"SO(10) GUT model and predictions on neutrino parameters and rare decays."

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Motivations.

- Construct a predictive GUT model.
- Consider SO(10) GUT.

  16 chiral fermions including ν_R.
- Minimum Higgs scalars in Yukawa coupling. 10 $\overline{126}$
- Quark-Lepton (and M_R) mass can be expressed in terms of two mass matrices.

- Data fitting

input Quark masses and CKM
charged lepton masses

$\xrightarrow[\text{(bottom up)}]{RGE}$ GUT scale and fit the mass relations at GUT scale

$$\boxed{\begin{array}{lll} M_u = C_{10} M_{10} + C_{126} M_{126} & & \text{up type quark} \\ M_d = M_{10} + M_{126} & & \text{down} \quad " \\ M_D = C_{10} M_{10} - 3 C_{126} M_{126} & & \text{Dirac neutrino} \\ M_e = M_{10} - 3 M_{126} & & \text{charged lepton} \\ M_R = C_R M_{126} & & \text{RH heavy Majorana} \end{array}}$$

and construct light neutrino mass matrix at GUT scale

$$M_\nu = M_D^T M_R^{-1} M_D$$

$\xrightarrow[\text{(top down)}]{RGE}$ Electro-Weak scale and fit with the oscillation data.

§2. Minimal SO(10) model

§3 one-loop RGE (省略)

§4. Numerical analysis and results

§5 The other observations

Jarlskog parameter

$\langle m_\nu \rangle_{ee}$ in $(\beta\beta)_{0\nu}$

CP violating parameter ϵ in leptogenesis

Branching Ratio of $\mu \to e\gamma$

§6. Superpotential and Proton Decay.

参考文献： Phys. Rev. D64 053015 (2001)
Phys. Rev. D65 033008 (2002)
JHEP 0211:011 (2002)
(hep-ph/0205066)

§2. Minimal SO(10) model and fermion masses.

$16 \times 16 = 10_s + 120_a + 126_s$

one Higgs $\longrightarrow$ $m_d = m_e$, $M_u = M_D$ X

two Higgs $\rightarrow$ heavy neutrino ν_R O

10, $\overline{126}$

$$W_Y = Y_{10}^{ij}\, 16_i\, H_{10}\, 16_j + Y_{126}^{ij}\, 16\, H_{126}\, 16_j$$

Y_{10}, Y_{126} : complex sym. 3×3 matrices

Symmetry breaking pattern

$$SO(10) \longrightarrow G_{422} \longrightarrow G_{std.}$$

$$10 \longrightarrow (6,1,1) + (1,2,2) \longrightarrow$$

$$\overline{126} \longrightarrow (6,1,1) + (10,3,1) + \underline{(\overline{10},1,3)} + (15,2,2) \longrightarrow G_{std}$$

$$16 \longrightarrow (4,2,1) + (\overline{4},1,2) \longrightarrow G_{std}$$

Two pairs of doublets at m_Z
one pair from $(1, 2, 2) \subset 10$
" from $(\overline{15}, 2, 2) \subset 126$.

$$W_Y = \overline{u}_i (Y_{10}^{ij} H_{10}^u + Y_{126}^{ij} H_{126}^u) q_j$$
$$+ \overline{d}_i (Y_{10}^{ij} H_{10}^d + Y_{126}^{ij} H_{126}^d) q_j$$
$$+ \overline{\nu}_i (Y_{10}^{ij} H_{10}^u - 3 Y_{126}^{ij} H_{126}^u) l_j$$
$$+ \overline{e}_i (Y_{10}^{ij} H_{10}^d - 3 Y_{126}^{ij} H_{126}^d) l_j$$
$$+ \overline{\nu}_i (Y_{126}^{ij} v_R) \nu_j ,$$

where
$$v_R \equiv \langle (\overline{10}, 1, 3) \rangle_0 .$$

Gauge coupling unification succeeds with only the MSSM particle contents.
$\Rightarrow$ one, (pair of) Higgs doublet remain light.
H_u, H_d

