第17回「宇宙ニュートリノ」研究会
２００５年6月２９日

DIHEDRAL FLAVOR SYMMETRY

AND

MIXING OF FERMIONS

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Plan

I. Motivation

II. Dihedral symmetry

III. An Example: Q6 model

VI. Conclusion

I. Motivation

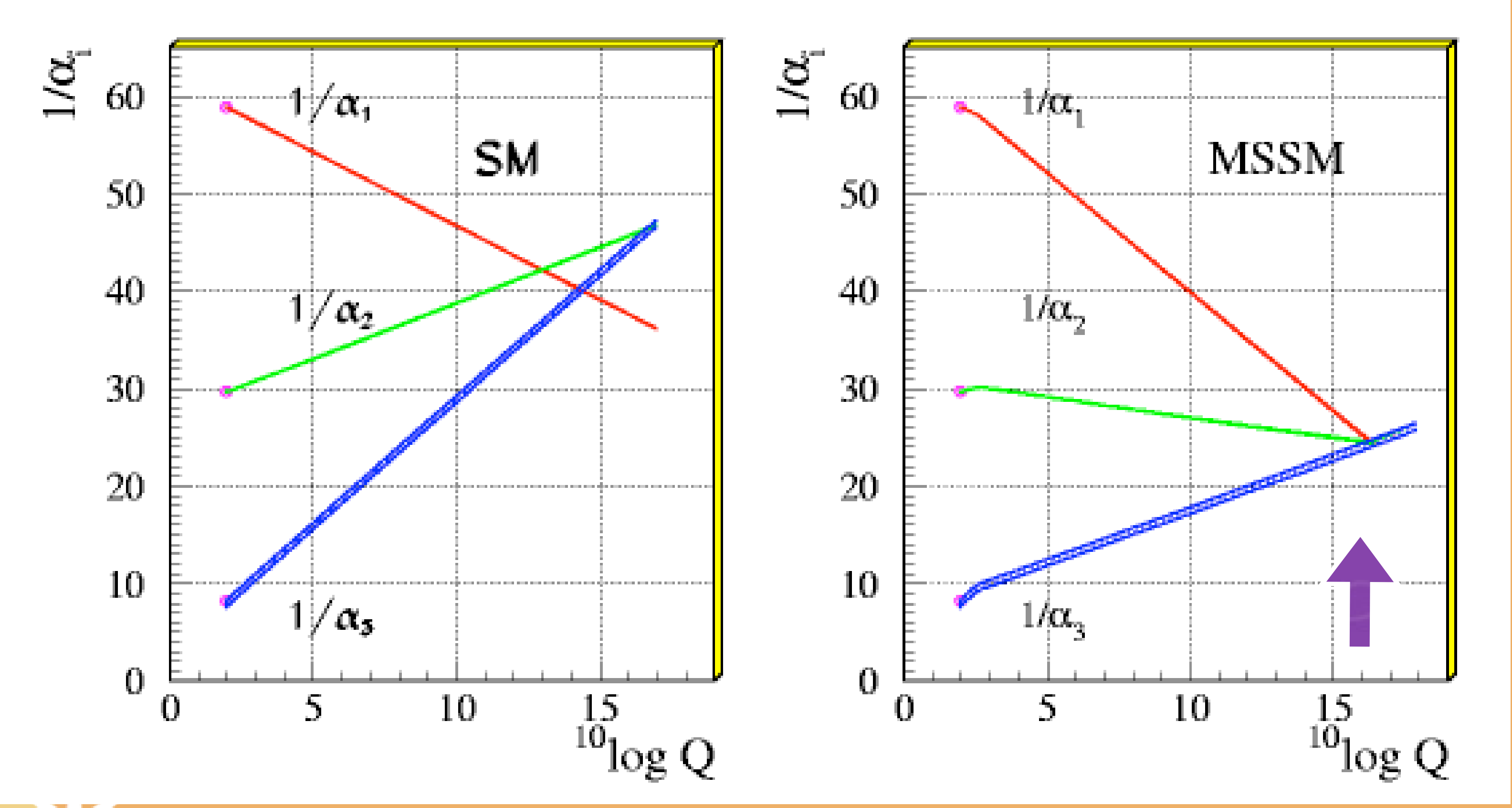
SUSY

Do you like SUSY?

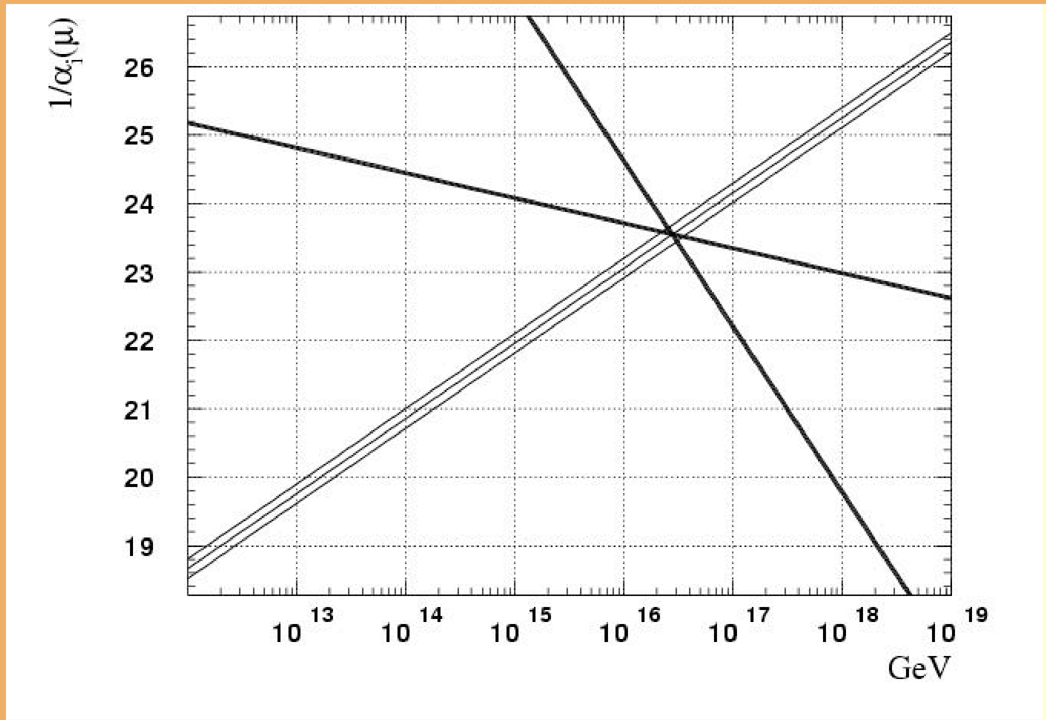
The most convincing argument for SUSY is the exact <u>Gauge Coupling Unification</u>.

A critical look at the facts.

Gauge Coupling Unification



If you require for the MSSM



you might have problems.

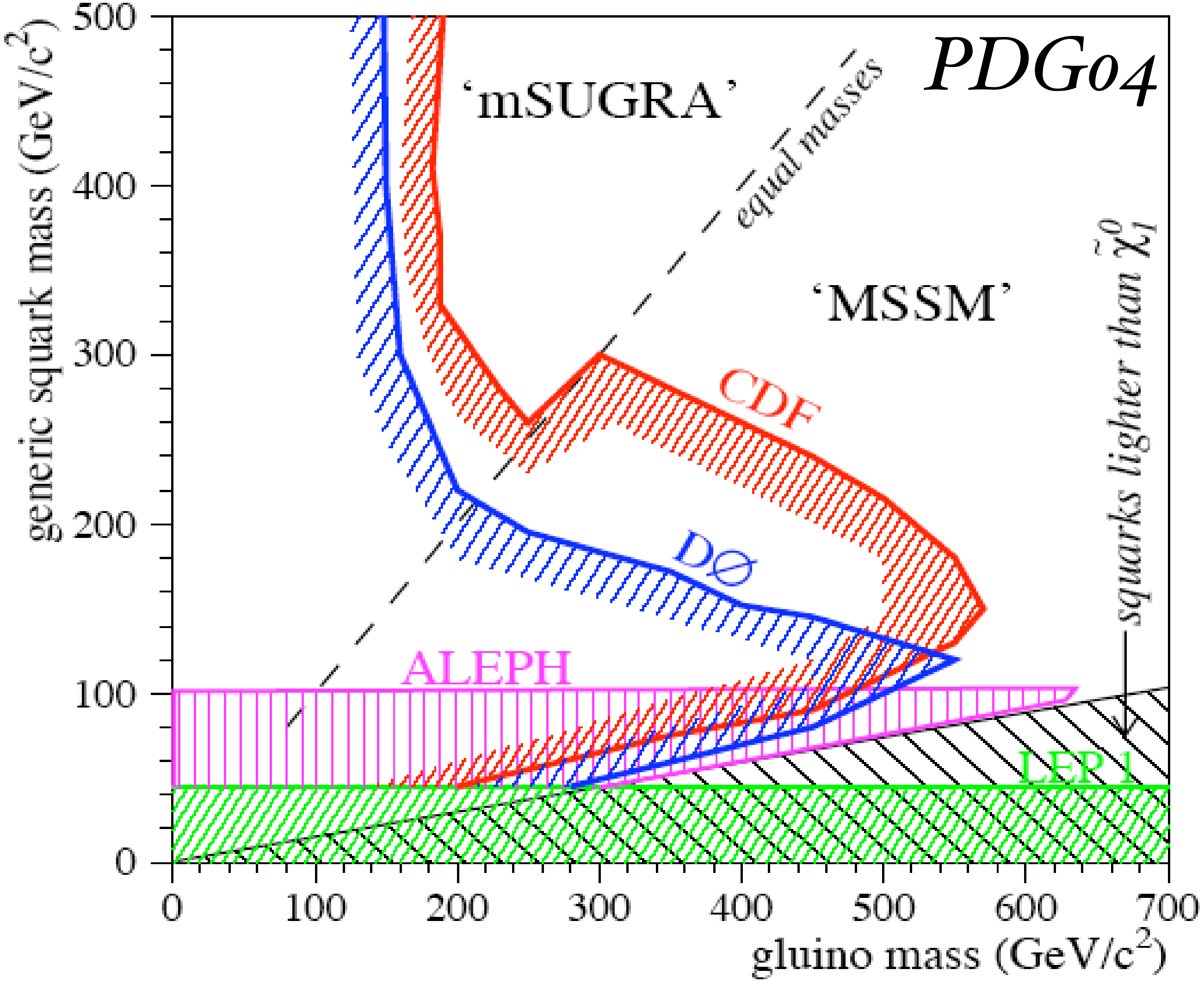
EW Global fit 2004: $\alpha_S(M_Z) = 0.1213 \pm 0.018$

