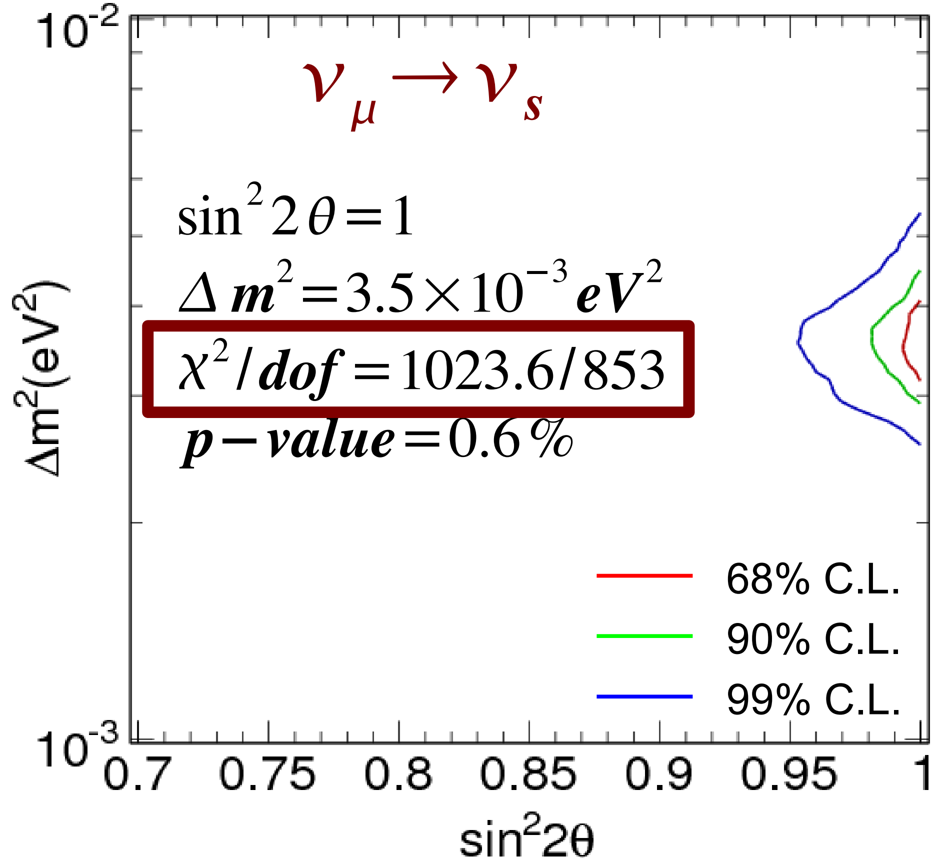
  ⇒ matter effect ⇒ oscillation is suppressed

Survival probability of ν_μ crossing Earth: $\Delta m^2 = 2.5\times10^{-3}\,eV^2$, $\sin^2 2\theta = 1$



Tau Neutrino vs Sterile Neutrino





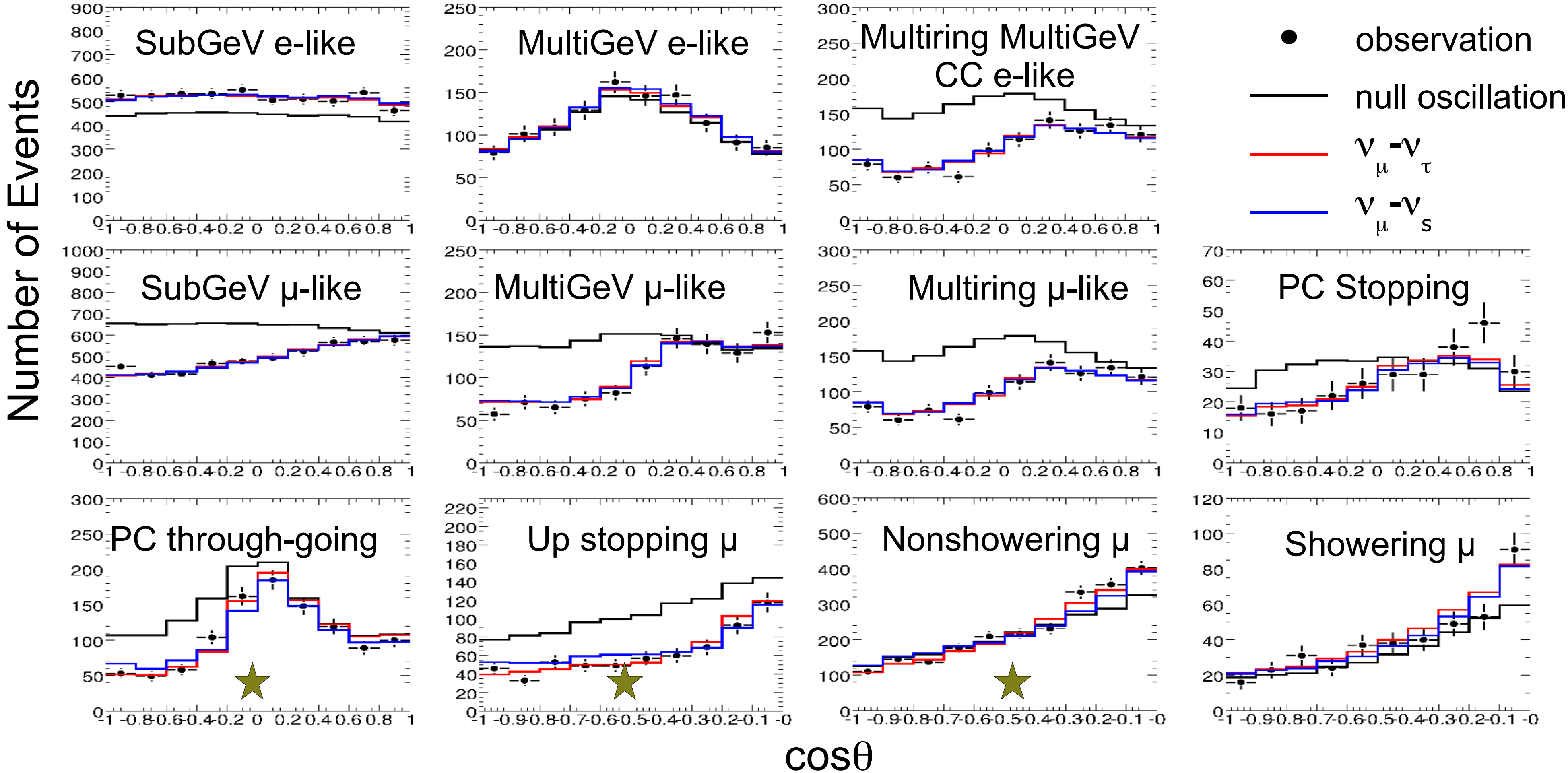
P-values calculated using toy MC method.