(This can be realized by constructing superpotential and use Dimopoulos - Wilczek mechanism)

$$H_u = \tilde{\alpha}_u H_{10}^u + \tilde{\beta}_u H_{126}^u + \cdots$$

$$H_d = \tilde{\alpha}_d H_{10}^d + \tilde{\beta}_d H_{126}^d + \cdots$$

So the low energy superpotential is

$$\begin{aligned} W_Y = & \overline{u}_i (\alpha^u Y_{10}^{ij} + \beta^u Y_{126}^{ij}) H_u q_j \\ & + \overline{d}_i (\alpha^d Y_{10}^{ij} + \beta^d Y_{126}^{ij}) H_d q_j \\ & + \overline{\nu}_i (\alpha^u Y_{10}^{ij} - 3\beta^u Y_{126}^{ij}) H_u l_j \\ & + \overline{e}_i (\alpha^d Y_{10}^{ij} - 3\beta^d Y_{126}^{ij}) H_d l_j \\ & + \overline{\nu}_i (Y_{126}^{ij} v_R) \nu_j \end{aligned}$$

Providing the VEV's

$$H_u = v \sin\beta \quad H_d = v\cos\beta, \quad v = 174\,\mathrm{GeV}$$

$$M_{10} = Y_{10}\alpha^d v \cos\beta, \quad M_{126} = Y_{126}\beta^d v\cos\beta$$

$$C_{10} \equiv \frac{\alpha^u}{\alpha^d}\tan\beta \quad C_{126} = \frac{\beta^u}{\beta^d}, \quad C_R \equiv \frac{v_R}{\beta^d v\cos\beta}$$

counting of free parameters.

M_{10} is real diagonal 3.

M_{126} real symmetric 6 + 1

two complex C_{10}. C_{126} $\quad$ β^d 4

$\overline{\quad 14 \quad}$

In put data

six quark masses 4 CKM parameters

three $\underbrace{\text{lepton}}_{\text{charged}}$ masses

$14 - 13 = 1$ remain free parameters! *

$$M_e = C_d (M_d + \kappa M_u)$$

$$C_d \equiv - \frac{3C_{10} + C_{126}}{C_{10} - C_{126}} , \quad \kappa \equiv - \frac{3C_{10} + C_{126}}{C_{10} - C_{126}}$$

eliminate $|C_d|$ and determine κ.

$$\left(\frac{\mathrm{tr}[\tilde{M}_e^{+} \tilde{M}_e]}{m_e^2 + m_\mu^2 + m_\tau^2} \right)^2 = \frac{\mathrm{tr}[(\tilde{M}_e^{+} \tilde{M}_e)^2]}{m_e^4 + m_\mu^4 + m_\tau^4}$$

$$\left(\frac{\mathrm{tr}[\tilde{M}_e^{+} \tilde{M}_e]}{m_e^2 + m_\mu^2 + m_\tau^2} \right)^3 = \frac{\det[\tilde{M}_e^{+} \tilde{M}_e]}{m_e^2 m_\mu^2 m_\tau^2}$$

where $\tilde{M}_e \equiv V_{CKM}^{*} D_d V_{CKM}^{+} + \kappa D_u$.

* Using this one free parameter (and $|C_R|$) we can realize all ν data and the others.

16'

