Studies of Neutrino Oscillations with and without Mass using Super-K

Wei Wang, Boston University
Neutrino Workshop, Kashiwa, Feb 20, 2007
Preaching Buddhism to Buddha

- Teaching Grandmas to Suck Eggs
- 班門弄斧

Ban-Men Nong-Fu

Syaka ni Seppou
An Era of Discovery in Neutrino Physics

\[ \nu_\alpha = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \nu_{m_a} \]

\[ \nu_\beta = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \nu_{m_b} \]

\[ \rightarrow P_{\nu_\alpha \rightarrow \nu_\alpha} = 1 - \sin^2 2\theta \sin^2 \frac{\Delta m^2 L}{4E} \]

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Massive and Massless Neutrino Osc
Outline

• Atmospheric neutrinos, Super-K experiment and events
• Standard $\nu_\mu - \nu_\tau$ oscillation analysis using zenith distributions
• Sterile neutrino as a alternative to tau neutrino
  → $\nu_\mu - \nu_\tau$ mixing vs $\nu_\mu - \nu_s$ mixing
  → An admixture analysis
• Neutrino oscillations induced by the violations of Lorentz (LIV) invariance and CPT (CPTV) symmetry
  → Fit LIV and CPTV induced oscillation against Super-K data
  → Allowed limits of LIV and CPTV
• Summary and conclusions
Atmospheric Neutrinos

- A large uncertainty on the absolute flux
- Good knowledge on flavor ratio 😊
- Up-down symmetric 😊

Abs flux: ~20% uncertainty
Well predicted ratio
Up-down symmetry
Super-Kamiokande Collaboration

- 140 collaborators from 35 institutes of 5 countries


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Super-Kamiokande Experiment

- A 50 kt water Cherenkov detector
  - Inner detector and outer detector optically separated
  - ID: 25in PMTs; gaps filled by black sheet
  - OD: 8in PMTs with wavelength shifters, wall covered by reflective Tyvek

- Operating periods
  - **SK-I: 1996 – 2001**
    - 1489 days livetime
    - ~40% ID coverage
  - **SK-II: 2003 – 2005**
    - 804 days livetime
    - Half ID tubes (acrylic&FRP)
      - ~20% coverage
  - **SK-III: since Summer 2006**
    - ID tubes (acrylic&FRP)
      - fully recovered
Super-K Neutrino Events

- Neutrino interaction
  → charged particles
  → Cherenkov radiation
  → recorded by PMTs

- Neutrino event categories
  - Fully contained
  - Partially contained
  - Upward going $\mu$
Event Reconstruction

- Vertex finding
- Ring recognition
- PID (e-/μ-like)
- Momentum reconstruction

FC Single Ring Events

<table>
<thead>
<tr>
<th>Event Type</th>
<th>CCνμ</th>
<th>CCνe</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sub-GeV</td>
<td>94.5%</td>
<td>82.6%</td>
</tr>
<tr>
<td>Multi-GeV</td>
<td>99.4%</td>
<td>88.0%</td>
</tr>
</tbody>
</table>
Five Decades of Energy

Number of events vs. $E_{\nu}(\text{GeV})$

- e-like
- single ring $\mu$
- multi-ring $\mu$
- PC through

- Fully Contained (FC)
- Multi-Ring (MC)
- PC STOP
- PC THRU
- UP$_\mu$ STOP
- UP$_\mu$ THRU
- UP$_\mu$ SHOWER

Energy ranges:
- $\sim 1\text{GeV}$
- $\sim 3\text{GeV}$
- $\sim 5\text{GeV}$
- $\sim 10\text{GeV}$
- $\sim 100\text{GeV}$
- $\sim 1\text{TeV}$
Four Decades of Pathlengths

- Large ranges of $L$ and $E$
- Various matter densities

$\Rightarrow$ great advantages for studying exotic phenomena
Atmospheric Neutrino Observations

SubGeV e-like
MultiGeV e-like
Multiring MultiGeV CC e-like
Null oscillation prediction

