# Flavor structure in string models

Tatsuo Kobayashi

1. Introduction
2. Yukawa in heterotic orbifold models
3. Discrete flavor symmetry
4. Yukawa in intersecting D-brane models
5. Summary

based on

Ko, T.K. and Park, hep-ph/0406041, hep-ph/0503029

T.K., Raby and Zhang, hep-ph/0409098,

Higaki, Kitazawa, T.K. and Takahashi, hep-th/0504019

# 1. Introduction

The origin of fermion masses and mixing angles is one of important issues in particle physics.

Fermion masses ← Yukawa couplings between fermions and Higgs fields

e.g. in the Standard Model

O(1) of Yukawa couplings are in a sense natural.

From this viewpoint, how to derive suppressed Yukawa couplings is a key-point in understanding fermion masses.

# Quark masses and mixing angles

$M_t = 174 \quad \text{GeV}, \qquad M_b = 4.3 \quad \text{GeV}$

$M_c = 1.2 \quad \text{GeV}, \qquad M_s = 117 \quad \text{MeV}$

$M_u = 3 \quad \text{MeV}, \qquad M_d = 6.8 \quad \text{MeV}$

$V_{us} = 0.22, \qquad V_{cb} = 0.04, \qquad V_{ub} = 0.004$

RG effects are not drastic in usual cases except PR-fixed points or superconformal fixed points.

# Lepton masses and mixing angles

$$M_e = 0.5 \quad \text{MeV}, \qquad M_\mu = 106 \quad \text{MeV}$$
$$M_\tau = 1.8 \quad \text{GeV},$$

$$\Delta M_{21}^2 = 8\times 10^{-5} \quad \text{eV}^2, \qquad \Delta M_{31}^2 = 2\times 10^{-3} \quad \text{eV}^2$$
$$\sin^2\theta_{12} = 0.3, \qquad \sin^2\theta_{23} = 0.5, \qquad \sin^2\theta_{13} = 0.00,$$

RG effects are not drastic in usual cases except PR-fixed points or superconformal fixed points.

# Superstring theory

Superstring theory :

promising candidate for unified theory

including gravity

It is important to study flavor structure and Yukawa structure from string theories.

Superstring theory → 4D space-time

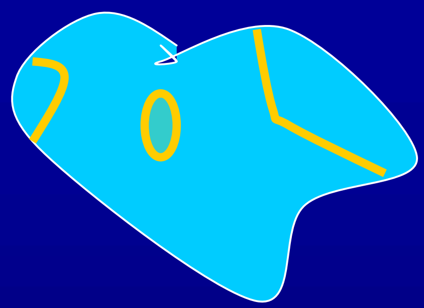
+ 6D compact space

# String models (String phenomenology)

Superstring ⇒ 10D space-time

= 4D space-time

+ 6D compact space

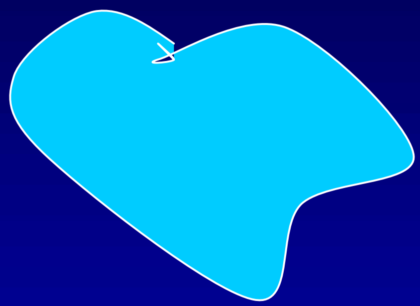


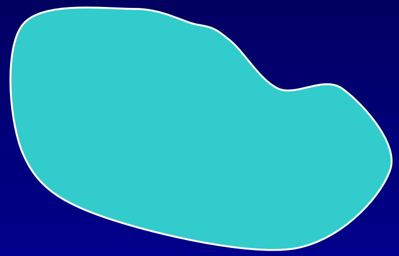
Origin of gauge groups SU(3), SU(2), U(1) ?

Origin of flavor structure 6/2 = 3 ?

Origin of parameters, g, Y, … ?

6D compact space





Several types of string models have been constructed.

(Several 6D compact spaces consistent with superstring theory)

Heterotic models on Calabi-Yau manifold, Orbifolds,
fermionic construction,
Gepner, . . . . . . . . . . .

Intersecting D-brane models
(magnetized D-branes)

These models lead to different flavor structures.

# Flavor structure from 6D compact space

6D compact space consistent with string theory

How to count the family number is well-known.

Flavor (Yukawa) structure

→ selection rule for allowed (Yukawa) couplings

calculations of their magnitude

flavor symmetry

→ Classify possible patterns of Yukawa matrices for quarks and leptons

Heterotic orbifold models and Intersecting D-brane models have similarlities.

# Localized modes
# (in extra dimensional field theory)

Couplings among bulk modes

→ O(1) Yukawa couplings

( N=2,4 sector → g = Y )

Couplings among (quasi-) localized modes

→ suppressed Yukawa couplings

String models with localized modes

Heterotic orbifold models

Intersecting D-brane models

. . . .

# Localized modes

Heterotic orbifold models and

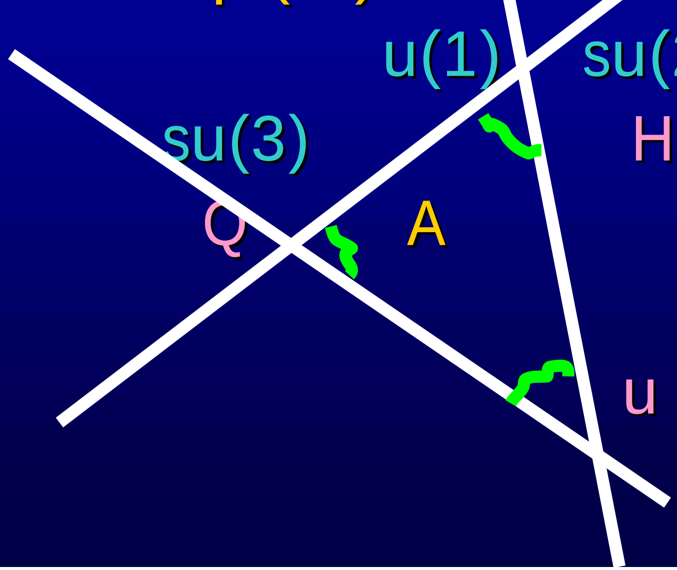
Intersecting (magnetized) D-brane models

are interesting to derive suppressed Yukawa couplings, because they have localized modes.