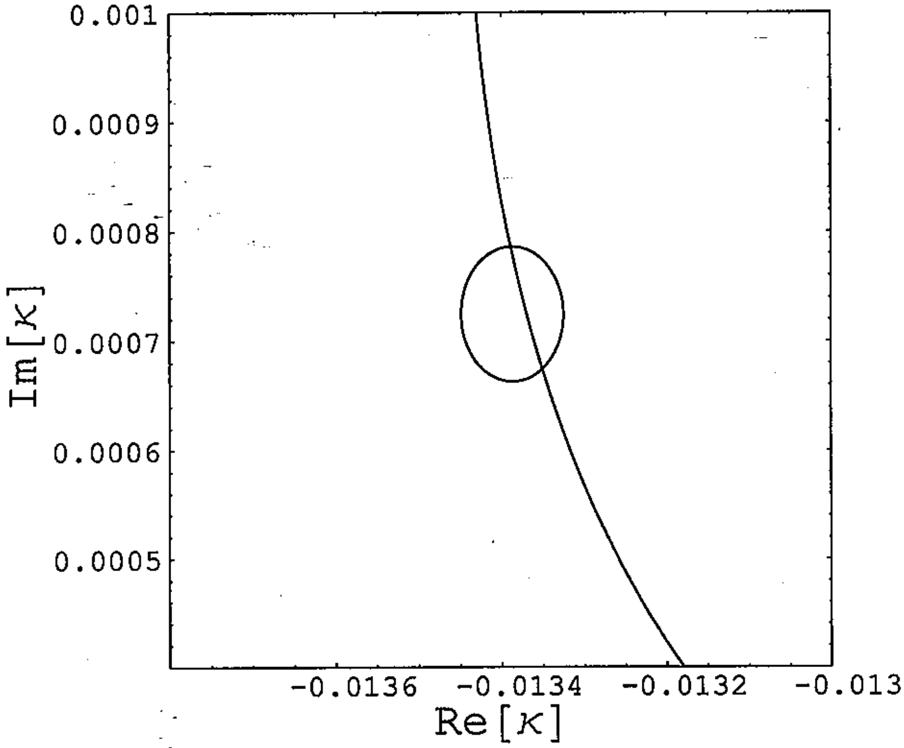

Figure 1: Contour plot on complex κ-plane. The vertical line and the circle correspond to the solutions of Eqs. (8) and (9), respectively.

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Numerical Analysis.

data at $\mu = M_Z$ [GeV]

$$m_u = 0.00233\,, \quad m_c = 0.677\,, \quad m_t = 176$$
$$m_d = 0.00469\,, \quad m_b = 3.00$$
$$m_e = 0.000487\,, \quad m_\mu = 0.103\,, \quad m_\tau = 1.75\,.$$

$$s_{12} = \frac{0.219 + 0.226}{2}\,, \quad s_{23} = \frac{0.037 + 0.043}{2}\,, \quad s_{13} = \frac{0.002 + 0.005}{2}\,.$$

$$V_{KM}(M_Z) = \begin{pmatrix} 0.975 & 0.222 & 0.000220 - 0.00349i \\ -0.222 - 0.000136i & 0.974 - 0.0000311i & 0.0400 \\ 0.00869 - 0.00340i & -0.0390 - 0.000777i & 0.999 \end{pmatrix}$$

$\xrightarrow{\text{RGE}}$ $\mu = M_{GUT}$

$$m_u = 0.00103\,, \quad m_c = 0.301\,, \quad m_t = 134$$
$$m_d = 0.00171\,, \quad m_s = 0.0265\,, \quad m_b = 1.56$$
$$m_e = 0.000413\,, \quad m_\mu = 0.0872\,, \quad m_\tau = 1.69$$

$$V_{KM}(M_G) = \begin{pmatrix} 0.975 & 0.222 & 0.000175 - 0.00279i \\ -0.222 - 0.000121i & 0.974 + 0.000129i & 0.0320 \\ 0.00695 - 0.00272i & -0.0312 - 0.000626i & 0.999 \end{pmatrix}$$