$$M_{SUSY} \simeq 16 \pm 10 \text{ GeV}$$

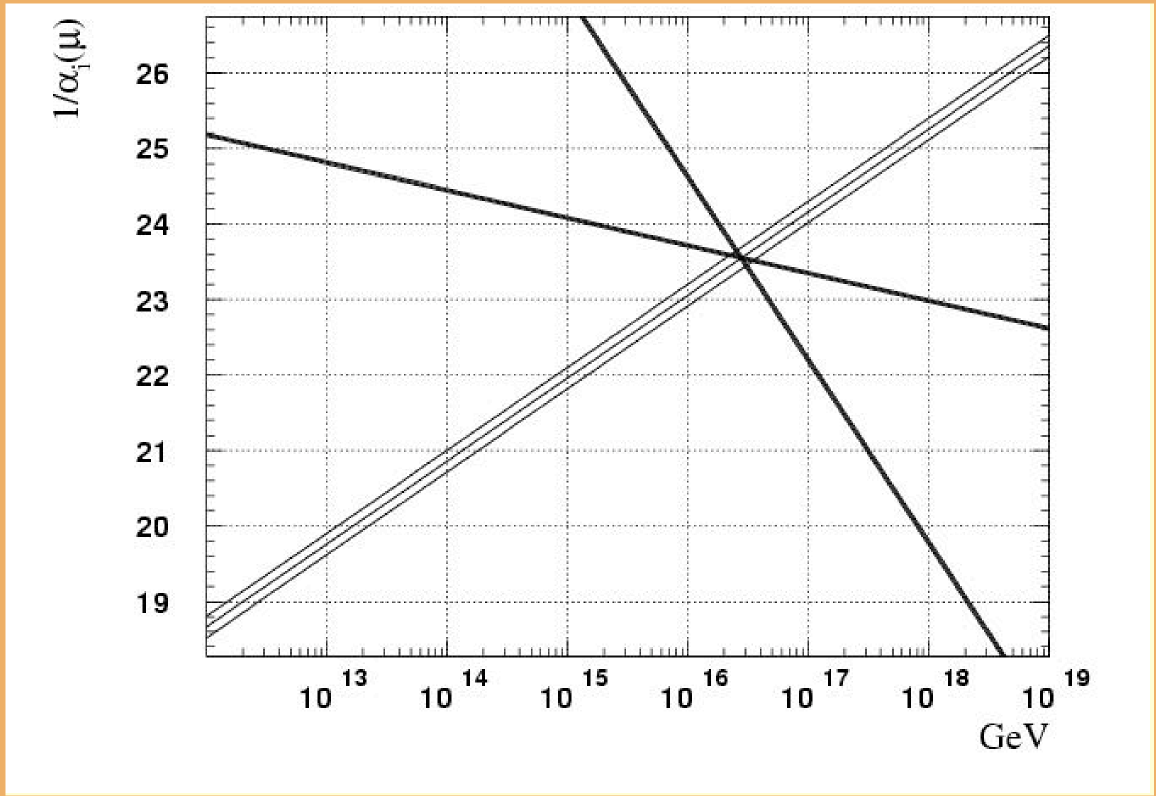
RPP QCD section: $\alpha_S(M_Z) = 0.1187 \pm 0.020$

$$M_{SUSY} \simeq 160 \pm 120 \text{ GeV}$$

Bourikov, '04



$M_{SUSY} \simeq 160 \pm 120$ GeV *may be problematic.*



may be accidental ???

What about

the Proton Decay ?

Super-K nucleon decay search summary

Summary of Nucleon Decay Searches

mode	exposure (kt· yr)	εB_m (%)	observed event	B.G.	τ/B limit (10^{32} yrs)
$p \rightarrow e^+ + \pi^0$	92	40	0	0.2	54
$p \rightarrow \mu^+ + \pi^0$	92	32	0	0.2	43
$p \rightarrow e^+ + \eta$	92	17	0	0.2	23
$p \rightarrow \mu^+ + \eta$	92	9	0	0.2	13
$n \rightarrow \bar{\nu} + \eta$	45	21	5	9	5.6
$p \rightarrow e^+ + \rho$	92	4.2	0	0.4	5.6
$p \rightarrow e^+ + \omega$	92	2.9	0	0.5	3.8
$p \rightarrow e^+ + \gamma$	92	73	0	0.1	98
$p \rightarrow \mu^+ + \gamma$	92	61	0	0.2	82
$p \rightarrow \bar{\nu} + K^+$	92				22
$K^+ \rightarrow \nu\mu^+$ (spectrum)		34	–	–	4.2
prompt $\gamma + \mu^+$		8.6	0	0.7	11
$K^+ \rightarrow \pi^+\pi^0$		6.0	0	0.6	7.9
$n \rightarrow \bar{\nu} + K^0$	92				2.0
$K^0 \rightarrow \pi^0\pi^0$		6.9	14	19.2	3.0
$K^0 \rightarrow \pi^+\pi^-$		5.5	20	11.2	0.8
$p \rightarrow e^+ + K^0$	92				10.7
$K^0 \rightarrow \pi^0\pi^0$		9.2	1	1.1	8.7
$K^0 \rightarrow \pi^+\pi^-$					
2-ring		7.9	5	3.6	4.0
3-ring		1.3	0	0.1	1.7
$p \rightarrow \mu^+ + K^0$	92				13.9
$K^0 \rightarrow \pi^0\pi^0$		5.4	0	0.4	7.1
$K^0 \rightarrow \pi^+\pi^-$					
2-ring		7.0	3	3.2	4.9
3-ring		2.8	0	0.3	3.7

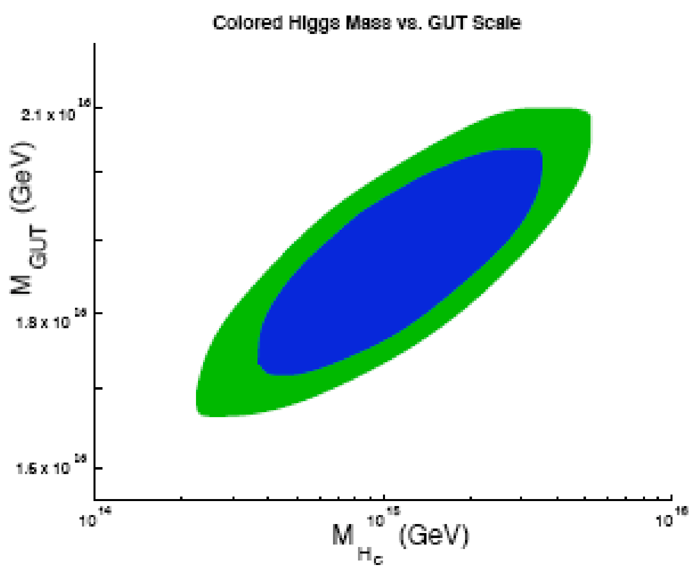


FIG. 2. Plot showing 68% and 90% contours allowed by the renormalization group analysis for the color Higgs triplet mass, M_{H_C}, and the GUT scale, $M_{GUT} \equiv (M_\Sigma M_V^2)^{1/3}$.

Murayama and Pierce, '02

$$\tau(p \to K^+\overline{\nu}) \leq 2.9 \times 10^{30} \text{ yrs.}$$

combined limit $\tau_p/B(p\to\nu K^+)$ > 2.2×10^{33} years (90% CL) *at superK*

The natural proton lives tooooo long for the minimal SUSY SU(5) GUT.

