Leptogenesis

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Where did anti-matter disappear ?

$$n_B/n_\gamma = 10^{-9\sim-10}$$

- Baryon density of the universe
  $\rho_B = 5 \times 10^{-30\sim-31}(g/cm^3)$
  $\rightarrow n_B = N_A \times \rho_B = 3 \times 10^{-6\sim-7}/cm^3$

- Photon number density of the universe
  $n_\gamma = 2.404\frac{T^3}{\pi^2} = O(750/cm^3)$
  $T = 3$ K background black body radiation.

Particle Physics Model for generating Baryon number asymmetry

$\rightarrow$ Three Ingredients: Sakharov

1. B-L violation at high energy

2. CP violation

3. Thermal non-equilibrium:

Why Baryogenesis through Leptogenesis ?

(Fukugita and Yanagida)

A simple baryogenesis scenario based on GUT does not work under the presence of anomaly. $B+L$ is not conserved.(The effect is significant for $T > 1$(TeV).)

$$\partial_\mu(j_B^\mu + j_L^\mu) \neq 0 \tag{1}$$

Primordial $B+L$ will be washed out as $(B+L)_{primordial}\exp[-t/\tau]$ and the present baryon number is proportional to $B-L$.

$$\begin{aligned} B_{now} &= \frac{8N_g + 4N_H}{22N_g + 13N_H}(B-L)_{primordial} \\ &\sim \frac{1}{3}(B-L)_{primordial} \end{aligned}$$

Particle Physics model must break $B-L$ at high energy. $B-L$ is gauge symmetry spontaneously broken or it must be explicitly broken.

CP violation

Standard model can not account for baryon number though it is good at explaining the present measured CP violation phenomena observed in Kaon and B meson system.

$\longrightarrow$ some new source of CP violation other than Kobayashi Maskawa phase is required.

The Seesaw Models (=the standard model + heavy Majorana neutrinos),

$B-L$ is broken and the other two conditions (CP violation and thermal non-equilibrium) can be satisfied.

This talk is about:

1. How well can we predict the baryon number asymmetry based on a specific model ?

2. How can we test the model in the laboratory experiments ?

Three conditions in the seesaw baryogenesis

Heavy Majorana neutrinos N_R with $M_R = (M_1, M_2, ...M_N)$.

$\nu_{Li}(i = 1 \sim 3)$: left-handed neutrinos.

$l = e, \mu, \tau$: charged leptons with $m_l = (m_e, m_\mu, m_\tau)$.

$$\mathcal{L} = -[\overline{\nu_L} m_D N_R + \frac{1}{2}\overline{(N_R)^c} M_R N_R + \overline{l_L} m_l l_R] - h.c.[\overline{N_R} m_D^\dagger \nu_L ...] \quad (2)$$

1. Lepton number violating mass term.

$N_R \to \exp(i\theta) N_R$:

$\overline{(N_R)^c} M_R N_R \to \exp(i2\theta)\overline{(N_R)^c} M_R N_R$.

2. CP violating Dirac mass terms

m_D

$Im(m_D) \neq 0 \to$ CP violation:

$CP[\overline{\nu_L} m_D N_R] CP^{-1} = \overline{N_R} m_D^T \nu_L$.

$L_{primordial} \sim \Gamma[N \to l^-\phi^+] - \Gamma[N \to l^+\phi^-]$

$L_{primordial}$ is generated with the mechanism known as "Direct CP violation" in K and B physics.

Direct CP violation

$\Gamma[B^+ \to K^+\pi^0] - \Gamma[B^- \to K^-\pi^0]$
$\sim Im(a_0 a_1{}^*) \sin(\delta_0 - \delta_1)$

$$\begin{aligned} Amp.(B^+ \to K^+\pi^0) &= a_0 exp(i\delta_0) + a_1 exp(i\delta_1) \\ Amp.(B^- \to K^-\pi^0) &= a_0^* exp(i\delta_0) + a_1^* exp(i\delta_1) \end{aligned} \quad (3)$$

$Im(a_0 a_1{}^*)$: weak interaction CP violating phase (In SM, KM phase).
δ: strong phase.(phase due to threshold effect of intermediate or final states interactions).