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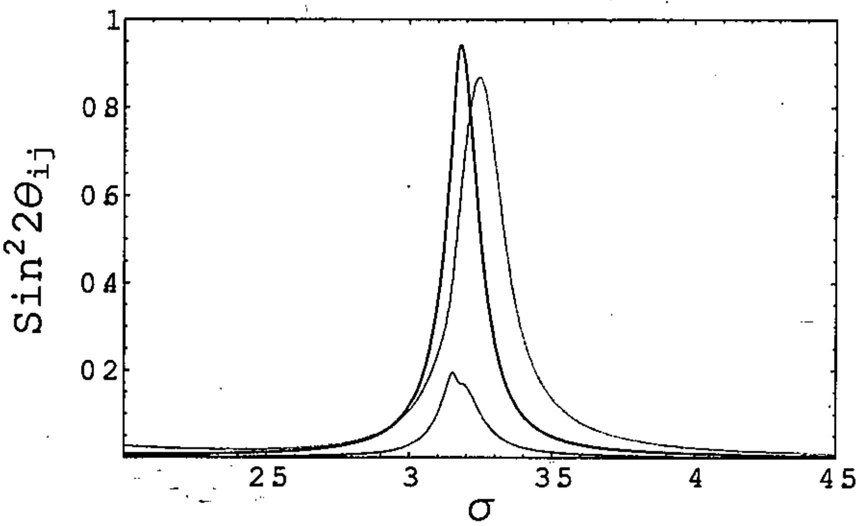

Figure 2: Three mixing angles in the PMNS matrix as functions of σ[rad]. The graphs with the highest, middle and lowest peaks are correspond to $\sin^2 2\theta_{23}$, $\sin^2 2\theta_{12}$ and $\sin^2 2\theta_{13}$, respectively. The plots of $\sin^2 2\theta_{23}$ and $\sin^2 2\theta_{13}$ have the sharp peaks at $\sigma \sim 3.2$[rad], while $\sin^2 2\theta_{12}$ has the sharp peak at $\sigma \sim 3.3$[rad].

more precise θ_{23}
→ more affirmative
large U_{e3}!

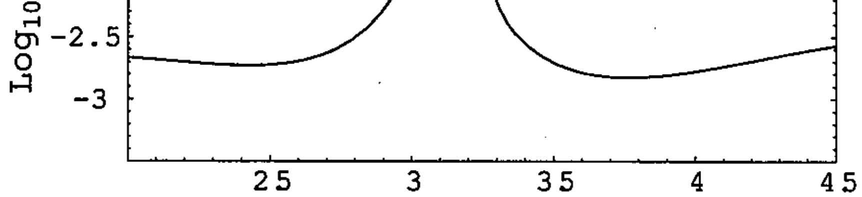

Figure 3: The ratio ($\mathrm{Log}_{10}[\Delta m^2_{\odot}/\Delta m^2_{\oplus}]$) as the function of σ[rad].

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very sensitive to σ. For $\sigma = 3.198$[rad], we obtain $\sin^2 2\theta_{12} = 0.722$, $\sin^2 2\theta_{23} = 0.881$ and $\sin^2 2\theta_{13} = 0.164$ at the GUT scale. After running this results back to the electroweak scale according to RGE of eq. (3.5), we find

$$\sin^2 2\theta_{12} = 0.723\,, \qquad \sin^2 2\theta_{23} = 0.895\,, \qquad \sin^2 2\theta_{13} = 0.164\,. \tag{4.7}$$

Note that RGE running effects are almost negligible. he ratio $\Delta m^2_{\odot}/\Delta m^2_{\oplus}$ is also independent of c_R, where $\Delta m^2_{\odot}$ and $\Delta m^2_{\oplus}$ are the oscillation parameters relevant for the solar and the atmospheric neutrino deficits, respectively. In figure 3, the ratio at the GUT scale is depicted as a function of σ. After the RGE running, we find, at the electroweak scale,

$$\frac{\Delta m^2_{\odot}}{\Delta m^2_{\oplus}} = 0.188\,. \tag{4.8}$$

RGE running effects are almost negligible also for the ratio. The neutrino mass matrix at the electroweak scale and the PMNS matrix which lead to the above results are as follows:

$$M_\nu = c_R^{-1} \begin{pmatrix} 14.7 + 48.0i & -46.9 - 96.5i & -446 - 370i \\ -46.9 - 96.5i & -347 + 90.3i & 1030 + 702i \\ -446 - 370i & 1030 + 702i & -497 - 914i \end{pmatrix} (\,\mathrm{GeV})\,, \tag{4.9}$$

and

$$U_{PMNS} = \begin{pmatrix} 0.168 + 0.838i & -0.467 + 0.0940i & -0.00508 + 0.207i \\ 0.0519 + 0.498i & 0.651 - 0.0473i & 0.0189 - 0.569i \\ 0.0745 + 0.116i & 0.450 - 0.381i & 0.431 + 0.669i \end{pmatrix}. \tag{4.10}$$

