Leptonic CP violation search with a neutrino factory

Toshihiko Ota,
Department of Physics Kyushu University.

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■Our points:
- ▶Matter density profile effect.
- ▶Test statistic.
- ▶Statistical method.

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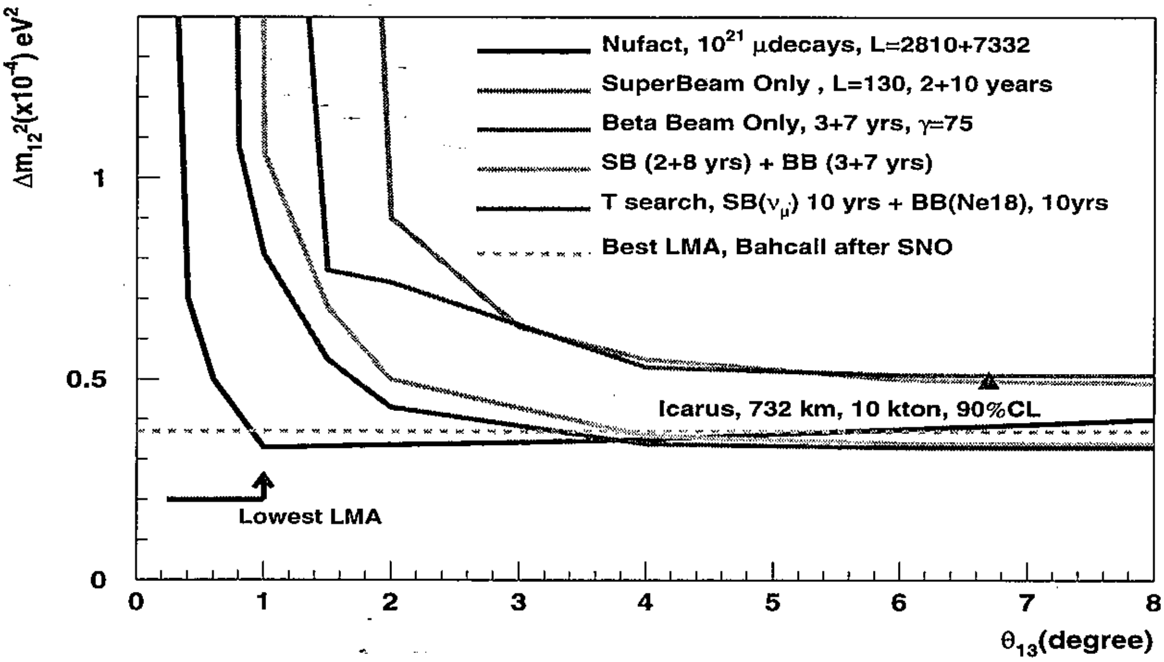
■Introduction: Works so far

■ Over the past few years a considerable number of studies have been made on the CP-violation search with neutrino factory.

▶ For example,

■ Parameter region which we can observe the CP-violation effect when nature takes the value of $\delta = \pi/2$.



A. Cervera *et al.*, Nucl. Phys. **B579** (2000) 17.

▶ We re-consider that

(i) matter effect: 3% uncertainty → 3% + extra.

(ii) test statistic: $\chi_1^2 \rightarrow \chi_3^2$.

(iii) statistical method:

d.o.f = # of parameters → # of bins.

■ Shortly speaking ...

- ▶ Why is our optimal baseline so short?
- ▶ Why do we use the less familiar test statistic?
- ▶ Why is the single-bin method advantageous?

■ We point out the correlation concerning the matter effect.

▶ The matter effect becomes serious than that has been considered so far.

▼

The short baseline option becomes better.

- We make the muon energy lower as the baseline length.
- However, we loss the event rate.

▼

■ We consider the method to collect the information with efficient in low energy region.

▼

We propose the test statistic, χ_3^2, which is mainly sensitive to $\sin\delta$.

The degree of freedom is the number of the bins.

▲

The best-fit regions do not distribute only around the nature values.

◀ ■ We reconsider the procedure to test the hypothesis.

■Introduction: Summary of our former work

■We investigated the matter density profile effect. (Phys. ReV. **D63** (2001) 093004)

▶We showed the conditions to enlarge the matter profile effect, analytically.

