ニュートリノの
ゼロテクスチャー質量行列

hep-ph/0212242

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bi-large mixing の起源は？

$$\boxed{U_{MNS} = U_\ell^\dagger U_\nu}$$

large mixing $\rightarrow M_\ell$

$\searrow M_\nu$

① $M_\ell \longrightarrow \boxed{\theta_{\mu\tau},\ \theta_{e\mu},\ \theta_{e3}}$

$M_\nu \longrightarrow \boxed{m_{\nu_\tau},\ m_{\nu_\mu},\ m_{\nu_e}}$

② $M_\nu \longrightarrow \begin{array}{|l|} \hline \theta_{\mu\tau},\ \theta_{e\mu},\ \theta_{e3} \\ m_{\nu_\tau},\ m_{\nu_\mu},\ m_{\nu_e} \\ \hline \end{array}$

我々はこの場合について調べた

SU(5)以上

GUTにおいて

$$\boxed{M_D \longleftrightarrow M_\ell} \Longleftrightarrow \boxed{M_U \longleftrightarrow M_{\nu_D}}$$

$$\boxed{\text{Georgi-Jarlskog relation}}$$

?

unknown

$$m_\tau = m_b$$

$$m_\mu = 3 m_s$$

$$m_e = m_d / 3$$

SO(10)

$$M_D, M_\ell : \begin{pmatrix} 0 & 10 & 0 \\ 10 & 126 & 10 \\ 0 & 10 & 10 \end{pmatrix} \qquad M_U, M_{\nu_D}$$

$$M_D = \begin{pmatrix} 0 & A_d & 0 \\ A_d & B_d & C_d \\ 0 & C_d & D_d \end{pmatrix} \longleftrightarrow M_\ell = \begin{pmatrix} 0 & A_d & 0 \\ A_d & -3B_d & C_d \\ 0 & C_d & D_d \end{pmatrix}$$

❀ Yukawaの比

	クォーク	レプトン
10	1 :	1
126	1 :	−3

Introduction

最近のニュートリノ振動実験の結果

Mixing angles

$$\boxed{\begin{array}{l} \sin^2 2\theta_{atm} > 0.83 \\ 0.29 \leq \tan^2\theta_{sol} \leq 0.86 \end{array}}$$

Mass differences

$$\boxed{\begin{array}{l} \Delta m^2_{atm} \sim 2.5 \times 10^{-3}\ \mathrm{eV}^2 \\ \Delta m^2_{sol} \sim 5 \times 10^{-5}\ \mathrm{eV}^2 \end{array}}$$

MNS行列は bi-large mixing

ニュートリノの bi-large mixingと質量を説明できるフェルミオンの質量行列のテクスチャーはどうなっているのか？

➡ ゼロテクスチャー

symmetric four zero texture

$$\begin{pmatrix} 0 & A & 0 \\ A & B & C \\ 0 & C & D \end{pmatrix} \longrightarrow M_U, M_D$$

$$\boxed{V_{CKM} = U_U^{\dagger} U_D}$$

$U_U^T M_U U_U = \mathrm{diag}(-m_u, m_c, m_t)$

$U_D^T M_D U_D = \mathrm{diag}(-m_d, m_s, m_b)$

$$V_{CKM} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ & 1 & \lambda^2 \\ & & 1 \end{pmatrix}$$

$\downarrow$

M_ν, M_ℓ

$$\boxed{U_{MNS} = U_\ell^{\dagger} U_\nu}$$

$U_\ell^T M_\ell U_\ell = \mathrm{diag}(-m_e, m_\mu, m_\tau)$

$U_\nu^T M_\nu U_\nu = \mathrm{diag}(-m_{\nu_e}, m_{\nu_\mu}, m_{\nu_\tau})$

16タイプのテクスチャー

$$\begin{pmatrix} 0 & 126 & 0 \\ 126 & 126 & 126 \\ 0 & 126 & 126 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 10 & 0 \\ 10 & 126 & 126 \\ 0 & 126 & 126 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 126 & 0 \\ 126 & 10 & 126 \\ 0 & 126 & 126 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 10 & 0 \\ 10 & 10 & 126 \\ 0 & 126 & 126 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 126 & 0 \\ 126 & 126 & 10 \\ 0 & 10 & 126 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 10 & 0 \\ 10 & 126 & 10 \\ 0 & 10 & 126 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 126 & 0 \\ 126 & 10 & 10 \\ 0 & 10 & 126 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 10 & 0 \\ 10 & 10 & 10 \\ 0 & 10 & 126 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 126 & 0 \\ 126 & 126 & 126 \\ 0 & 126 & 10 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 10 & 0 \\ 10 & 126 & 126 \\ 0 & 126 & 10 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 126 & 0 \\ 126 & 10 & 126 \\ 0 & 126 & 10 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 10 & 0 \\ 10 & 10 & 126 \\ 0 & 126 & 10 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 126 & 0 \\ 126 & 126 & 10 \\ 0 & 10 & 10 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 10 & 0 \\ 10 & 126 & 10 \\ 10 & 10 & 10 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 126 & 0 \\ 126 & 10 & 10 \\ 0 & 10 & 10 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 10 & 0 \\ 10 & 10 & 10 \\ 0 & 10 & 10 \end{pmatrix}$$

126表現のヒッグスがどの成分にcoupleするのか

⇩

16タイプのテクスチャー

どのタイプのテクスチャーがニュートリノのbi-large mixingとmass differenceをreproduceできるのかについて調べた。

⇒ $\left\{ \begin{array}{l} \bullet\ |U_{e3}| \text{の予言値} \\ \bullet\ \text{ニュートリノの質量の絶対値} \end{array} \right.$

Best Texture

$$\boxed{M_u = \begin{pmatrix} 0 & 126 & 0 \\ 126 & 10 & 10 \\ 0 & 10 & 126 \end{pmatrix}}$$

$$M_R = \begin{pmatrix} 0 & r & 0 \\ r & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} m_R$$

$$r = \frac{1}{9}\sqrt{\frac{m_u^2 m_c}{m_t^3}} \sim 10^{-7}$$

$\Downarrow$

~ 0.2

$$M_\nu = \begin{pmatrix} 0 & -3\sqrt{\frac{m_c}{m_t}} & 0 \\ -3\sqrt{\frac{m_c}{m_t}} & \frac{2m_c}{\sqrt{m_u m_t}} & 1 \\ 0 & 1 & 1 \end{pmatrix} \frac{9m_t^2}{m_R}$$