If $C_R = 3.01 \times 10^{13}$

$\longrightarrow \Delta M_{\odot}^2 = 3.76 \times 10^{-4}\,\mathrm{eV}^2$

$\Delta M_{\oplus}^2 = 2.00 \times 10^{-3}\,\mathrm{eV}^2$

(Kamland best-fit

$\Delta M_{\odot}^2 = 6.9 \times 10^{-5}\,\mathrm{eV}^2$

$\Delta M_{\oplus}^2 = 2.8 \times 10^{-3}\,\mathrm{eV}^2$

$M_{R_1} = 1.16 \times 10^{11}$ $M_{R_2} = 1.77 \times 10^{12}$

$M_{R_3} = 8.3 \times 10^{12}$ [GeV]

$\tan\beta$	$m_s(M_Z)$	δ	σ	$\sin^2 2\theta_{12}$	$\sin^2 2\theta_{23}$	$\sin^2 2\theta_{13}$	$\Delta m_\odot^2/\Delta m_\oplus^2$
40	0.0718	93.6°	3.190	0.738	0.900	0.163	0.205
45	0.0729	86.4°	3.198	0.723	0.895	0.164	0.188
50	0.0747	77.4°	3.200	0.683	0.901	0.164	0.200
55	0.0800	57.6°	3.201	0.638	0.878	0.152	0.198

Table 1: ??? .

§4 The Other Predictions

$$J_{CP} = \mathrm{Im}\left[U_{e2}U_{\mu 2}^* U_{e3}^* U_{\mu 3}\right],$$

where U_{fi} is the PMNS matrix element.

$\tan\beta$	$\langle m_\nu\rangle_{ee}$ (eV)	J_{CP}	ϵ
40	0.00172	0.00110	0.00107
45	0.00167	−0.00429	0.00119
50	0.00168	−0.00631	0.00127
55	0.00167	−0.00612	0.00128

Table 3: ??? .

$\boxed{\text{Leptogenesis}}$

Mod. Phys. Lett. A17 1725 (2002)

$$\epsilon \sim \sum_{j=2,3} \frac{\mathrm{Im}\left[(M_D M_D^\dagger)_{1j}^2\right]}{v^2 \sin^2\beta (M_D M_D^\dagger)_{11}} \frac{M_{R_1}}{M_{R_j}}.$$ for $M_{R_j} \gg M_{R_1}$

$$m_D = h_1 \langle H_{10} \rangle + h_2 \langle H_{126} \rangle$$
$$\equiv (h_1 \cos\beta + h_2 \sin\beta)\, v$$

usual $m_D \longrightarrow \overline{m_D} = \dfrac{m_D}{\sin\beta}$

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We considered the typical case in Fig. 1.2.

In the present talk

We have fixed M_D and for that case

it will be checked to be consistent or not

soon.

