DEPARTMENT OF PHYSICS
NAGOYA UNIVERSITY

DATE
2003. No.10
at Niigata Univ.

# Precocious Gauge Symmetry Breaking in $SU(6) \times SU(2)_R$ Model

hep-ph/0209004

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# §1. Introduction

The 4-D eff. theory from string compactification :

constrained by topol. and geom. structure of $K$

far more constrained than ordinary field theory

Perturbative Heterotic Superstring (K-M level 1)

- Gauge symmetry : $G \subset E_6$
- Matter contents of massless sector

  no adjoint and higher representation !
- Generation structure of massless sector
- Flavor symmetry

In GUT-type model from string :

GUT scale : distinct from the string scale $M_S$

Can the GUT scale be generated by radiative effects ?

(á la the electroweak scale )

> In $SU(6) \times SU(2)_R$ string-inspired model
> the GUT scale can be generated by radiative effects
> due to a large Yukawa coupling

# §2. $SU(6) \times SU(2)_R$ Model

(i). Gauge symmetry : $SU(6) \times SU(2)_R = G \subset E_6$

via $\mathbf{Z}_2 = \{U_g\}$ flux breaking on $K$

maximal subgroup of $E_6$

$G \to G_{SM}$ possibly occurs via Higgs mechanism

$\langle \mathbf{27} \rangle, \quad \langle \mathbf{27}^* \rangle$ without adjoint

(ii). Matter contents (minimal) :

$$\Phi(\mathbf{27}) = \begin{cases} \phi(\mathbf{15}, \mathbf{1}) & : \quad Q, L, g, g^c, S, \\ \psi(\mathbf{6}^*, \mathbf{2}) & : \quad (U^c, D^c), (N^c, E^c), (H_u, H_d), \end{cases}$$

colored Higgs $g$, $g^c$ and doublet Higgs $H_u$, $H_d$

belong to different irr. repr. of $G$

natural solution to triplet-doublet splitting problem

$$3 \times \begin{pmatrix} \phi(\mathbf{15},\ \mathbf{1}) \\ \psi(\mathbf{6}^*,\ \mathbf{2}) \end{pmatrix}_i + \left\{ \begin{pmatrix} \phi(\mathbf{15},\ \mathbf{1}) \\ \psi(\mathbf{6}^*,\ \mathbf{2}) \end{pmatrix}_0 + \begin{pmatrix} \overline{\phi}(\mathbf{15}^*,\ \mathbf{1}) \\ \overline{\psi}(\mathbf{6},\ \mathbf{2}) \end{pmatrix} \right\}$$

$\Phi_i \ (i = 1, 2, 3)$ $\qquad$ $\Phi_0$ $\qquad$ $\overline{\Phi}$

(iii). Flavor symmetry :

We introduce non-anomalous flavor symmetry :

$$\mathbf{Z}_{19}(R) \times \mathbf{Z}_{18}(\text{non}-R) \times \tilde{D}_4$$

$$\mathbf{Z}_{19} \times \mathbf{Z}_{18} \longrightarrow \mathbf{Z}_{342}$$

Grassmann $\theta$ : $(-1, 0) \longrightarrow q_\theta = 18 \mod 342$

Assignment of $\mathbf{Z}_{342}$ charges

| | $\Phi_1$ | $\Phi_2$ | $\Phi_3$ | $\Phi_0$ | $\overline{\Phi}$ |
|---|---|---|---|---|---|
| $\phi(\mathbf{15}, \mathbf{1})$ | 126 | 102 | 46 | 12 | $-16$ |
| $\psi(\mathbf{6}^*, \mathbf{2})$ | 120 | 80 | 16 | $-14$ | $-67$ |

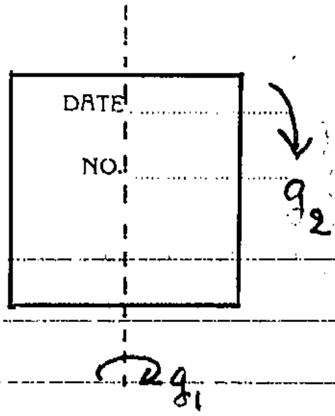
(PTP 108 (2002) 965)

Binary dihedral group $\tilde{D}_4$

- Theoretical motivation : nontrivial bg. $B$-field
  coordinates : non-commuting op.
  $\Rightarrow$ projective representation of disc. sym.
- Phenomenological motivation :
  R-handed Maj. mass scale

$$M_{R33} \sim \frac{m_\tau^2}{m_{\nu_3}} \sim \sqrt{M_{Pl} M_Z}$$

colored Higgs : $\mathcal{O}(10^{17\sim18})$GeV,
doublet Higgs : $\mathcal{O}(10^{2\sim3})$GeV