We would like to study somehow systematically.

e.g., intersecting D-brane configuration

$Y = \exp(-A)$   A: area



# Flavor structure in string models

Ko, K.T., Park, '04, '05

Systematical analyses on Yukawa matrices in heterotic orbifold models

T.K., Raby, Zhang, '04, (in preparation)

Nonabelian discrete flavor symmetry controlling higher dimensional operators (Orbifold)

Higaki, Kitazawa, T.K.、Takahashi, '05

Generic study on Yukawa structure in Intersecting D-brane models

# 2. Yukawa in hetero. orbifolds

The number of 6D orbifolds is finite,

Z3,Z4,Z6,……

(crystallographic) space group

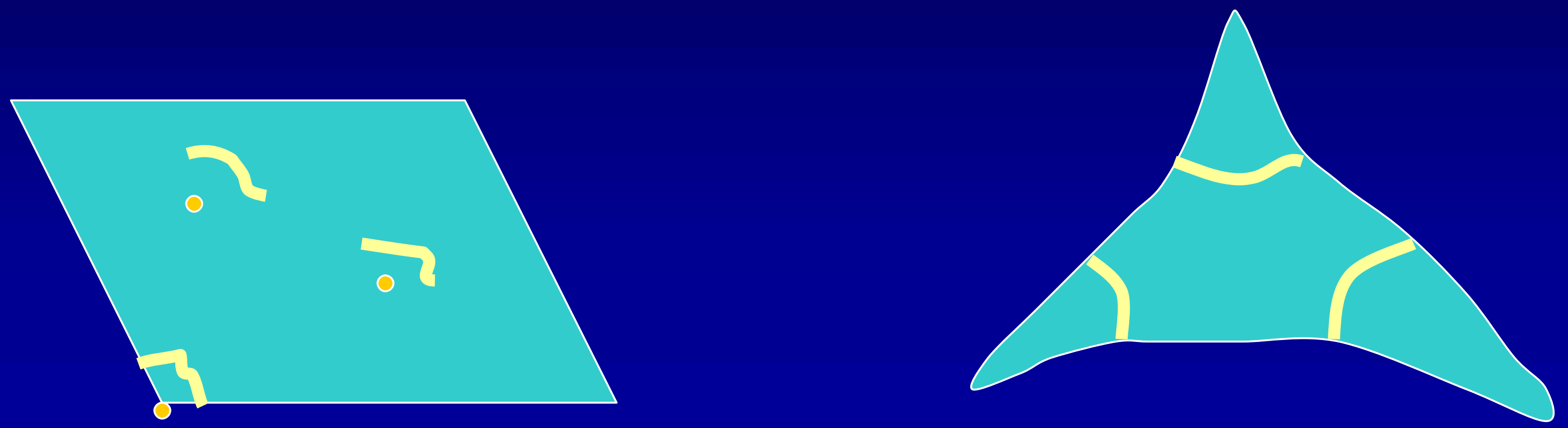
For an orbifold all of fixed points, where twisted string is localized, are known,

and its number is finite.

We know selection rules for allowed Yukawa couplings, and their magnitudes.

In principle, systematic study is possible.

# Orbifolds



Fixed points on 2D $Z_3$ orbifold

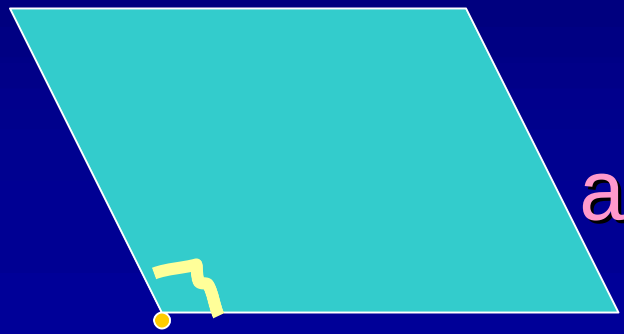
(0,0), (2/3,1/3), (1/3,2/3) in su(3) root basis

Twisted strings are associated with these fixed points.

The flavor structure and the selection rule are determined by geometrical aspect when we fix orbifold.

# 2D Z6 orbifold

First twisted states T1

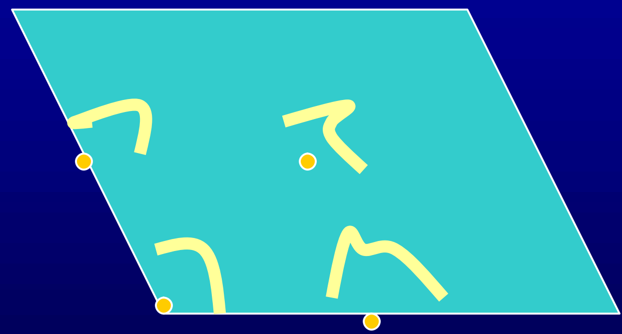


a single fixed point (0,0)

Second twisted states T2

(0,0), (2/3,1/3), (1/3,2/3)

third ones T3



(0,0), (1/2,0),(1/2,1/2)(0,1/2)

# 6D orbifold

6D Z3  orbifold = a product of 3 (2D Z3)

27 fixed points

6D Z6-I orbifold =

a product of 2 (2D Z6) and (2D Z3)