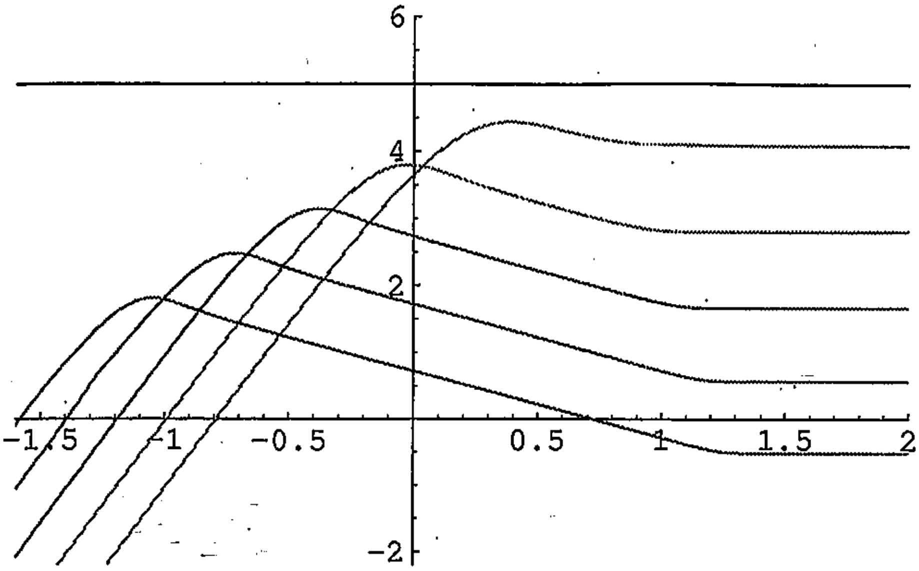


Figure 1: The solution of the Boltzmann equation ($\mathrm{Log}_{10}(Y_{B-L} \times 10^{10})$) with $\epsilon/g_* = 10^{-5}$ (the upper horizontal line), $M_1 = 10^{10}$ GeV and $\overline{m}_{\nu 3} = 0.2$ eV. The solutions for $\overline{m}_{\nu 1} = 4 \times 10^{-3}$, 0.04, 0.4, 4 and 40 eV are plotted from above at $z = 10^2$.

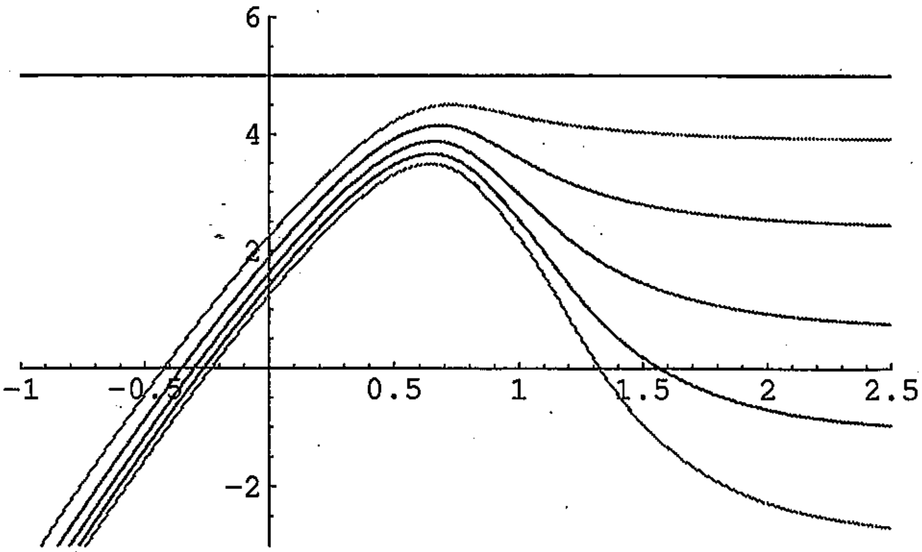


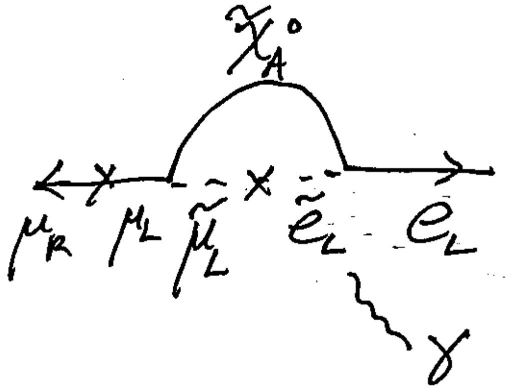
Figure 2: The solution of the Boltzmann equation ($\mathrm{Log}_{10}(Y_{B-L} \times 10^{10})$) with $\epsilon/g_* = 10^{-5}$, $M_1 = 10^{10}$ GeV and $\overline{m}_{\nu 1} = 4 \times 10^{-4}$ eV. The solutions for the input values $\overline{m}_{\nu 3} = 7$, 12, 17, 22 and 27 eV are plotted from above.