- Exclusion Level: 7.2σ

Comparison of Zenith Distributions

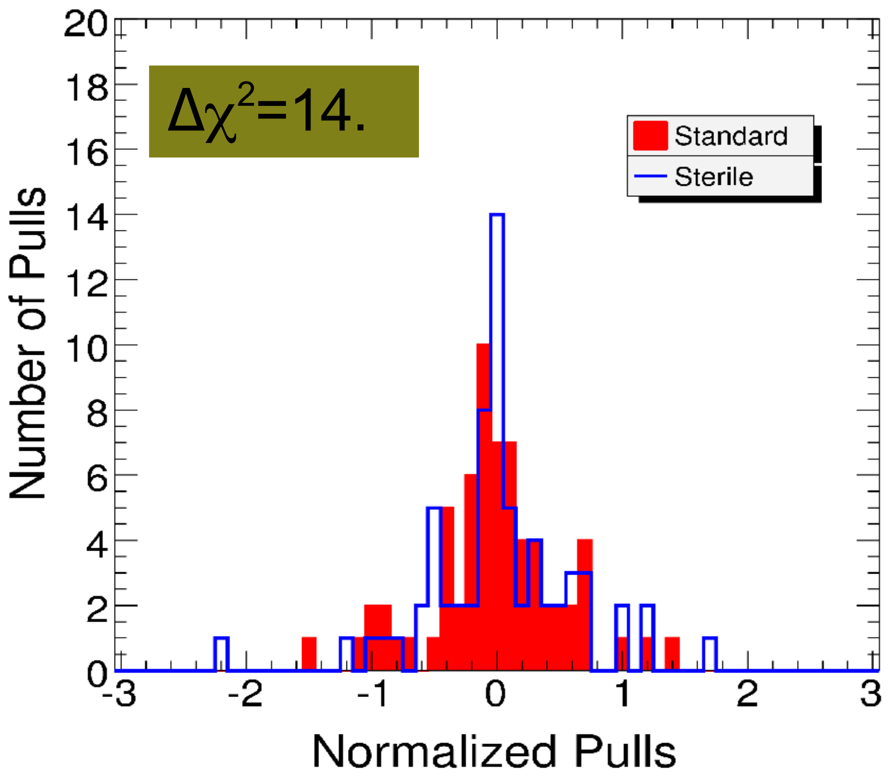
χ^2 Breakdown

	Data bins	Systematics
χ^2 (Standard)	952.9	18.3
χ^2 (Sterile)	990.7	32.9
Difference	37.8	14.6



$\Delta\chi^2$ Contribution Breakdown

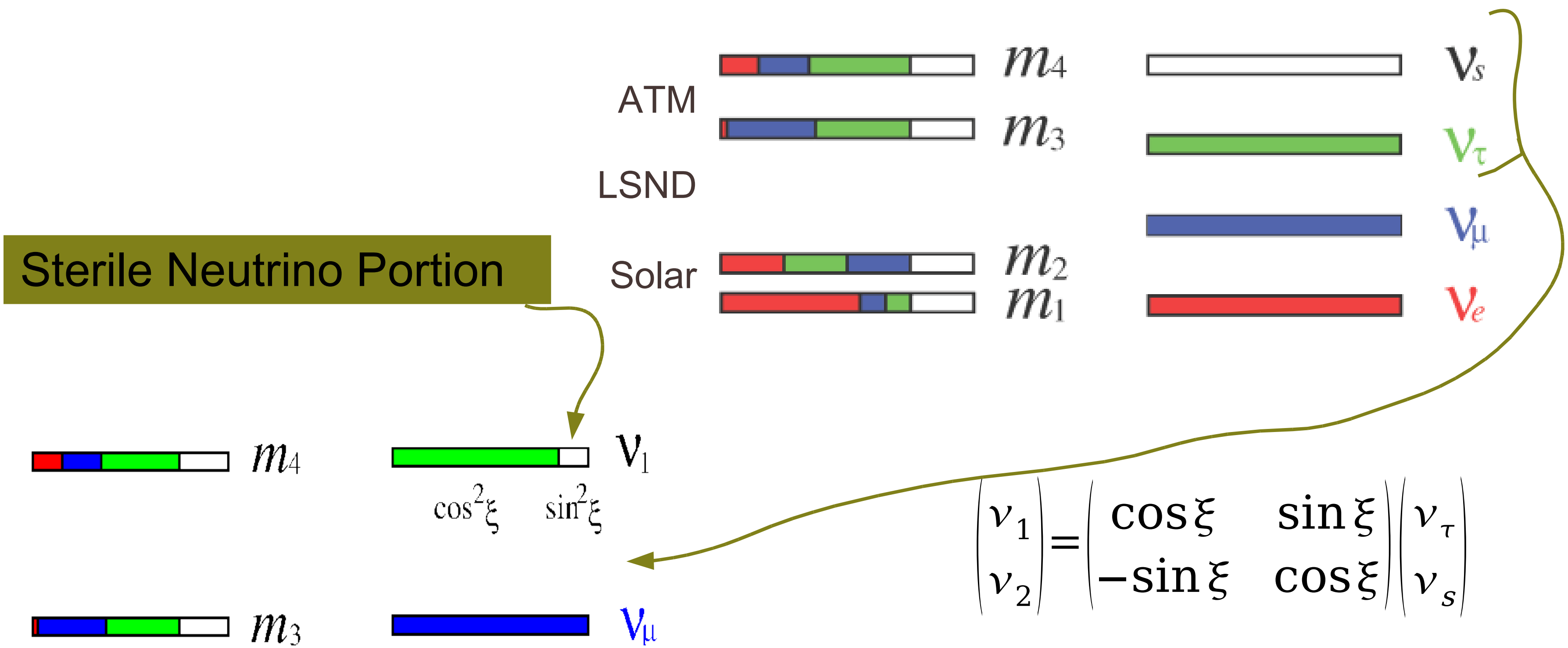
Single Ring SubGeV e-like	0.8
Single Ring MultiGeV e-like	-2.1
Multi-Ring MultiGeV CC e-like	0.8
Single Ring SubGeV μ-like	-1.3
Single Ring MultiGeV μ-like	-2
Multi-Ring μ-like	3.8
NC-Enhanced SubGeV	5
NC-Enhanced MultiGeV	1.2
PC Stopping μ	2.9
PC Through-Going μ	12.3
Upward Stopping μ	7.2
Upward NonShowering μ	11.2
Upward Showering μ	-1.5
TOTAL	37.8