T1 3 twisted states

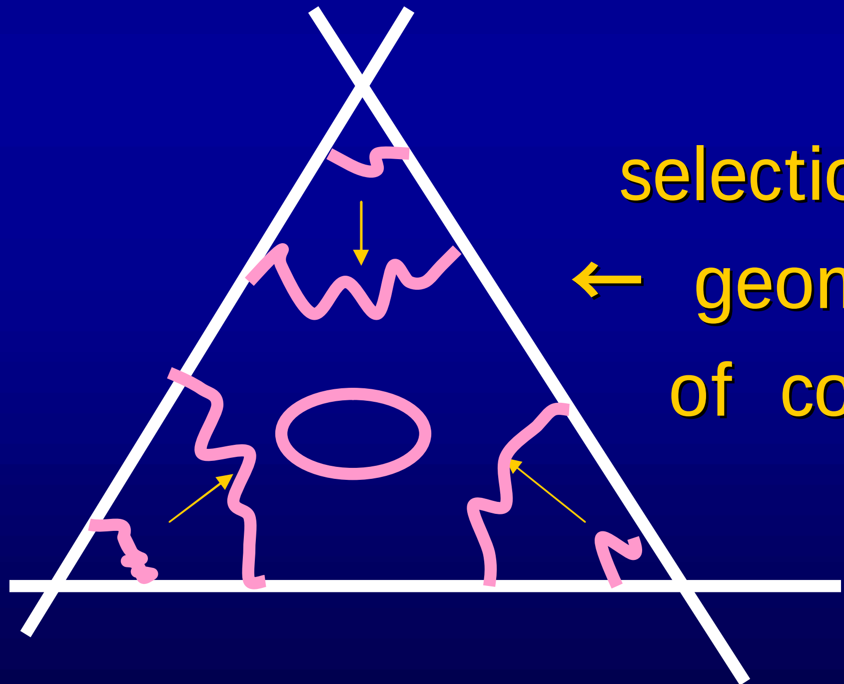
T2 27 twisted states

T3 16 twisted states

# Coupling selection rule

Yukawa couplings are allowed when three twisted strings combine into a closed string, which shrink.

We have to consider equivalence on the torus/orbifold.



selection rule ← geometrical aspects of compact space

⇒ in general complicated

# Space group selection rule

$$X(\sigma = \pi) = \Theta X(\sigma = 0) + e$$

$$(\Theta, e)(\Theta', e')(\Theta'', e'') = (1,0)$$

Conjugacy class

$$X + \Lambda = \Theta(X + \Lambda) + e$$

$$(\Theta, e) = (\Theta, e + (1 - \Theta)\Lambda)$$

In some case, only diagonal couplings are allowed, and in other case off-diagonal couplings are also allowed.

# Space group selection rule

2D $Z_3$ Only T1 T1 T1 couplings are allowed.

$f_k$ k=0,1,2

$i + j + k = 0 \mod 3$ diagonal

We can not get non-vanishing mixing angles.

This corresponds to Z3 symmetry, and its charge is equal to k.

2D Z6 orbifold

Only T1 T2 T3 and T2 T2 T2 couplings are allowed.

Off-diagonal couplings are allowed for T1T2T3.

T2 T2 T2 couplings are diagonal.

For example, T1T1T1 couplings are not allowed.

# Yukawa couplings on orbifold

CFT calculations → Dixon, et al '87, Hamidi, Vafa, '87,

Burwick, et al '91, Erler, et al '93, Casas, et al '93……

……………… T.K., Lebedev '03

$\hat{T}_1 \hat{T}_2 \hat{T}_3$ couplings

$$Y = \exp[-\sqrt{3}/(4\pi)(f_2 - f_3)^T M (f_2 - f_3)]$$

$$M = \begin{pmatrix} (R_1)^2 & -3(R_1)^2/2 & 0 & 0 \\ -3(R_1)^2/2 & 3(R_1)^2 & 0 & 0 \\ 0 & 0 & (R_2)^2 & -3(R_2)^2/2 \\ 0 & 0 & -3(R_2)^2/2 & 3(R_2)^2 \end{pmatrix}$$

$\hat{T}_2 \hat{T}_2 \hat{T}_2$ couplings

$$Y = \exp[-\sqrt{3}/(16\pi)(f_2 - f_3)^T M (f_2 - f_3)]$$

# Systematical study on Z6-I: Quark sector

Ko, T.K., Park '04

We study systematically all of possible assignments of 2nd and 3rd families to fixed points,

assume one pair of up and down Higgs fields,

examine allowed entries by the selection rule,

try to fit the mass ratios $m_c/m_t$, $m_s/m_b$ and mixing angle $V_{cb}$ by varying $R_1$ and $R_2$.

$$[m_c / m_t]_{exp} = 0.0038, \qquad [m_s / m_b]_{exp} = 0.025$$

$$[V_{cb}]_{exp} = 0.041$$

# Assignment 1

$Q_2, Q_3,$ $u_2, u_3$ $d_2, d_3$ $H_u, H_d$

$\hat{T}_2^{(2)}, \hat{T}_2^{(4)}$ $\hat{T}_3^{(3)}, \hat{T}_3^{(2)}$ $\hat{T}_3^{(1)}, \hat{T}_3^{(3)}$ $\hat{T}_1, \hat{T}_1$

$T_1 = 27.8,$ $T_2 = 107$

$$Y_u = \begin{pmatrix} 0.0416 & 0.718 \\ 0.0557 & 0.848 \end{pmatrix} \qquad Y_d = \begin{pmatrix} 0.0313 & 0.0416 \\ 0.0370 & 0.0557 \end{pmatrix}$$

$$m_c / m_t = 0.0038, \qquad m_s / m_b = 0.029, \qquad V_{cb} = 0.041$$

There are several examples leading to similar results.

# Assignment 2

$Q_2, Q_3,$ $\quad u_2, u_3$ $\quad d_2, d_3$ $\quad H_u, H_d$

$\hat{T}_3^{(2)}, \hat{T}_3^{(4)}$ $\quad \hat{T}_2^{(3)}, \hat{T}_2^{(2)}$ $\quad \hat{T}_2^{(1)}, \hat{T}_2^{(3)}$ $\quad \hat{T}_1, \hat{T}_1$

$T_1 = 24.0, \quad T_2 = 150$

$$Y_u = \begin{pmatrix} 0.0281 & 0.439 \\ 0.0371 & 0.665 \end{pmatrix} \qquad Y_d = \begin{pmatrix} 0.0199 & 0.0281 \\ 0.0302 & 0.0371 \end{pmatrix}$$

$$m_c / m_t = 0.0038, \qquad m_s / m_b = 0.032, \qquad V_{cb} = 0.041$$

There are several examples leading to similar results.