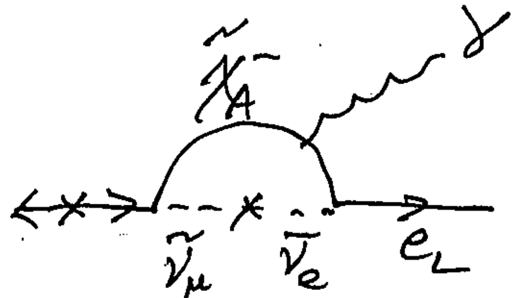
Lepton Flavor Violation

$\mu \to e \gamma$

We have the all informations including right-handed heavy neutrinos!

mass matrices



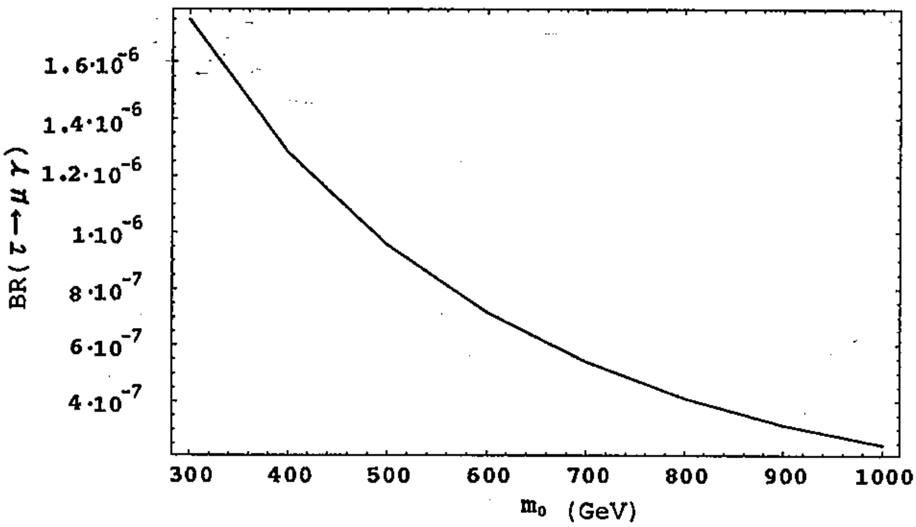


$+ \cdots$

$$\mu \frac{d}{d\mu} (m^2_{\tilde{L}})_i^{\,j} = \left(\mu \frac{d}{d\mu} (m^2_{\tilde{L}})_i^{\,j}\right)_{MSSM} + \frac{1}{16\pi^2} \Big[(m^2_{\tilde{L}} f_\nu^\dagger f_\nu + f_\nu^\dagger f_\nu m^2_{\tilde{L}})_i^{\,j} + 2 (f_\nu^\dagger m^2_{\tilde{\nu}} f_\nu + \tilde{m}_h^2 f_\nu^\dagger f_\nu + A_\nu^\dagger A_\nu)_i^{\,j} \Big]$$

The ambiguities come from those of f_ν. However we have fixed f_ν and M_R and large part of ambiguities are removed!

Very Preliminary.



m_0 : universal soft SUSY mass term;

$$m_{\tilde{q}}^2 = m_{\tilde{\ell}}^2 = m_0^2 \mathbb{1}$$

$$m_{H_u}^2 = m_{H_d}^2 = m_0^2$$

Too large (for also $\mu \to e\gamma$)

現在再計算中.

§ 6. Superpotential

We must incorporate several Higgses in order to realize Doublet – Triplet and Doublet – Doublet splitting problems, which is accomplished via Dimopoulos – Wilczek Mechanism.

Incorporated are the following Higgses.

$\{45\}$, $\{54\}$, $\{126\}$, $\{\overline{126}\}$

two $\{10\}$, and $\{210\}$.

Proton Decay — 現在考慮中.