Gauge-Higgs unification in E_6, E_7, E_8 5D SUSYGUTs

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☺ Plan of talk

1. Introduction
   - 1-1. motivation
   - 1-2. 5D N=1 SUSY GUT on S_1/Z_2
   - 1-3. gauge-Higgs unification in SU(6)
   - 1-4. 4D GUT
2. E_6 GUT
3. E_7 GUT
4. E_8 GUT
5. summary & discussion

1.Introduction

1-1: motivation

☆ motivation of introducing extraD

1. weakness of gravity → large extraD (ADD)
2. solution of GUT problems → extraD GUT
3. origin of adjoint Higgs → Hosotani mech.

S^1

- $A_5 = \Sigma_{24}$ dynamics of 5D gauge theory $\rightarrow \langle \Sigma_{24} \rangle$

S^1/Z_2

- $H_D \subset \Sigma_{24} \rightarrow$ doublet Higgs?
- $\psi^c_{5D} A_5 \psi_{5D} \rightarrow$ Yukawa int?

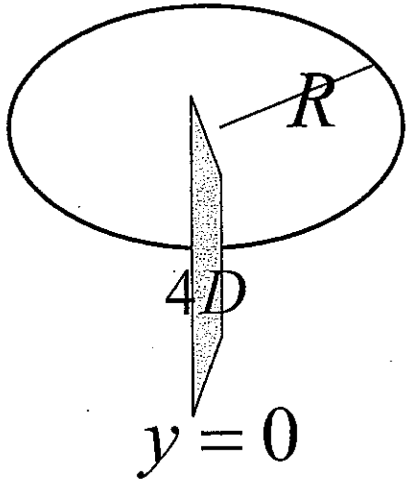
⇒ origin of Higgs doublets & Yukawa int.
(gauge-Higgs unification)

☆extra D⇔5D S_1/Z_2

(1): $M^4 \otimes S^1$

$$T:\ \phi(x^\mu, y + 2\pi R) = T\phi(x^\mu, y)$$

$[T \in U(N)]$



$$\phi(x^\mu, y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{\infty} \phi^{(n)}(x^\mu)\ e^{i\frac{n}{R}y}$$

$$\int_{y=0}^{2\pi R} dy \int dx^\mu \,|(\partial^\mu + \partial^y + ig_5 A^\mu + ig_5 A^5)\phi(x^\mu, y)|^2$$

$$\left(g_4 = \frac{g_5}{\sqrt{2\pi R}}\right)$$

$$\Rightarrow \int dx^\mu \,|(\partial^\mu + ig_4 A^\mu + ig_4 A^5)\phi^{(n)}(x^\mu)|^2 + \left(\frac{n}{R}\right)^2 |\phi^{(n)}(x^\mu)|^2$$

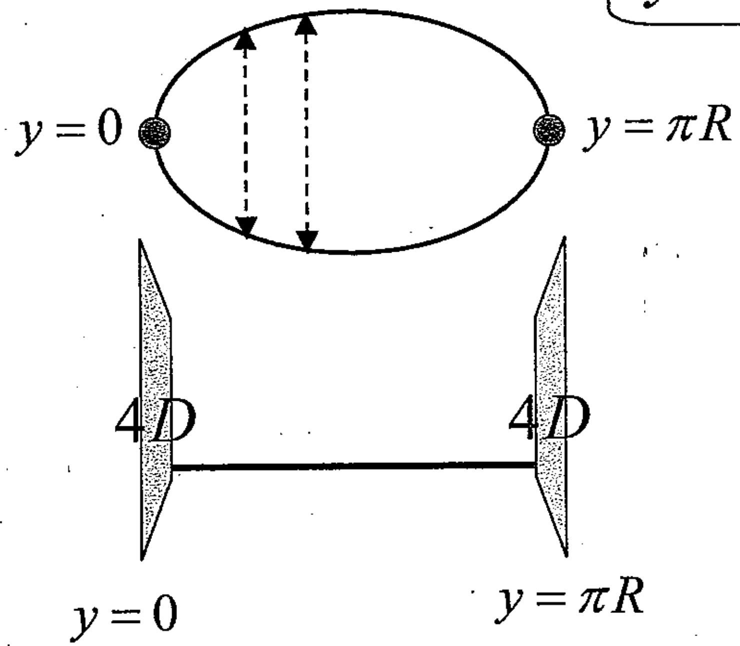
⇧ adjoint scalar ⇧ KK mass

(2): $M^4 \otimes S^1 / Z^2$

$y = -y$

$$P: \ \phi(x^\mu, -y) = P\phi(x^\mu, y)$$

$[P^2 = 1 \ \because \phi(y) = P\phi(-y) = P^2\phi(y)]$



$$\phi_+(x^\mu, y) = \frac{1}{\sqrt{2^{\delta_{n,0}}\pi R}} \sum_{n=0}^{\infty} \phi_+^{(n)}(x^\mu) \ \cos(\frac{ny}{R})$$

$$\phi_-(x^\mu, y) = \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \phi_-^{(n)}(x^\mu) \ \sin(\frac{ny}{R})$$

$$A_\mu(x^\mu, -y) = PA_\mu(x^\mu, y)P$$
$$A_5(x^\mu, -y) = -PA_5(x^\mu, y)P$$
$$\psi_L(x^\mu, -y) = P\psi_L(x^\mu, y)$$
$$\psi_R(x^\mu, -y) = -P\psi_R(x^\mu, y)$$

$[\psi(x^\mu, -y) = Pi\gamma^y\psi(x^\mu, y)]$

$5D: \ \gamma^M = (\gamma^\mu, i\gamma^5)$

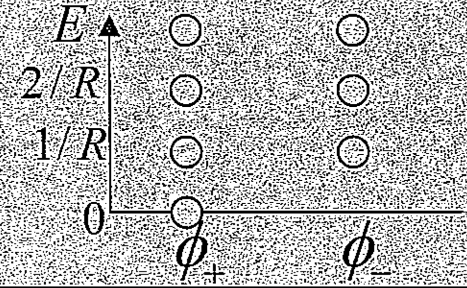
$$\int_{y=0}^{2\pi R} dy \int dx^\mu \, |(\partial^M + ig_5 A^M)\phi(x^\mu, y)|^2 \quad (g_4 = \frac{g_5}{\sqrt{2\pi R}})$$

$$\int_{y=0}^{2\pi R} dy \cos(\frac{ny}{R})\cos(\frac{my}{R}) = \int_{y=0}^{2\pi R} dy \frac{1}{2}[\cos(\frac{(n+m)y}{R}) + \cos(\frac{(n-m)y}{R})] = \frac{1}{2}(\delta_{n,m} + \delta_{n,-m})$$

$$\int_{y=0}^{2\pi R} dy \cos(\frac{ny}{R})\cos(\frac{my}{R})\cos(\frac{ky}{R}) = \frac{1}{4}(\delta_{n,m,k} + \delta_{n,-m,k} + \delta_{n,m,-k} + \delta_{n,-m,-k})$$

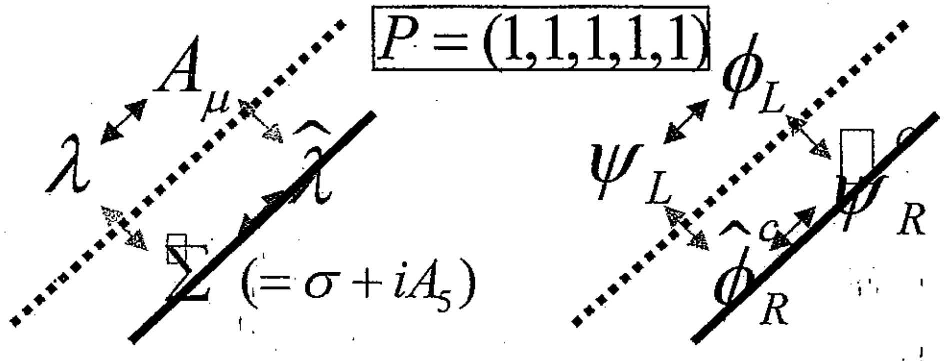
$$\phi^{(n)} - \phi^{(n)} - A^{(0)} : \ g_4$$

$$\phi^{(n)} - \phi^{(n)} - A^{(n)} : \ \frac{g_4}{\sqrt{2}}$$



1-2. 5D N=1 SUSY GUT on S_1/Z_2

$$P:\ \phi(x^\mu,-y)=P\phi(x^\mu,y)$$
$$A_\mu(x^\mu,-y)=PA_\mu(x^\mu,y)P$$
$$A_5(x^\mu,-y)=-PA_5(x^\mu,y)P$$
$$\psi_L(x^\mu,-y)=P\psi_L(x^\mu,y)$$
$$\psi_R(x^\mu,-y)=-P\psi_R(x^\mu,y)$$