# Assignment 5

$Q_2, Q_3,$ $u_2, u_3$ $d_2, d_3$ $H_u, H_d$

$\hat{T}_2^{(2)}, \hat{T}_2^{(4)}$ $\hat{T}_2^{(2)}, \hat{T}_2^{(4)}$ $\hat{T}_2^{(3)}, \hat{T}_2^{(2)}$ $\hat{T}_2^{(4)}, \hat{T}_2^{(4)}$

$T_1 = 180,$ $T_2 = 180$

$$Y_u = \begin{pmatrix} 0 & 0.0309 \\ 0.0309 & 0.500 \end{pmatrix} \qquad Y_d = \begin{pmatrix} 0.00132 & 0 \\ 0.0214 & 0.0309 \end{pmatrix}$$

$m_c / m_t = 0.0038,$ $m_s / m_b = 0.029,$ $V_{cb} = 0.041$

# Results

We have found examples leading to a realistic mixing angle as well as mass ratios between the $2^{nd}$ and $3^{rd}$ families of quarks.

Our results is the first examples to show the possibilities for leading to realistic mixing angles by use of stringy renomalizable couplings with one pair of the up and down Higgs fields.

Results for three families of quarks are not perfectly good.

# Systematical study on Z6-I: Lepton sector

Ko, T.K., Park '05

We study systematically all of possible assignments of three families to fixed points,

assume one pair of up and down Higgs fields,

examine allowed entries by the selection rule,

try to fit the lepton mass ratios and

mixing angles by varying $R_1$ and $R_2$.

$$[m_e / m_\tau]_{exp} = 0.000288, \qquad [m_\mu / m_\tau]_{exp} = 0.0595$$

$$[\Delta m^2_{31} / \Delta m^2_{21}]_{exp} = 27, \qquad [\sin^2 \theta_{12}]_{exp} = 0.30$$

$$[\sin^2 \theta_{23}]_{exp} = 0.50, \qquad [\sin^2 \theta_{13}]_{exp} = 0.000$$

# Dirac neutrino mass: Assignment 4

$L_1, L_2, L_3,$ $N_1, N_2, N_3$ $e_1, e_2, e_3$ $H_u, H_d$

$\hat{T}_2^{(2)}, \hat{T}_2^{(3)}, \hat{T}_2^{(4,-1)}$ $\hat{T}_2^{(2)}, \hat{T}_2^{(4,1)}, \hat{T}_2^{(4,-1)}$ $\hat{T}_3^{(1)}, \hat{T}_3^{(2)}, \hat{T}_3^{(4,1)}$ $\hat{T}_2^{(2)}, \hat{T}_1$

$T_1 = 26,$ $T_2 = 21$

$m_e / m_\tau = 0.0003,$ $m_\mu / m_\tau = 0.06,$ $\Delta m_{31}^2 / \Delta m_{21}^2 = 14,$

$\sin^2\theta_{12} = 0.38,$ $\sin^2\theta_{23} = 0.70,$ $\sin^2\theta_{13} = 0.000,$

There are several examples leading to similar results、 but smaller ratios of neutrino mass difference.

# Seesaw scenario: Assignment 4

We assume the right-handed Majorana neutrino mass to be proportional to identity matrix for simplicity.

| | | | |
|---|---|---|---|
| $L_1, L_2, L_3,$ | $N_1, N_2, N_3$ | $e_1, e_2, e_3$ | $H_u, H_d$ |
| $\hat{T}_2^{(2)}, \hat{T}_2^{(3)}, \hat{T}_2^{(4,1)}$ | $\hat{T}_2^{(2)}, \hat{T}_2^{(3)}, \hat{T}_2^{(4,1)}$ | $\hat{T}_3^{(1)}, \hat{T}_3^{(2)}, \hat{T}_3^{(4,1)}$ | $\hat{T}_2^{(2)}, \hat{T}_1$ |

$T_1 = 23, \qquad T_2 = 26$

$m_e / m_\tau = 0.0003, \qquad m_\mu / m_\tau = 0.06, \qquad \Delta m_{31}^2 / \Delta m_{21}^2 = 29,$

$\sin^2\theta_{12} = 0.32, \qquad \sin^2\theta_{23} = 0.48, \qquad \sin^2\theta_{13} = 0.000,$

There are several examples leading to similar results.

# Results

We have found examples leading to a realistic values of charged lepton mass ratios, neutrino mass difference ratio and mixing angles for three families.

It is quite non-trivial to fit six observables only by two parameters.

Our results is the first examples to show the possibilities for leading to realistic mixing angles by use of stringy renomalizable couplings with one pair of the up and down Higgs fields.

# 教訓

弦模型において、

3点結合の選択則、その強さは、コンパクト空間の幾何学で決まっている。

→　ある程度現実的な湯川行列を導くコンパクト空間もあれば、そうでないものもある。

# 3. Discrete flavor symmetry

Effective Yukawa

← higher dim. op. through symmetry breaking → small Yukawa

e.g. Froggatt-Nielsen

Symmetries are important to control higher dim. Op.

e.g. U(2), S3, D4, A4, U(1), ZN,………

(Symmetrical approach)

What is the origin of these symmetries ?

# Discrete Flavor Symmetry

What are their origins of discrete flavor symmetry like S3, D4, A4 …… ?

They are symmetries of geometric solids.

So, they may be originated from geometry of extra dimensional space.

Actually, we have found explicit string models, whose three families = singlet + doublet under D4.

T.K., Raby, Zhang, '04

This is the first explicit models leading to D4 flavor structure in string models.

# D4 flavor

Our model is from Z6-II orbifold, Z6=Z2*Z3 .

The essence that our model leads to D4 flavor structure can be understood by the simplified extra dim. space, S1/Z2.