~ 1.2

得られた関係式

$$\boxed{\begin{aligned} \tan^2 2\theta_{e\mu} &\simeq \frac{9m_c}{m_t}\tan^2 2\theta_{\mu\tau} \\ \sin^2\theta_{e3} &\simeq \frac{9m_c}{m_t}\sin^2\theta_{\mu\tau}\cos^2\theta_{e\mu} \end{aligned}}$$

symmetric four zero texture

Nishiura, Matsuda & Fukuyama
PRD 60 (1999) 013006

$$M_D = \begin{pmatrix} 0 & \sqrt{\frac{m_d m_s m_b}{m_b - m_d}} & 0 \\ \sqrt{\frac{m_d m_s m_b}{m_b - m_d}} & m_s & \sqrt{\frac{m_d m_b (m_b - m_s - m_d)}{m_b - m_d}} \\ 0 & \sqrt{\frac{m_d m_b (m_b - m_s - m_d)}{m_b - m_d}} & m_b - m_d \end{pmatrix}$$

$$\simeq m_b \begin{pmatrix} 0 & \frac{\sqrt{m_d m_s}}{m_b} & 0 \\ \frac{\sqrt{m_d m_s}}{m_b} & \frac{m_s}{m_b} & \sqrt{\frac{m_d}{m_b}} \\ 0 & \sqrt{\frac{m_d}{m_b}} & 1 \end{pmatrix} \qquad (m_b \gg m_s \gg m_d)$$

$$M_U \simeq m_t \begin{pmatrix} 0 & \frac{\sqrt{m_u m_c}}{m_t} & 0 \\ \frac{\sqrt{m_u m_c}}{m_t} & \frac{m_c}{m_t} & \sqrt{\frac{m_u}{m_t}} \\ 0 & \sqrt{\frac{m_u}{m_t}} & 1 \end{pmatrix} \equiv m_t \begin{pmatrix} 0 & a_u & 0 \\ a_u & b_u & c_u \\ 0 & c_u & 1 \end{pmatrix}$$

16 type

16 type

$$M_{\nu_D} = \begin{pmatrix} 0 & *a_u & 0 \\ *a_u & *b_u & *c_u \\ 0 & *c_u & * \end{pmatrix} \equiv m_t \begin{pmatrix} 0 & a & 0 \\ a & b & c \\ 0 & c & d \end{pmatrix}$$

$\boxed{* : 1 \text{ or } -3}$

See-Saw mechanism

$$M_\nu = M_{\nu_D}^T M_R^{-1} M_{\nu_D}$$

$$M_R = v_{\mathbf{126}} \begin{pmatrix} 0 & A_R & 0 \\ A_R & 0 & 0 \\ 0 & 0 & D_R \end{pmatrix} \equiv m_R \begin{pmatrix} 0 & r & 0 \\ r & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\boxed{M_\nu = \begin{pmatrix} 0 & \frac{a^2}{r} & 0 \\ \frac{a^2}{r} & 2\frac{ab}{r} + c^2 & c(\frac{a}{r}+1) \\ 0 & c(\frac{a}{r}+1) & d^2 \end{pmatrix} \frac{m_t^2}{m_R}}$$

Type	Texture	$\sin^2 2\theta_{atm}$	$\tan^2 \theta_{sol}$	h
C_1	$\begin{pmatrix} 0 & \overline{126} & 0 \\ \overline{126} & 10_2 & \overline{126} \\ 0 & \overline{126} & \overline{126} \end{pmatrix}$	○	×	none
C_2	$\begin{pmatrix} 0 & 10 & 0 \\ 10 & 10 & \overline{126} \\ 0 & \overline{126} & \overline{126} \end{pmatrix}$	○	×	none
C_3	$\begin{pmatrix} 0 & 10 & 0 \\ 10 & 10 & \overline{126} \\ 0 & \overline{126} & 10 \end{pmatrix}$	○	×	none
C_4	$\begin{pmatrix} 0 & \overline{126} & 0 \\ \overline{126} & 10 & \overline{126} \\ 0 & \overline{126} & 10 \end{pmatrix}$	○	×	none
F_1	$\begin{pmatrix} 0 & \overline{126} & 0 \\ \overline{126} & \overline{126} & 10 \\ 0 & 10 & \overline{126} \end{pmatrix}$	×	×	none
F_2	$\begin{pmatrix} 0 & 10 & 0 \\ 10 & \overline{126} & 10 \\ 0 & 10 & \overline{126} \end{pmatrix}$	×	×	none
F_3	$\begin{pmatrix} 0 & 10 & 0 \\ 10 & \overline{126} & 10 \\ 0 & 10 & 10 \end{pmatrix}$	×	×	none
F_4	$\begin{pmatrix} 0 & \overline{126} & 0 \\ \overline{126} & \overline{126} & 10 \\ 0 & 10 & 10 \end{pmatrix}$	×	×	none

	Type	Texture	$\sin^2 2\theta_{atm}$	$\tan^2\theta_{sol}$	h	m_R (GeV)	$r m_R$
Best →	S_1	$\begin{pmatrix} 0 & \overline{126} & 0 \\ \overline{126} & 10 & 10 \\ 0 & 10 & \overline{126} \end{pmatrix}$	○	◎	$0.4-1.4$	2×10^{15}	10^8
	S_2	$\begin{pmatrix} 0 & \overline{126} & 0 \\ \overline{126} & 10 & 10 \\ 0 & 10 & 10 \end{pmatrix}$	○	◎	$0.4-1.4$	2×10^{14}	
	A_1	$\begin{pmatrix} 0 & \overline{126} & 0 \\ \overline{126} & \overline{126} & \overline{126} \\ 0 & \overline{126} & \overline{126} \end{pmatrix}$	○	○	$0.5-1.3$	2×10^{15}	
	A_2	$\begin{pmatrix} 0 & 10 & 0 \\ 10 & 10 & 10 \\ 0 & 10 & \overline{126} \end{pmatrix}$	○	○	$0.5-1.3$	2×10^{15}	
	A_3	$\begin{pmatrix} 0 & \overline{126} & 0 \\ \overline{126} & \overline{126} & \overline{126} \\ 0 & \overline{126} & 10 \end{pmatrix}$	○	○	$0.5-1.3$	2×10^{14}	
	A_4	$\begin{pmatrix} 0 & 10 & 0 \\ 10 & 10 & 10 \\ 0 & 10 & 10 \end{pmatrix}$	○	○	$0.5-1.3$	2×10^{14}	
	B_1	$\begin{pmatrix} 0 & 10 & 0 \\ 10 & \overline{126} & \overline{126} \\ 0 & \overline{126} & \overline{126} \end{pmatrix}$	○	△	$0.6-1.2$	2×10^{15}	
	B_2	$\begin{pmatrix} 0 & 10 & 0 \\ 10 & \overline{126} & \overline{126} \\ 0 & \overline{126} & 10 \end{pmatrix}$	○	△	$0.6-1.2$	2×10^{14}	

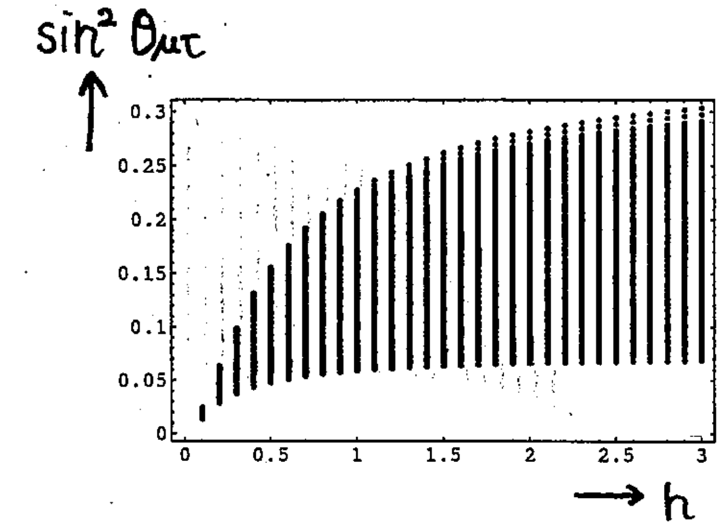

Figure 1: Calculated values of $\sin^2 2\theta_{\mu\tau}$ versus h in class F_1.