- The right energies and baselines of those events give the strongest matter effects

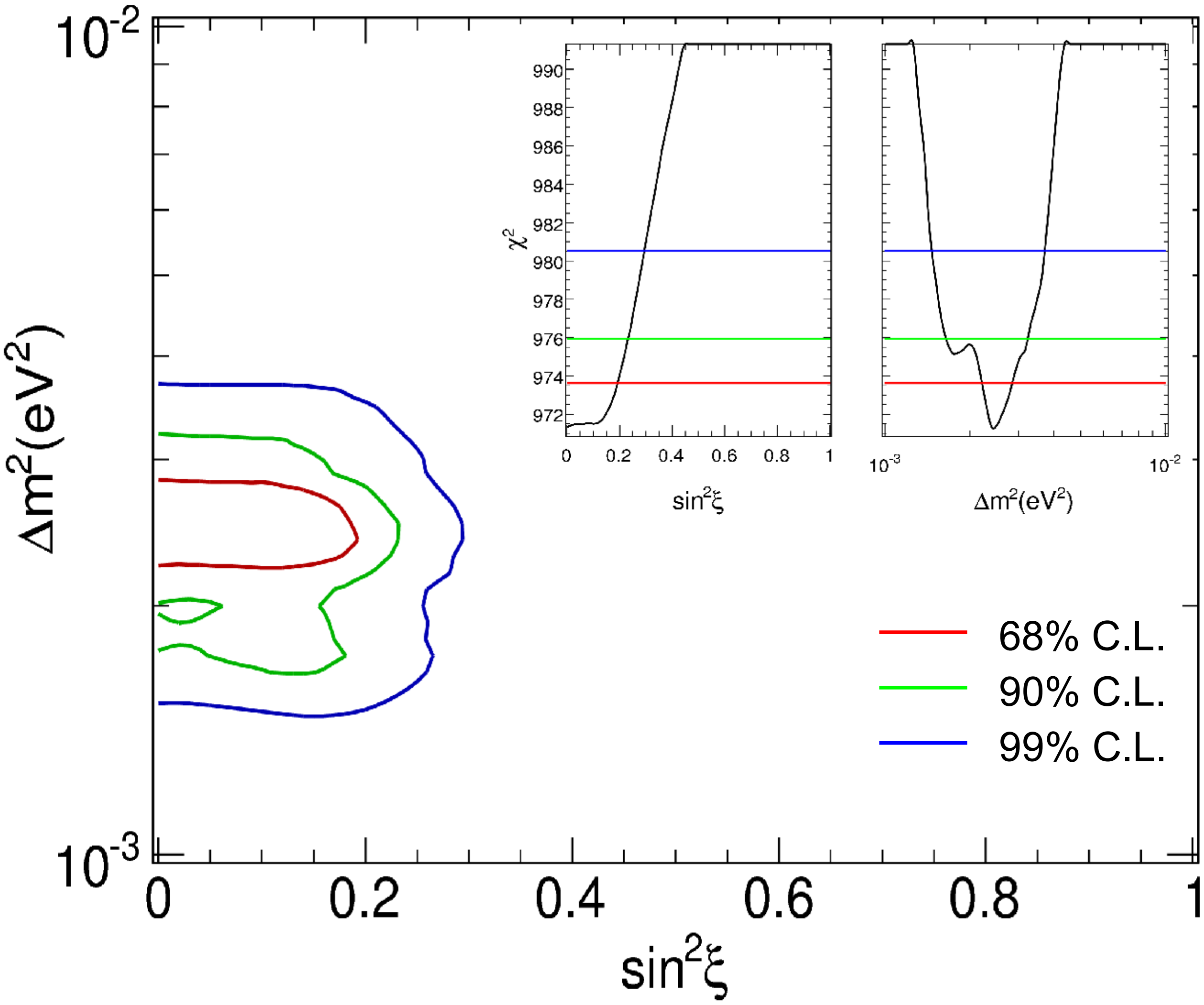
An Admixture Case

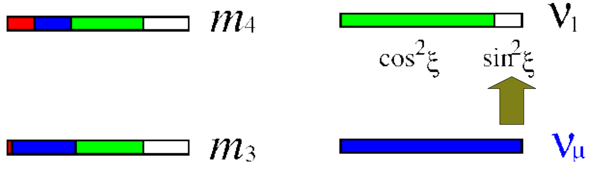
- Admixtures are model dependent
- This analysis is base on Fogli *et al* PRD 63(053008), 2001
  - A 2+2 mass hierarchy model
  - Constructing two superposition states of ν_s and ν_τ → two flavor mixing



$$\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}=\begin{pmatrix}\cos\xi & \sin\xi\\ -\sin\xi & \cos\xi\end{pmatrix}\begin{pmatrix}\nu_\tau\\ \nu_s\end{pmatrix}$$

Admixture Allowance





- Allowed sterile neutrino admixture limit at 90% C.L.: $\sin^2\xi$< 23%

Why Violations of Lorentz and CPT?