GUTs other than the minimal one

Extra Dimensions

String-like Unification

Intersecting Branes

etc

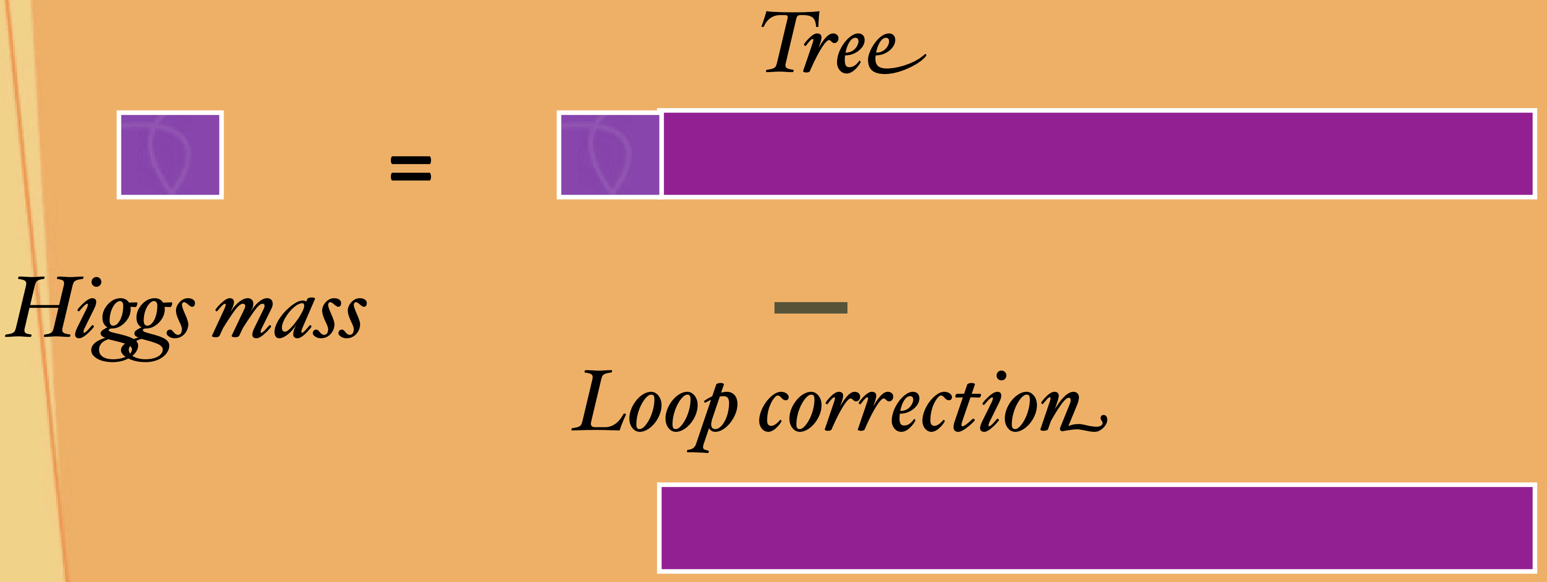
Nevertheless we like SUSY.

A. The maximal extension of the Poincaré algebra.
(Coleman and Mandula)

B. She can increase the natural scale ('t Hooft and Veltman)

Schön, schön, schön

Fine tuning of the Higgs mass



if the Higgs mass is quadratic divergent.

→ We like SUSY.

But ~~*SUSY*~~.

Soft ~~*SUSY*~~ *at low-energy*

The SUSY flavor problem.

More than 100 soft parameters into the SM.

They induce FCNCs and $\cancel{CP}$ *that are extremely suppressed in the SM.*

$$\mu \to e + \gamma\ , \quad b \to s + \gamma\ ,$$

$$\Delta M_K\ , \quad \Delta M_B\ , \quad \epsilon_K\ , \quad \epsilon'/\epsilon\ ,$$ *and EDMs.*

A parametrization of FCNCs and ~~CP~~ in the superCKM basis:

$$\Delta^a_{LL,RR} = U^\dagger_{aL,R}\, \tilde{\mathbf{m}}^{\mathbf{2}}_{aLL,RR}\, U_{aL,R} \text{ and } \Delta^a_{LR} = U^\dagger_{aL}\, \tilde{\mathbf{m}}^{\mathbf{2}}_{aLR}\, U_{aR}$$

$$\delta^a_{LL,RR,LR} = \Delta^a_{LL,RR,LR}/m^2_{\tilde{a}} \quad (a = \ell, q)$$

(*Gbbiani et al, Abel, Khalil and Lebedev, Endo, Kakizaki and Yamaguchi, Hisano and Shimizu*)

	Exp. bound
$\lvert(\delta^l_{12})_{LL}\rvert$	$4.0\times10^{-5}\ \tilde m^2_{\tilde l}$
$\lvert(\delta^l_{12})_{RR}\rvert$	$9\times10^{-4}\ \tilde m^2_{\tilde l}$
$\lvert(\delta^l_{13})_{LL}\rvert$	$2\times10^{-2}\ \tilde m^2_{\tilde l}$
$\lvert(\delta^l_{23})_{LL}\rvert$	$2\times10^{-2}\ \tilde m^2_{\tilde l}$
$\lvert(\delta^l_{12})_{LR}\rvert$	$8.4\times10^{-7}\ \tilde m^2_{\tilde l}$
$\lvert(\delta^l_{13})_{LR}\rvert$	$1.7\times10^{-2}\ \tilde m^2_{\tilde l}$
$\lvert(\delta^l_{23})_{LR}\rvert$	$1\times10^{-2}\ \tilde m^2_{\tilde l}$

	Exp. bound
$\sqrt{\lvert\mathrm{Re}(\delta^d_{12})^2_{LL,RR}\rvert}$	$4.0\times10^{-2}\ \tilde m_{\tilde q}$
$\sqrt{\lvert\mathrm{Re}(\delta^d_{12})_{LL}(\delta^d_{12})_{RR}\rvert}$	$2.8\times10^{-3}\ \tilde m_{\tilde q}$
$\sqrt{\lvert\mathrm{Re}(\delta^d_{12})^2_{LR}\rvert}$	$4.4\times10^{-3}\ \tilde m_{\tilde q}$
$\sqrt{\lvert\mathrm{Re}(\delta^d_{13})_{LL}(\delta^d_{13})_{RR}\rvert}$	$1.8\times10^{-2}\ \tilde m_{\tilde q}$
$\sqrt{\lvert\mathrm{Re}(\delta^d_{13})^2_{LR}\rvert}$	$3.3\times10^{-3}\ \tilde m_{\tilde q}$
$\sqrt{\lvert\mathrm{Re}(\delta^u_{12})_{LL}(\delta^u_{12})_{RR}\rvert}$	$1.7\times10^{-2}\ \tilde m_{\tilde q}$
$\sqrt{\lvert\mathrm{Re}(\delta^u_{12})^2_{LR}\rvert}$	$3.1\times10^{-3}\ \tilde m_{\tilde q}$
$\lvert(\delta^d_{23})_{LR}\rvert$	$1.6\times10^{-2}\ \tilde m^2_{\tilde q}$

	Exp. bound		Exp. bound
$\lvert\mathrm{Im}(\delta^d_{12})_{LL,RR}\rvert$	$4.8\times10^{-1}\ \tilde m^2_{\tilde q}$	$\lvert\mathrm{Im}(\delta^d_{12})_{LR}\rvert$	$2.0\times10^{-5}\ \tilde m^2_{\tilde q}$
$\lvert\mathrm{Im}(\delta^d_{11})_{LR}\rvert$	$6.7\times10^{-8}\ \tilde m^2_{\tilde q}$	$\lvert\mathrm{Im}(\delta^u_{11})_{LR}\rvert$	$6.7\times10^{-8}\ \tilde m^2_{\tilde q}$
$\lvert\mathrm{Im}(\delta^\ell_{11})_{LR}\rvert$	$3.7\times10^{-8}\ \tilde m^2_{\tilde\ell}$		

$$\tilde m_{\tilde l}=m_{\tilde l}/100\ \mathbf{GeV},\ \tilde m_{\tilde q}=m_{\tilde q}/500\ \mathbf{GeV}$$

Why are δ's so small?

The SUSY flavor problem

A popular assumption is:

Hidden Sector Scenario

$$\cancel{\text{SUSY}} \xrightarrow[\text{Mediator}]{} \text{MSSM}$$

Gravity, Gauge, Anomaly, Gaugino, etc

We use

"Low-energy discrete Flavor Symmetry"

to constrain the Yukawa sector, and simultaneously
to soften the SUSY flavor problem.

No hidden sector scenario

Testable predictions

II. Dihedral Symmetry

(平面対称性)

The classification of the finite groups has been completed 1981 (Gorenstein); about 100 years later than the case of the continues group.

g= order of a finite group
= # of the group elements

1. No non-abelian finite group exists for odd g.

2. For smaller g there exist only three types.

a) Permutation groups

$S_N\ ,\ N=3,4,5,\ldots\ ,\ A_N\ ,\ N=4,5,\ldots$

b) Dihedral groups

$D_N\ ,\ N=3,4,5,\ldots$

and

Binary dihedral (Dicyclic) group

$Q_{2N}\ ,\ N=2,3,4,\ldots$

c) Twisted products of $Z_M \times Z_N$

3. The smallest non-abelian finite group is $S_3=D_3$.

g	
6	$D_3 \equiv S_3$
8	$D_4, Q = Q_4$
10	D_5
12	D_6, Q_6, T
14	D_7
16	$D_8, Q_8, Z_2 \times D_4, Z_2 \times Q$
18	$D_9, Z_3 \times D_3$
20	D_{10}, Q_{10}
22	D_{11}
24	$D_{12}, Q_{12}, Z_2 \times D_6, Z_2 \times Q_6, Z_2 \times T,$
	$Z_3 \times D_4, Z_3 \times Q, Z_4 \times D_3, S_4$
26	D_{13}
28	D_{14}, Q_{14}
30	$D_{15}, D_5 \times Z_3, D_3 \times Z_5$

Classification of Finite Groups

(Frampton and Kephart, '01)

Twisted products $([Z_M\ ,\ Z_N] \neq 0)$

g	
16	$Z_2 \tilde{\times} Z_8$ (two, excluding D_8), $Z_4 \tilde{\times} Z_4$, $Z_2 \tilde{\times} (Z_2 \times Z_4)$ (two)
18	$Z_2 \tilde{\times} (Z_3 \times Z_3)$
20	$Z_4 \tilde{\times} Z_5$
21	$Z_3 \tilde{\times} Z_7$
24	$Z_3 \tilde{\times} Q, Z_3 \tilde{\times} Z_8, Z_3 \tilde{\times} D_4$
27	$Z_9 \tilde{\times} Z_3, Z_3 \tilde{\times} (Z_3 \times Z_3)$