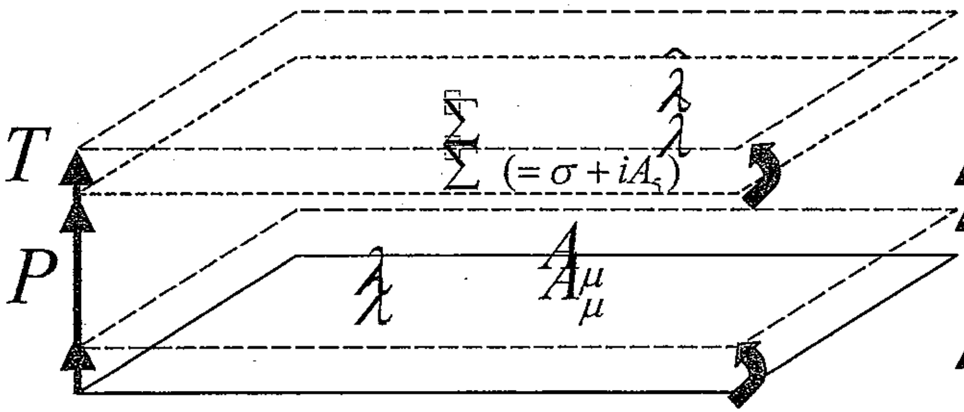


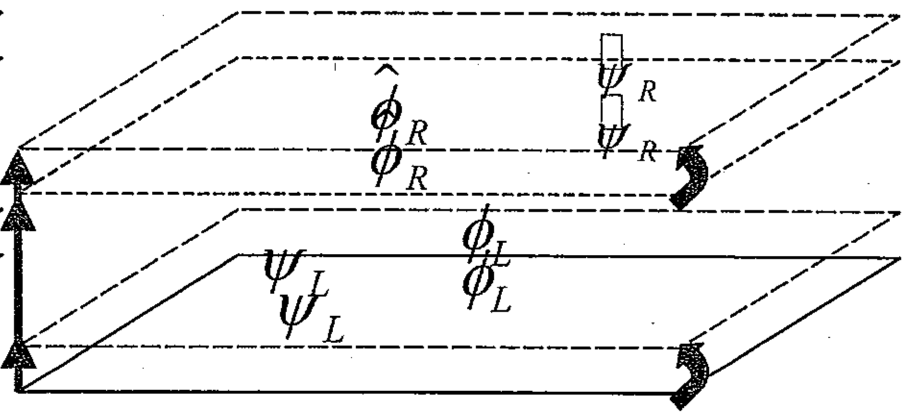
(4D N=1 SUSY)

$$\begin{pmatrix} g\ (\gamma) & X,Y \\ X,Y & W,Z \end{pmatrix}\ T:\phi(x^\mu,y+2\pi R)=T\phi(x^\mu,y)\ \begin{pmatrix} H_{T,\bar{T}} \\ H_{u,d} \end{pmatrix}$$

$$T=(-1,-1,-1,\ 1,\ 1)$$

(gauge reduction) (TD-splitting)





1-3. gauge-Higgs unification in SU(6)

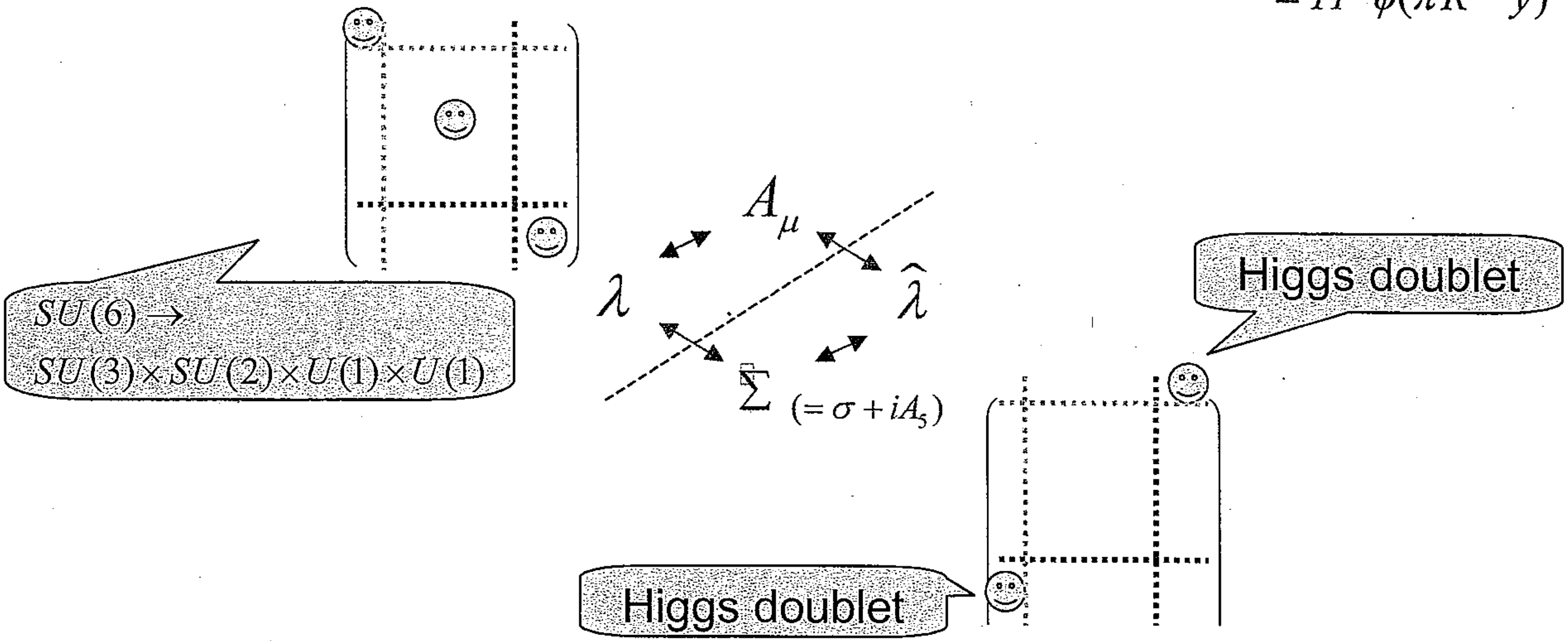
$$P=\begin{pmatrix}1&&&&&\\&1&&&&\\&&1&&&\\&&&1&&\\&&&&-1&\\&&&&&-1\end{pmatrix}\qquad P'=\begin{pmatrix}1&&&&&\\&-1&&&&\\&&-1&&&\\&&&-1&&\\&&&&-1&\\&&&&&-1\end{pmatrix}$$