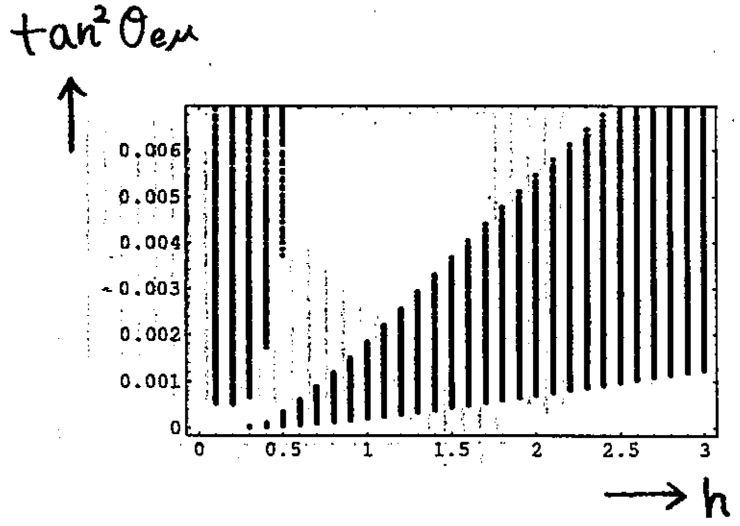

Figure 2: Calculated values of $\tan^2\theta_{e\mu}$ versus h in class F_1.

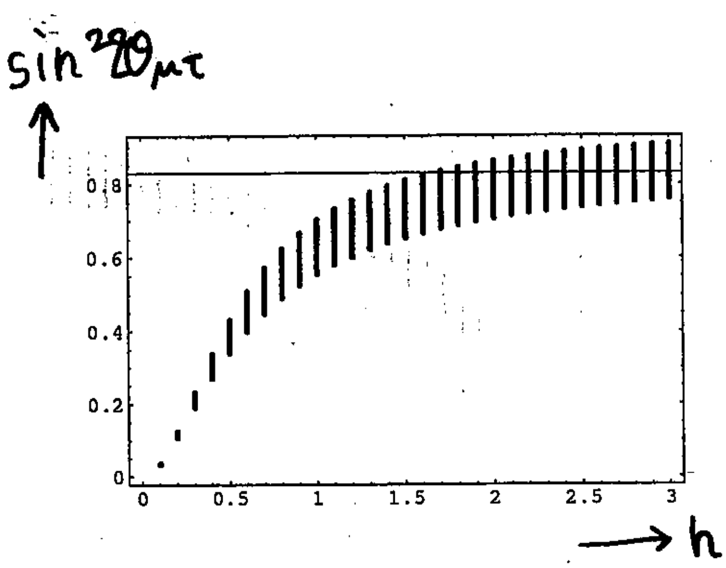

Figure 1: Calculated values of $\sin^2 2\theta_{\mu\tau}$ versus h in class C_1.

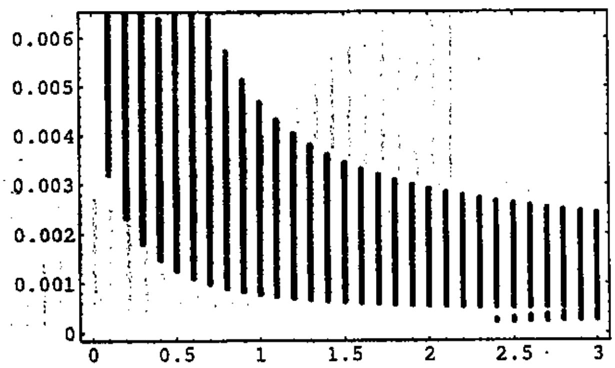

Figure 2: Calculated values of $\tan^2 \theta_{e\mu}$ versus h in class C_1.

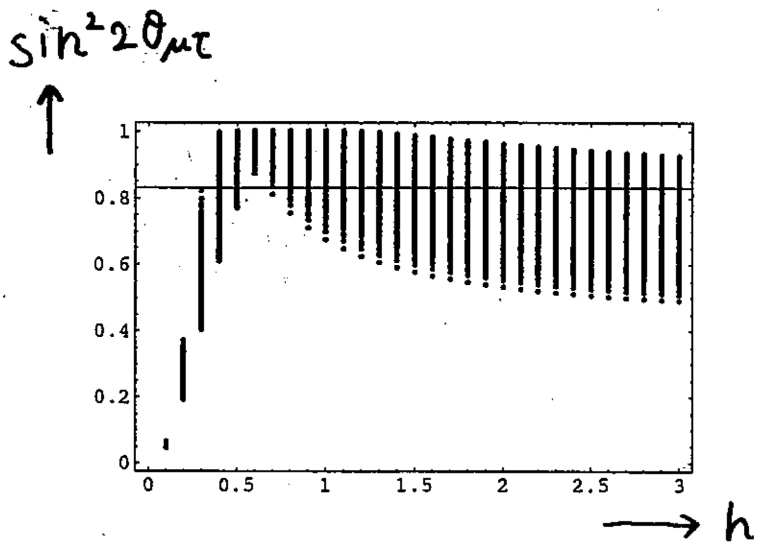

Figure 1: Calculated values of $\sin^2 2\theta_{\mu\tau}$ versus h in class S_1.

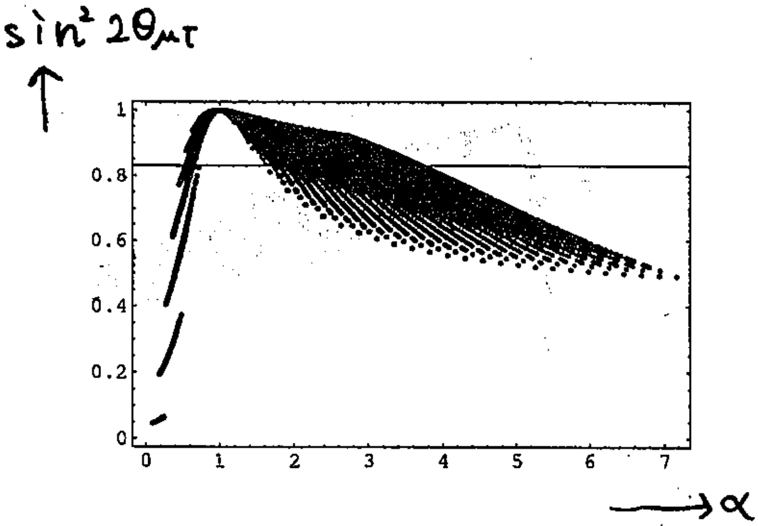

Figure 2: Calculated values of $\sin^2 2\theta_{\mu\tau}$ versus α in class S_1.