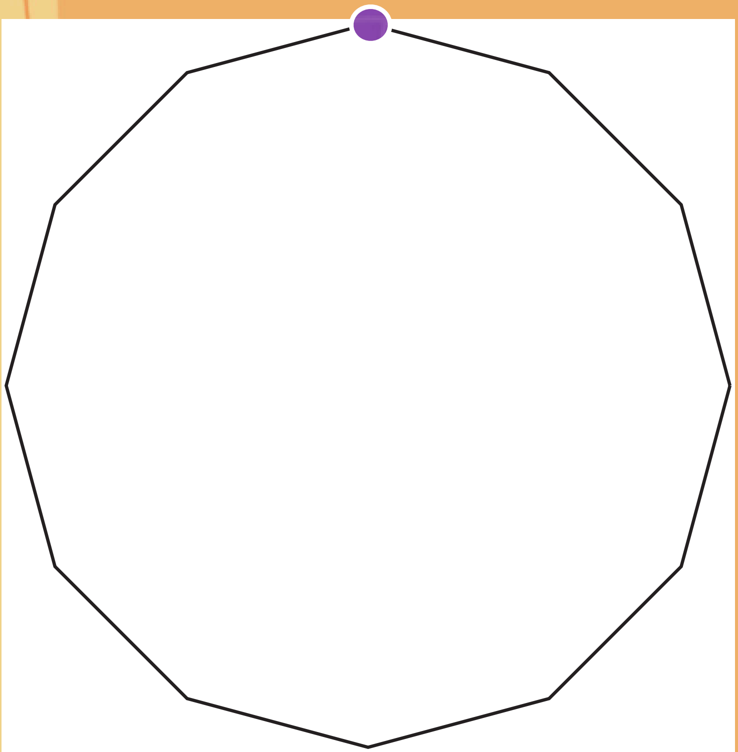
Literature on Finite Groups: Landau and Lifschitz," Quantum mechanics"

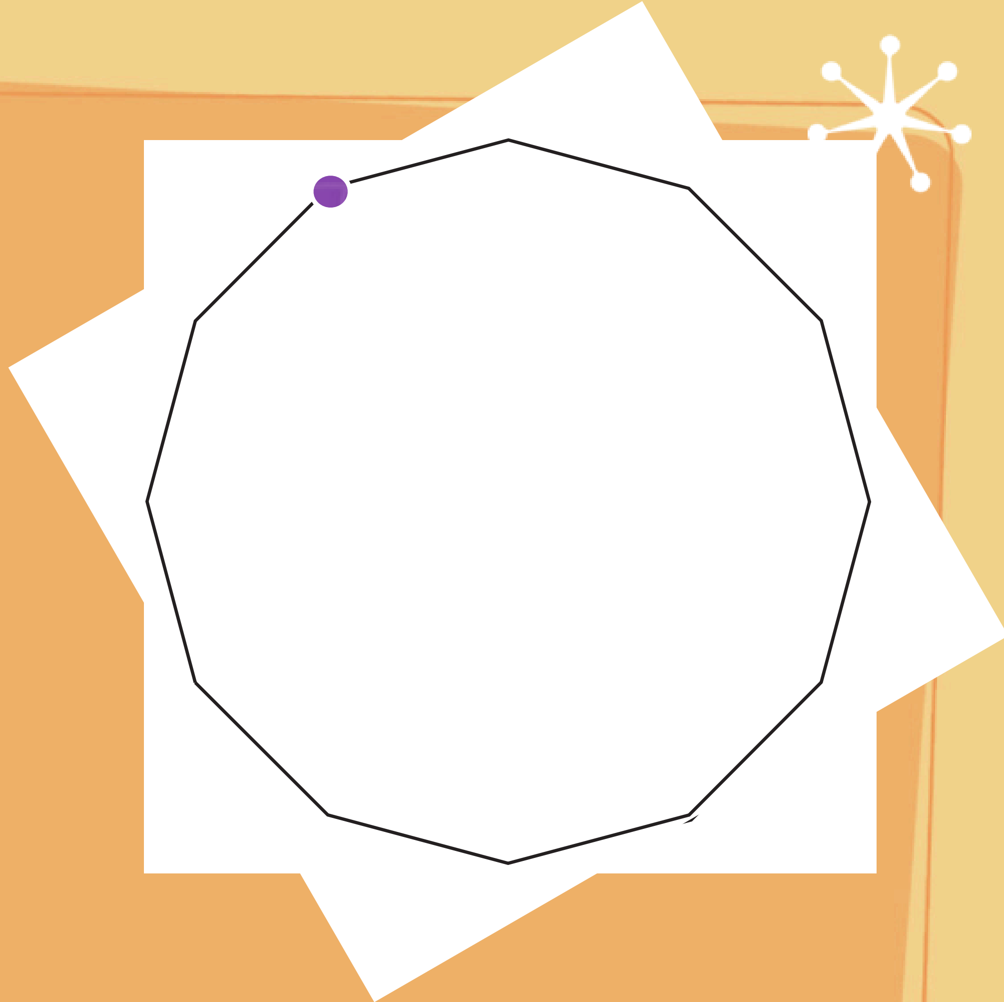
Dihedral Symmetry (平面対称性)

||

Symmetry of a regular polygon (正多角形)

(Babu and Kubo. Phys. Rev. D71,056006 (2005); Kubo, hep-ph/0506043)





$\tilde{P}_D \equiv$ Parity transformation

$R_N \equiv$ Rotation about $2\pi/N$ $(N = 12)$

24 Elements of D_N

$$
\begin{aligned}
\mathcal{G}_{D_N} &= \{\tilde{R}_N, (\tilde{R}_N)^2, \ldots, (\tilde{R}_N)^N = \mathbf{1}, \\
&\quad \tilde{R}_N\tilde{P}_D, (\tilde{R}_N)^2\tilde{P}_D, \ldots, (\tilde{R}_N)^N\tilde{P}_D = \tilde{P}_D\} \\
&= \{2\pi/N, 4\pi/N, \ldots, 2\pi, 2 \\
&\quad \pi/N \text{ with } \tilde{P}_D, 4\pi/N \text{ with } \tilde{P}_D, \ldots, \tilde{P}_D\}\text{rotations}
\end{aligned}
$$

Two-dimensional representations of R and P

$$D_N \qquad (\theta_N = 2\pi/N)$$

$$\tilde{R}_N = \begin{pmatrix} \cos\theta_N & \sin\theta_N \\ -\sin\theta_N & \cos\theta_N \end{pmatrix}, \quad \tilde{P}_D = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\tilde{R}_N \to \begin{pmatrix} \cos\theta_N & \sin\theta_N & 0 \\ -\sin\theta_N & \cos\theta_N & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tilde{P}_D \to \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\det \tilde{R}_N = \det \tilde{P}_D \to 1 \quad \Rightarrow \quad D_N \subset S0(3)$$

$$Q_{2N} \quad (\theta_N = 2\pi/N)$$

$$\tilde{R}_{2N} = \begin{pmatrix} \cos\frac{\theta_N}{2} & \sin\frac{\theta_N}{2} \\ -\sin\frac{\theta_N}{2} & \cos\frac{\theta_N}{2} \end{pmatrix}, \quad \tilde{P}_Q = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$$

$$\det \tilde{R}_{2N} = \det \tilde{P}_Q = 1 \quad \Rightarrow \quad Q_{2N} \subset SU(2)$$

Q_{2N} *contains real as well as complex irreps.*

Some group theoretical features for Q_{2N}

1. All the real irreps are identical with those of D_N

$\Rightarrow$ Q_{2N} is the 'covering group' of D_N

2. Only one- and two-dimensional irreps.

3. There exist N-1 two-dimensional irreps:

$$\mathbf{2}_k = \begin{cases} \text{real for } k = \text{even} \\ \text{pseudoreal for } k = \text{odd} \end{cases}$$

4. One-dimensional irreps can be characterized according to $Z_2 \times Z_4$ charge:

real

$$
\begin{array}{llllll}
\mathbf{1}_{+,0}, & \mathbf{1}_{-,0}, & \mathbf{1}_{+,2}, & \mathbf{1}_{-,2} & \text{for} & N = 2, 4, 6, \ldots, \\
\mathbf{1}_{+,0}, & \mathbf{1}_{-,1}, & \mathbf{1}_{+,2}, & \mathbf{1}_{-,3} & \text{for} & N = 3, 5, 7, \ldots,
\end{array}
$$

real *complex*

5. Tensor Products (Q6)

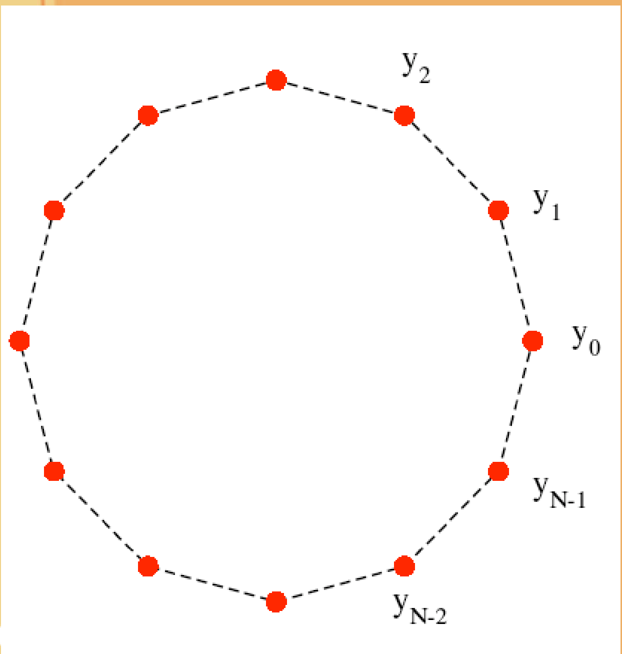
$$
\begin{array}{ccccccccc}
\mathbf{2}_1 & \times & \mathbf{2}_2 & = & \mathbf{1}_{-,3} & + & \mathbf{1}_{-,1} & + & \mathbf{2}_1 \\
\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} & \times & \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} & = & (x_1 a_2 + x_2 a_1) & & (x_1 a_1 - x_2 a_2) & & \begin{pmatrix} x_1 a_1 + x_2 a_2 \\ x_1 a_2 - x_2 a_1 \end{pmatrix}
\end{array}
$$