$$P'=T\ P$$

P': *parity at* $y'\to -y'$

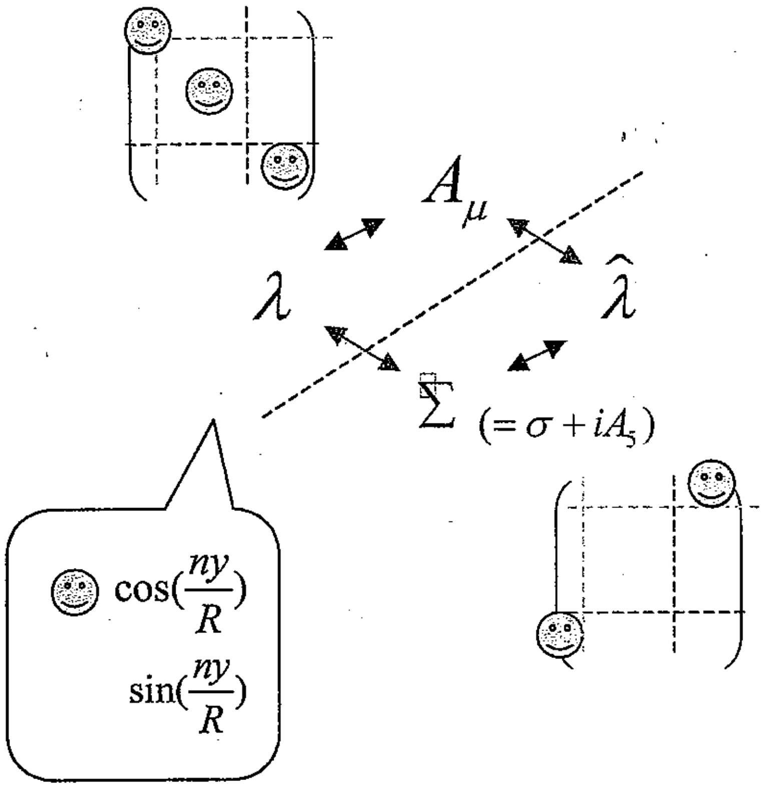
$(y'=\pi R+y)$

$$\phi(\pi R+y)=T\ \phi(-\pi R+y)$$
$$=TP\ \phi(\pi R-y)$$

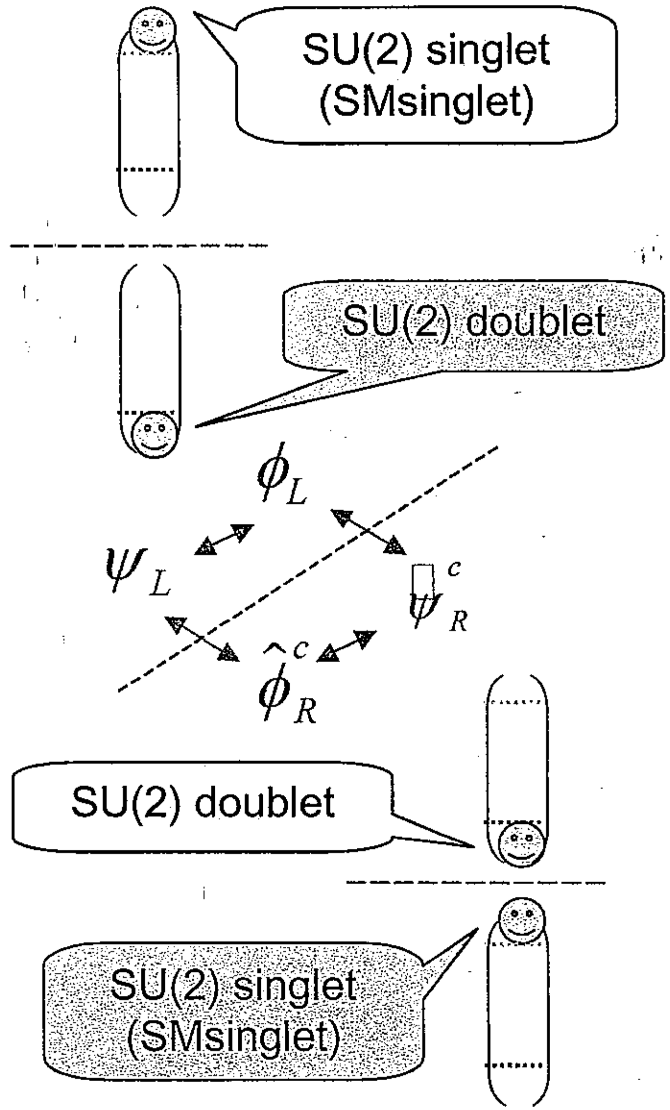


Yukawa?

$$\supset L = [\psi^c_{5D} \Sigma_5 \ \psi_{5D}]_{\theta^2} + h.c.$$



fund. rep. bulk matter



$\rightarrow$only ν_D Yukawa

($m_{d,e}$:15_{rep}. m_u :20_{rep}.) Burdman-Nomura

1-4. 4D GUT

charge quantization quark ⟺ lepton

$$SU(3)_C \times SU(2)_L \times U(1)_Y \subset SU(5) \subset SO(10) \subset E_6 \subset E_7 \subset E_8$$

$SU(5)$: $10 = (Q, \overline{U}, \overline{E})$, $\overline{5} = (\overline{D}, L)$, $1 = (\overline{N})$ $H_5, \overline{H_5}$

$$W_Y = \underset{m_u}{\underline{10 \cdot 10 \cdot H_5}} + \underset{m_d, m_e}{\underline{10 \cdot \overline{5} \cdot \overline{H_5}}} + \underset{m_\nu^D}{\underline{\overline{5} \cdot 1 \cdot H_5}} + \underset{M_R}{\underline{M \cdot 1 \cdot 1}}$$

$SO(10)$: $16 = 10 + \overline{5} + 1$ $10_H (+10'_H)$

$$W_Y = 16 \cdot 16 \cdot 10_H + 16 \cdot 16 \cdot 10'_H$$

$$V:\ 45 = [24_0 + 1_0] + [10_4 + \overline{10}_{-4}] \quad (SU(5) \times U(1))$$

E_6: $27 = 16 + 10 + 1$

$$W_Y = 27 \cdot 27 \cdot 27_H$$

$$V:\ 78 = [45_0 + 1_0] + [16_{-3} + \overline{16}_3] \quad (SO(10) \times U(1))$$

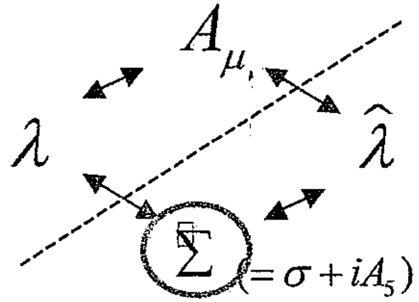
E_7: $133 = [78_0 + 1_0] + [27_1 + \overline{27}_{-1}]$ $(E_6 \times U(1))$

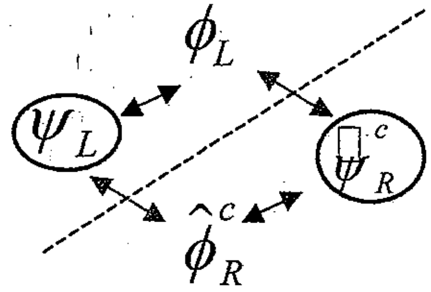
E_8: $248 = [(78,1) + (1,8)] + [(27,3) + (\overline{27}, \overline{3})]$ $(E_6 \times SU(3))$

1-4 (5D) GUT

charge quantization   quark ⟺ lepton

$$L = [\underline{\psi^c_{5D}}\ \underline{\Sigma_5}\ \underline{\psi_{5D}}]_{\theta^2} + h.c.$$





E_6 : $27 = 16 + 10 + 1$

$$W_Y = 27^c \cdot \Sigma_{78} \cdot 27 + 78^c \cdot \Sigma_{78} \cdot 78 + \cdots$$

V : $78 = [45_0 + 1_0] + [16_{-3} + \overline{16}_3]$

E_7 :

$$W_Y = 133^c \cdot \Sigma_{133} \cdot 133$$

$(E_6 \times U(1))$

E_8 :

$$W_Y = 248^c \cdot \Sigma_{248} \cdot 248$$

$(E_6 \times SU(3))$

2. E_6 GUT

S^1/Z_2 $\quad (V_{78}, \Sigma_{78})$

(0) ——— (πR)

$SU(5)\times U(1)^2$ $\quad E_6$ $\quad SU(5)'\times U(1)'^2$ $\quad \rightarrow SU(5)\times U(1)_V \times U(1)_X$