The formulae for
"Direct CP violation" (asymmetry).
$N = 2$ case.

$$\epsilon_1 = \frac{\Gamma_1 - \overline{\Gamma_1}}{\Gamma_1 + \overline{\Gamma_1}} = -\frac{3M_1}{2M_2V^2}\frac{Im[\{(m_D{}^\dagger m_D)_{12}\}^2]}{(m_D{}^\dagger m_D)_{11}},$$

where $V = \sqrt{4\pi}v$ with $v = 246$ GeV.

$$-\mathcal{L} = y_D \bar{\psi}_L \tilde{\phi} N_R$$

$$y_D \sim \frac{m_D}{v}$$

$L_{primordial}$ may be only generated in the early hot universe. $T > M_R$.

3. $L_{primodial}$ **is produced under the non-equilibrium condition.** In the thermal equilibrium, the decay process and production process are balanced and the net production of lepton number is zero. $N \leftrightarrow l^- \phi^+$
In the expanding universe,e.g.,in the Friedmann Universe:
$ds^2 = dt^2 - a(t)^2(dx^2 + dy^2 + dz^2)$.
Time evolution of the number density of heavy Majorana neutrino $n(t)$ and lepton number density $L(t)$.

$$\frac{dn(t)}{dt} + 3Hn(t) = -\Gamma[N](n - n_{eq.}).$$
$$\frac{dL(t)}{dt} + 3HL(t) = (\Gamma[N \to l^- \phi^+] - \Gamma[N \to l^+ \phi^-])(n - n_{eq}) \quad (4)$$

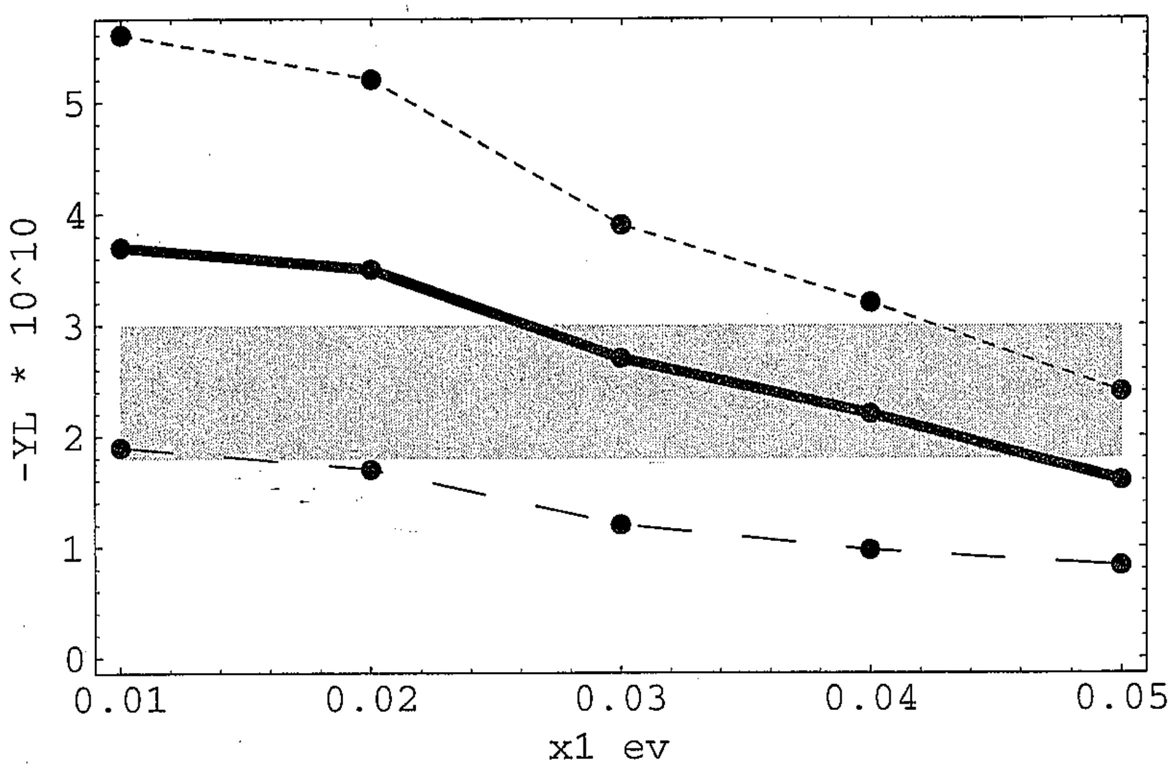
$t = \sqrt{\frac{45}{16\pi^3 g^*} \frac{M_{pl}}{T^2}}$, **Initial condition:@** $T =$ 10^{16} **(GeV)**

$$n = n_{eq}, L = 0$$

The result depends on $\Gamma[N]$,
" Direct CP violation" $\epsilon = \frac{\Gamma[N \to l^- \phi^+] - \Gamma[N \to l^+ \phi^-]}{\Gamma[N \to l^- \phi^+] + \Gamma[N \to l^+ \phi^-]}$
and expansion rate of the universe (Hubble) $H = \frac{\dot{a}}{a}$. $L_{primodial}$ may be easily produced if the non-equilibrium condition is satisfied.