SubGeV μ-like
MultiGeV μ-like
Multiring μ-like

PC Stopping

PC through-going
Up stopping μ
Nonshowing μ
Showering μ

Number of Events

$\cos \theta$
Data Analysis: Binning

\[ P_{\nu_{\mu} \rightarrow \nu_{\mu}} = 1 - \sin^2 2\theta \sin^2 \frac{\Delta m^2 L}{4E} \]

- For SK-I and SK-II, 2×38 energy bins ×10 zenith bins = 760 bins
Data Analysis: Pull Method

$$\chi^2 = \sum_{i=1}^{N} 2(N_i^{\text{exp}} - N_i^{\text{obs}} - N_i^{\text{obs}} \ln \frac{N_i^{\text{obs}}}{N_i^{\text{exp}}}) + \sum_{j=1}^{M} \left( \frac{\epsilon_j}{\sigma_j} \right)^2$$

Data bins: likelihood ratio  
Systematic uncertainties: Gaussian

$$N_i^{\text{exp0}} = P_{\text{survival}}(\text{model x with parameters } \vec{x}) \cdot N_i^{\text{nosc}}$$

$$N_i^{\text{exp}} = (1 + \sum_{j=1}^{M} f_i^j \epsilon_j) \cdot N_i^{\text{exp0}}$$

Expected number of events without considering systematics

Plug in different models and find the minimum chi-squares
1. Minimize wrt systemic terms
   - Solving a linear equation set
2. Minimize wrt model parameters
   - searching on a grid in parameter space

Predicted events based on $\nu$ flux
Combining SK-I and SK-II

• Data bins are considered as independent observations

• Systematic uncertainties

  ➔ Identical for SK-I and SK-II
  - Atm neutrino flux (14)
  - Neutrino interaction (12)
  - Solar activity (1)

  ➔ Independent for SK-I and SK-II
  - Data selection and event reconstruction (21)

➔ In total, 70 systematic uncertainties in SK-I and SK-II combined analysis
Zenith Distributions of $\nu_\mu - \nu_\tau$ Oscillation

**Number of Events**

- **SubGeV e-like**
- **MultiGeV e-like**
- **Multiring MultiGeV CC e-like**
- **SubGeV $\mu$-like**
- **MultiGeV $\mu$-like**
- **Multiring $\mu$-like**
- **PC Stopping**
- **PC through-going**
- **Up stopping $\mu$**
- **Nonshowering $\mu$**
- **Showering $\mu$**

- $\bullet$ observation
- $\nu_\mu - \nu_\tau$
- null oscillation

$$\cos \theta$$

Feb 20, 2007

Massive and Massless Neutrino Osc
Standard Mixing Parameters

$$\nu_\mu \rightarrow \nu_\tau$$

$$\sin^2 2\theta = 1$$
$$\Delta m^2 = 2.5 \times 10^{-3} \text{ eV}^2$$
$$\chi^2 / \text{dof} = 839.7 / 755$$
$$p-value = 18\%$$

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Must It Be Tau Neutrino?

- LEP experiments: Z decay cross section indicates there are only three neutrino flavors: \( N_\nu = 2.992 \pm 0.020 \)

- If only three flavors of neutrinos, it must be tau neutrino
  - \( \nu_\mu \rightarrow \nu_e \) oscillation does not explain the Super-K observation
  - Chooz and Palo Verde experiments
    \[ \Rightarrow \text{NO} \; \bar{\nu}_e \rightarrow \bar{\nu}_x \; \text{oscillation at the scale of} \; \Delta m^2 \sim 10^{-3} \text{eV}^2 \]
Sterile Neutrinos Are Possible

- Sterile neutrino ($\nu_s$: no electric, strong or weak charge) is not charged under Standard Model $\rightarrow$ a potential candidate of atmospheric neutrino oscillation
  - Some theoretical models do predict the existence of sterile neutrinos
    - e.g. right-handed neutrino to explain neutrino mass
  - Some observation are in favor of the existence of sterile neutrinos
    - Sterile neutrino helps to solve the LSND anomaly
    - Sterile neutrino helps to solve the nuclear synthesis problem during the supernova R-process