$P=1 \quad T=1$

$24_{(0,0)} + 1_{(0,0)V} + 1_{(0,0)X}$

$$V_{(gauge)}: \; 78 = [45_0 + 1_0] + [16_{-3} + \overline{16}_3] \quad (SO(10)\times U(1))$$

$$78 = [(35,1) + (1,3)] + [(20,3)] \quad (SU(6)\times SU(2))$$

$$\Sigma: \; 78 = [45_0 + 1_0] + [16_{-3} + \overline{16}_3] \quad (SO(10)\times U(1))$$

$$78 = [(35,1) + (1,3)] + [(20,3)] \quad (SU(6)\times SU(2))$$

Higgs

$5^H_{(-3,3)}, \; \bar{5}^H_{(3,-3)}, \; 1^H_{(-5,-3)}, \; 1^H_{(5,3)}$

S^1/Z_2 $\boxed{78:(\psi_L,\psi_R^c)\times 2}$

0 ——— πR

$SU(5)\times U(1)^2$ $\quad E_6 \quad$ ◄ $\quad SU(5)'\times U(1)'^2$

$$L=[\psi^c_{5D}\,\Sigma_5\,\psi_{5D}]_{\theta^2}+h.c.$$

$P=1 \quad T=1$

$\psi_L:\ 78=[45_0+1_0]+[16_{-3}+\overline{16}_3]$ $(SO(10)\times U(1))$

$78=[(35,1)+(1,3)]+[(20,3)]$ $(SU(6)\times SU(2))$

$\rightarrow\ 10_{(-1,-3)},\overline{10}_{(1,3)}$

$\psi_R^c:\ 78^c=[45_0+1_0]+[16_{-3}+\overline{16}_3]$ $(SO(10)\times U(1))$

$78^c=[(35,1)+(1,3)]+[(20,3)]$ $(SU(6)\times SU(2))$

$\rightarrow\ 10_{(4,0)},\overline{10}_{(-4,0)}$

$\psi_L:\ 78'=[45'_0+1'_0]+[16'_{-3}+\overline{16}'_3]$ $(SO(10)\times U(1))$

$78'=[(35,1)'+(1,3)']+[(20,3)']$ $(SU(6)\times SU(2))$

$\rightarrow\ 5'_{(-3,3)},\bar{5}'_{(3,-3)}$, $1'_{(-5,-3)},1'_{(5,3)}$

$\psi_R^c{}':\ 78^c{}'=[45'_0+1'_0]+[16'_{-3}+\overline{16}'_3]$ $(SO(10)\times U(1))$

$78^c{}'=[(35,1)'+(1,3)']+[(20,3)']$ $(SU(6)\times SU(2))$

$\rightarrow\ 24'_{(0,0)},1'_{(0,0)},1'_{(0,0)}$

S^1/Z_2 $78:(\psi_L,\psi_R^c)\times 2$

(0) —— (πR)

$SU(5)\times U(1)^2$ E_6 ◄ $SU(5)'\times U(1)'^2$

$P=1 \quad T=1$

$$L=[\psi^c_{5D}\,\Sigma_5\ \psi_{5D}]_{\theta^2}+h.c.$$

ψ_L :

ψ_R^c :

$$W_Y \supset \frac{10_{(4,0)}5^H_{(-3,3)}10_{(-1,-3)}}{m_u} + \overline{10}_{(-4,0)}\overline{5}^H_{(3,-3)}\overline{10}_{(1,3)}$$

$10_{(-1,-3)},\overline{10}_{(1,3)}$

↕

$10_{(4,0)},\overline{10}_{(-4,0)}$

$m_{d,e}$?

ψ_L :

$\psi_R^c{}'$:

$$W_Y \supset [24'_{(0,0)}+1'_{V(0,0)}+1'_{X(0,0)}]\times \frac{(5^H_{(-3,3)}\overline{5}'_{(3,-3)}+\overline{5}^H_{(3,-3)}5'_{(-3,3)})}{m^D_\nu} + \frac{[1'_{V(0,0)}+1'_{X(0,0)}](1^H_{(5,3)}1'_{(-5,-3)}+1^H_{(-5,-3)}1'_{(5,3)})}{M^\nu_R}$$

$5'_{(-3,3)},\overline{5}'_{(3,-3)}$
$1'_{(-5,-3)},1'_{(5,3)}$

↕

$24'_{(0,0)},1'_{(0,0)},1'_{(0,0)}$

S^1/Z_2 $27:(\psi_L,\psi_R^c)$

(0) ——— (πR)

$SU(5)\times U(1)^2$ E_6 ◂ $SU(5)'\times U(1)'^2$

$P=1 \quad T=1$

$$L=[\psi^c_{5D}\Sigma_5\ \psi_{5D}]_{\theta^2}+h.c.$$

$\psi_L:\ 27=[16^f_1]+[10^f_{-2}+1^f_4]$ $(SO(10)\times U(1))$ → $10^f_{(-1,1)}$

$27=[(15^f,1^f)]+[(\bar{6}^f,2^f)]$ $(SU(6)\times SU(2))$ → $10^f_{(-1,1)}$

$\psi_R^c:\ 27^c=[\overline{16}^f_{-1}]+[\overline{10}^f_2+1^f_{-4}]$ $(SO(10)\times U(1))$ → $\bar{5}^f_{(-2,2)}$

$27^c=[(\overline{15}^f,1^f)]+[(6^f,2^f)]$ $(SU(6)\times SU(2))$ → $\bar{5}^f_{(-2,2)}$

origin of chiral

S^1/Z_2 $27 : (\psi_L, \psi_R^c)$

(0) —— (πR)

$SU(5)\times U(1)^2$ E_6 ◄ $SU(5)'\times U(1)'^2$

$P=1 \quad T=1$

$$L = [\psi^c_{5D} \Sigma_5 \ \psi_{5D}]_{\theta^2} + h.c.$$

ψ_L :

$$W_Y \supset \frac{\bar{5}^f_{(-2,2)} \bar{5}^H_{(3,-3)} 10^f_{(-1,1)}}{m_{d,e}}$$

ψ_R^c :

$10^f_{(-1,1)}$ ↕ $\bar{5}^f_{(-2,2)}$

origin of chiral

$$SU(5)\times U(1)_V \times U(1)_X \quad (V_{24}+V_{1_V}+V_{1_X}, \quad 5^H+\bar{5}^H+1^H+\bar{1}^H)$$

(0) ———————————————— (πR)

$SU(5)\times U(1)^2$ $\qquad$ $SU(5)'\times U(1)'^2$

$$78: (\psi_L,\psi_R^c)+(\psi_L^{'},\psi_R^{c'}),\ \ 27:(\psi_L^f,\psi_R^{fc})$$

$$W=\psi_R^c\sum\psi_L$$

$$\rightarrow \underbrace{10_{(4,0)}5^H_{(-3,3)}10_{(-1,-3)}}_{m_u}+\overline{10}_{(-4,0)}\bar{5}^H_{(3,-3)}\overline{10}_{(1,3)}$$

$$+[24'_{(0,0)}+1'_{V(0,0)}+1'_{X(0,0)}](\underbrace{5^H_{(-3,3)}\bar{5}'_{(3,-3)}}_{m_\nu^D}+\bar{5}^H_{(3,-3)}5'_{(-3,3)})$$

$$+[1'_{V(0,0)}+1'_{X(0,0)}](1^H_{(5,3)}1'_{(-5,-3)}+\underbrace{1^H_{(-5,-3)}1'_{(5,3)}}_{M_\nu})$$

$$+\underbrace{\bar{5}^f_{(-2,2)}\bar{5}^H_{(3,-3)}10^f_{(-1,1)}}_{m_{d,e}}$$

$$(matter)\ \ \underline{(16_1 + \overline{16}_{-1}) \times 2},\ \ 10_{-2} + 10_2,\ \ \hat{1}_0$$

$$(higgs)\ \ 16_1^{\,h} + \overline{16}_{-1}^{\,h},\ \ \hat{1}_{-4}^{\,h} + \hat{1}_4^{\,h}$$