$$H > \Gamma[N] \tag{5}$$

Fig.1 $\to -Y_L = \frac{L}{s}$ (s entropy density).
$x_1 = \frac{(m_D^\dagger m_D)_{11}}{M_1} = \frac{\Gamma[N] V^2}{M_1^2}$. $V = \sqrt{4\pi v}, v = 256 GeV$.



The maximum possible lepton number density $(-Y_L)$ as a function of x_1 for three different M_1. From up to bottom, $M_1 = 3, 2, 1 \times 10^{11}$ GeV.

Is the CP violation of baryogenesis related to CP violation in low energy ?

$$P(\nu_\mu \to \nu_e) - P(\bar{\nu_\mu} \to \bar{\nu_e}) =$$

$$4J[\sin(\frac{\Delta m_{12}^2 L}{2E}) + \sin(\frac{\Delta m_{23}^2 L}{2E}) + \sin(\frac{\Delta m_{31}^2 L}{2E})].$$

$$J = Im\left(U_{e1}U_{\mu 1}^* U_{e2}^* U_{\mu 2}\right).$$

Branco Morozumi Nobre and Rebelo (Nucl.Phys.B617) show, in general, there may be some relation.

$$\mathcal{L} = \bar{l^i} m_{li} l^i + \bar{\nu^i} m_{Dij} N_{Rj} + \frac{1}{2}(\bar{N_{Rj}})^c M_j N_{Rj}, \quad (6)$$

$(3, N)$model.

The basis all flavor mixing and CP origin are in the Dirac mass term of neutrino m_D.

$M_j = Diag.(M_1, M_2, ..M_N), m_l = Diag.(m_e, m_\mu, m_\tau)$.

$$L_{ini} \sim Im(m_D^\dagger m_D)_{ij}^2, (i \neq j):$$

$$-U^\dagger m_D \frac{1}{M} m_D{}^T U^* = \begin{pmatrix} n_1 & 0 & 0 \\ 0 & n_2 & 0 \\ 0 & 0 & n_3 \end{pmatrix}$$

$$L \leftarrow Im(m_D) \rightarrow Im(U) \rightarrow J.$$

Both J and L are related to CP phases in m_D.

- How are J and $B(L)$ related ? Identifying the low energy CP phase which is related to leptogenesis.
  - For quantitative analysis on the correlation, we must fix the parameters of the model. What kind of physical observables are needed to fix all parameters ? Write J in the parametrization which apply for the most general case.

- Give a specific example which shows a direct link " correlation" between J and B.
  - The general case and numerical analysis:

Independent number of parameters in neutrino sector of seesaw model.

model	(3,N)	(3,3)	(3,2)
M	N	3	2
$Re(m_D)$	3 N	9	6
$Im(m_D)$	3N-3	6(3)	3 (1)
total	7N-3	18	11

($blue$) Independent number of CP phases for leptogenesis. $({m_D}^{\dagger} m_D)_{ij} (i \neq j)$ Low energy observables: $= 7 \ll 11$ (Minimal model:$N = 2$).

mixing angles Δm^2 *neutrinoless double* β *CP violation in oscillation*	$2 + \lvert U_{e3} \rvert$ 2 (solar, atm.) $\lvert (m_{eff})_{ee} \rvert$ 1 (ΔP)
total	7

We adopt four high energy physical quantities as input. Heavy Majorana masses (M_1, M_2) and their decay widths $(\Gamma_1, \Gamma_2.)$

We can fix 11 parameters of the minimal seesaw (3,2) model.