→ The other side of the story: neutrino oscillation without mass

- Recall: mass eigenstate mixing
  → $E_i = pc + m_i^2/2p$ → neutrino oscillation
- Violations of Lorentz invariance and CPT symmetry
  → modified dispersion relation
  → different energies for the same momentum
  → neutrino oscillation

→ Important fundamental symmetries: are they broken at some high energy level? (predicted by some Quantum Gravity theories)

- Not practical to reach ~M_P yet
  → Seek for small effects at low energy
  → Neutrino oscillation (interferometry) provides a promising ground

✔ SK neutrino energy and pathlength coverage has great advantages for this study

Minimal Standard Model Extension

- Minimal Standard Model Extension, Kostelecky et al, hep-ph/0403088
  - $L_{mSME} = \frac{i}{2} c_{AB\mu\nu} (\bar{L}_A \gamma^\mu D^\nu L_B + D^\nu \bar{L}_A \gamma^\mu L_B) - a_{AB\mu} \bar{L}_A \gamma^\mu L_B$
  - The first term only violates Lorentz invariance (LIV); the second term violates both CPT (CPTV) and Lorentz invariance
- Two rotationally invariant cases of LIV and CPTV (only time components are considered)
  - Coleman and Glashow, PRD 59(116008), 1999
    - LIV-induced oscillation
  - Barger *et al*, PRL 85(5055), 2000
    - CPTV-induced oscillation

Oscillations Induced by LIV and CPTV

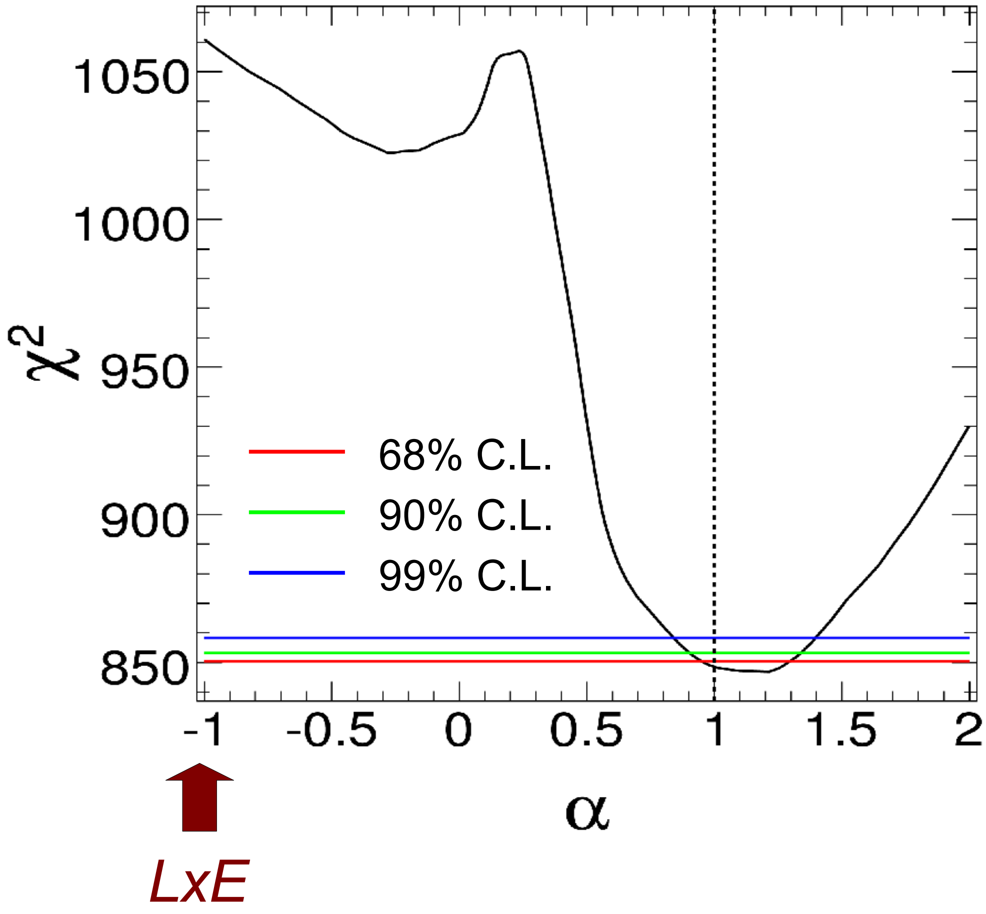
- Rotationally invariant cases: keeping only temporal components
  - $H_{\mathrm{int}} = -c_{AB}^{00}(\bar{L}_A \gamma^0 \partial^0 L_B + \partial^0 \bar{L}_A \gamma^0 L_B)$
    - The eigenstates by diagonalizing (rotating by θ_v) c_{AB} are defined as "maximum attainable velocity" eigenstates
    - Modified dispersion relation: $E_i = pc - pc_i$

    $$\Rightarrow P_{\mathrm{osc}} = \sin^2 2\theta_v \sin^2(c^{TT} L E), \; c^{TT} \equiv c_A - c_B$$
  - $H_{\mathrm{int}} = a_{AB}^{00} \bar{L}_A \gamma^0 L_B$

    $$\Rightarrow P_{\mathrm{osc}} = \sin^2 2\theta_a \sin^2(\pm \Delta a L), \; \Delta a \equiv a_A - a_B$$
    - Neutrino oscillation does depend on energy

Lorentz Invariance Violation

Try a more general form: $P_{\nu_\mu \to \nu_\mu} = 1 - \sin^2 2\theta_v \sin^2 \kappa L / E^\alpha$



- *LxE* oscillation is strongly disfavored
  - Excluded at ~14σ
- *L/E* is within the $1^{st}\sigma$
  - 1.16+0.14/-0.21
- A natural question: what is the scale LIV might appear?