⓪ $SU(5)\times U(1)^2$ ... $(1)'^2$

$$W = \psi_R^c \Sigma \psi_L$$

$$\rightarrow 10_{(4,0)} 5^H_{(-3,3)} 10_{(-1,-3)} + \overline{10}_{(-4,0)} \overline{5}^H_{(3,-3)} \overline{10}_{(1,3)}$$

$$+ 1'_{X(0,0)} (5^H_{(-3,3)} \overline{5}'_{(3,-3)} + \overline{5}^H_{(3,-3)} 5'_{(-3,3)})$$

$$+ 1'_{X(0,0)} (1^H_{(5,3)} 1'_{(-5,-3)} + 1^H_{(-5,-3)} 1'_{(5,3)})$$

$$+ \overline{5}^f_{(-2,2)} \overline{5}^H_{(3,-3)} 10^f_{(-1,1)}$$

→ **one 10 remains in the low energy**

$SU(5)\times U(1)^2$ ⓪ (1)'^{2}

(*matter*) $(16_1 + \overline{16}_{-1})\times 2,\ \underline{10_{-2} + 10_2},\ \hat{1}_0$

(*higgs*) $16^h_1 + \overline{16}^h_{-1},\quad \hat{1}^h_{-4} + \hat{1}^h_4$

$$W = \psi^c_R \Sigma \psi_L$$

$$\rightarrow 10_{(4,0)} 5^H_{(-3,3)} 10_{(-1,-3)} + \overline{10}_{(-4,0)} \bar{5}^H_{(3,-3)} \overline{10}_{(1,3)}$$

$$+ 1'_{X(0,0)} (5^H_{(-3,3)} \bar{5}'_{(3,-3)} + \bar{5}^H_{(3,-3)} 5'_{(-3,3)})$$

$$+ 1'_{X(0,0)} (1^H_{(5,3)} 1'_{(-5,-3)} + 1^H_{(-5,-3)} 1'_{(5,3)})$$

$$+ \bar{5}^f_{(-2,2)} \bar{5}^H_{(3,-3)} 10^f_{(-1,1)}$$

→ **one $\bar{5}$ remains in the low energy**

(0) $SU(5)\times U(1)^2$

(*matter*) $(16_1 + \overline{16}_{-1})\times 2,\ 10_{-2} + 10_2,\ \hat{1}_0$

(*higgs*) $16^h_1 + \overline{16}^h_{-1},\ \hat{1}^h_{-4} + \hat{1}^h_4$

$(1)'^2$

$$W = \psi^c_R \Sigma \psi_L$$

$$\to 10_{(4,0)} 5^H_{(-3,3)} 10_{(-1,-3)} + \overline{10}_{(-4,0)} \bar{5}^H_{(3,-3)} \overline{10}_{(1,3)}$$

$$+ 1'_{X(0,0)} (5^H_{(-3,3)} \bar{5}'_{(3,-3)} + \bar{5}^H_{(3,-3)} 5'_{(-3,3)})$$

$$+ 1'_{X(0,0)} (1^H_{(5,3)} 1'_{(-5,-3)} + 1^H_{(-5,-3)} 1'_{(5,3)})$$

$$+ \bar{5}^f_{(-2,2)} \bar{5}^H_{(3,-3)} 10^f_{(-1,1)}$$

→ **one 1 remains in the low energy**

⓪

$SU(5)\times U(1)^2$ … $U(1)'$

(*matter*) $(16_1 + \overline{16}_{-1})\times 2,\ 10_{-2} + 10_2,\ 1_0$

(*higgs*) $\langle 16_1^h \rangle + \langle \overline{16}_{-1}^h \rangle,\ \langle 1_{-4}^h \rangle + \langle 1_4^h \rangle$

$\Rightarrow 1^h_{(-5,1)} + \bar{1}^h_{(5,-1)},\ 1^h_{(0,-4)} + 1^h_{(0,4)}$

$$W = \psi_R^c \Sigma \psi_L$$

$$\rightarrow 10_{(4,0)} 5^H_{(-3,3)} 10_{(-1,-3)} + \overline{10}_{(-4,0)} \bar{5}^H_{(3,-3)} \overline{10}_{(1,3)}$$

$$W^{4D} \supset \overline{16}_{-1_k}\left(m_{10_k}\langle 1_4^h\rangle 16_{-3} + m_{10_k^c}\langle 16_1^h\rangle 45_0 + m_{10_k^f} 16_1^f\right)$$

$$+16_{1_k}\left(m_{10_k}\langle 1_{-4}^h\rangle \overline{16}_3 + m_{10_k^c}\langle \overline{16}_1^h\rangle 45_0\right) + M_{10_k} 16_{1_k} \overline{16}_{-1_k}$$

$$10^0 ;\ \cos\phi_1 \cos\phi_3 10_{(-1,-3)} + \sin\phi_1 \cos\phi_3 10_{(4,0)} + \sin\phi_3 10^f_{(-1,1)}$$

$$\tan\phi_1 = \frac{\langle 1_4^h\rangle (m_{10_2} m^f_{10_1} - m_{10_1} m^f_{10_2})}{\langle 16_1^h\rangle (m_{10_1^c} m^f_{10_2} - m_{10_2^c} m^f_{10_1})}, \ldots$$

(0) $SU(5)\times U(1)^2$ … $(1)'^2$

(matter) $(16_1 + \overline{16}_{-1})\times 2,\ 10_{-2} + 10_2,\ 1_0$

(higgs) $\langle 16_1^h \rangle + \langle \overline{16}_{-1}^h \rangle\ \ \langle 1_{-4}^h \rangle + \langle 1_4^h \rangle$

$\Rightarrow 1^h_{(-5,1)} + \bar{1}^h_{(5,-1)},\ \ 1^h_{(0,-4)} + 1^h_{(0,4)}$

$$W^{eff} \supset y_u 10^0 10^0 5^H$$

$$y_u \equiv \frac{g}{2}\sin 2\phi_1 \cos^2\phi_3$$

$$W^{4D} \supset \overline{16}_{-1_k}\left(m_{10_k}\langle 1_4^h\rangle 16_{-3} + m_{10_k^c}\langle 16_1^h\rangle 45_0 + m_{10_k^f} 16_1^f\right)$$

$$+16_{1_k}\left(\bar{m}_{10_k}\langle 1_{-4}^h\rangle \overline{16}_3 + \bar{m}_{10_k^c}\langle \overline{16}_1^h\rangle 45_0\right) + M_{10_k} 16_{1_k}\overline{16}_{-1_k}$$

$$10^0 \sim \cos\phi_1\cos\phi_3 10_{(-1,-3)} + \sin\phi_1\cos\phi_3 10_{(4,0)} + \sin\phi_3 10^f_{(-1,1)}$$

$$\tan\phi_1 = \frac{\langle 1_4^h\rangle (m_{10_2} m^f_{10_1} - m_{10_1} m^f_{10_2})}{\langle 16_1^h\rangle (m_{10_1^c} m^f_{10_2} - m_{10_2^c} m^f_{10_1})}, \ldots$$

⓪ $SU(5)\times U(1)^2$

(matter) $(10_1 + \overline{10}_{-1})\times 2,\ 10_{-2} + 10_2,\ 1_0$

(higgs) $\langle 10^h_1\rangle + \langle \overline{10}^h_{-1}\rangle,\ \langle 1^h_{-4}\rangle + \langle 1^h_4\rangle$

$$\Rightarrow 1^h_{(-5,1)} + \bar{1}^h_{(5,-1)},\quad 1^h_{(0,-4)} + 1^h_{(0,4)}$$