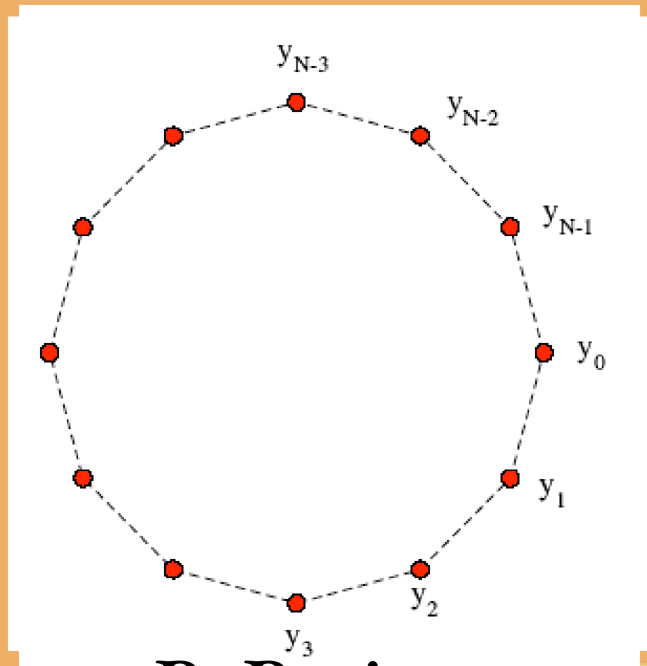
and so on.

A possible origin of the dihedral symmetry

Dihedral Invariance of a deconstructed extra dimension

(Kubo, hep-ph/0506043)





R: Discrete Rotations

P: Parity

Dihedral Transformation

$$\phi'(x,y) \;\; = \;\; \tilde{Q}_{2N}\ \phi(x, \tilde{D}_N^{-1} y),\ \ \tilde{Q}_{2N} \in Q_{2N},\ \ \tilde{D}_N \in D_N$$

Orbifold *B.C.s break Dihedral Invariance.*
But the Q_2N as a global , internal symmetry is intact.

$$\phi(x,y) \to \phi'(x,y) = \tilde{Q}_{2N}\ \phi(x,y),\ \ \tilde{Q}_{2N} \in Q_{2N}$$

from SuperStrings → *Tatsuo*

III. An Example

A Q_6 *Model*

(Babu and Kubo, '04)

	A_4 Babu et al	S_3 Kubo et al	D_4 Grimus +Lavoura	Q_4 Frigerio et al	Q_6 Babu+Kubo	D_7 Chen +Ma
Lepton	$\theta_{23} = \pi/4$ $m_0 > 0.3$ eV	$\theta_{23} \simeq \pi/4$ $\sin\theta_{13} = 0.034$	$\theta_{23} = \pi/4$ $\theta_{13} = 0$	$\theta_{23} = \pi/4(?)$ $\theta_{13} = 0(?)$	later	$\triangle$
Quark	$\triangle$	o	$\triangle$	$\triangle$	later	$\sin 2\beta \simeq 0.733$
nonSUSY	x	o	o	o	?	o
SUSY	o	o	?	?	o	?
SUSY Flavor	x	o Kobayashi, Terao+jk	x	x	o	x?

	A_4 Babu et al	S_3 Kubo et al	D_4 Grimus +Lavoura	Q_4 Frigerio et al	Q_6 Babu+Kubo	D_7 Chen +Ma
Anomalies	?	O	?	?	O	?
FCNC by Higgses	X	X	O	X	X	X
Leptogenesis	?	O Araki,jk+Paschos	O	?	△	?
Scale	$\mathcal{H}$	$\mathcal{L}$	$\mathcal{L}$	$\mathcal{L}$	$\mathcal{L}$	$\mathcal{L}$
GUT	O	△	X	X	OX	X

Q_6 *Model* $(Q_{2N}$ with $N = 3, 5, 7, \ldots)$

The original motivation:

To obtain the mass matrix of the nearest neighbor type

$$M = \begin{pmatrix} 0 & C & 0 \\ \pm C & 0 & B \\ 0 & B' & A \end{pmatrix}$$

(Weinberg, Wilczek, Fritzsch)

purely from a discrete symmetry.

The conditions have to be met:

a. Real and complex irreps

b. Type-II Higgs sector
(up- and down-type Higgses)

The smallest group is Q6, which is the covering group of the smallest non-abelian finite group S3.

Q6 assignment

Quarks

	Q	U^c, D^c, H^u, H^d	Q_3	$U^c_3, D^c_3, H^u_3, H^d_3$
Q_6	$\mathbf{2}_1$	$\mathbf{2}_2$	$\mathbf{1}_{+,2}$	$\mathbf{1}_{-,1}$

Leptons

I

	L	E^c, N^c	L_3	E^c_3, N^c_3
Q_6	$\mathbf{2}_1$	$\mathbf{2}_2$	$\mathbf{1}_{+,2}$	$\mathbf{1}_{-,1}$

II

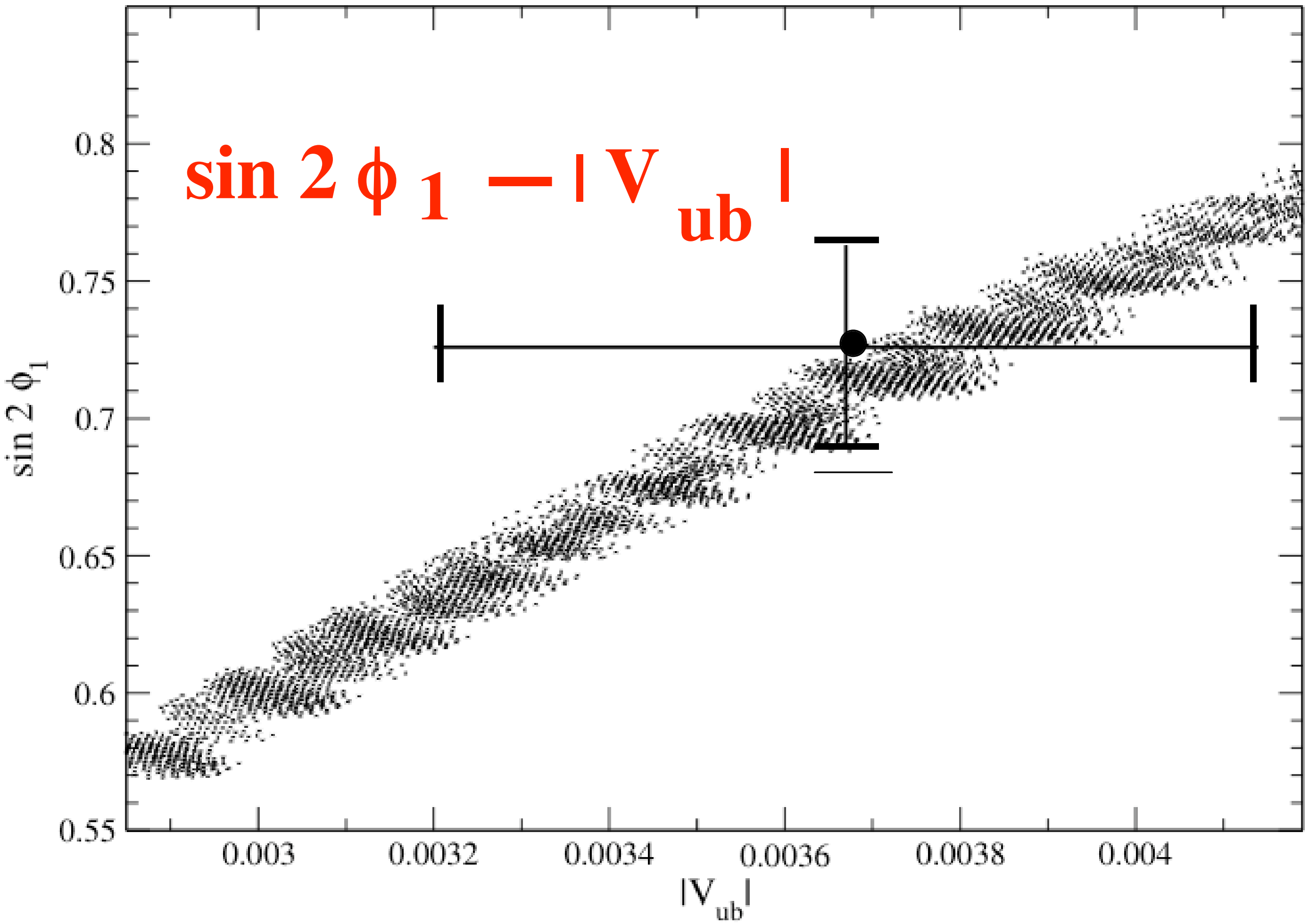
	L, E^c, N^c	L_3, E^c_3	N^c_3
Q_6	$\mathbf{2}_2$	$\mathbf{1}_{+,0}$	$\mathbf{1}_{-,3}$

Spontaneous $\not{CP}$ is possible.