7 (low energy observables)+ 4 (high energy observables)=11. Besides M_1, M_2, 9 parameters in m_D is parametrized as:

$$
\begin{aligned}
m_D &= \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \\ m_{31} & m_{32} \end{pmatrix} \\
&= U_L \begin{pmatrix} 0 & 0 \\ m_2 & 0 \\ 0 & m_3 \end{pmatrix} V_R (= U_L m V_R).
\end{aligned}
$$

U_L: 3×3 , V_R:2×2 unitary matrix.

$$
\begin{aligned}
U_L &= O(\theta_{L23}) U(\theta_{L13}, \delta_L) O(\theta_{L12}) \\
&\quad \times diag.(1, \exp[-i\frac{\gamma_L}{2}], \exp[i\frac{\gamma_L}{2}]) \\
V_R &= \begin{pmatrix} c_R & s_R \\ -s_R & c_R \end{pmatrix} \begin{pmatrix} \exp(-i\frac{\gamma_R}{2}) & 0 \\ 0 & \exp(i\frac{\gamma_R}{2}) \end{pmatrix}.
\end{aligned}
$$

Among 3 CP phases$(\delta_L, \gamma_L, \gamma_R)$ $\gamma_R \to$ leptogenesis.

$$\begin{aligned}\epsilon_1 \quad &\sim \Gamma[N_1 \to l^- \phi^+] - \Gamma[N_1 \to l^+ \phi^-] \\ &\sim -Im[(m_D^\dagger m_D)_{12}^2] \\ &\sim -({m_3}^2 - {m_2}^2)^2 {s_R}^2 {c_R}^2 \sin 2\gamma_R.\end{aligned}$$

γ_R has impact on low energy CP J through MNS, U. U is determined as a unitary matrix diagonalizing $m_{eff} = -m_D \frac{1}{M} m_D^T$.

$$U^\dagger m_{eff} U^* = diag.(n_1, n_2, n_3)$$
$$m_{eff} = -U_L (m V_R \frac{1}{M} {V_R}^T m^T) {U_L}^T$$
$$U = U_L K_R,$$
$$K_R^\dagger \begin{pmatrix} 0 & 0 & 0 \\ 0 & Z_{22} & Z_{23} \\ 0 & Z_{32} & Z_{33} \end{pmatrix} K_R^* = \begin{pmatrix} 0 & 0 & 0 \\ 0 & n_2 & 0 \\ 0 & 0 & n_3 \end{pmatrix}.$$

$Z_{ij} = Z_{ij}(M_i, m_i, \gamma_R, \theta_R)$ $n_1 = 0$ one neutrino is massless.

K_R low energy manifestation of high energy physics. $(\gamma_R, \theta_R, M_i, m_i)$.

$$
\begin{aligned}
K_R &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \exp[-i\phi] \\ 0 & -\sin\theta\exp[i\phi] & \cos\theta \end{pmatrix} \\
&\times \begin{pmatrix} 1 & 0 & 0 \\ 0 & \exp[i\alpha] & 0 \\ 0 & 0 & \exp[-i\alpha] \end{pmatrix}.
\end{aligned}
$$

$$
\begin{aligned}
2\theta &= Arc.tan\left(\frac{2|Z_{22}^* Z_{23} + Z_{23}^* Z_{33}|}{|Z_{33}|^2 - |Z_{22}|^2}\right) \Leftrightarrow \theta_R \\
\phi &= Arg.(Z_{22}^* Z_{23} + Z_{23}^* Z_{33}) \Leftrightarrow \gamma_R \\
2\alpha &= Arg.\left(\cos^2\theta Z_{22} + \sin^2\theta Z_{33}\exp(-2i\phi)\right. \\
&\left. -\sin 2\theta Z_{23}\exp(-i\phi)\right) \Leftrightarrow \gamma_R
\end{aligned}
$$

All three CP phases contribute to $J = Im[U_{e1}U_{\mu 1}^* U_{e2}^* U_{\mu 2}]$.

$$
\begin{aligned}
J &= \frac{1}{8}\sin 2\theta_{L12}\sin 2\theta_{L13}[c_{L13}\cos 2\theta \sin\delta_L \sin 2\theta_{L23} \\
&+ c_{L12}\sin 2\theta \sin(\delta_L - \gamma_L - \phi)\cos 2\theta_{L23} \\
&- \frac{1}{2} s_{L12} s_{L13}\sin 2\theta \sin 2\theta_{L23}\sin(2\delta_L - \gamma_L - \phi)] \\
&+ \frac{1}{8}\sin 2\theta \sin 2\theta_{L23}\sin(\gamma_L + \phi) \\
&\times(\sin 2\theta_{L12} c_{L13} s_{L12} - \sin 2\theta_{L13} s_{L13} c_{L12}).
\end{aligned}
$$