There are two fixed points, on which two types of states exist.

In string models, these states are degenerate in massless spectra. These correspond to D4 doublets and bulk modes correspond to singlets.

# D4 Flavor Symmetry

Stringy symmetries require that

Lagrangian has the permutation symmetry between

1 and 2, and each coupling is controlled by Z2 symmetry.

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

D4 elements

$$\pm 1, \qquad \pm\sigma_1, \qquad \pm i\sigma_2, \qquad \pm\sigma_3$$

Geometry of compact space

→ origin of finite flavor symmetry

# Z6 example

Gauge shift 6V=(22200000)(11000000)

Wilson lines 3W=(-11000000)(00200000)

Wilson line 2W=(10000111)(00000000)

→

Gauge group SU(4) SU(2) SU(2) (hidden group)

Matter 3 [(4,2,1) + ($\bar{4}$,1,2)] + (1,2,2) + exotics

Pati-Salam model with 3 families + exotics

3 families = (D4 doublets + D4 singlets)

Electroweak higgs = D4 singlet

# Phenomenological aspects

D4 flavor structure has phenomenological aspects on Yukawa couplings, SUSY breaking terms and so on.

Possibilities for realizing other non-abelian discrete flavor symmetries ?

→ Work in progress

# 教訓

弦模型において、

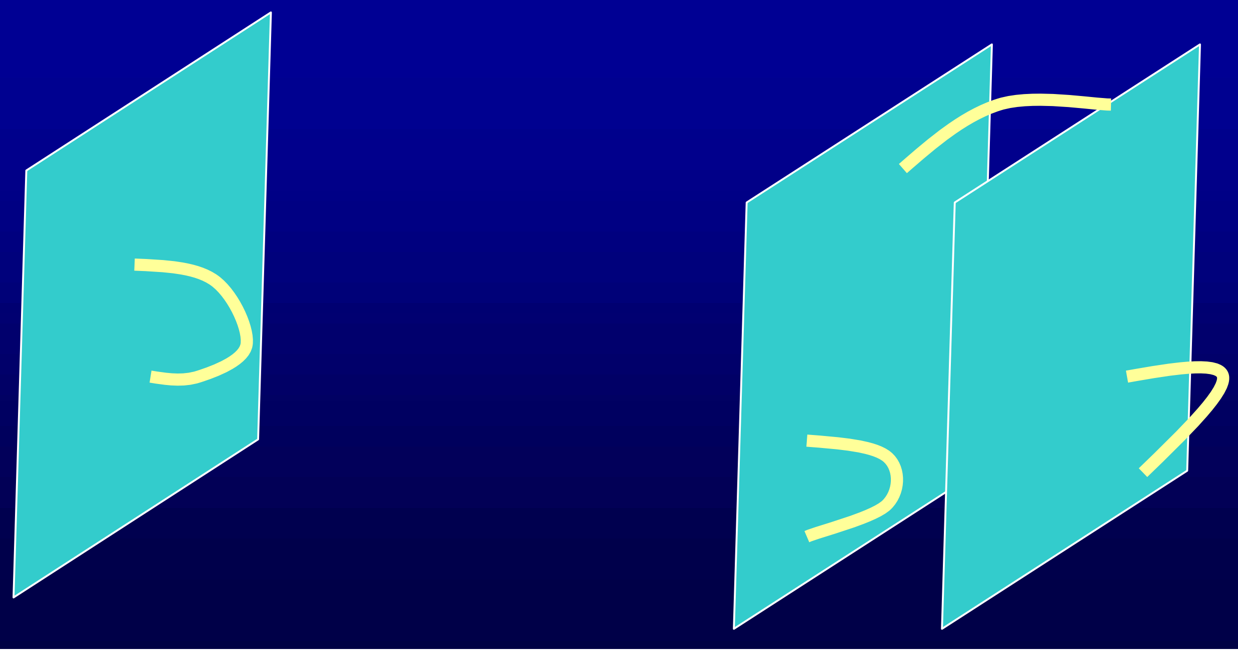
高次の結合の選択則、その強さは、コンパクト空間の幾何学で決まっている。

→ 場の理論の模型で仮定されてきた
フレーバー対称性の起源を与える。

# 4. Flavor in intersecting D-branes

Dp-brane: (p+1) dimensional extended object,
where open string can end

gauge boson: open string, whose two end-points
are on the same (set of) D-brane(s)