LIV as a Sub-Dominant Effect

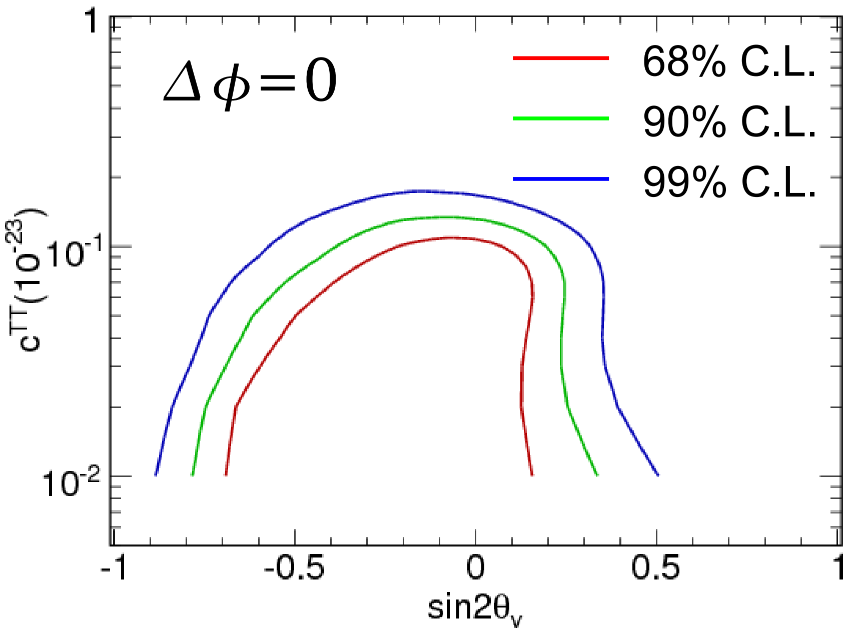
- Considering LIV as a sub-dominant effect
- Assuming best-fit parameter values for the standard oscillation

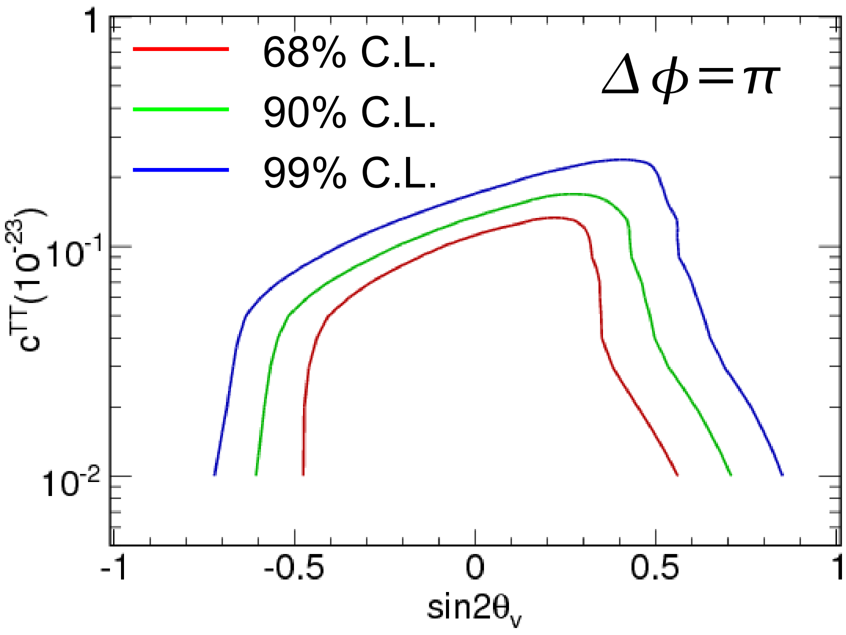
$$P_{\nu_\mu \to \nu_\mu} = 1 - \sin^2 2\Theta \sin^2 \Omega$$

$$\begin{cases} \tan 2\Theta = \dfrac{1 + (E/E_c)^2 \sin 2\theta_v}{(E/E_c)^2 \cos 2\theta_v} \\ \Omega = 1.27\sqrt{(\Delta m^2 L/E)^2 \pm 4 c^{TT} \sin 2\theta_v L E + 4(c^{TT} L E)^2} \\ E_c = \sqrt{\dfrac{\Delta m^2}{2 c^{TT}}} \end{cases}$$

- c^{TT}: the difference of maximum attainable velocities
- θ_v: mixing angle between two different maximal attainable velocity eigenstates
- "+/-": the 0/π phase difference between the mass mixing matrix and the maximum attainable velocity mixing matrix

Limits on LIV





- $\Delta\phi=0$: $c^{TT}<1.2\times10^{-24}$ at 90% C.L.
  - $\sin 2\theta_v=-0.12$; $c^{TT}=0.05\times10^{-23}$
- $\Delta\phi=\pi$: $c^{TT}<1.3\times10^{-24}$ at 90% C.L.
  - $\sin 2\theta_v=-0.02$; $c^{TT}=0.06\times10^{-23}$

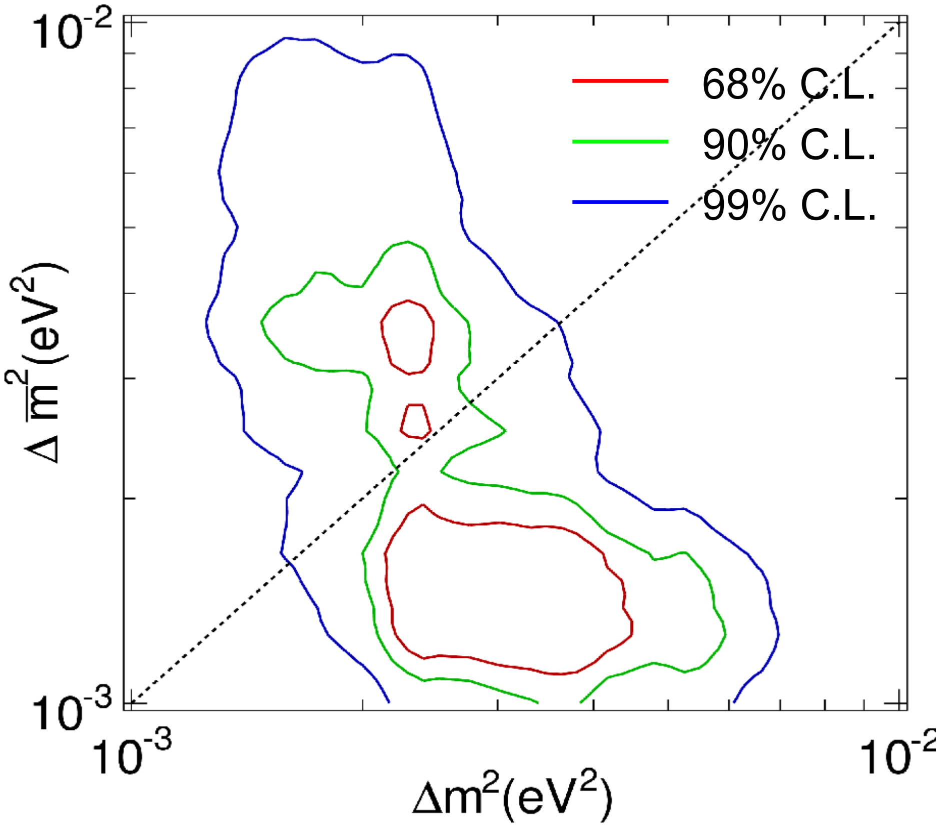
- Limits from other experiments
  - Cosmic ray spectrum: $\sim10^{-15}(\gamma)$, $\sim10^{-23}(p)$
  - Nuclear magnetic resonance frequencies: $\sim10^{-21}(e)$, $\sim10^{-30}(n)$