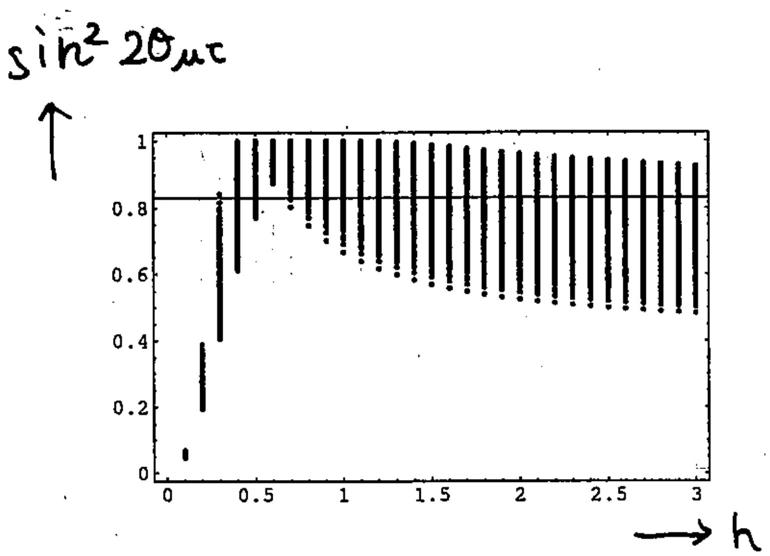

Figure 1: Calculated values of $\sin^2 2\theta_{\mu\tau}$ versus h in class B_1.

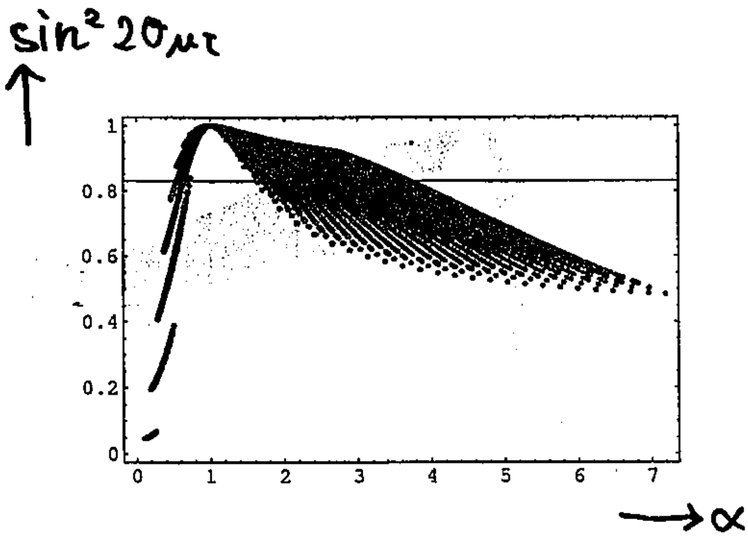

Figure 2: Calculated values of $\sin^2 2\theta_{\mu\tau}$ versus α in class B_1.

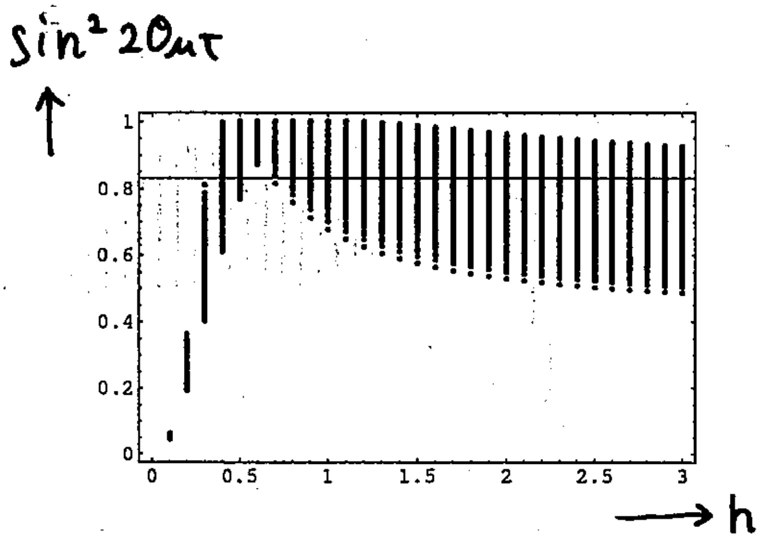

Figure 1: Calculated values of $\sin^2 2\theta_{\mu\tau}$ versus h in class A_1.

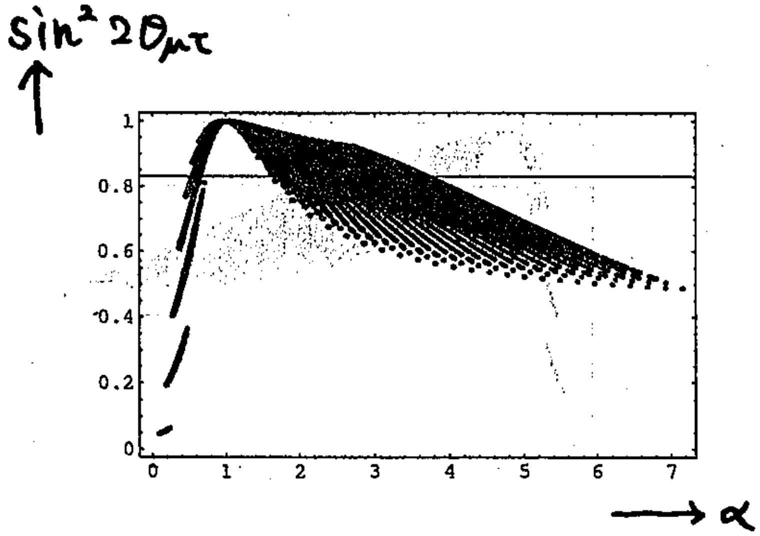

Figure 2: Calculated values of $\sin^2 2\theta_{\mu\tau}$ versus α in class A_1.