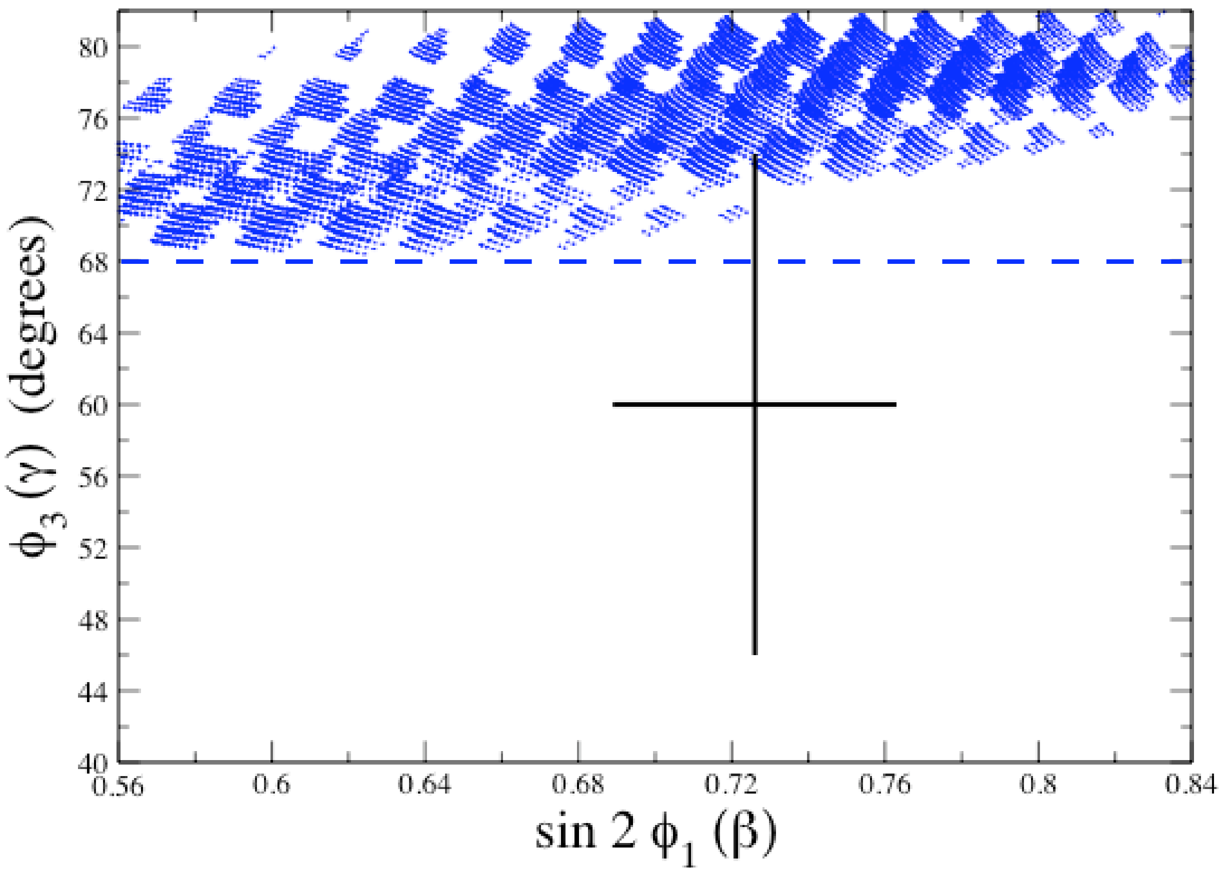
Prediction in the quark sector

8 real and one phase parameters to describe 6 quark masses and 4 parameters of V_{CKM}.

One prediction

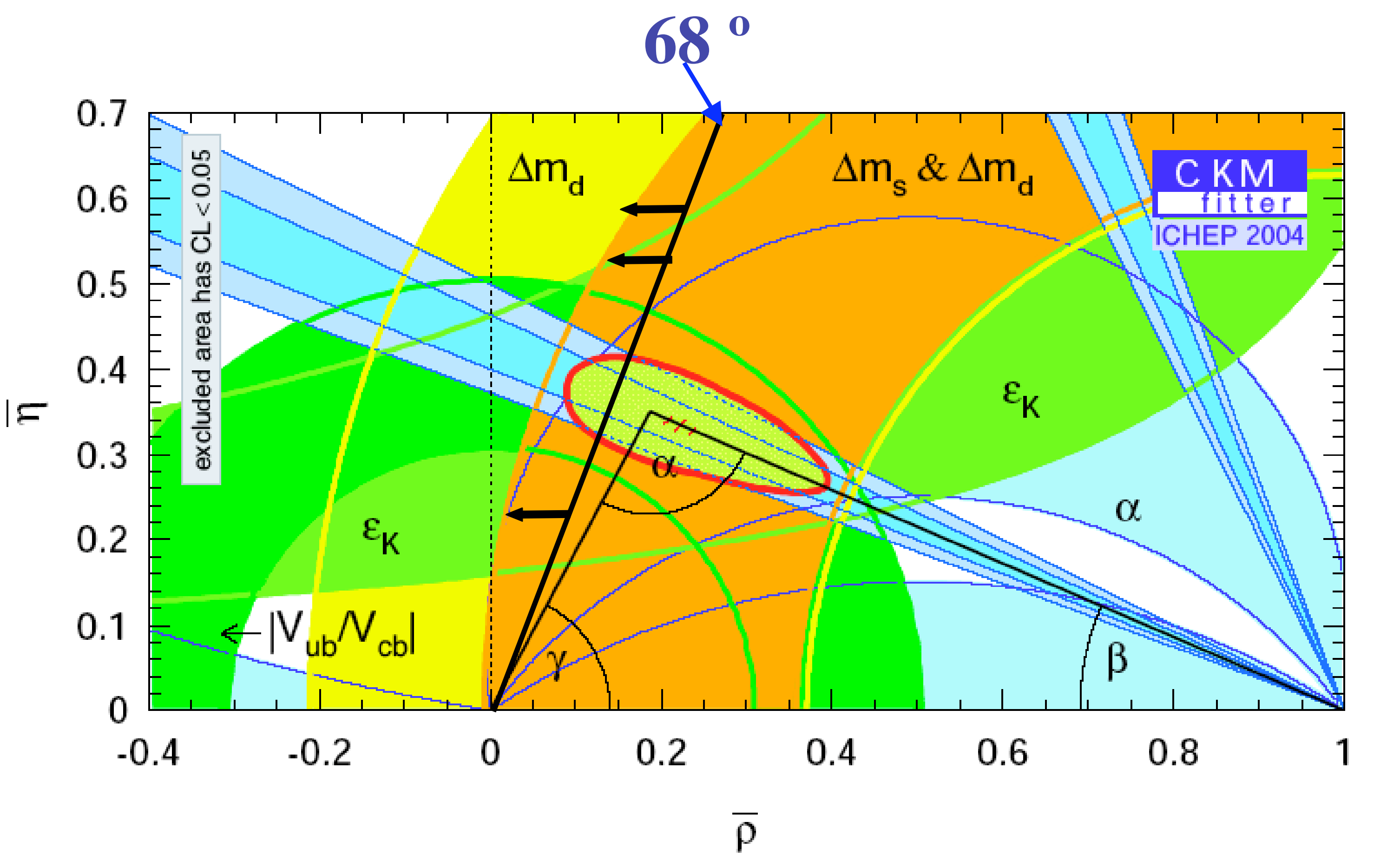


sin 2 ϕ 1 — | V ub |
0.8
0.75
0.7
0.65
0.6
0.55
sin 2 ϕ1
0.003
0.0032
0.0034
0.0036
0.0038
0.004
|Vub|



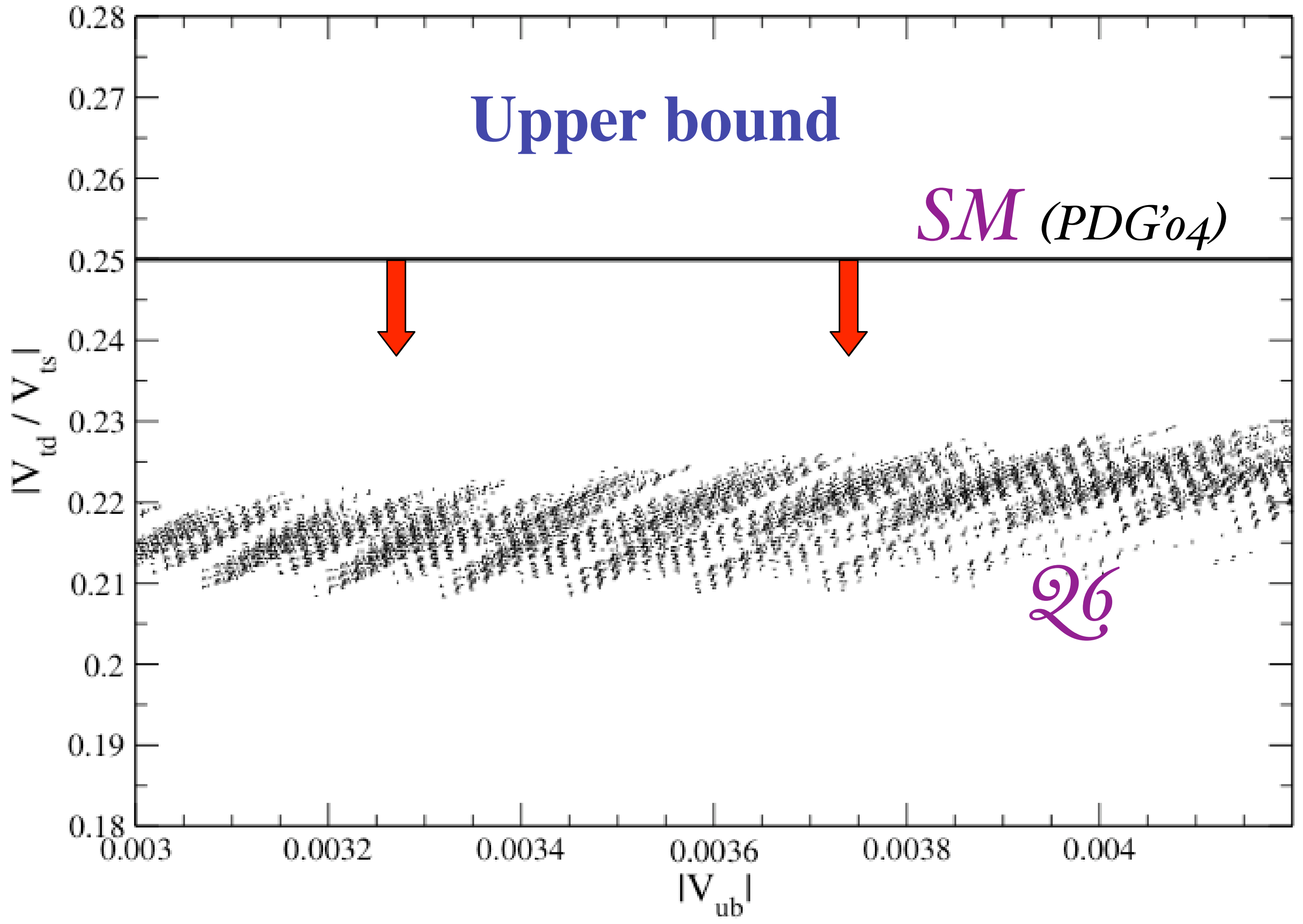
80
76
72
68
64
60
56
52
48
44
40
0.56
0.6
0.64
0.68
0.72
0.76
0.8
0.84
ϕ_3 (γ) (degrees)
sin 2 ϕ_1 (β)