An *ad hoc* CPT Violation Test

- Simple assumption: neutrinos and antineutrinos could have different mass squared splittings

$$P_{\nu_\mu \to \nu_\mu / \bar{\nu}_\mu \to \bar{\nu}_\mu} = 1 - \sin^2 2\theta \sin^2 \left(\frac{(\Delta m^2 / \Delta \bar{m}^2) L}{4E} \right)$$

- Question: is this allowed by SK?



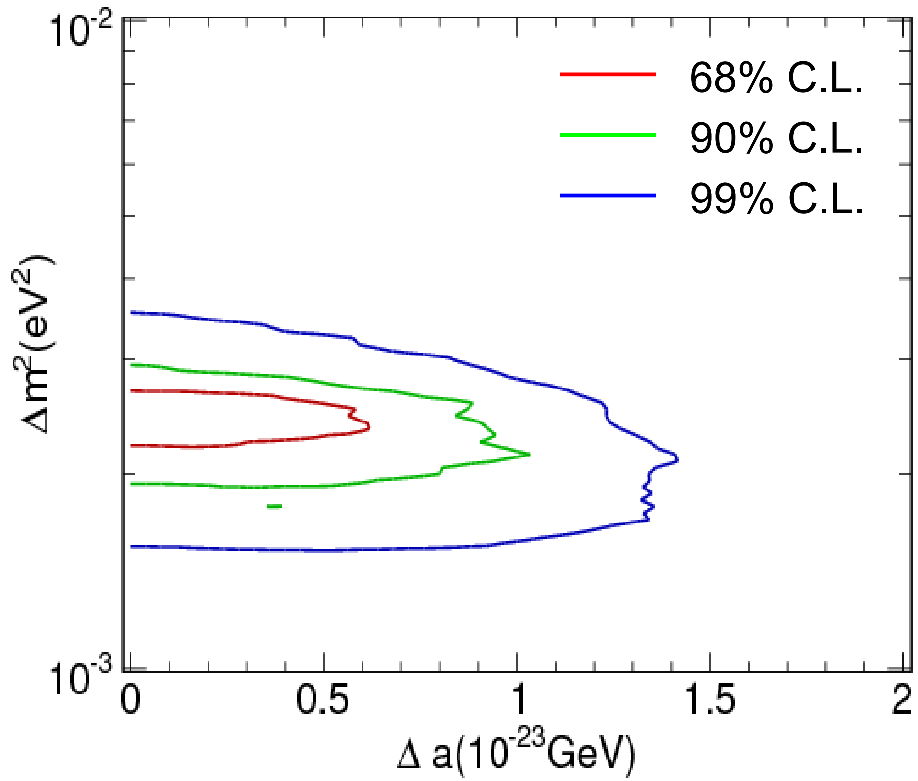
- Best-fit:

$$\begin{cases} \sin 2\theta = 1 \\ \Delta m^2 = 3.7 \times 10^{-3}\, eV^2 \\ \Delta \bar{m}^2 = 1.5 \times 10^{-3}\, eV^2 \end{cases}$$

Super-K best-fit is far away from the LSND scale
→ then, what is the limit on CPTV?

Limit on CPT Violation

- $-a_{AB} L_A \gamma^0 L_B \Rightarrow \Delta a L$ oscillation
- As a sub-dominant effect $\Rightarrow P_{\nu_\mu \to \nu_\mu} = 1 - \sin^2 2\theta \sin^2 (\frac{\Delta m^2}{4E} \pm \Delta a) L$
- Assuming maximal mixing for the mass eigenstates



- At 90% C.L.: $\Delta a < 1.05 \times 10^{-23}$ GeV

- Limits from other experiments

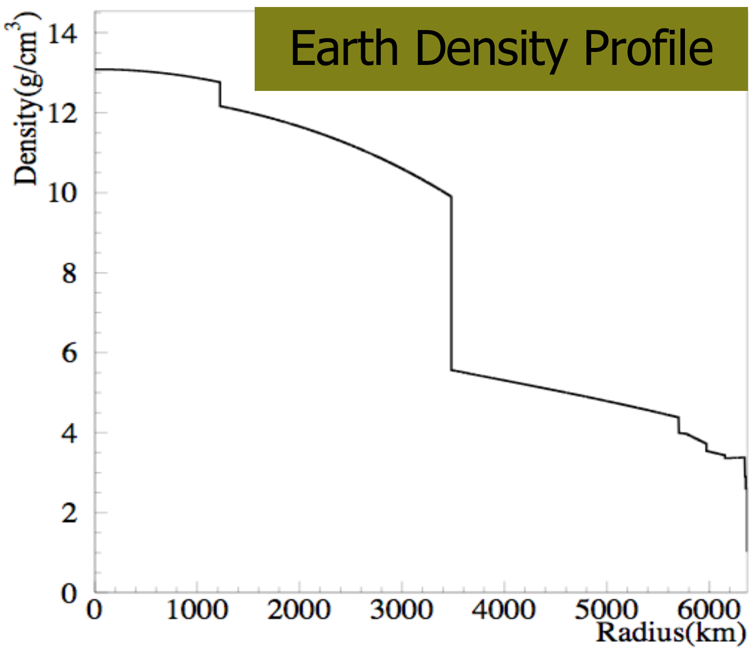
  Barger *et al*, PRL 85(5055), 2000

  - *g-2*: $\sim 10^{-23}$ GeV
  - $K^0 - \bar{K}^0$: $\sim 0.44 \times 10^{-18}$ GeV