$(1)'^2$

$$W^{4D} \supset \overline{10}_2\left(m_{5'}\langle 10^h_{-1}\rangle 16_{-3} + m_{5^f} 10^f_{-2}\right)$$

$$+10_{-2}\left(m_{5'}\langle 10^h_{-1}\rangle \overline{16}_3\right) + M_5 10_2 10_{-2}$$

$$\bar{5}^0 \,;\ -\sin\theta_5 \bar{5}'_{(3,-3)} + \cos\theta_5 \bar{5}^f_{(-2,2)} \qquad \tan\theta_5 = \frac{\langle 10^h_1\rangle m_{5'}}{m^f_5}$$

$$W^{eff} \supset y_{d,e} 10^0 \bar{5}^0 \bar{5}^H$$

$$y_{d,e} \equiv -g\sin\phi_3 \sin\theta_5$$

$\textcircled{0}$

$SU(5)\times U(1)^2$ … $U(1)'^2$

(matter) $(\tilde{16}_1+\overline{\tilde{16}}_{-1})\times 2,\ \tilde{16}_{-2}+\tilde{16}_2,\ \tilde{1}_0$

(higgs) $\langle\tilde{16}_1^h\rangle+\langle\overline{\tilde{16}}_{-1}^h\rangle,\ \langle\tilde{1}_{-4}^h\rangle+\langle\tilde{1}_4^h\rangle$

$\Rightarrow \tilde{1}^h_{(-5,1)}+\bar{\tilde{1}}^h_{(5,-1)},\quad \tilde{1}^h_{(0,-4)}+\tilde{1}^h_{(0,4)}$

$$W^{4D}\supset \tilde{1}_0(m_{1'c}\langle\tilde{1}_4^h\rangle\langle\overline{\tilde{16}}_{-1}^h\rangle 16'_{-3}+m_{1'}1'_0)+m_{1'c}\tilde{1}_0\langle\tilde{1}_{-4}^h\rangle\langle\tilde{16}_1^h\rangle 16'_{-3}+M_1\tilde{1}_0^2$$

$$1^0;\ -\sin\theta_1 1'_{(-5,-3)}+\cos\theta_1 1'_{X(0,0)}\qquad \tan\theta_1=\frac{\langle\tilde{1}_0^h\rangle\langle\tilde{16}_1^h\rangle m_{1'c}}{m_{1'}}$$

$$+1'_{X(0,0)}(5^H_{(-3,3)}\bar{5}'_{(3,-3)}+\bar{5}^H_{(3,-3)}5'_{(-3,3)})$$

$$+1'_{X(0,0)}(1^H_{(5,3)}1'_{(-5,-3)}+1^H_{(-5,-3)}1'_{(5,3)})$$

$$W^{eff}\supset y_\nu\bar{5}^0 1^0 5^H+y_M 1^0 1^0 1^H$$

$$y_\nu\equiv -g\sin\theta_5\cos\theta_1,\quad y_M\equiv -g\sin 2\theta_1/2$$

1-generation model

$SU(5)\times U(1)_V \times U(1)_X \quad (V_{24}+V_{1_V}+V_{1_X}, \quad 5^H+\overline{5}^H+1^H+\overline{1}^H)$

(0) ——————————————— (πR)

$SU(5)\times U(1)'^2$ $\quad$ $78: (\psi_L, \psi_R^c)+(\psi_L', \psi_R^{c'}), \; 27: (\psi_L^f, \psi_R^{fc})$ $\quad$ $SU(5)\times U(1)'^2$

$(matter) \quad (16_1+\overline{16}_{-1})\times 2, \; 10_{-2}+10_2, \; 45_0, \; 1_0$

$(higgs) \quad 16_1^h + \overline{16}_{-1}^h, \quad 1_{-4}^h + 1_4^h$

$$W^{eff} = y_u 10^0 10^0 5^H + y_{d,e} 10^0 \overline{5}^0 \overline{5}^H + y_\nu \overline{5}^0 1^0 5^H + y_M 1^0 1^0 1^H$$

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3-generation model

$$SU(5)\times U(1)_V\times U(1)_X \quad (V_{24}+V_{1_V}+V_{1_X}, \quad 5^H+\bar{5}^H+1^H+\bar{1}^H)$$

0 ——— πR

$SU(5)\times U(1)'^2$

$$78: (\psi_L,\psi_R^c)+(\psi_L',\psi_R^{c'}),\ 27: (\psi_L^f,\psi_R^{fc}) \times 3$$

$SU(5)\times U(1)'^2$

$$(matter)\ \left[(\mathbf{16}_1+\overline{\mathbf{16}}_{-1})\times 3,\ \mathbf{10}_{-2}+\mathbf{10}_2,\ \mathbf{45}_0,\ \mathbf{1}_0\right]\times 3$$

$$(higgs)\quad \mathbf{16}_1^h + \overline{\mathbf{16}}_{-1}^h,\quad \mathbf{1}_{-4}^h + \mathbf{1}_4^h$$

$$W^{eff} = y_u 10^0 10^0 5^H + y_{d,e} 10^0 \bar{5}^0 \bar{5}^H + y_\nu \bar{5}^0 1^0 5^H + y_M 1^0 1^0 1^H$$

$$(M_{10})_1/m:\ \lambda^4,\ (M_{10})_2/m:\ \lambda^2,\ (M_{10})_3/m:\ 1$$

$$m \equiv m_{10_k}V:\ m_{10_k^c}V:\ m_{10_k^f}:\ \bar{m}_{10_k}V:\ \bar{m}_{10_k^c}V$$

$$W^{eff} = y_u 10 \cdot 10 \cdot 5^H + y_{d,e} 10 \cdot \bar{5} \cdot \bar{5}^H + y_\nu \bar{5} \cdot 1 \cdot 5^H + M_R \cdot 1 \cdot 1$$

$$10_3 \to 10_3,\ 10_2 \to \lambda^2 10_2,\ 10_1 \to \lambda^4 10_1$$

$$10 = (Q, \overline{U}, \overline{E}),\ \bar{5} = (\overline{D}, L),\ 1 = (\overline{N})$$

$$W = y^u 10\cdot 10\cdot H_5 + y^{d/e} 10\cdot \overline{5}\cdot \overline{H_5} + y^{\nu}\overline{5}\cdot 1\cdot H_5 + M_R\cdot 1\cdot 1$$

$$10_i = (Q,\overline{U},\overline{E})_i \quad [10_3 \to 10_3,\ 10_2 \to \lambda^2 10_2,\ 10_1 \to \lambda^4 10_1]$$

$$m_u\ ;\ \overset{L\qquad\qquad R}{\begin{pmatrix} \lambda^8 & \lambda^6 & \lambda^4 \\ \lambda^6 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix}} \langle H_u\rangle \Longrightarrow L\begin{pmatrix} 1 & \lambda^2 & \lambda^4 \\ \lambda^2 & 1 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix}\begin{pmatrix} \lambda^8 & & \\ & \lambda^4 & \\ & & 1 \end{pmatrix}\begin{pmatrix} 1 & \lambda^2 & \lambda^4 \\ \lambda^2 & 1 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} R\ \langle H_u\rangle$$

$$m_d\ ;\ \begin{pmatrix} \lambda^4 & \lambda^4 & \lambda^4 \\ \lambda^2 & \lambda^2 & \lambda^2 \\ 1 & 1 & 1 \end{pmatrix} \langle H_d\rangle \Longrightarrow \begin{pmatrix} 1 & \lambda^2 & \lambda^4 \\ \lambda^2 & 1 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix}\begin{pmatrix} \lambda^4 & & \\ & \lambda^2 & \\ & & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \langle H_d\rangle$$

$$m_l\ ;\ \begin{pmatrix} \lambda^4 & \lambda^2 & 1 \\ \lambda^4 & \lambda^2 & 1 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} \langle H_d\rangle \Longrightarrow \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} \lambda^4 & & \\ & \lambda^2 & \\ & & 1 \end{pmatrix}\begin{pmatrix} 1 & \lambda^2 & \lambda^4 \\ \lambda^2 & 1 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} \langle H_d\rangle$$

$$m_\nu\ ;\ \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \frac{\langle H_u\rangle^2}{M_R} \Longrightarrow \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \frac{\langle H_u\rangle^2}{M_R}$$