Dihedral group : $D_4 = \mathbf{Z}_2 \ltimes \mathbf{Z}_4$

$$\mathbf{Z}_2 = \{1,\ g_1\}$$

$$\mathbf{Z}_4 = \{1,\ g_2,\ g_2^2,\ g_2^3\}$$

$$g_1 g_2 g_1^{-1} = g_2^{-1}$$

($\mathbf{Z}_2$ : reflection, $\mathbf{Z}_4$ : rotation )

Binary ($\tilde{D}_4$) : projective representation of $D_4$

$$\gamma(g_1)\,\gamma(g_2)\,\gamma(g_1)^{-1} = i\,\gamma(g_2)^{-1}$$

$$\gamma(g_1) = \sigma_1, \qquad \gamma(g_2) = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} = \sigma_4$$

"$\tilde{D}_4$-charges" $\quad (\theta\ :\ \gamma(g_1) = \sigma_1)$

| | $\phi_i,\ \psi_i\ (i = 1,2,3)$ | $\phi_0,\ \psi_0$ | $\overline{\phi},\ \overline{\psi}$ |
|---|---|---|---|
| $\phi(\mathbf{15},\ \mathbf{1})$ | $\sigma_1$ | 1 | 1 |
| $\psi(\mathbf{6}^*,\ \mathbf{2})$ | $\sigma_2$ | $\sigma_3$ | $\sigma_4$ |

$\tilde{D}_4$ symmetry : extension of $R$-parity

$R$-parity transformation : $\gamma(g_2^2) = \sigma_3$

$$\sigma_3\,\sigma_{1,2}\,\sigma_3^{-1} = -\sigma_{1,2} \qquad \sigma_3\,\sigma_{3,4}\,\sigma_3^{-1} = +\sigma_{3,4}$$

(iv). Superpotential $W$ : gauge inv. trilinear term

$$(\phi(\mathbf{15},\mathbf{1}))^3 \sim QQg + Qg^cL + g^cgS$$

$$\phi(\mathbf{15},\mathbf{1})(\psi(\mathbf{6}^*,\mathbf{2}))^2 \sim QH_dD^c + QH_uU^c + LH_dE^c + LH_uN^c + SH_uH_d + gN^cD^c + gE^cU^c + g^cU^cD^c$$

The gauge sym. and flavor sym. require :

$$\begin{aligned} W_Y = & \frac{1}{3!} z_0 \left(\frac{\phi_0\overline{\phi}}{M_1^2}\right)^{\zeta_0} (\phi_0)^3 + \frac{1}{3!} \overline{z} \left(\frac{\phi_0\overline{\phi}}{M_1^2}\right)^{\overline{\zeta}} (\overline{\phi})^3 \\ & + \frac{1}{2} \sum_{i,j=1}^{3} z_{ij} \left(\frac{\phi_0\overline{\phi}}{M_1^2}\right)^{\zeta_{ij}} \phi_0\phi_i\phi_j \\ & + \frac{1}{2} h_0 \left(\frac{\phi_0\overline{\phi}}{M_1^2}\right)^{\eta_0} \phi_0\psi_0\psi_0 \\ & + \frac{1}{2} \overline{h} \left(\frac{\phi_0\overline{\phi}}{M_1^2}\right)^{\overline{\eta}} \left(\frac{\psi_0\overline{\psi}}{M_2^2}\right)^{2} \overline{\phi}\,\overline{\psi}\,\overline{\psi} \\ & + \frac{1}{2} \sum_{i,j=1}^{3} h_{ij} \left(\frac{\phi_0\overline{\phi}}{M_1^2}\right)^{\eta_{ij}} \phi_0\psi_i\psi_j \\ & + \sum_{i,j=1}^{3} m_{ij} \left(\frac{\phi_0\overline{\phi}}{M_1^2}\right)^{\mu_{ij}} \psi_0\phi_i\psi_j . \end{aligned}$$

$$M_1,\ M_2 = \mathcal{O}(1) \times M_S$$

$\mathcal{O}(1)$ factor : from volume of compact space
in which the matter fields reside