Unitarity Triangle



(S. Bosch, Moriond '05)

from $\Delta M_{B_s}/\Delta M_{B_d}$



Prediction in the lepton sector (type II)

6 real and one phase parameters to describe 6 lepton masses and 6 parameters of V_{MNS}.

Many predictions

1. Inverted neutrino mass spectrum, i.e., $m_{\nu_3} < m_{\nu_1}, m_{\nu_2}$.

2. $m_{\nu_2}^2/\Delta m_{23}^2 = \frac{(1+2t_{12}^2+t_{12}^4-rt_{12}^4)^2}{4t_{12}^2(1+t_{12}^2)(1+t_{12}^2-rt_{12}^2)\cos^2\phi_\nu} - \tan^2\phi_\nu$

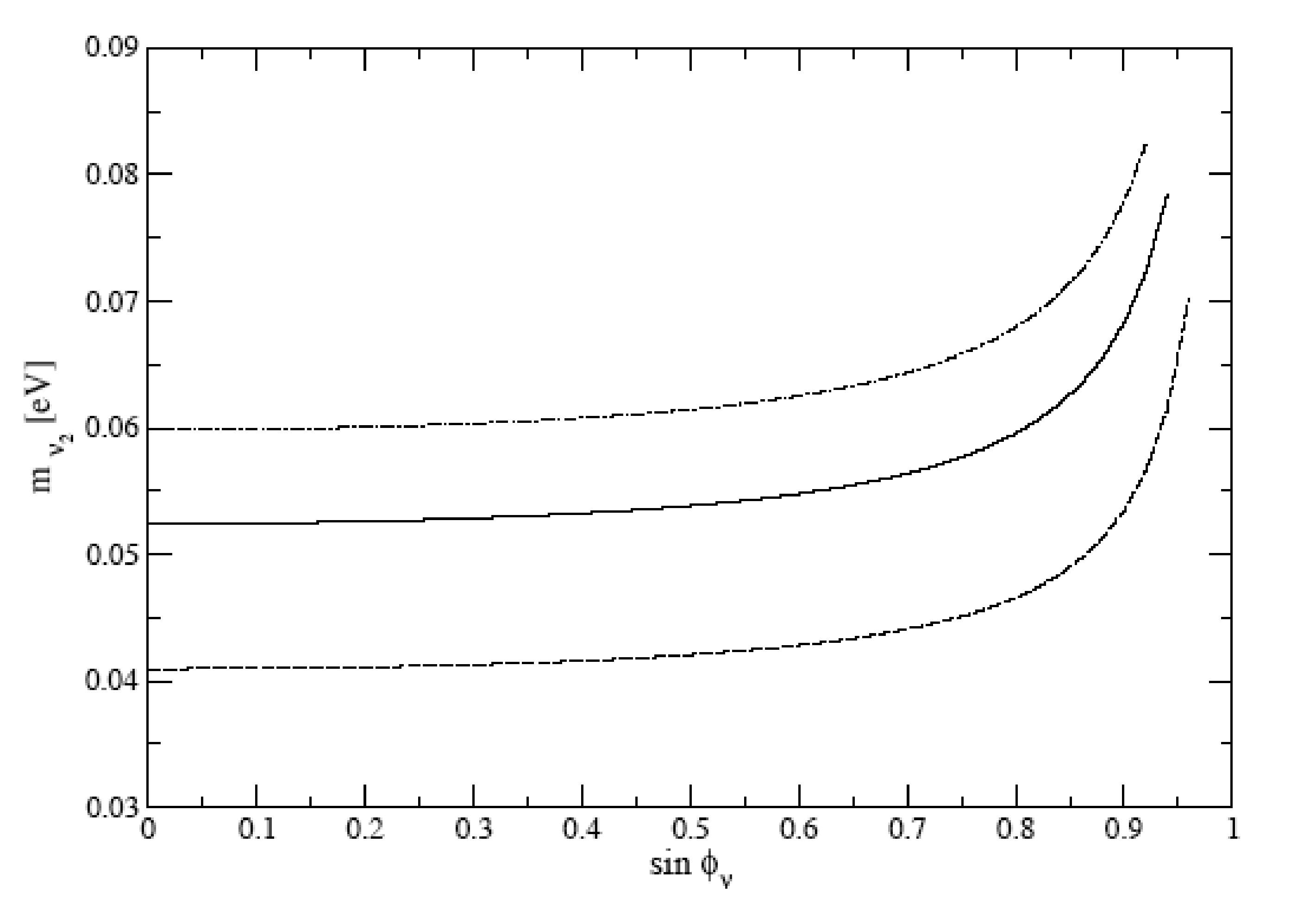
$(r = \Delta m_{21}^2/\Delta m_{23}^2, t_{12} = \tan\theta_{12})$,

where ϕ_ν is an independent phase.

3. $\sin\theta_{13} \simeq m_e/\sqrt{2}m_\mu \simeq 3.4\times10^{-3}$

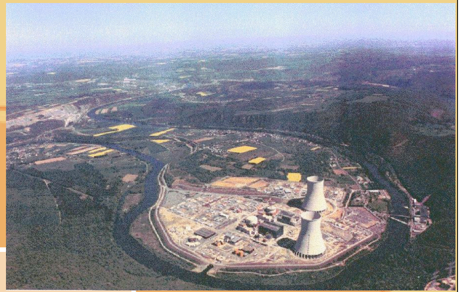
$\tan\theta_{23} \simeq 1-(m_e/\sqrt{2}m_\mu)^2 = 1-O(10^{-5})$.

4. The prediction of $< m_{ee} >$.



$$\sin^2\theta_{12} = 0.3 \text{ and } \Delta m^2_{21} = 6.9 \times 10^{-5}\ \mathrm{eV}^2$$

$$\Delta m^2_{23} = 1.4, 2.3 \text{ and } 3.0 \times 10^{-3}\ \mathrm{eV}^2$$



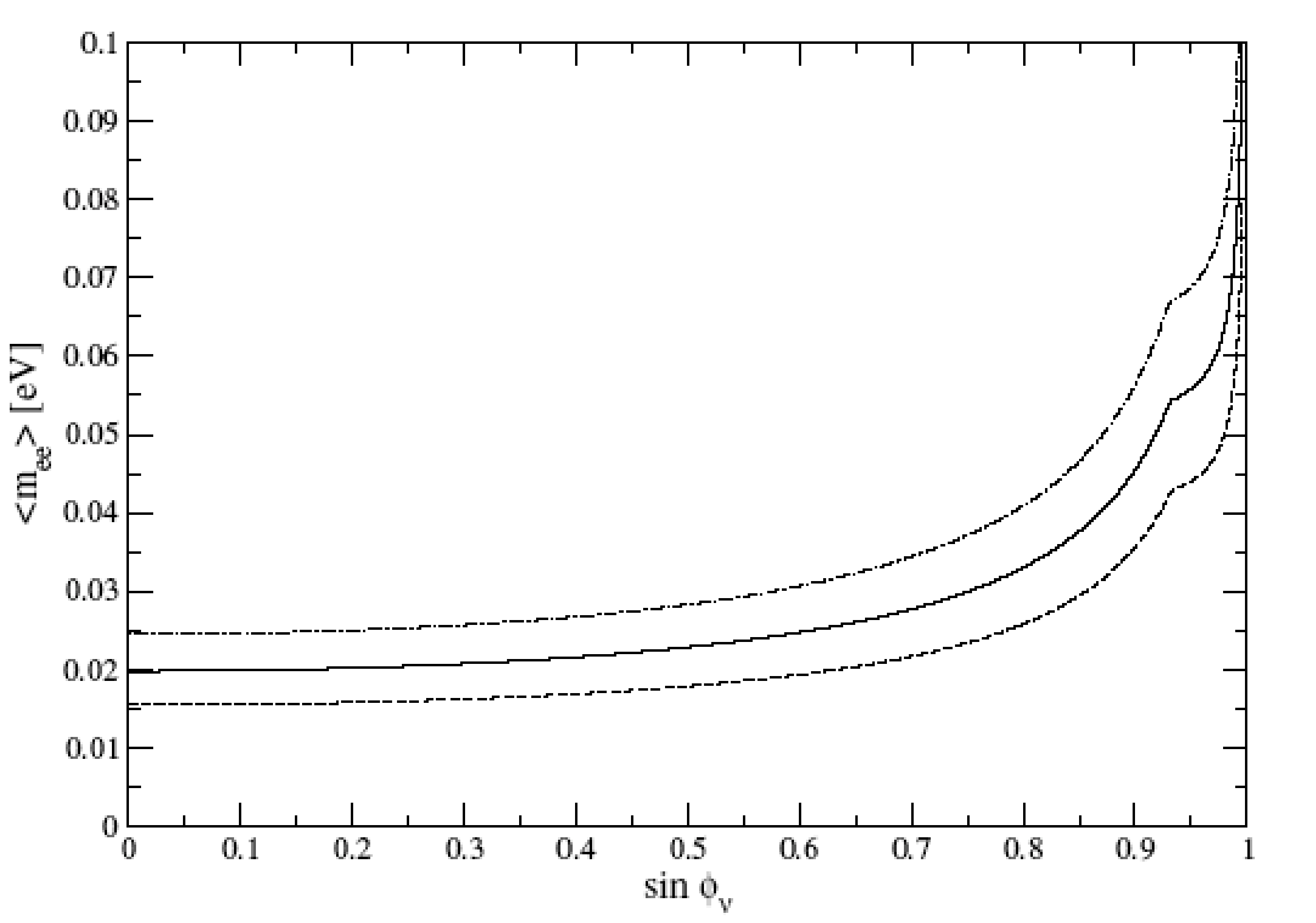
Presumably too small to be measured in a laboratory.