# Intersecting D6 models

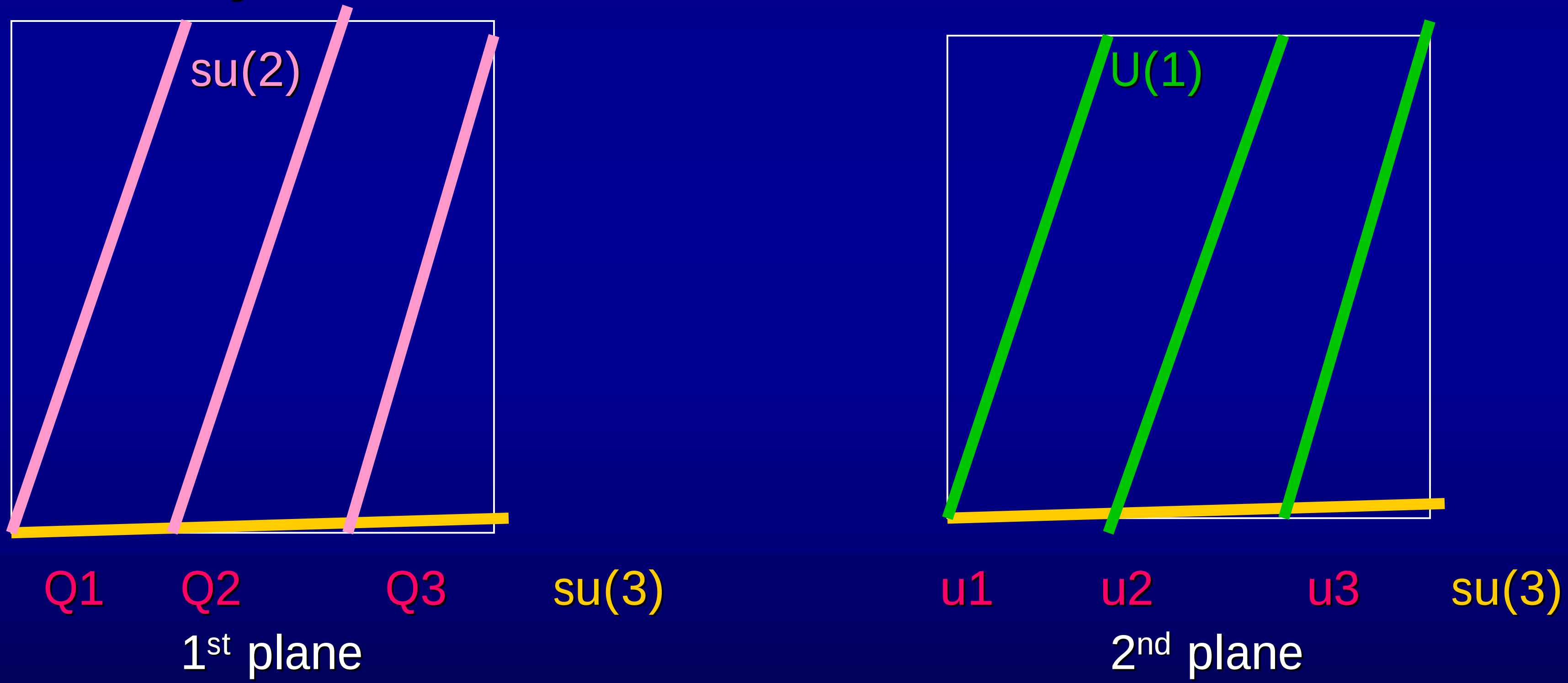
Setup of intersecting D6 models

6D compact space: a direct product of three 2D tori

D6-branes wrap one cycle of each 2D torus

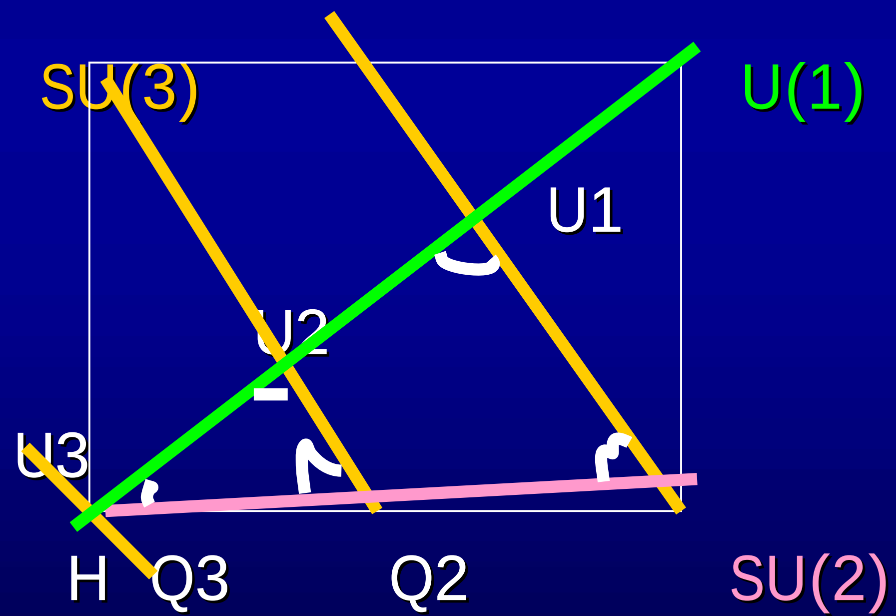
# Origin of flavor

Family number = intersection number



# Yukawa in intersecting D-brane models

Flavor number = intersecting number



# Flavor structure

Higaki, Kitazawa, T.K.,Takahashi, '05

Analyses on flavor structure have been done model by model so far.

We need to formulate model-independent analysis on selection rules for allowed couplings like heterotic orbifold models.

selection rules for 3-point couplings

Cremades, Ibanez, Marchesano, '03

Our approach is different.

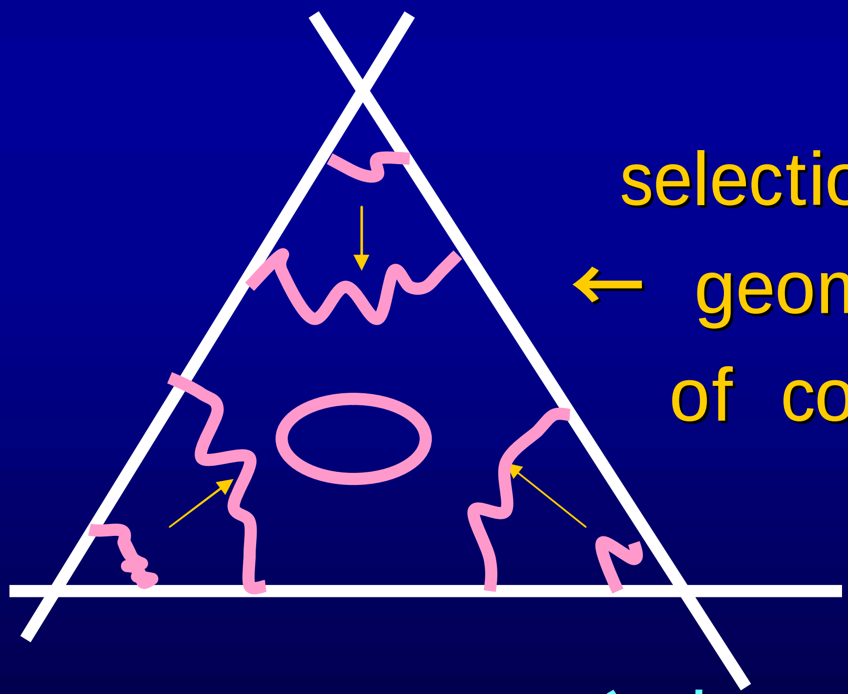
We would like to study somehow systematically

flavor structure, which can be realized in intersecting D-brane configuration.