→ Compare $\nu_\mu - \nu_\tau$ oscillation and $\nu_\mu - \nu_s$ oscillation
Signatures of Sterile Neutrinos

Based on the definition of sterile neutrino:

\[ \nu_\mu, \nu_\tau \]

\[ \nu_s: \text{no interaction} \]

\[ \begin{array}{c}
\text{pions} \\
\text{eg: } \pi^0 \rightarrow \gamma \gamma
\end{array} \]

Difference between \( \nu_\mu - \nu_\tau \) oscillation and \( \nu_\mu - \nu_s \) oscillation:

1. Inside the detector: less neutral current events
2. During the propagation: Matter Effect
1. NC Events at Super-K

- Multi-ring events: neutral pions are the NC signature at SK
- Brightest ring e-like: to remove CC $\nu_\mu$ events
- $E_{\text{vis}} > 400\text{MeV}$: low energy events do not point well

![Graphs showing NC and CC events](image-url)

- For $400\text{MeV} < E_{\text{vis}} < 1330\text{MeV}$:
  - NC: 37%
  - CC $\nu_\mu$: 18%

- For $E_{\text{vis}} > 1330\text{MeV}$:
  - NC: 24%
  - CC $\nu_\mu$: 31%
2. Matter Effect

- If two neutrino flavors interact differently in matter

\[ P_{\text{osc}} = \sin^2 2\theta_M \sin^2 \frac{\Delta m^2_M L}{4E} \]

\[ \sin^2 2\theta_M = \frac{\sin^2 2\theta}{(2E \Delta V / \Delta m^2 - \cos 2\theta)^2 + \sin^2 2\theta} \]

\[ \Delta m^2_M = \Delta m^2 \sqrt{(2E \Delta V / \Delta m^2 - \cos 2\theta)^2 + \sin^2 2\theta} \]

- \( \nu_\mu - \nu_\mu \) and \( \nu_s \) interact with matter differently

\[ \Rightarrow \text{matter effect} \Rightarrow \text{oscillation is suppressed} \]

Survival probability of \( \nu_\mu \) crossing Earth: \( \Delta m^2 = 2.5 \times 10^{-3} \text{eV}^2 \), \( \sin^2 2\theta = 1 \)
Tau Neutrino vs Sterile Neutrino

\[ \nu_\mu \rightarrow \nu_\tau \]

\[ \sin^2 2\theta = 0.995 \]

\[ \Delta m^2 = 2.5 \times 10^{-3} \text{eV}^2 \]

\[ \chi^2 / \text{dof} = 971.2 / 853 \]

\[ p\text{-value} = 7.3\% \]

\[ \nu_\mu \rightarrow \nu_s \]

\[ \sin^2 2\theta = 1 \]

\[ \Delta m^2 = 3.5 \times 10^{-3} \text{eV}^2 \]

\[ \chi^2 / \text{dof} = 1023.6 / 853 \]

\[ p\text{-value} = 0.6\% \]

- **Exclusion Level**: 7.2\(\sigma\)

P-values calculated using toy MC method.
Comparison of Zenith Distributions

$\chi^2$ Breakdown

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<thead>
<tr>
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<th>Systematics</th>
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<td>18.3</td>
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Number of Events

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$\cos \theta$

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### $\Delta \chi^2$ Contribution Breakdown