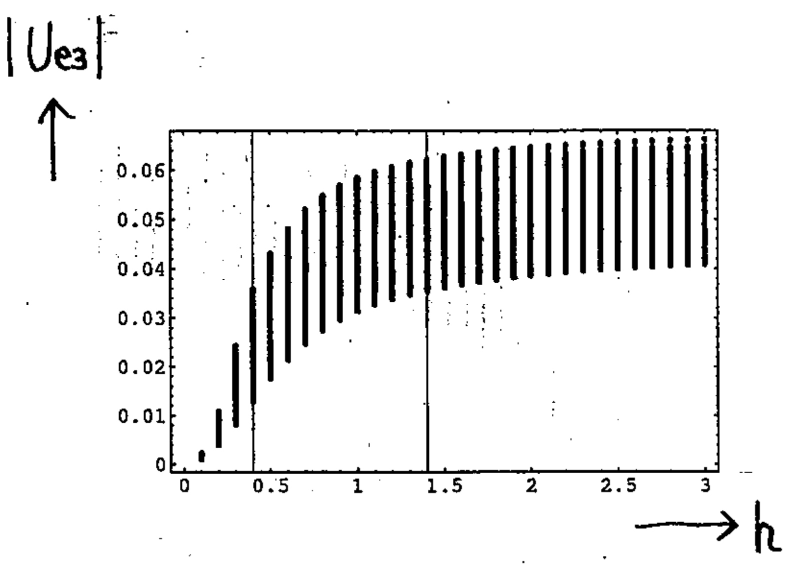

Figure 1: Predicted values of $|U_{e3}|$ versus h in class S_1.

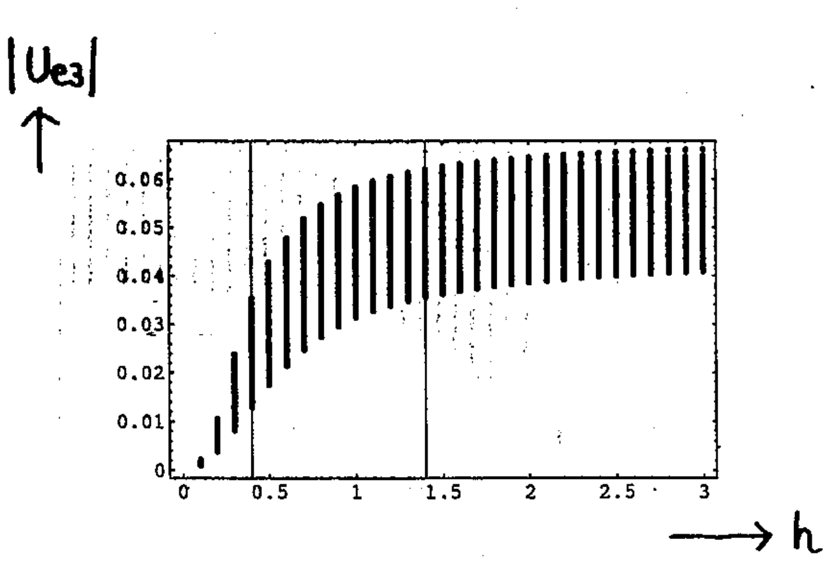

Figure 2: Predicted values of $|U_{e3}|$ versus h in class S_2.

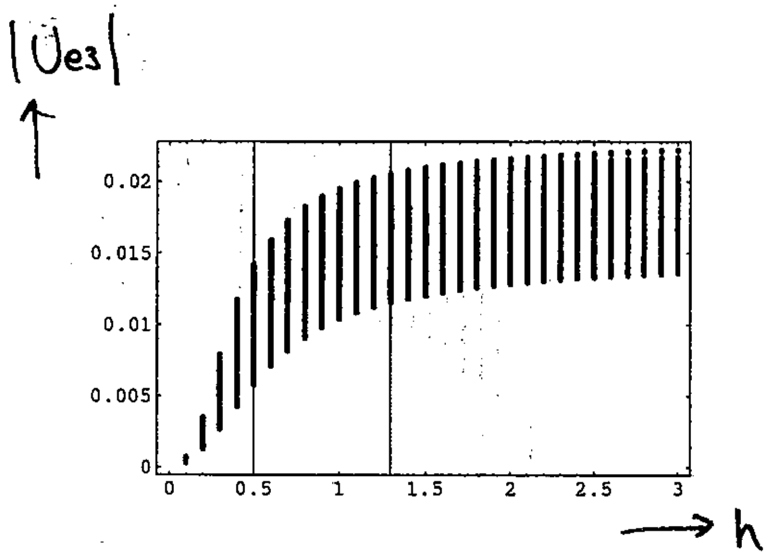

Figure 1: Predicted values of $|U_{e3}|$ versus h in class A_1.

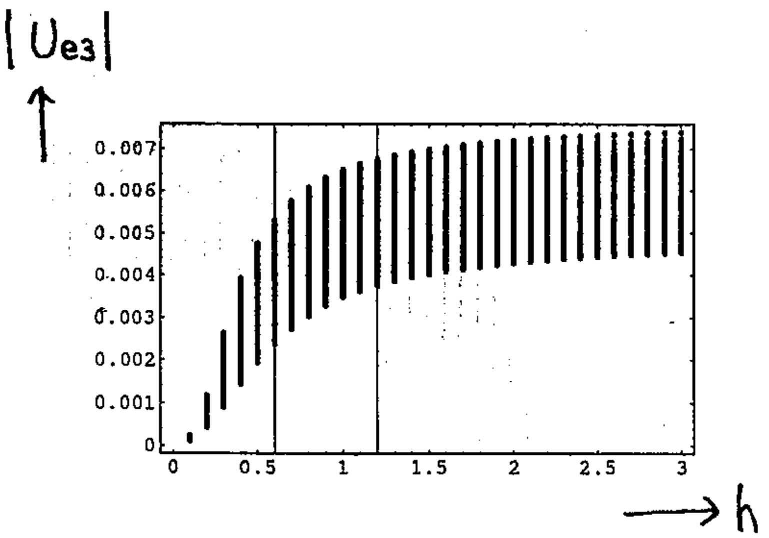

Figure 2: Predicted values of $|U_{e3}|$ versus h in class B_1.

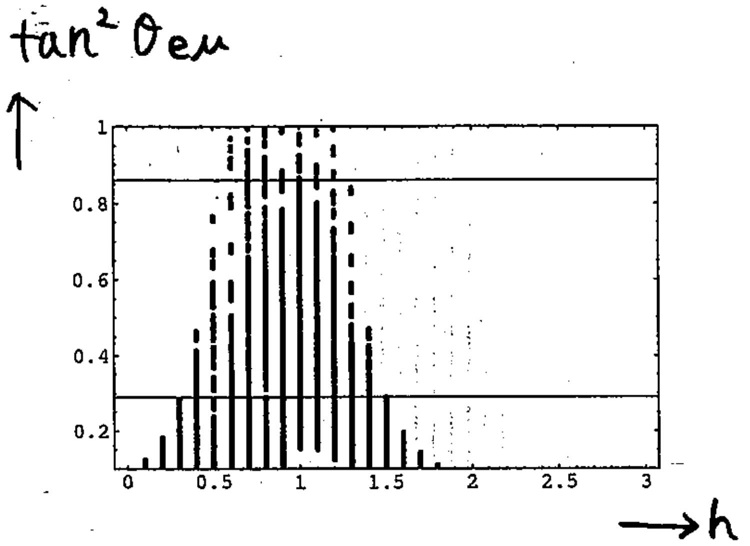

Figure 1: Calculated values of $\tan^2\theta_{e\mu}$ versus h in class S_1.