$$W = y^u 10\cdot 10\cdot H_5 + y^{d/e} 10\cdot \bar{5}\cdot \overline{H_5} + y^\nu \bar{5}\cdot 1\cdot H_5 + M_R \cdot 1\cdot 1$$

$$10_i = (Q,\overline{U},\overline{E})_i \quad [10_3 \to 10_3,\ 10_2 \to \lambda^2 1\ldots$$

$$m_u ;\ \overset{L\qquad\qquad R}{\begin{pmatrix} \lambda^8 & \lambda^6 & \lambda^4 \\ \lambda^6 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix}} \langle H_u \rangle \Rightarrow \overset{L}{\begin{pmatrix} 1 & \lambda^2 & \lambda^4 \\ \lambda^2 & 1 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix}}^{\dagger} \begin{pmatrix} \lambda^8 & & \\ & \ldots & \\ & & \ldots \end{pmatrix}$$

$$m_d ;\ \begin{pmatrix} \lambda^4 & \lambda^4 & \lambda^4 \\ \lambda^2 & \lambda^2 & \lambda^2 \\ 1 & 1 & 1 \end{pmatrix} \langle H_d \rangle \Rightarrow \begin{pmatrix} 1 & \lambda^2 & \lambda^4 \\ \lambda^2 & 1 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} \begin{pmatrix} \lambda^4 & & \\ & \lambda^2 & \\ & & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \langle H_d \rangle$$

$$m_l ;\ \begin{pmatrix} \lambda^4 & \lambda^2 & 1 \\ \lambda^4 & \lambda^2 & 1 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} \langle H_d \rangle \Rightarrow \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}^{\dagger} \begin{pmatrix} \lambda^4 & & \\ & \ldots & \\ & & \ldots \end{pmatrix} \begin{pmatrix} 1 & \lambda^2 & \lambda^4 \\ \ldots & \ldots & \ldots \end{pmatrix}$$

$$m_\nu ;\ \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \frac{\langle H_u \rangle^2}{M_R} \Rightarrow \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & & \\ & \ldots & \\ & & \ldots \end{pmatrix}$$

$$V_{CKM} ;\ \begin{pmatrix} 1 & \lambda^2 & \lambda^4 \\ \lambda^2 & 1 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix}$$

$$V_{MNS} ;\ \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

coefficients of O(1):

determination of O(1) coefficients

↓

high rep. Higgs & vector-like fields at high energy

[16 vector like, 45 Higgs ……(Babu-Barr)]

↓

integrating out heavy fields

$$m_u \;;\; \begin{pmatrix} 0 & -4d\lambda^6 & 0 \\ -4d\lambda^6 & c\lambda^4 & 0 \\ 0 & b\lambda^2 & 1 \end{pmatrix} \langle H_u \rangle, \quad m_d \;;\; \begin{pmatrix} 4d\lambda^4 & d\lambda^4 & d\lambda^4 \\ d/5\lambda^2 & d\lambda^2 & d\lambda^2 \\ c/2 & b & 1 \end{pmatrix} \langle H_d \rangle,$$

$$m_l \;;\; \begin{pmatrix} \lambda^4 & 0 & 0 \\ b\lambda^4 & -2c\lambda^2 & 1 \\ 0 & -b\lambda^2 & 5 \end{pmatrix} \langle H_d \rangle, \quad m_\nu \;;\; \begin{pmatrix} e & e & 0 \\ 0 & c & 2.5 \\ 0 & 2.5 & 5 \end{pmatrix} \frac{\langle H_u \rangle^2}{M_R}$$

$$(b = 4,\; c = 3.6,\; d = 2,\; e = 1)$$

(Maru,Nakamura,NH)

When $10_i = (Q, \overline{U}, \overline{E})_i$ produce hierarchy,

Good Points:

$$m_u : m_c : m_t : \ \lambda^8 : \lambda^4 : 1$$

$$m_d : m_s : m_b : \ m_e : m_\mu : m_\tau : \ \lambda^4 : \lambda^2 : 1$$

$$m_{\nu_e} : m_{\nu_\mu} : m_{\nu_\tau} : \ 1:1:1$$

small flavor mixing in Quark $\Leftrightarrow$ large flavor mixing in Lepton

Bad Points:

$m_d, m_e, \quad m_\mu : m_s$

too large U_{e3}, too small V_{us} *in order*

modification

$+$*one more* $[\mathbf{10}_2 + \overline{\mathbf{10}}_{-2}]$ *at* $y = 0$

$$(M_5)_1 / m : \lambda^2,\ (M_5)_{2,3} / m : 1$$

$$m_5 \equiv m_{5^f_l} : m_{5'_l} V : \overline{m}_{5'_l}$$

$$V_{CKM} ; \begin{pmatrix} 1 & \lambda^2 & \lambda^4 \\ \lambda^2 & 1 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix}$$

$$m_u ; \begin{pmatrix} \lambda^8 & \lambda^6 & \lambda^4 \\ \lambda^6 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} \langle H_u \rangle, m_d ; \begin{pmatrix} \lambda^4 & \lambda^4 & \lambda^4 \\ \lambda^2 & \lambda^2 & \lambda^2 \\ 1 & 1 & 1 \end{pmatrix} \langle H_d \rangle$$

$$m_l ; \begin{pmatrix} \lambda^4 & \lambda^2 & 1 \\ \lambda^4 & \lambda^2 & 1 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} \langle H_d \rangle, m_\nu ; \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \frac{\langle H_u \rangle^2}{M_R}$$

$$V_{MNS} ; \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

modification

+one more $[\mathbf{10}_2 + \overline{\mathbf{10}}_{-2}]$ at $y = 0$

$$(M_5)_1 / m : \lambda^2, \; (M_5)_{2,3} / m : 1$$

$$m_5 \equiv m_{5_l^f} : m_{5'_l} V : \overline{m}_{5'_l}$$

$$V_{CKM} ; \begin{pmatrix} 1 & \lambda^2 & \lambda^4 \\ \lambda^2 & 1 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix}$$

$$m_u ; \begin{pmatrix} \lambda^8 & \lambda^6 & \lambda^4 \\ \lambda^6 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} \langle H_u \rangle, m_d ; \begin{pmatrix} \lambda^6 & \lambda^4 & \lambda^4 \\ \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & 1 & 1 \end{pmatrix} \langle H_d \rangle$$

$$m_l ; \begin{pmatrix} \lambda^6 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} \langle H_d \rangle, m_\nu ; \begin{pmatrix} \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & 1 & 1 \\ \lambda^2 & 1 & 1 \end{pmatrix} \frac{\langle H_u \rangle^2}{M_R}$$

$$V_{MNS} ; \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & \lambda^2 \\ 1/2 & -1/2 & 1/\sqrt{2} \\ -1/2 & 1/2 & 1/\sqrt{2} \end{pmatrix}$$

3. E_7GUT

S^1/Z_2 $\quad (V_{133}, \Sigma_{133})$

0 ——— πR

$SO(10)\times U(1)^2$ $\quad E_7$ $\quad SO(10)'\times U(1)'^2$

$\rightarrow SO(10)\times U(1)_X \times U(1)_Z$

$P=1 \quad T=1$

$$45_{(0,0)} + 1_{(0,0)X} + 1_{(0,0)Z}$$

$$V_{(gauge)}: \; 133 = [78_0 + 1_0] + [27_1 + \overline{27}_{-1}] \qquad (E_6\times U(1)_Z)$$

$$133 = [(66,1) + (1,3)] + [(32',2)] \qquad (SO(12)\times SU(2))$$

$$\Sigma: \; 133 = [78_0 + 1_0] + [27_1 + \overline{27}_{-1}] \qquad (E_6\times U(1)_Z)$$

$$133 = [(66,1) + (1,3)] + [(32',2)] \qquad (SO(12)\times SU(2))$$

Higgs

$$10^H_{(-2,1)},\; \overline{10}^H_{(2,-1)},\; 1^H_{(4,1)},\; 1^H_{(-4,-1)}$$

S^1/Z_2 $\boxed{133:(\psi_L,\psi_R^c)}$

(0) ———— (πR)