<table>
<thead>
<tr>
<th>Category</th>
<th>$\Delta \chi^2$</th>
</tr>
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<tbody>
<tr>
<td>Single Ring SubGeV e-like</td>
<td>0.8</td>
</tr>
<tr>
<td>Single Ring MultiGeV e-like</td>
<td>-2.1</td>
</tr>
<tr>
<td>Multi-Ring MultiGeV CC e-like</td>
<td>0.8</td>
</tr>
<tr>
<td>Single Ring SubGeV $\mu$-like</td>
<td>-1.3</td>
</tr>
<tr>
<td>Single Ring MultiGeV $\mu$-like</td>
<td>-2</td>
</tr>
<tr>
<td>Multi-Ring $\mu$-like</td>
<td>3.8</td>
</tr>
<tr>
<td>NC-Enhanced SubGeV</td>
<td>5</td>
</tr>
<tr>
<td>NC-Enhanced MultiGeV</td>
<td>1.2</td>
</tr>
<tr>
<td>PC Stopping $\mu$</td>
<td>2.9</td>
</tr>
<tr>
<td>PC Through-Going $\mu$</td>
<td>12.3</td>
</tr>
<tr>
<td>Upward Stopping $\mu$</td>
<td>7.2</td>
</tr>
<tr>
<td>Upward NonShowering $\mu$</td>
<td>11.2</td>
</tr>
<tr>
<td>Upward Showering $\mu$</td>
<td>-1.5</td>
</tr>
<tr>
<td>TOTAL</td>
<td>37.8</td>
</tr>
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</table>

- The right energies and baselines of those events give the strongest matter effects

$\Delta \chi^2 = 14.$
An Admixture Case

- Admixtures are model dependent
- This analysis is based on Fogli et al. PRD 63(053008), 2001
  - A 2+2 mass hierarchy model
  - Constructing two superposition states of $\nu_s$ and $\nu_\tau \rightarrow$ two flavor mixing

\[
\begin{pmatrix}
\nu_1 \\
\nu_2
\end{pmatrix} =
\begin{pmatrix}
\cos \xi & \sin \xi \\
-\sin \xi & \cos \xi
\end{pmatrix}
\begin{pmatrix}
\nu_\tau \\
\nu_s
\end{pmatrix}
\]

Sterile Neutrino Portion
Admixture Allowance

- Allowed sterile neutrino admixture limit at 90% C.L.: $\sin^2\xi < 23\%$
Why Violations of Lorentz and CPT?

→ The other side of the story: neutrino oscillation without mass
  
  • Recall: mass eigenstate mixing
    → $E_i = pc + \frac{m_i^2}{2} \, p$ → neutrino oscillation
  
  • Violations of Lorentz invariance and CPT symmetry
    → modified dispersion relation
    → different energies for the same momentum
    → neutrino oscillation

→ Important fundamental symmetries: are they broken at some high energy level? (predicted by some Quantum Gravity theories)

  • Not practical to reach $\sim M_P$ yet
    → Seek for small effects at low energy
    → Neutrino oscillation (interferometry) provides a promising ground
  
  ✔ SK neutrino energy and pathlength coverage has great advantages for this study
Minimal Standard Model Extension

  \[ L_{\text{SME}} = \frac{i}{2} c_{AB\mu\nu} (\bar{L}_A \gamma^\mu D^\nu L_B + D^\nu L_A \gamma^\mu L_B) - a_{AB\mu} \bar{L}_A \gamma^\mu L_B \]
  - The first term only violates Lorentz invariance (LIV);
  - the second term violates both CPT (CPTV) and Lorentz invariance

- Two rotationally invariant cases of LIV and CPTV (only time components are considered)
  - Coleman and Glashow, PRD 59(116008), 1999
    - LIV-induced oscillation
  - Barger et al, PRL 85(5055), 2000
    - CPTV-induced oscillation
Oscillations Induced by LIV and CPTV

• Rotationally invariant cases: keeping only temporal components

\[ H_{\text{int}} = -c_{AB}^{00}(\bar{L}_A \gamma^0 \partial^0 L_B + \partial^0 \bar{L}_A \gamma^0 L_B) \]

• The eigenstates by diagonalizing (rotating by \( \theta_v \)) \( c_{AB} \)
  are defined as “maximum attainable velocity” eigenstates