(Minakata, Sugiyama, Yasuda, Inoue and Suekane)

HOWEVER,

May be observable in the flavor conversion inside Supernova (Supernova Neutrino Oscillation).

(Ando, Sato)



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$$\Delta m^2_{23} = 1.4, 2.3 \text{ and } 3.0 \times 10^{-3}\ \mathrm{eV}^2$$

How is the SUSY Flavor problem softened?

A. CP violations induced by the soft terms

Q_6 *and spontaneous* $\cancel{CP}$

↓

Phase alignment

$$\Delta_{LL,RR} = U^{\dagger}_{L,R}\, \tilde{\mathbf{m}}^{\mathbf{2}}_{LL,RR}\, U_{L,R} = \text{real}$$

$$\Delta_{LR} = U^{\dagger}_{aL}\, \tilde{\mathbf{m}}^{\mathbf{2}}_{LR}\, U_{R} = \text{real}$$

The <u>most stringent</u> constraints coming from EDMs are satisfied.

B. <u>FCNCs induced by the soft terms</u>

Lepton sector

(Kobayashi, Terao and Kubo, '04)

	Exp. bound	Q_6 Model
$\lvert(\delta^l_{12})_{LL}\rvert$	$4.0 \times 10^{-5}\ \tilde{m}^2_{\tilde{l}}$	$4.8 \times 10^{-3}\Delta_L$
$\lvert(\delta^l_{12})_{RR}\rvert$	$9 \times 10^{-4}\ \tilde{m}^2_{\tilde{l}}$	$8.4 \times 10^{-8}\Delta_e$
$\lvert(\delta^l_{13})_{LL}\rvert$	$2 \times 10^{-2}\ \tilde{m}^2_{\tilde{l}}$	$1.710^{-5}\Delta_L$
$\lvert(\delta^l_{23})_{LL}\rvert$	$2 \times 10^{-2}\ \tilde{m}^2_{\tilde{l}}$	$8.4 \times 10^{-8}\Delta_L$
$\lvert(\delta^l_{23})_{LL}(\delta^d_{13})_{LL}\rvert$	$1 \times 10^{-4}\ \tilde{m}^2_{\tilde{l}}$	$1.4 \times 10^{-12}\sqrt{\Delta_L\Delta_e}$
$\lvert(\delta^l_{23})_{LL}(\delta^l_{13})_{RR}\rvert$	$2 \times 10^{-5}\ \tilde{m}_{\tilde{q}}$	$5 \times 10^{-9}\sqrt{\Delta_L\Delta_e}$
$\lvert(\delta^l_{23})_{RR}(\delta^l_{13})_{RR}\rvert$	$9 \times 10^{-4}\ \tilde{m}^2_{\tilde{l}}$	$8.3 \times 10^{-8}\sqrt{\Delta_L\Delta_e}$
$\lvert(\delta^l_{23})_{RR}(\delta^l_{13})_{LL}\rvert$	$2 \times 10^{-5}\ \tilde{m}^2_{\tilde{l}}$	$2.4 \times 10^{-11}\sqrt{\Delta_L\Delta_e}$
$\lvert(\delta^l_{12})_{LR}\rvert$	$8.4 \times 10^{-7}\ \tilde{m}^2_{\tilde{l}}$	$\sim 10^{-6}\tilde{m}^{-1}_{\tilde{l}}$
$\lvert(\delta^l_{13})_{LR}\rvert$	$1.7 \times 10^{-2}\ \tilde{m}^2_{\tilde{l}}$	$\sim 10^{-7}\tilde{m}^{-1}_{\tilde{l}}$
$\lvert(\delta^l_{23})_{LR}\rvert$	$1 \times 10^{-2}\ \tilde{m}^2_{\tilde{l}}$	$\sim 10^{-9}\tilde{m}^{-1}_{\tilde{l}}$

$$\frac{m_e}{m_\mu}$$

$$\frac{m_e m_\mu}{m_\tau^2}$$

Table 2: Experimental bounds on δ's, where the parameter $\tilde{m}_{\tilde{l}}$ denote $m_{\tilde{l}}/100$ GeV.

Quark sector

	Exp. bound	Q_6 Model
$\sqrt{\lvert \mathrm{Re}(\delta^d_{12})^2_{LL,RR}\rvert}$	$4.0\times 10^{-2}\ \tilde{m}_{\tilde{q}}$	$(LL)1.2\times 10^{-4}\Delta_Q, (RR)1.7\times 10^{-1}\Delta_d$
$\sqrt{\lvert \mathrm{Re}(\delta^d_{12})_{LL}(\delta^d_{12})_{RR}\rvert}$	$2.8\times 10^{-3}\ \tilde{m}_{\tilde{q}}$	$4.5\times 10^{-3}\sqrt{\Delta_Q\Delta_d}$
$\sqrt{\lvert \mathrm{Re}(\delta^d_{12})^2_{LR}\rvert}$	$4.4\times 10^{-3}\ \tilde{m}_{\tilde{q}}$	$\sim 10^{-4}\tilde{m}_{\tilde{q}}^{-1}$
$\sqrt{\lvert \mathrm{Re}(\delta^d_{13})^2_{LL,RR}\rvert}$	$9.8\times 10^{-2}\ \tilde{m}_{\tilde{q}}$	$(LL)7.9\times 10^{-3}\Delta_Q, (RR)1.4\times 10^{-1}\Delta_d$
$\sqrt{\lvert \mathrm{Re}(\delta^d_{13})_{LL}(\delta^d_{13})_{RR}\rvert}$	$1.8\times 10^{-2}\ \tilde{m}_{\tilde{q}}$	$3.4\times 10^{-2}\sqrt{\Delta_Q\Delta_d}$
$\sqrt{\lvert \mathrm{Re}(\delta^d_{13})^2_{LR}\rvert}$	$3.3\times 10^{-3}\ \tilde{m}_{\tilde{q}}$	$\sim 10^{-4}\tilde{m}_{\tilde{q}}^{-1}$
$\sqrt{\lvert \mathrm{Re}(\delta^u_{12})^2_{LL,RR}\rvert}$	$1.0\times 10^{-1}\ \tilde{m}_{\tilde{q}}$	$(LL)1.2\times 10^{-4}\Delta_Q, (RR)4.4\times 10^{-4}\Delta_u$
$\sqrt{\lvert \mathrm{Re}(\delta^u_{12})_{LL}(\delta^u_{12})_{RR}\rvert}$	$1.7\times 10^{-2}\ \tilde{m}_{\tilde{q}}$	$2.3\times 10^{-4}\sqrt{\Delta_Q\Delta_u}$
$\sqrt{\lvert \mathrm{Re}(\delta^u_{12})^2_{LR}\rvert}$	$3.1\times 10^{-3}\ \tilde{m}_{\tilde{q}}$	$\sim 10^{-4}\tilde{m}_{\tilde{q}}^{-1}$
$\lvert(\delta^d_{23})_{LL,RR}\rvert$	$8.2\ \tilde{m}^2_{\tilde{q}}$	$(LL)1.6\times 10^{-2}\Delta_Q, (RR)4.7\times 10^{-1}\Delta_d$
$\lvert(\delta^d_{23})_{LR}\rvert$	$1.6\times 10^{-2}\ \tilde{m}^2_{\tilde{q}}$	$\sim 10^{-2}\tilde{m}_{\tilde{q}}^{-1}$

Table 1: Experimental bounds on δ's, where the parameter $\tilde{m}_{\tilde{q}}$ denote $m_{\tilde{q}}/500$ GeV.

(Kajiyama)

FCNCs mediated by the neutral Higgses at the tree level

(one + two neutral scaler Higgses and two pseudo scaler Higgses)

$\Delta M_{B_d}, \Delta M_K$ ⇒ $M_H > 5\ \mathrm{TeV}$

(Okada, Sakamaki and Kifune)

IV. Conclusion

"Low-energy discrete Flavor Symmetry"

constrains the flavor structure of the SM,

reducing the number of the redundant parameters of the SM.

softens the SUSY flavor problem.

No assumption on the universality of the soft terms is needed.