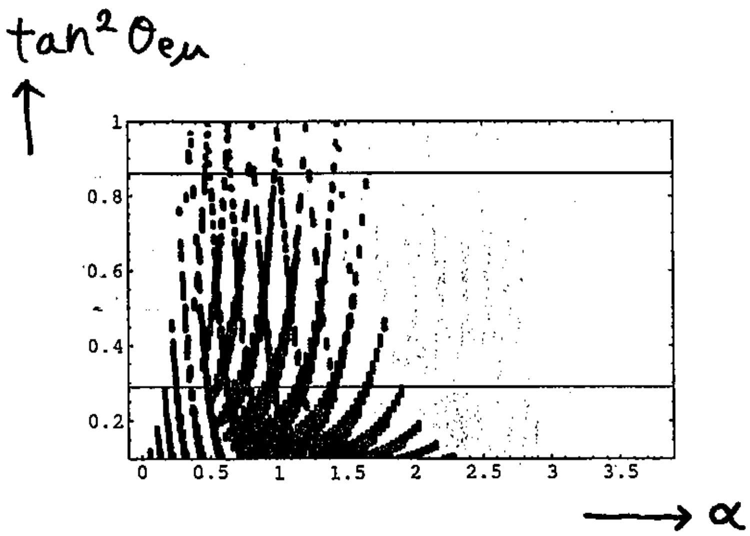

Figure 2: $\tan^2\theta_{e\mu}$ versus α in class S_1.

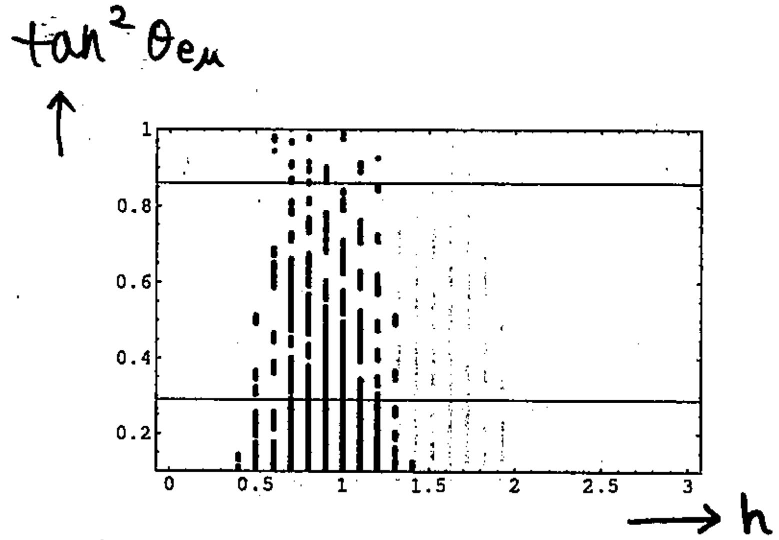

Figure 1: Calculated values of $\tan^2\theta_{e\mu}$ versus h in class A_1.

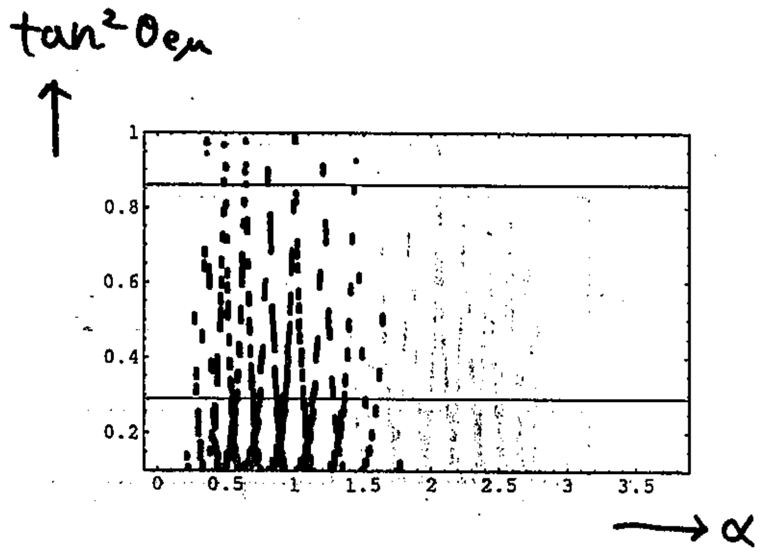

Figure 2: $\tan^2\theta_{e\mu}$ versus α in class A_1.

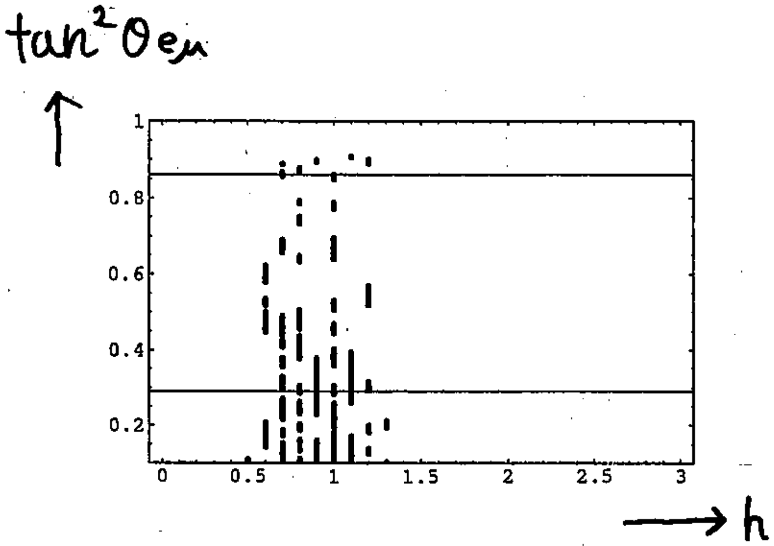

Figure 1: Calculated values of $\tan^2\theta_{e\mu}$ versus h in class B_1.

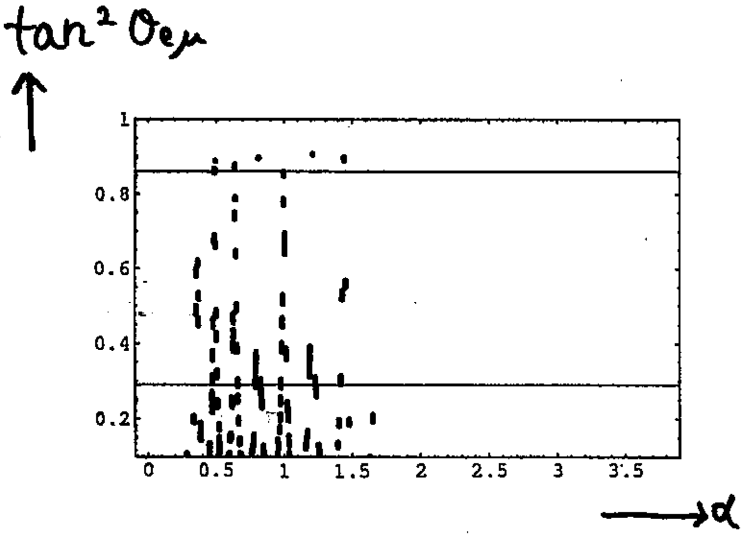

Figure 2: $\tan^2\theta_{e\mu}$ versus α in class B_1.