# Coupling selection rule

Yukawa couplings are allowed when three twisted strings combine into a closed string, which shrink.

We have to consider equivalence on the torus.

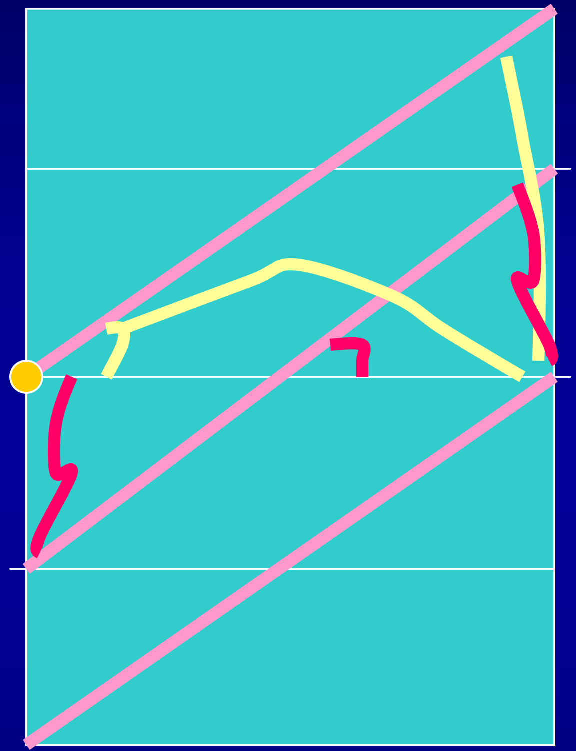


selection rule

← geometrical aspects of compact space

⇒ in general complicated

# Open string at intersecting D-brane



How can we describe open strings at different intersecting points and open strings equivalent up to torus ?

# Intersecting points

Two D-brane Da, Db (winding numbers)

$$W_a = (n_a, m_a), \qquad W_b = (n_b, m_b),$$

Intersecting number $I_{ab} = | n_a m_b - n_b m_a |$

Equivalent open strings at intersecting points

$$(0,0) = kW_a + \ell W_b$$

Independent open strings at intersecting points

are described by shift vectors Vab corresponding to

coset representatives $\Lambda / \Lambda_{ab}$

$\Lambda_{ab} = \{W_a, W_b\}, \qquad \Lambda = \{(1,0),(0,1)\}$ sub-lattice

$I_{ab} = Vol(\Lambda_{ab}) / Vol(\Lambda)$

# Example (Coset representatives)

Two D-brane Da, Db (winding numbers)

$$W_a = (1,0), \qquad W_b = (1,3),$$

Intersecting number Iab=3

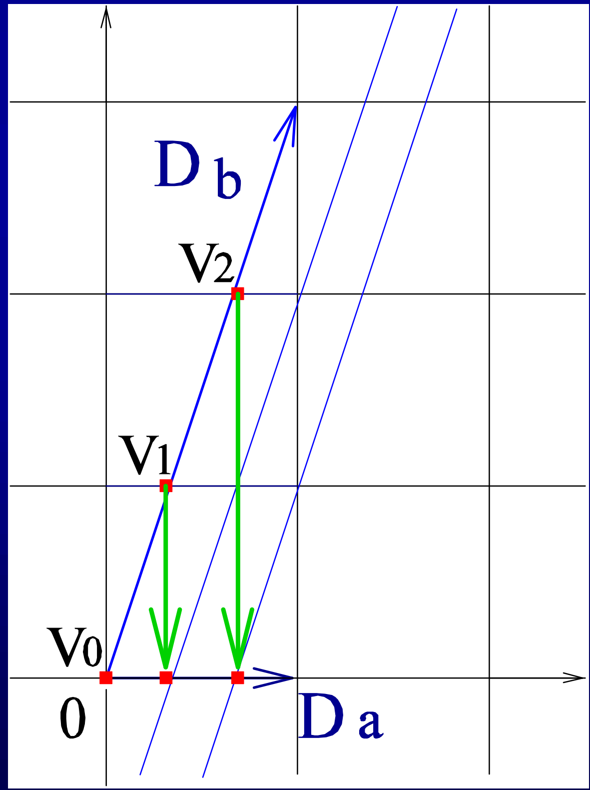
Intersecting points

$$(k/3)W_a, \qquad k = 0,1,2$$

Shift vectors represent three intersecting points as coset rep..

$$V_{ab} = (0,0), (0,1), (0,2),$$

$$\Lambda_{ab} = \{(1,0), (0,3)\},$$



# Open strings and selection rule

End points of open string

$$X_a - X_b = V_{ab}$$

Three sets of intersecting D-branes

$$W_a = (n_a, m_a), \quad W_b = (n_b, m_b), \quad W_c = (n_c, m_c),$$

Condition for Yukawa couplings among three open strings between Da-Db, Db-Dc and Dc-Da branes

$$V_{ab} + V_{bc} + V_{ca} = 0$$

$\mathrm{mod} \quad \Lambda_{ab}, \Lambda_{bc}, \Lambda_{ca}$ sub-lattices

only diagonal couplings or off-diagonal couplings ?

# Flavor structure on 2D torus

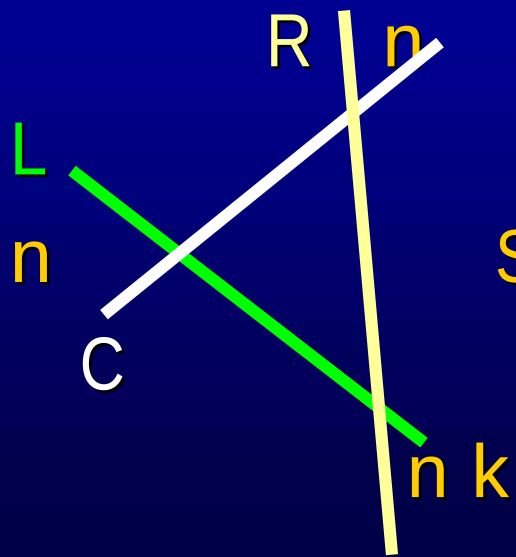
Which types of flavor structure can be realized from intersection D-brane configurations ?