Summary and Conclusions

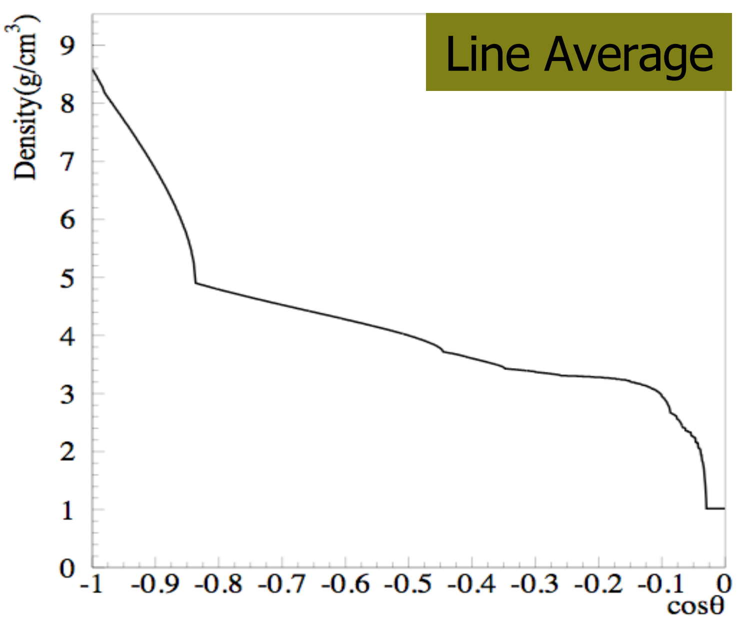
- Neutrino oscillations can happen with or without mass
- ν_μ-ν_τ oscillation is compared with 2 kinds of alternatives: massive neutrino oscillation and massless neutrino oscillation
  - Mass-induced ν_μ-ν_s oscillation: excluded at 7.2σ
  - Oscillations induced by two isotropic cases of LIV and CPTV are not able to explain Super-K atmospheric observation
- Atmospheric neutrino data provide valuable constraints on the scales of new physics beyond the Standard Model
  - An admixture 23% of ν_s is allowed at 90% C.L. (2+2 mass hierarchy)
  - LIV and CPTV limits are set by considering them as sub-dominant effects:
    - $c^{TT}\sim 10^{-24}$ at 90% C.L.
    - $\Delta a\sim 10^{-23}$ GeV at 90% C.L.