We can get the values of θ and ϕ from
1) high energy physics input $(M_1, M_2, \Gamma_1, \Gamma_2)+$
2) neutrino mass eigenvalue eq.
$det.(m_{eff}m_{eff}^{\dagger} - n^2) = 0$, $(n_2 = \sqrt{\Delta m^2_{sol.}} = 7 \times 10^{-3}(eV), n_3 = \sqrt{\Delta m^2_{atm.}} = 5 \times 10^{-2})$.
With the input we can determime $(m_2, m_3, \gamma_R, \theta_R)$ which leads to the prediction of θ, ϕ and also lepton number asymmetry ϵ_1.

$$\cos 2\gamma_R = \frac{n_2^2 + n_3^2 - x_1^2 - x_2^2}{2(x_1 x_2 - n_2 n_3)}, \quad x_i = \frac{(m_D{}^{\dagger} m_D)_{ii}}{M_i} = \Gamma_i \left(\frac{V}{M_i}\right)^2$$

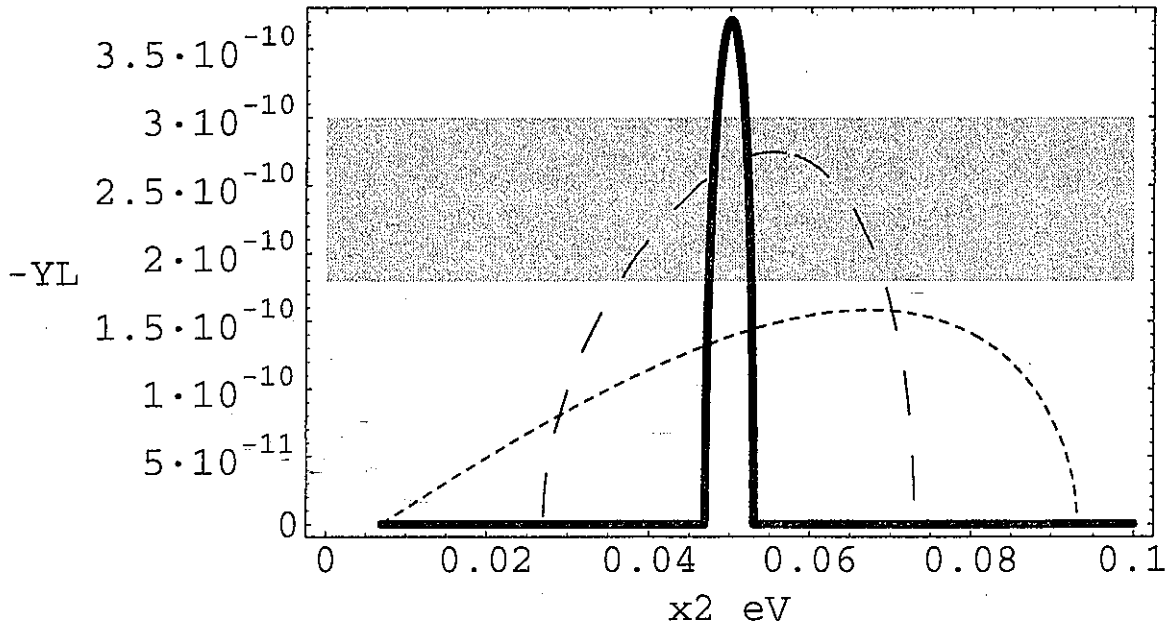
$$\epsilon_1 = -\frac{3M_1}{4x_1 V^2}\sqrt{\left((n_-)^2 - (x_-)^2\right)\left((x_+)^2 - (n_+)^2\right)},$$

where $n_{\pm} = n_3 \pm n_2$ and $x_{\pm} = x_1 \pm x_2$.

The range of (θ, ϕ) which is consistent with baryogenesis is limited.

The lower bound of M_1 is obtained from lepton number asymmetry.

The decay width Γ_1 (or x_1) has upper bound from non-equilibrium condition (obtained from solving Boltzman equation.)