→ infinite number of varieties ?

There is a rule for intersecting numbers.

Left-right (n,n) symmetric generation on torus

→ the number of Higgs scalars = n k




for k=1

$$\Lambda_{ab} = \Lambda_{bc} = \Lambda_{ca}$$

Selection rule ← Zn symmetry

diagonal couplings

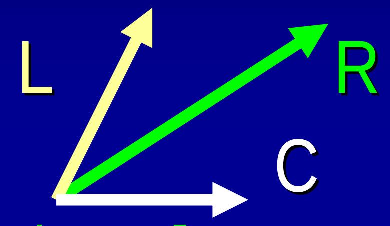
# Flavor structure

Winding vectors

$$W_C = (n_C, m_C), \qquad W_L = (n_L, m_L), \qquad W_R = (n_R, m_R),$$

Intersecting numbers

$$| I_{CL} | = | I_{RC} | = n$$




→ $W_L \pm W_R = kW_C, \qquad | I_{LR} | = nk$

for k=1 $\Lambda_{CL} = \Lambda_{CR} = \Lambda_{LR}$

Selection rule Zn symmetry

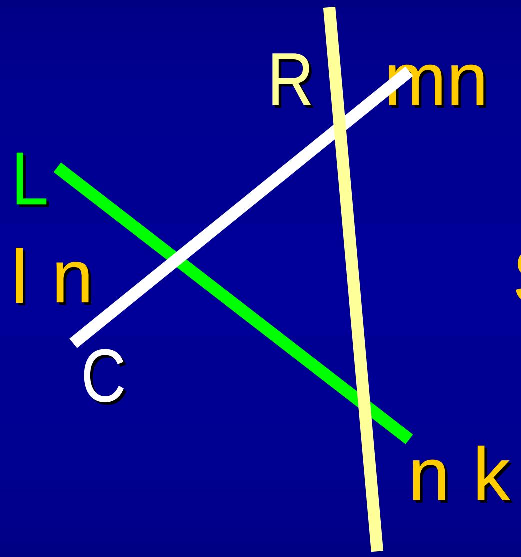
for k > 1 $\mathrm{Vol}(\Lambda_{LR}) = k\mathrm{Vol}(\Lambda_{CR} = \Lambda_{LR})$

Selection rule still Zn

Higgs (LR) sector has Zk charges, but those are irrelevan to coupling selection rule.

# Generic rule for intersecting numbers

Similarly, we can study generic case.




Selection rule ← Zn symmetry

g.c.d.(k,l) = g.c.d.(l,m) = g.c.d.(m,k) = 1

# Flavor structure

e.g. 3 families → 3 Higgs

Left-handed Q

Right-handed Q

Higgs

$$V_{ab} = (0,0), (0,1), (0,2)$$

$$V_{bc} = (0,0), (0,1), (0,2)$$

$$V_{ca} = (0,0), (0,1), (0,2)$$

$$V_{ab} + V_{bc} + V_{ca} = (0,0) \quad \text{mod} \quad (0,3)$$

Selection rule for Yukawa coupling ← Z3 symmetry

diagonal couplings like Z3 heterotic orbifold models

# Yukawa matrix

$$Y = \begin{pmatrix} H_0 & \varepsilon H_2 & \varepsilon H_1 \\ \varepsilon H_2 & H_1 & \varepsilon H_0 \\ \varepsilon H_1 & \varepsilon H_0 & H_2 \end{pmatrix}$$

Quark masses

$$(m_u, m_c, m_t) \propto (v_0^u, v_1^u, v_2^u)$$

$$(m_d, m_s, m_b) \propto (v_0^d, v_1^d, v_2^d)$$

Mixing angles

$$V_{12} : V_{13} : V_{23} = m_t / m_c : m_s / m_b : m_d / m_b$$

→ Not realistic

# Asymmetic flavor structure

Intersecting number

Left-handed Q

Right-handed Q

Higgs

$$I_{CL} = (3,1,1)$$

$$I_{CR} = (1,3,1)$$

$$I_{LR} = (1,1,1)$$

$$Y_{ij} = a_i b_j$$

Rank one Yukawa matrix not realistic

# Result

We have formulated the selection rule for allowed Yukawa couplings as well as generic n-point couplings.

There is a rule of intersecting numbers for generic configurations. → selection rule : Zn symmetry

Left-right symmetric 3 families → 3 or more Higgs

← Selection rule Z3 symmetry

Left-right asymmetric 3 families

→Rank one matrix

Asymmetric structure with more families

→ non-trivial forms, but …

We may need another type of flavor structures.

# One of different points

Heterotic orbifold models seem better than Intersecting D-brane models, in order to realize realistic Yukawa matrices (by the present knowledge).

In heterotic orbifold models, discrete wilson lines can break degenerate spectrum of fixed points.

If one can resolve degenerate spectrum of intersecting points,

one may derive more realistic models.

# Summary

We have studied flavor structure in string models.

Heterotic orbifold models have possibilities for realizing realistic Yukawa matrices for quarks and leptons.

Other orbifolds ?

We have just started how to derive non-abelian discrete flavor symmetries from string models.

We have formulated selection rule of Yukawa couplings in intersecting D-brane models.

We have just started the classification of flavor structure derived from D-brane configurations.

# Future study

How to stabilize moduli VEVs at proper values is an important issue to study.

Flavor structure → (realistic) Yukawa matrices

Such flavor structure affects on SUSY breaking terms, e.g. scalar masses and A-matrices.

Each model would have certain pattern of SUSY breaking terms.

It is important to study such prediction on SUSY breaking terms.