Table 2: h, m_R, $|m_{\nu_\tau}|$, $|m_{\nu_\mu}|$ and $|m_{\nu_e}|$ for each type

Type	h	m_R (GeV)	$\lvert m_{\nu_\tau}\rvert$ ([redacted]eV)	$\lvert m_{\nu_\mu}\rvert$ ([redacted]eV)	$\lvert m_{\nu_e}\rvert$ ([redacted]eV)
S_1	$0.4 - 1.4$	2×10^{15}	$0.005 - 0.17$	$0.001 - 0.015$	$3 \times 10^{-5} - 4 \times 10^{-3}$
S_2	$0.4 - 1.4$	2×10^{14}	$0.005 - 0.17$	$0.001 - 0.015$	$3 \times 10^{-5} - 4 \times 10^{-3}$
A_1	$0.5 - 1.3$	2×10^{15}	$0.006 - 0.15$	$0.002 - 0.014$	$3 \times 10^{-6} - 1 \times 10^{-3}$
A_2	$0.5 - 1.3$	2×10^{15}	$0.006 - 0.15$	$0.002 - 0.014$	$3 \times 10^{-6} - 1 \times 10^{-3}$
A_3	$0.5 - 1.3$	2×10^{14}	$0.006 - 0.15$	$0.002 - 0.014$	$3 \times 10^{-6} - 1 \times 10^{-3}$
A_4	$0.5 - 1.3$	2×10^{14}	$0.006 - 0.15$	$0.002 - 0.014$	$3 \times 10^{-6} - 1 \times 10^{-3}$
B_1	$0.6 - 1.2$	2×10^{15}	$0.006 - 0.13$	$0.002 - 0.014$	$5 \times 10^{-7} - 3 \times 10^{-3}$
B_2	$0.6 - 1.2$	2×10^{14}	$0.006 - 0.13$	$0.002 - 0.015$	$6 \times 10^{-7} - 4 \times 10^{-3}$

$|U_{e3}|$

S $0.01 - 0.06$

A $0.006 - 0.02$

B $0.002 - 0.007$

まとめ

ニュートリノの bi-large mixing と mass difference を reproduce できるテクスチャーについて調べた。

⇩

Best texture

$$M_U, M_{\nu_D} : \begin{pmatrix} 0 & 126 & 0 \\ 126 & 10 & 10 \\ 0 & 10 & 126 \end{pmatrix} \quad S_1$$

with Λ

$$M_R = \begin{pmatrix} 0 & r & 0 \\ r & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} m_R \qquad r = \frac{1}{9}\sqrt{\frac{m_u^2 m_c}{m_t^3}} \sim 10^{-7}$$

$$M_\nu = \begin{pmatrix} 0 & -3\sqrt{\frac{m_c}{m_t}} & 0 \\ -3\sqrt{\frac{m_c}{m_t}} & \frac{2m_c}{\sqrt{m_u m_t}} & 1 \\ 0 & 1 & 1 \end{pmatrix} \frac{9m_t^2}{m_R}$$

~ 0.2

2

1.2

$$\boxed{\begin{aligned} \tan^2 2\theta_{e\mu} &\simeq \frac{9m_c}{m_t}\tan^2 2\theta_{\mu\tau} \\ \sin^2\theta_{e3} &\simeq \frac{9m_c}{m_t}\sin^2\theta_{\mu\tau}\cos^2\theta_{e\mu} \end{aligned}}$$

S, A, B

- 質量の絶対値についてはほとんど差はない

- $|U_{e3}|$について…

S 0.01 − 0.06

A 0.006 − 0.02

B 0.002 − 0.007

at GUT scale

Fusaoka & Koide
PRD 57 (1998) 3986

$$\boxed{\begin{aligned} m_u &= 1.04^{+0.19}_{-0.20}\ \text{MeV} \\ m_c &= 302^{+25}_{-27}\ \text{MeV} \\ m_t &= 129^{+196}_{-40}\ \text{GeV} \end{aligned}}$$

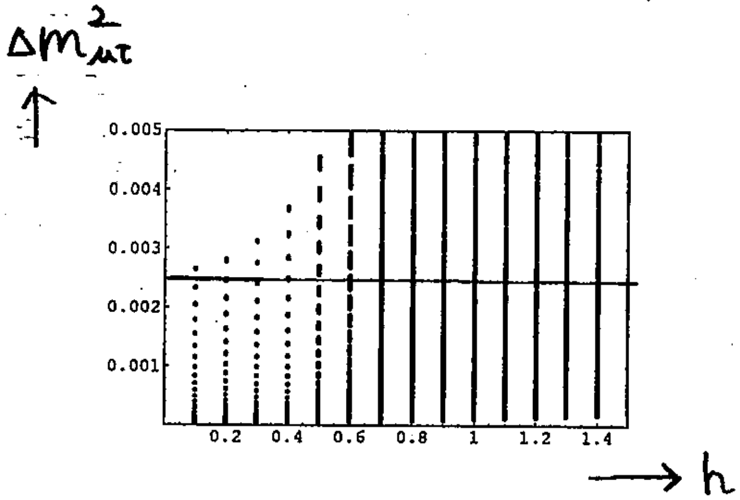

Figure 1: Calculated values of $\Delta m^2_{\mu\tau}$ versus h in class S_1.

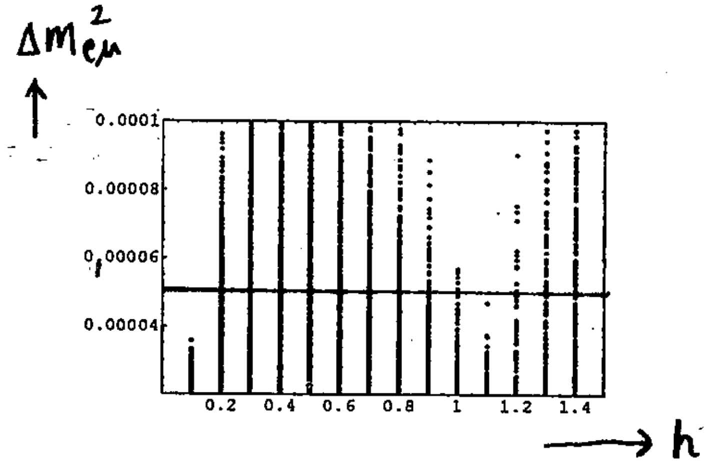

Figure 2: Calculated values of $\Delta m^2_{e\mu}$ versus h in class S_1.