The exponents :

$$(\zeta_0,\ \overline{\zeta},\ \eta_0,\ \overline{\eta}) = (\underline{0},\ 150,\ 158,\ 84)$$

$$\zeta_{ij} = \begin{pmatrix} 57 & 51 & 37 \\ 51 & 45 & 31 \\ 37 & 31 & 17 \end{pmatrix}_{ij}$$

$$\eta_{ij} = \begin{pmatrix} 54 & 44 & 28 \\ 44 & 34 & 18 \\ 28 & 18 & 2 \end{pmatrix}_{ij}$$

$$\mu_{ij} = \begin{pmatrix} 49 & 39 & 23 \\ 43 & 33 & 17 \\ 29 & 19 & 3 \end{pmatrix}_{ij}$$

Renormalizable interaction in $W_Y$ :

$$W_{Y0} = \frac{1}{3!}\, z_0(\phi_0)^3$$

large Yukawa coupling

scalar mass squared of $\phi_0$

can be driven negative slightly below $M_S$

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## Gauge inv. term in $R$-parity even sector

$$W_1 = M_S^3 \left[ \lambda_0 \left( \frac{\phi_0 \overline{\phi}}{M_1^2} \right)^{2n} + \lambda_1 \left( \frac{\phi_0 \overline{\phi}}{M_1^2} \right)^{n} \left( \frac{\psi_0 \overline{\psi}}{M_2^2} \right)^{m} + \lambda_2 \left( \frac{\psi_0 \overline{\psi}}{M_2^2} \right)^{2m} \right]$$

$$n = 81, \quad m = 4 \quad \text{and} \quad \lambda_i = \mathcal{O}(1)$$

## Scalar potential

$$\begin{aligned} V &= (\text{F term}) + (\text{D term}) + V_{\text{soft}} + \Delta V_{1-\text{loop}} \\ V_{\text{soft}} &= \tilde{m}_{\phi 0}^2 |\phi_0|^2 + \tilde{m}_{\overline{\phi}}^2 |\overline{\phi}|^2 \\ &\quad + \tilde{m}_{\psi 0}^2 |\psi_0|^2 + \tilde{m}_{\overline{\psi}}^2 |\overline{\psi}|^2 + (\text{A term}) \end{aligned}$$

$$\tilde{m}_{\phi 0}^2 = \tilde{m}_{\overline{\phi}}^2 = \tilde{m}_{\psi 0}^2 = \tilde{m}_{\overline{\psi}}^2 = \tilde{m}_0^2 \qquad \text{at } M_S$$

## RG evolution of scalar masses down from $M_S$

If $\tilde{m}_{\phi 0}^2 + \tilde{m}_{\overline{\phi}}^2 < 0$, gauge sym. br. occurs

## Minimization of scalar potential :

$$\begin{aligned} |\langle \phi_0 \rangle| = |\langle \overline{\phi} \rangle| &\sim M_1 \, \rho^{1/322} \sim 10^{17.5} \text{GeV} \\ |\langle \psi_0 \rangle| = |\langle \overline{\psi} \rangle| &\sim M_2 \, \rho^{81/1288} \sim 10^{16.5} \text{GeV} \\ \rho = \tilde{m}_0 / M_1 &\sim 10^{-16} \end{aligned}$$

$$SU(6) \times SU(2)_R \xrightarrow{\langle \phi_0 \rangle} SU(4)_{\text{PS}} \times SU(2)_L \times SU(2)_R \xrightarrow{\langle \psi_0 \rangle} G_{SM},$$

( PL B337 (1994) 63,
PTP 92 (1994) 153 )

# §3. The RG evolution (one-loop level)

The RG equations for $g_6$ and $M_6$ : $t = \ln(Q/M_S)$

$$(4\pi)^2 \frac{dg_6^2}{dt} = -2b_6\, g_6^4 \qquad (b_6 = 3)$$

$$(4\pi)^2 \frac{dM_6}{dt} = -2b_6\, g_6^2\, M_6$$

Solution : $u = 1 + (3/8\pi^2) g_0^2\, t$

$$g_6^2(u) = \frac{g_0^2}{u}, \qquad M_6(u) = \frac{M_{1/2}}{u}$$

Renormalizable term of superpotential and its $A$-term :

$$W_{Y0} = \frac{1}{3!}\, z_0 (\phi_0)^3$$

$$(A \text{ term}) = \frac{1}{3!}\, A_0\, z_0 (\phi_0)^3$$

The RG equations for $z_0$, $A_0$ and $\tilde{m}^2$ :