Lepton number density ($-Y_L$) as a function of x_2 for $M_1 = 2 \times 10^{11}$ GeV. Solid line corresponds to $x_1 = 0.01$ eV. The long dashed line corresponds to $x_1 = 0.03$ eV and the dotted line corresponds to $x_1 = 0.05$ eV.

$$U_{MNS} \simeq$$

$$\begin{pmatrix} c_{L12} & s_{L12} & s_{L13}e^{-i\delta_L} + s_{L12}s_\theta e^{-i\phi'} \\ -s_{L12}c_{L23} & c_{L12}c_{L23} & s_{L23} \\ s_{L12}s_{L23} & -c_{L12}s_{L23} & c_{L23} \end{pmatrix}$$

$$\times P(\alpha', -\alpha')$$

$$|(U_{MNS})_{e3}| \simeq |s_{L13}e^{-i\delta} + \frac{s_\theta}{\sqrt{2}}e^{-i\phi'}|,$$

$$J \simeq \frac{1}{4}\left(s_{L13}\sin\delta_L + \frac{s_\theta}{\sqrt{2}}\sin\phi'\right),$$

$$|(m_{eff})_{ee}| \simeq |\frac{n_2}{2}e^{4i\alpha'} + n_3(s_{L13}e^{-i\delta_L} + \frac{s_\theta}{\sqrt{2}}e^{-i\phi'})^2|.$$

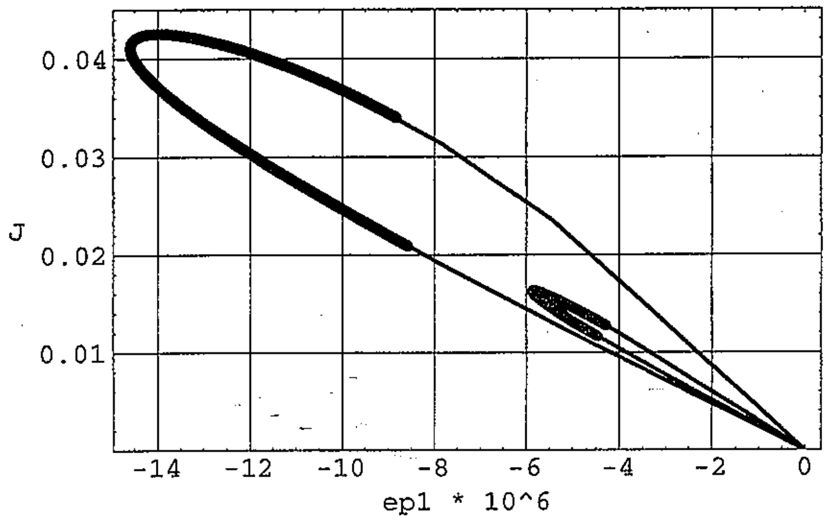
Here we give an example for the model in which J is determined by leptogenesis phase γ_R. Suppose U_L is a real orthogonal matrix as $U_L = O_{23}O_{12}$. MNS matrix U has the following form; Taking $\theta_{L23} = \pi/4$, $\theta_{L12} = \pi/4$, we have;

$$U = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{\cos\theta}{\sqrt{2}} & \frac{\sin\theta\exp[-i\phi]}{\sqrt{2}} \\ -\frac{1}{2} & \frac{\cos\theta-\sqrt{2}\sin\theta\exp[i\phi]}{2} & \frac{\sin\theta\exp[-i\phi]+\sqrt{2}\cos\theta}{2} \\ \frac{1}{2} & -\frac{\cos\theta+\sqrt{2}\sin\theta\exp[i\phi]}{2} & \frac{-\sin\theta\exp[-i\phi]+\sqrt{2}\cos\theta}{2} \end{pmatrix} \times P.(1,\alpha,-\alpha),$$

$$J = \sin 2\theta \sin\phi \frac{1}{8\sqrt{2}}$$

$$\gamma_R \neq 0 \Leftrightarrow J \neq 0.$$

The sign (excess of matter (antimatter)) has also correlation in the case.



Correlation between lepton number asymmetry ϵ_1 and neutrino CP violation J, for $M_1 = 2 \times 10^{11}$ GeV. The large (small) contour corresponds to $x_1 = 0.03(0.01)$ eV. The thick solid lines show the allowed region from baryogenesis.

Summary

- leptogenesis phase (γ_R) certainly affects the neutrino oscillation CP violation through (ϕ)

- However, in general ground, we can not distinguish the phase coming from leptogenesis (ϕ) and another phase γ_L in U_L because only a combination appear as $\phi + \gamma_L$. The isolation must be done using some other quantities, double β decay etc.

- Leptogenesis formule is written in terms of physical quantities: Heavy neutrino mass and decay width and light neutrino mass.