Line Average Approximation



Oscillation in uniform matter

- A well-defined phase expression
- A well-defined amplitude expression

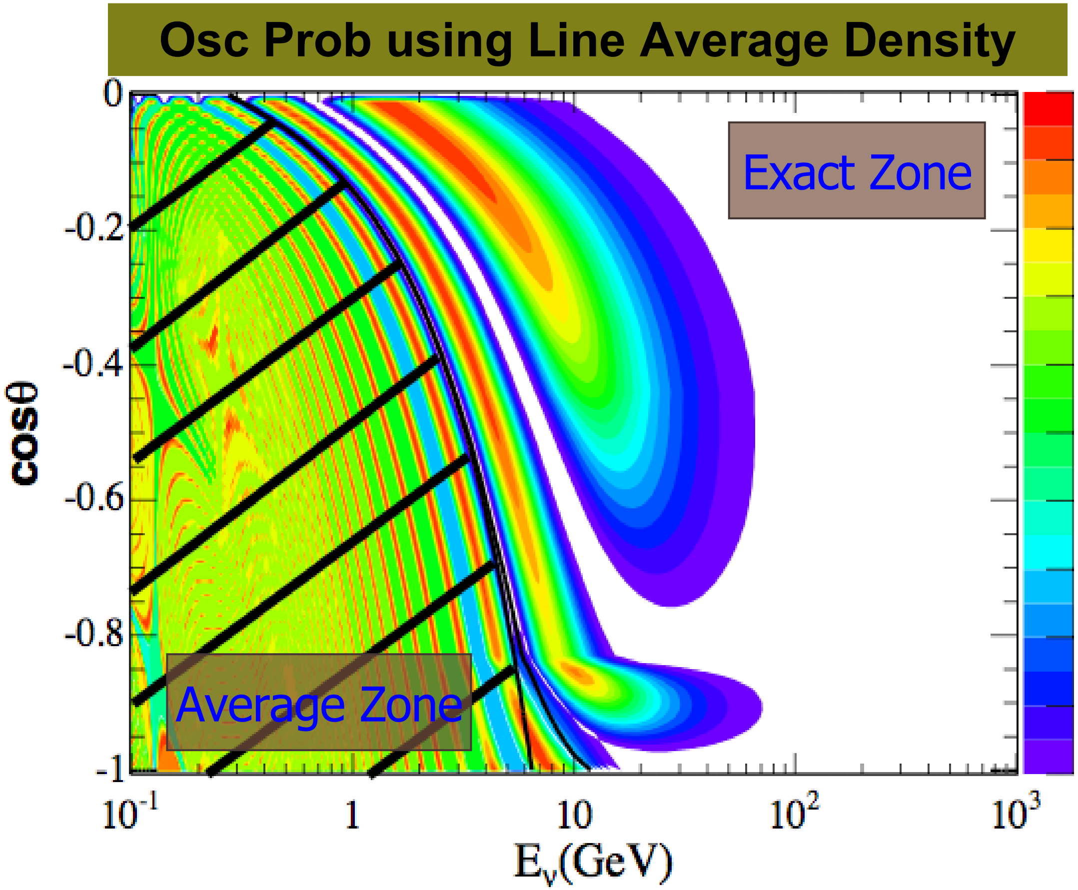


1. Integrate the density along the path
2. Take the average

Matter Effect Reconsidered

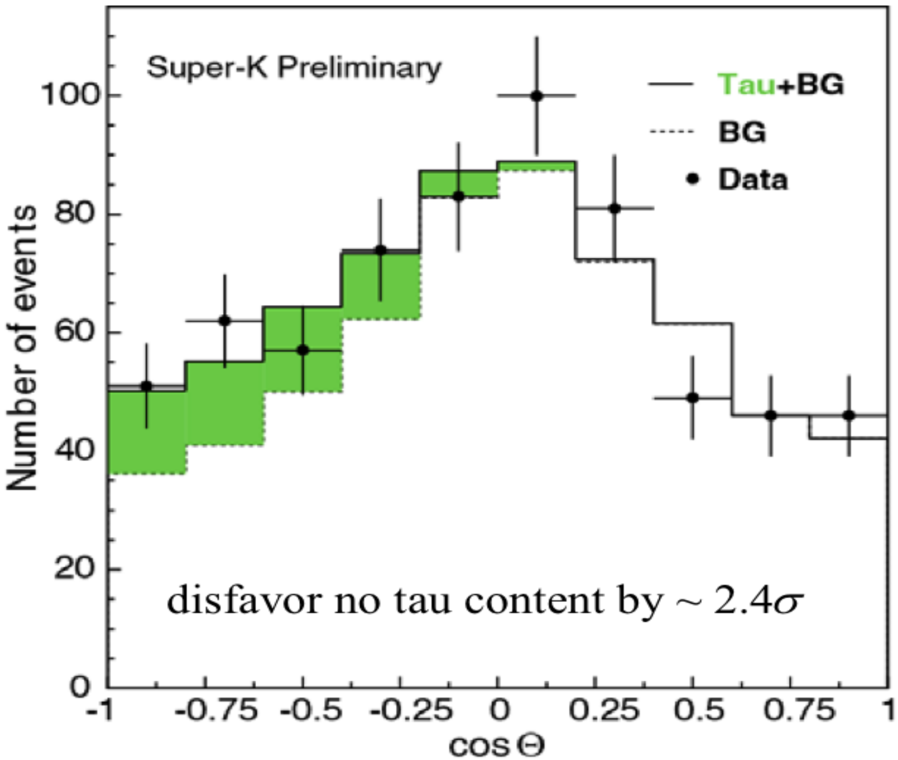
$$P_{osc}=\sin^2 2\theta_M \sin^2\frac{\Delta m_M^2 L}{4E}$$

$$\begin{cases} \sin^2 2\theta_M = \dfrac{\sin^2 2\theta}{(2E\Delta V/\Delta m^2 - \cos 2\theta)^2 + \sin^2 2\theta} \\ \Delta m_M^2 = \Delta m^2 \sqrt{(2E\Delta V/\Delta m^2 - \cos 2\theta)^2 + \sin^2 2\theta} \end{cases}$$



- Good approximation for oscillation cycles
- → $\Delta m_M^2 L/4E > 2\pi$?
  - YES:
    osc prob = $\sin^2 2\theta_M/2$
  - NO:
    propagate thru Earth
    → exact osc prob

Tau Event Searching



Statistically separate (NN & likelihood) tau-like events in high energy sample; look for up-down asymmetry (after accounting for oscillation)

- Expected: 79±28(sys)
- Found:
  - Likelihood: 145±48(stat)+15/-38(sys)
  - Neural Network: 152±47(stat)+17/-29(sys)

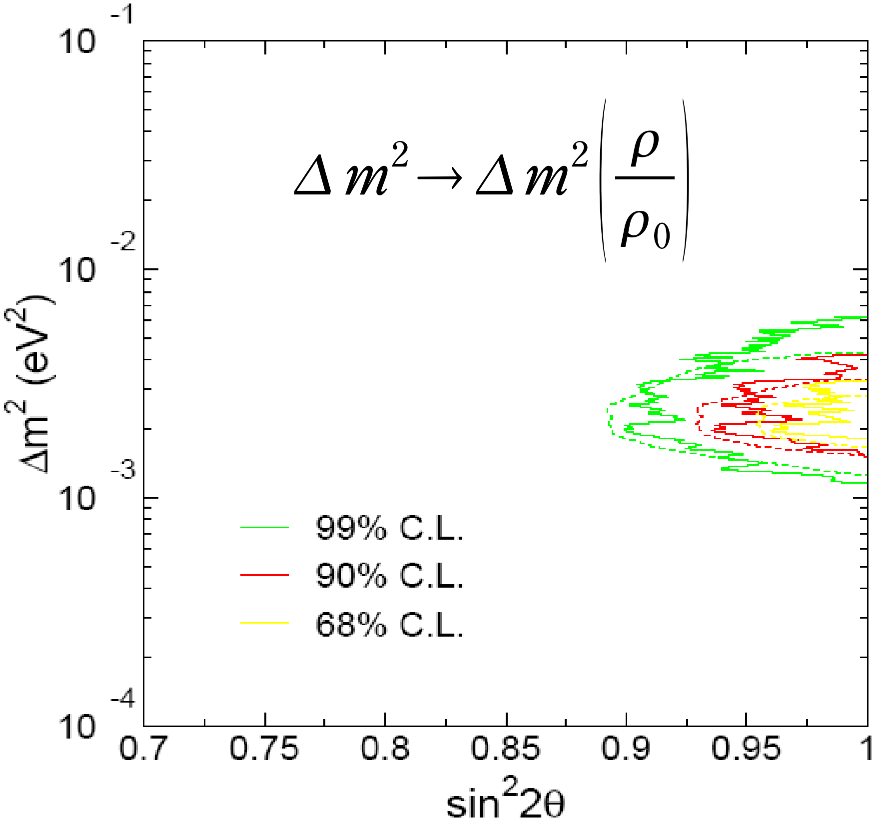
No tau events assumption is disfavored by ~2.4σ

Testing MaVaN

- Neutrinos gain mass only in high density matter (not in air or vacuun

- Best Fit:
  - χ^2_{MaVaN} = 194.4/178 d.o.f
    ($\sin^2 2\theta$, Δm^2) = (1.00, 2.19×10^{-3} eV^2)
  - $\chi^2_{Standard}$ = 174.97/178 d.o.f
    ($\sin^2 2\theta$, Δm^2) = (1.00, 2.11×10^{-3} eV^2)

- Excluded at 4.4σ level



Under study: $$\Delta m^2 \rightarrow \Delta m^2 \left(\frac{\rho}{\rho_0}\right)^n$$

------- Standard 2-flavor oscillations

—— MaVaN oscillations