$$(4\pi)^2 \frac{dz_0^2}{dt} = \left(-56\, g_6^2 + 18\, z_0^2\right) z_0^2$$

$$(4\pi)^2 \frac{dA_0}{dt} = 3A_0\, z_0^2 + 56 M_6\, g_6^2$$

$$(4\pi)^2 \frac{d\tilde{m}_{\phi 0}^2}{dt} = \underline{3\left(\tilde{m}_{\phi 0}^2 + \frac{1}{3} A_0^2\right) z_0^2 - \frac{112}{3} M_6^2\, g_6^2}$$

$$(4\pi)^2 \frac{d\tilde{m}_{\phi}^2}{dt} = -\frac{112}{3} M_6^2\, g_6^2$$

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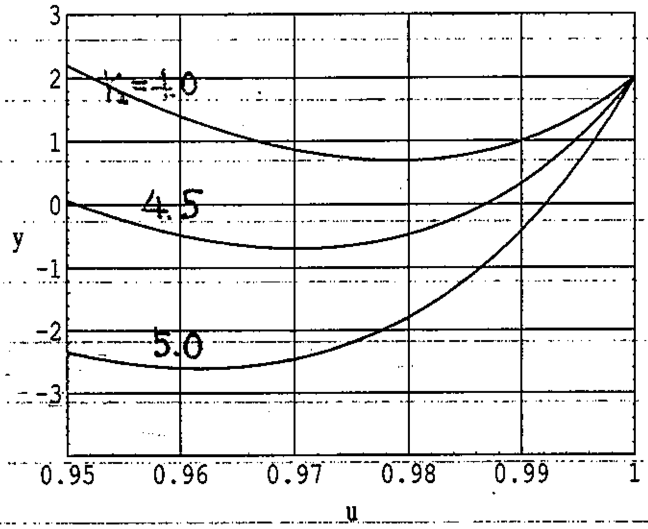
$y(u) \equiv (\tilde{m}^2_{\phi 0}(u) + \tilde{m}^2_{\phi}(u))/\tilde{m}^2_0$ vs. $u$.




$$u = 1 + \frac{3}{8\pi^2} g_0^2 \ln(Q/M_S)$$

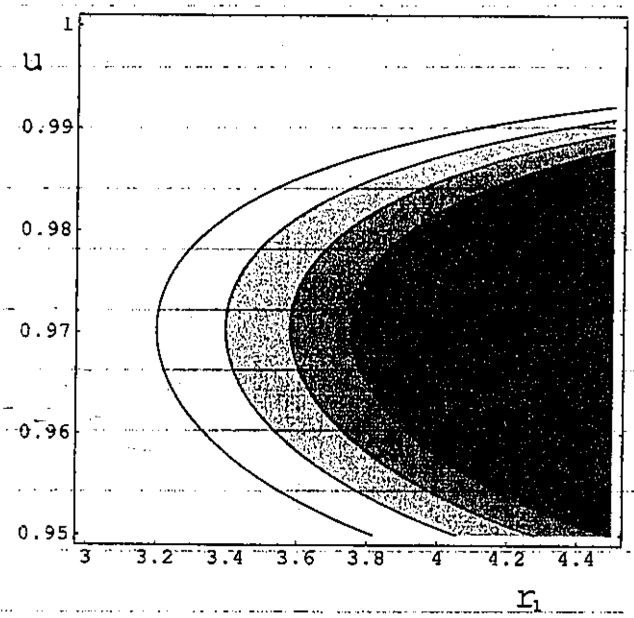
$$r_0 = \frac{z_0(1)}{g_0} = 3.0$$

$$r_1 = \frac{M_{1/2}}{\tilde{m}_0} = 3.5$$

$$r_2 = \frac{A_0(1)}{M_{1/2}} = 4.0,\ 4.5,\ 5.0$$

The upper, middle and lower solid curves are for the cases of $r_2 = 4.0,\ 4.5,\ 5.0$