• Modified dispersion relation: \( E_i = p c - p c_i \)
  \[ \Rightarrow P_{\text{osc}} = \sin^2 2\theta_v \sin^2 (c_{TT} L E), \quad c_{TT} \equiv c_A - c_B \]

\[ H_{\text{int}} = a_{AB}^{00} \bar{L}_A \gamma^0 L_B \]

  \[ \Rightarrow P_{\text{osc}} = \sin^2 2\theta_a \sin^2 (\pm \Delta a L), \quad \Delta a \equiv a_A - a_B \]

• Neutrino oscillation does depend on energy
Lorentz Invariance Violation

Try a more general form: \( P_{\nu_\mu \rightarrow \nu_\mu} = 1 - \sin^2 2\theta \sin^2 \kappa L/E^\alpha \)

- \( LxE \) oscillation is strongly disfavored
  - Excluded at \( \sim 14\sigma \)
- \( L/E \) is within the 1\(^{st}\)\(\sigma\)
  - 1.16\(+0.14/-0.21\)
- A natural question: what is the scale LIV might appear?

\[ \chi^2 \]

\[ LxE \]

\[ \alpha \]

\[ \text{68% C.L.} \]
\[ \text{90% C.L.} \]
\[ \text{99% C.L.} \]
LIV as a Sub-Dominant Effect

- Considering LIV as a sub-dominant effect
- Assuming best-fit parameter values for the standard oscillation

\[ P_{\nu_\mu \rightarrow \nu_\mu} = 1 - \sin^2 2\Theta \sin^2 \Omega \]

\[
\tan 2\Theta = \frac{1 + (E/E_c)^2 \sin 2\theta_v}{(E/E_c)^2 \cos 2\theta_v}
\]

\[
\Omega = 1.27 \sqrt{(\Delta m^2 L/E)^2 + 4c^{TT}\sin 2\theta_v L E + 4(c^{TT} L E)^2}
\]

\[
E_c = \sqrt{\frac{\Delta m^2}{2c^{TT}}}
\]

- \(c^{TT}\): the difference of maximum attainable velocities
- \(\theta_v\): mixing angle between two different maximal attainable velocity eigenstates
- “+/-”: the 0/\(\pi\) phase difference between the mass mixing matrix and the maximum attainable velocity mixing matrix
**Limits on LIV**

- **$\Delta \phi = 0$:** $c^{TT} < 1.2 \times 10^{-24}$ at 90% C.L.
  - $\sin^2 \theta_v = -0.12$; $c^{TT} = 0.05 \times 10^{-23}$
- **$\Delta \phi = \pi$:** $c^{TT} < 1.3 \times 10^{-24}$ at 90% C.L.
  - $\sin^2 \theta_v = -0.02$; $c^{TT} = 0.06 \times 10^{-23}$

**Limits from other experiments**
- Cosmic ray spectrum: $\sim 10^{-15}(\gamma), \sim 10^{-23}(p)$
- Nuclear magnetic resonance frequencies: $\sim 10^{-21}(e), \sim 10^{-30}(n)$
An *ad hoc* CPT Violation Test

- **Simple assumption:** neutrinos and antineutrinos could have different mass squared splittings
  
  \[ P_{\nu_\mu \rightarrow \nu_\mu / \bar{\nu}_\mu \rightarrow \bar{\nu}_\mu} = 1 - \sin^2 2\theta \sin^2 \left( \frac{\Delta m^2 / \Delta \bar{m}^2}{4E} L \right) \]

- **Question:** is this allowed by SK?

- **Best-fit:**
  
  \[
  \begin{align*}
  \sin 2\theta &= 1 \\
  \Delta m^2 &= 3.7 \times 10^{-3} \text{ eV}^2 \\
  \Delta \bar{m}^2 &= 1.5 \times 10^{-3} \text{ eV}^2
  \end{align*}
  \]

Super-K best-fit is far away from the LSND scale → then, what is the limit on CPTV?
Limit on CPT Violation

- $-a_{AB} L_A \gamma^0 L_B \Rightarrow \Delta a L$ oscillation
- As a sub-dominant effect $\Rightarrow P_{\nu_\mu \to \nu_\mu} = 1 - \sin^2 2\theta \sin^2(\frac{\Delta m^2}{4E} \pm \Delta a) L$
- Assuming maximal mixing for the mass eigenstates