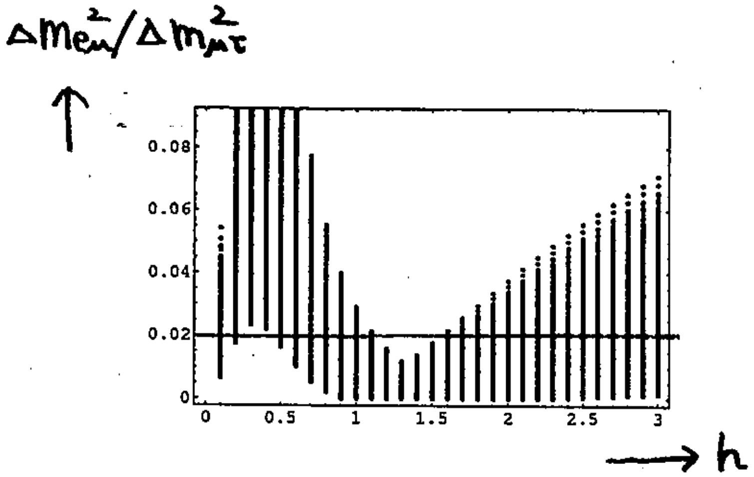

Figure 3: Calculated values of $\Delta m^2_{e\mu}/\Delta m^2_{\mu\tau}$ versus h in class S_1.

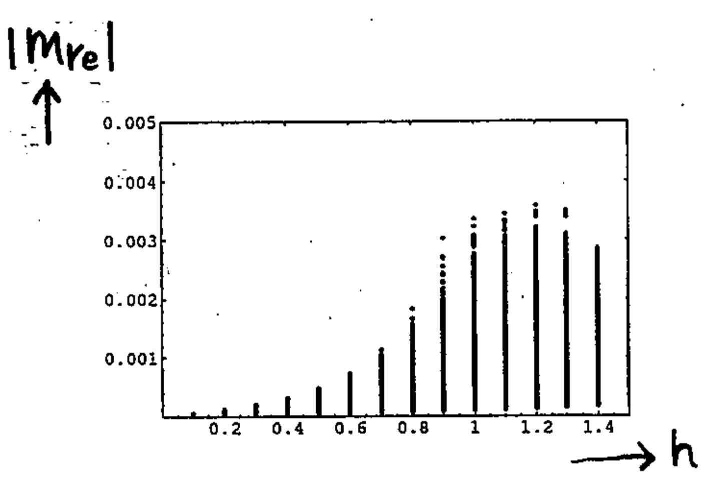

Figure 1: Calculated values of $|m_{\nu_e}|$ versus h in class S_1.

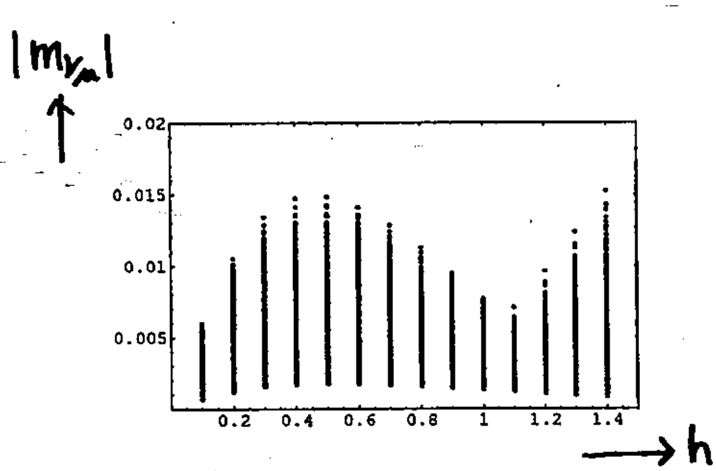

Figure 2: Calculated values of $|m_{\nu_\mu}|$ versus h in class S_1.

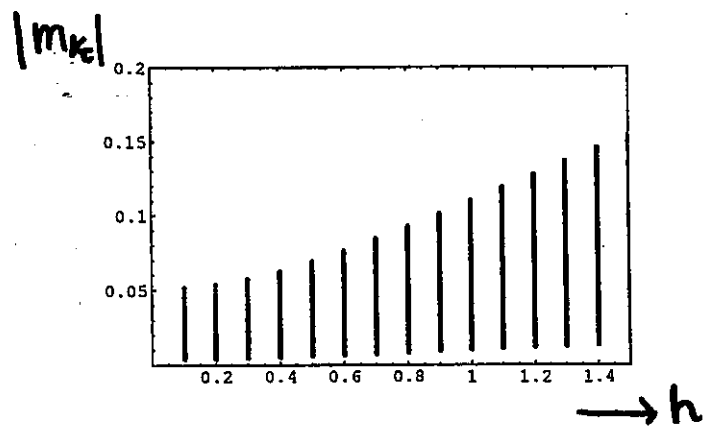

Figure 3: Calculated values of $|m_{\nu_\tau}|$ versus h in class S_1.

$\theta_{\mu\tau}$ が大きくなるためには

$$\boxed{r \sim \frac{ac}{d^2} \sim * \sqrt{\frac{m_u^2 m_c}{m_t^3}} \sim 10^{-7}}$$

$\Longrightarrow M_\nu$ が決まる!!

$$M_\nu = \begin{pmatrix} 0 & \frac{a^2}{r} & 0 \\ \frac{a^2}{r} & \frac{2ab}{r} & \frac{ac}{r} \\ 0 & \frac{ac}{r} & d^2 \end{pmatrix} \frac{m_t^2}{m_R} \equiv \begin{pmatrix} 0 & \beta & 0 \\ \beta & \alpha & h \\ 0 & h & 1 \end{pmatrix} \frac{d^2 m_t^2}{m_R}$$

with

$$h = \frac{ac}{rd^2}, \qquad \alpha = \frac{2ab}{rd^2}, \qquad \beta = \frac{a^2}{rd^2}$$

$h \sim \mathcal{O}(1)$

Table 3: values of h, α and β with the value of d for each type

Type	d	h in $\frac{ac}{r}$ unit	α in $\frac{2b}{cr}$ unit	β in $\frac{a}{cr}$ unit
S_1	9	$-1/3$	1	-3
S_2	1	-3	1	-3
A_1	1	1	1	1
A_2	1	9	1	1
A_3	9	$1/9$	1	1
A_4	1	1	1	1
B_1	9	$-1/3$	1	$-1/3$
B_2	1	-3	1	$-1/3$
C_1	9	1	$-1/3$	1
C_2	9	$-1/3$	$-1/3$	$-1/3$
C_3	1	-3	$-1/3$	$-1/3$
C_4	1	9	$-1/3$	$1/9$
F_1	9	$-1/3$	-3	-3
F_2	9	$1/9$	$-1/3$	9
F_3	1	1	-3	1
F_4	9	-3	-3	-3