The $u$- and $r_1$-dependences of $y(u) = (\tilde{m}^2_{\phi 0}(u) + \tilde{m}^2_{\bar{\phi}}(u))/\tilde{m}^2_0$




$$u = 1 + \frac{3}{8\pi^2} g_0^2 \ln(Q/M_S)$$

$$r_0 = \frac{z_0(1)}{g_0} = 3.0$$

$$r_1 = \frac{M_{1/2}}{\tilde{m}_0} = 3.0 \sim 4.5$$

$$r_2 = \frac{A_0(1)}{M_{1/2}} = 4.5$$

In the white region : $y(u) > 0$

In the region from gray to black : $y(u) = 0 \sim -3.0$

# §4. Summary

In $SU(6) \times SU(2)_R$ string-inspired model

with the non-anomalous flavor symmetry

$$\mathbf{Z}_{19}(R) \times \mathbf{Z}_{18}(\text{non}-R) \times \tilde{D}_4$$

we possibly have a large Yukawa coupling at $M_S$

scalar mass squared $\tilde{m}^2_{\phi 0} + \tilde{m}^2_{\overline{\phi}}$ :

possibly goes negative slightly below $M_S$

precocious breaking of $SU(6) \times SU(2)_R$ symmetry

can occur due to radiative effects

Large Yukawa coupling at $M_S$ :

identical with colored Higgs coupling

$$W_{Y0} = \frac{1}{3!} z_0 (\phi_0)^3 \ni z_0 S_0 g_0 g_0^c$$

Colored Higgs mass :

$$m_{g_0/g_0^c} = z_0 \langle \phi_0 \rangle \simeq 2.4 \times M_1 = \mathcal{O}(10^{18})\text{GeV}$$

$\longrightarrow$ proton life : more than $10^{34}$ yr

In the eff. theory below $\langle \phi_0 \rangle$

$g_0$, $g_0^c$, $Q_0$, $L_0$, $\overline{Q}$, $\overline{L}$, $(S_0 - \overline{S})/\sqrt{2}$ : decouple

remaining fields in $\phi_0$, $\overline{\phi}$ : $\overline{g}$, $\overline{g^c}$, $(S_0 + \overline{S})/\sqrt{2}$

## < Q/L mass matrix and mixing >

(1) $M_{ij}\, Q_i U^c_j H_{u0}$

$$M_{ij} = m_{ij}\, x^{\mu_{ij}}$$

$$m_{ij} = O(1), \qquad x = \langle \phi_0 \rangle \langle \bar{\Phi} \rangle / M_1^2$$

(2)
$$\begin{array}{c} \\ g \\ D \end{array} \begin{array}{c} \begin{array}{cc} g^c & D^c \end{array} \\ \begin{pmatrix} \langle \phi_0 \rangle Z & \langle \psi_0 \rangle M \\ 0 & v_u M \end{pmatrix} \end{array} \quad : \quad g^c\text{–}D^c \text{ mixing}$$

$$(m_u, m_c, m_t) \sim (\lambda^{7.8}, \lambda^{5.2}, \lambda^{0.5}) \times v_u$$

$$(m_d, m_s, m_b) \sim (\lambda^{7.8}, \lambda^{6.7}, \lambda^{3.5}) \times v_d$$

$$(x^{6.3} \simeq \lambda)$$

$$V_{CKM} \sim \begin{pmatrix} 1 & \lambda & \lambda^5 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

$$(3)\qquad \begin{array}{c c} & \begin{array}{cc} H_u^+ & E^{c+} \end{array} \\ \begin{array}{c} H_d^- \\ L^- \end{array} & \begin{pmatrix} \langle\phi_0\rangle H & 0 \\ \langle\psi_0\rangle M & v_d M \end{pmatrix} \end{array} \quad : \ L\text{-}H_d \text{ mixing}$$

(4) $L$-$H_d$ mixing + seesaw mechanism

$$M_2^{-1}\cdot\left(\frac{\phi_0\bar{\Phi}}{M_1^2}\right)^{\nu_{ij}} (N_i^c \overline{N^c})(N_j^c \overline{N^c}) \quad : \ \text{R-handed } \nu \text{ Maj.}$$

$$(m_e,\ m_\mu,\ m_\tau) \sim (\lambda^{7.8},\ \lambda^{5.9},\ \lambda^{2.7}) \times v_d$$

$$(m_{\nu_1},\ m_{\nu_2},\ m_{\nu_3}) \sim (\lambda^{1.9},\ \lambda^{0.6},\ 1) \times 10^{-1}\,\mathrm{eV}$$

$$\begin{cases} \tan\theta_{12} \sim \lambda^{0.7} \\ \tan\theta_{23} \sim \lambda^{0.3} \\ \tan\theta_{13} \sim \lambda \end{cases}$$

(LMA solution)

Ref.


PTP 96 (1996) 1249
99 (1998) 831
100 (1998) 107
PL B487 (2000) 104
B499 (2001) 287
PTP 106 (2001) 1275
108 (2002) 965