- At 90% C.L.: $\Delta a < 1.05 \times 10^{-23}$ GeV

- Limits from other experiments
  - Barger et al, PRL 85(5055), 2000
    - $g-2$: $\sim 10^{-23}$ GeV
    - $K^0 - \bar{K}^0$: $\sim 0.44 \times 10^{-18}$ GeV
Summary and Conclusions

- Neutrino oscillations can happen with or without mass
- $\nu_\mu \rightarrow \nu_\tau$ oscillation is compared with 2 kinds of alternatives: massive neutrino oscillation and massless neutrino oscillation
  - Mass-induced $\nu_\mu \rightarrow \nu_s$ oscillation: excluded at 7.2$\sigma$
  - Oscillations induced by two isotropic cases of LIV and CPTV are not able to explain Super-K atmospheric observation
- Atmospheric neutrino data provide valuable constraints on the scales of new physics beyond the Standard Model
  - An admixture 23% of $\nu_s$ is allowed at 90% C.L. (2+2 mass hierarchy)
  - LIV and CPTV limits are set by considering them as sub-dominant effects:
    - $c^{TT} \sim 10^{-24}$ at 90% C.L.
    - $\Delta a \sim 10^{-23}$ GeV at 90% C.L.
Line Average Approximation

1. Integrate the density along the path
2. Take the average

Earth Density Profile

Oscillation in uniform matter
- A well-defined phase expression
- A well-defined amplitude expression

Line Average
Matter Effect Reconsidered

\[ P_{osc} = \sin^2 2\theta_M \sin^2 \frac{\Delta m^2_M L}{4E} \]

\[
\sin^2 2\theta_M = \frac{\sin^2 2\theta}{(2E \Delta V/\Delta m^2 - \cos 2\theta)^2 + \sin^2 2\theta}
\]

\[
\Delta m^2_M = \Delta m^2 \sqrt{(2E \Delta V/\Delta m^2 - \cos 2\theta)^2 + \sin^2 2\theta}
\]

- Good approximation for oscillation cycles
  \[ \Delta m^2_M L/4E > 2\pi ? \]
  - YES:
    osc prob = \( \sin^2 2\theta_M /2 \)
  - NO:
    propagate thru Earth
    \[ \rightarrow \text{exact osc prob} \]
### Tau Event Searching

- **Expected:** $79 \pm 28\text{(sys)}$
- **Found:**
  - **Likelihood:** $145 \pm 48\text{(stat)} + 15/-38\text{(sys)}$
  - **Neural Network:** $152 \pm 47\text{(stat)} + 17/-29\text{(sys)}$

No tau events assumption is disfavored by $\sim 2.4\sigma$

Statistically separate (NN & likelihood) tau-like events in high energy sample; look for up-down asymmetry (after accounting for oscillation)
Testing MaVaN

• Neutrinos gain mass only in high density matter (not in air or vacuum)

• Best Fit:
  \[ \chi^2_{\text{MaVaN}} = 194.4/178 \text{ d.o.f} \]
  \[ (\sin^2 2\theta, \Delta m^2) = (1.00, 2.19 \times 10^{-3} \text{ eV}^2) \]
  \[ \chi^2_{\text{Standard}} = 174.97/178 \text{ d.o.f} \]
  \[ (\sin^2 2\theta, \Delta m^2) = (1.00, 2.11 \times 10^{-3} \text{ eV}^2) \]

• Excluded at 4.4\(\sigma\) level

Under study: \[ \Delta m^2 \rightarrow \Delta m^2 \left( \frac{\rho}{\rho_0} \right)^n \]

\[ \Delta m^2 \rightarrow \Delta m^2 \left( \frac{\rho}{\rho_0} \right) \]

99% C.L.
90% C.L.
68% C.L.

--- Standard 2-flavor oscillations
--- MaVaN oscillations