$SO(10)\times U(1)^2$ $\quad E_7 \quad \leftarrow \quad SO(10)'\times U(1)'^2$

$$L=[\psi^c_{5D}\,\Sigma_5\,\psi_{5D}]_{\theta^2}+h.c.$$

$P=1 \quad T=1$

$$\psi_L:\ 133=[78_0+1_0]+[27_1+\overline{27}_{-1}] \quad (E_6\times U(1)_Z)$$

$$133=[(66,1)+(1,3)]+[(32',2)] \quad (SO(12)\times SU(2))$$

$\rightarrow \boxed{16_{(-3,0)},\overline{16}_{(3,0)}}$

$$\psi_R^c:\ 133^c=[78_0+1_0]+[27_1+\overline{27}_{-1}] \quad (E_6\times U(1)_Z)$$

$$133^c=[(66,1)+(1,3)]+[(32',2)] \quad (SO(12)\times SU(2))$$

$\rightarrow \boxed{16_{(1,1)},\overline{16}_{(-1,-1)}}$

S^1/Z_2 $\boxed{133 : (\psi_L, \psi_R^c)}$

(0) ———————— (πR)

$SO(10)\times U(1)^2$ E_7 ◄ $SO(10)'\times U(1)'^2$

$P=1 \quad T=1$

$$L = [\psi^c_{5D} \Sigma_5 \ \psi_{5D}]_{\theta^2} + h.c.$$

ψ_L : $16_{(-3,0)}, \overline{16}_{(3,0)}$

ψ_R^c : $16_{(1,1)}, \overline{16}_{(-1,-1)}$

$$W_Y ; \ \underline{16_{(1,1)} 10^H_{(2,-1)} 16_{(-3,0)}} + \overline{16}_{(-1,-1)} 10^H_{(-2,1)} \overline{16}_{(3,0)}$$

$$+16_{(1,1)} 1^H_{(-4,-1)} \overline{16}_{(3,0)} + \overline{16}_{(-1,-1)} 1^H_{(4,1)} 16_{(-3,0)}$$

⓪ $SO(10)\times U(1)$ … $U(1)'^2$

(matter) 27_1, 78_0

$\langle 27^h_1\rangle + \langle \overline{27}^h_{-1}\rangle$, $\langle 1^h_1\rangle + \langle 1^h_{-1}\rangle$

(higgs) $\Rightarrow 1^h_{(4,1)} + 1^h_{(-4,-1)}$ $1^h_{(0,1)} + 1^h_{(0,-1)}$

flavor mixings
$\Leftarrow$ *add. bulk matter*
hierarchy
$\Leftarrow$ *4D phys.*

$$W = \psi^c_R \Sigma \psi_L$$

$$W^{eff} ;\ y_{10} 16^0 16^0 10^H$$

$$y_{10} \equiv -2g \sin 2\theta_{16}$$

$$W^{4D} ;\ 78_0 (m_E \langle \overline{27}^h_{-1}\rangle 27^H_1 + m'_E 78_0)$$

$$16^0 = -\sin\theta_{16} 16_{(1,1)} + \cos\theta_{16} 16_{(-3,0)}$$

$$\tan\theta_{16} = m'_E / (\langle \overline{27}^h_{-1}\rangle m_E)$$

4. E_8GUT

S^1/Z_2 $\quad (V_{248}, \Sigma_{248})$

(0) —— (πR) $\quad \rightarrow E_6 \times SU(2) \times U(1)$

$E_6 \times SU(2) \times U(1)$ $\quad E_8$ $\quad E_6 \times SU(2) \times U(1)$

$$(78,1)_0 + (1,3)_0 + (1,1)_0$$

$$P=1 \quad T=1$$

$$V_{(gauge)}: \; 248 = [(78,1)+(1,8)] + [(27,3)+(\overline{27},\overline{3})] \quad (E_6 \times SU(3))$$

$$248 = [(133,1)+(1,3)] + [(56,2)] \quad (E_7 \times SU(2))$$

$$\Sigma: \; 248 = [(78,1)+(1,8)] + [(27,3)+(\overline{27},\overline{3})] \quad (E_6 \times SU(3))$$

$$248 = [(133,1)+(1,3)] + [(56,2)] \quad (E_7 \times SU(2))$$

Higgs

$$(27,1)_2^H, \; (\overline{27},1)_{-2}^H$$

S^1/Z_2 $\boxed{248:(\psi_L,\psi_R^c)}$

0 ——— πR

$E_6\times SU(2)\times U(1)$ $\quad E_8 \quad$ ◄ $\quad E_6\times SU(2)\times U(1)$

$P=1 \quad T=1$

$$L=[\psi_{5D}^c \Sigma_5 \ \psi_{5D}]_{\theta^2}+h.c.$$

$$\psi_L:\ 248=[(78,1)+(1,8)]+[(27,3)+(\overline{27},\overline{3})]_{(E_6\times SU(3))}$$

$$248=[(133,1)+(1,3)]+[(56,2)] \quad (E_7\times SU(2))$$

→ $(78,1)_0,(1,3)_0,(1,1)_0$

$$\psi_R^c:\ 248^c=[(78,1)+(1,8)]+[(27,3)+(\overline{27},\overline{3})]_{(E_6\times SU(3))}$$

$$248^c=[(133,1)+(1,3)]+[(56,2)] \quad (E_7\times SU(2))$$

→ $(27,1)_2,(\overline{27},1)_{-2}$

S^1/Z_2 $\quad$ $248 : (\psi_L, \psi_R^c)$

0 —— πR

$E_6 \times SU(2) \times U(1)$ $\quad E_8 \quad$ ◂ $\quad E_6 \times SU(2) \times U(1)$

$P=1 \quad T=1$

$$L = [\psi_{5D}^c \Sigma_5 \ \psi_{5D}]_{\theta^2} + h.c.$$

ψ_L :

ψ_R^c :

$$W_Y \ ; \ [(78,1)_0 + (1,3)_0 + (1,1)_0](\overline{27},1)_{-2}^H (27,1)_2$$

$$+[(78,1)_0 + (1,3)_0 + (1,1)_0](27,1)_2^H (\overline{27},1)_{-2}$$

$(78,1)_0, (1,3)_0, (1,1)_0$

$(27,1)_2, (\overline{27},1)_{-2}$

$$\supset 16_{(-3,0)} 10^H_{(2,-2)} 16_{(1,2)}$$

$(SO(10) \times U(1)^2)$

many un-wanted fields

5.summary and discussion

5D E_6, E_7, E_8 GUTs on S^1/Z_2

- $H_D \subset \Sigma_{24} \rightarrow$ doublet Higgs
- $\psi^c_{5D} A_5\, \psi_{5D} \rightarrow$ Yukawa ints.

⇒ **origin of Higgs doublets & Yukawa int.**

gauge-Higgs unification!

E_6: bulk matters ⇒ **adjoint & fund.**

$E_{7,8}$: bulk matters ⇒ **adjoint**

☆SUSY br.

1. Giudice-Masiero: $[\Sigma^2]_D \Rightarrow \mu$ 2. Scherk-Schwarz: $\Rightarrow \mu$

☆TD-splitting:

1. non-local OP in W: $\Rightarrow P\exp(\int \Sigma\, dy)$
2. missing partner in bulk: (example E_6)

$$(1728, 1728^c) \supset (\overline{50}_{(3,1)}, 75_{(0,-4)}), \qquad (\overline{1728}, \overline{1728}^c) \supset (50_{(-3,-1)}, 75_{(0,4)})$$

$$1728^c\, \Sigma_{78}\, 1728 \supset \langle 75_{(0,-4)} \rangle\, 5^H_{(-3,3)}\, \underbrace{\overline{50}_{(3,1)}}_{\supset (3,1)_2}, \quad \overline{1728}^c\, \Sigma_{78}\, \overline{1728} \supset \langle 75_{(0,4)} \rangle\, \overline{5}^H_{(3,-3)}\, \underbrace{50_{(-3,-1)}}_{\supset (3,1)_{-2}}$$