Hamiltonian in flavor base: 2-generation

$$H = \frac{1}{2E_\nu}\left\{U\begin{pmatrix}0 & \\ & \Delta m^2\end{pmatrix}U^\dagger + \begin{pmatrix}a_0 + \delta a(x) & \\ & 0\end{pmatrix}\right\}$$

$$= \frac{1}{2E_\nu}\left\{\tilde{U}_0\begin{pmatrix}\lambda_- & \\ & \lambda_+\end{pmatrix}\tilde{U}_0^\dagger + \begin{pmatrix}\delta a(x) & \\ & 0\end{pmatrix}\right\}$$

*mass-square eigenvalues in const.(a_0) matter

$$\delta a(x) = \sum_{n=-\infty}^{\infty} a_n e^{-ip_n x}, \quad p_n \equiv \frac{2n\pi}{L}$$

(i) Fourier coefficients are large.

(ii)The resonance conditions are satisfied.

$$\lambda_+ - \lambda_- = 2p_n E_\nu$$

▶ Only first few modes are important.

▼

The conclusion was

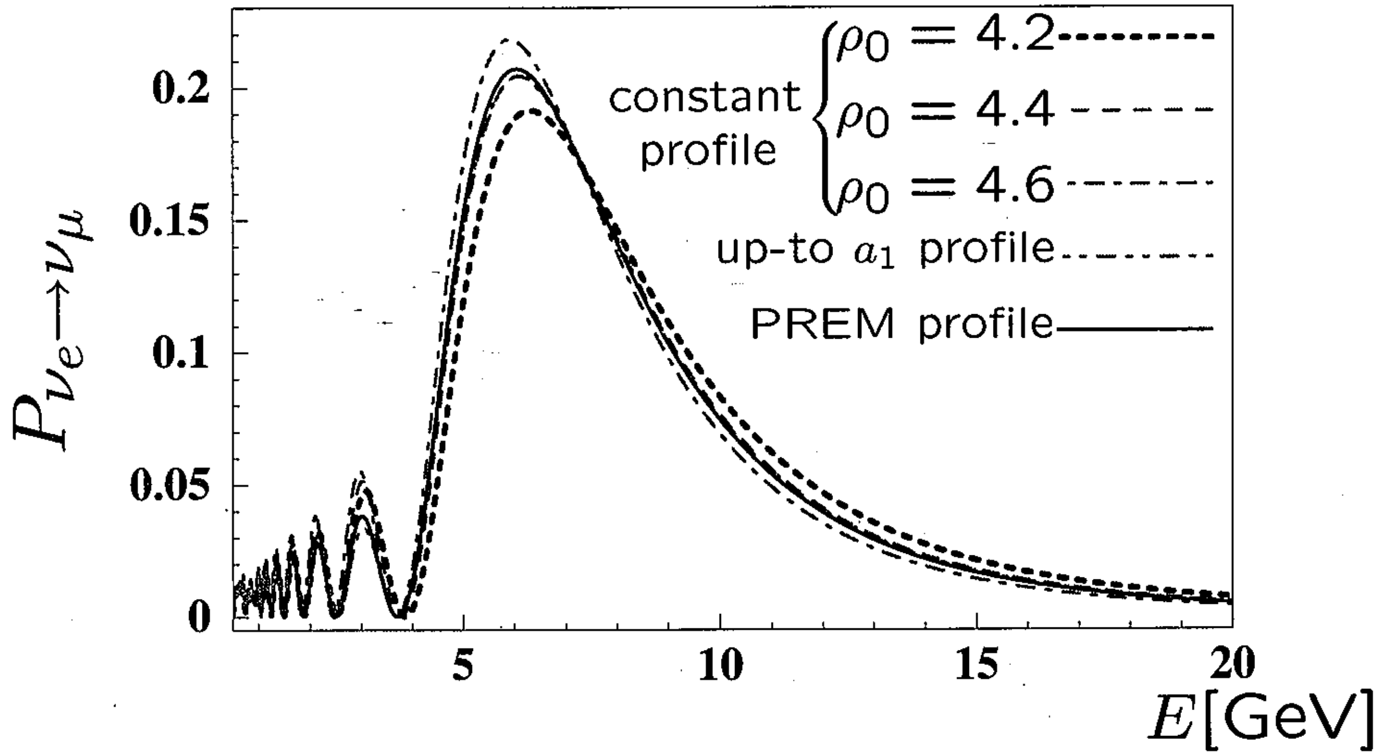
In $L \lesssim$5,000km,

the matter profile effect is not significant,

if PREM is trustworthy.

■Yet Another Correlation: *hep-ph/0211095.*

■ One example: L =7,332 km.

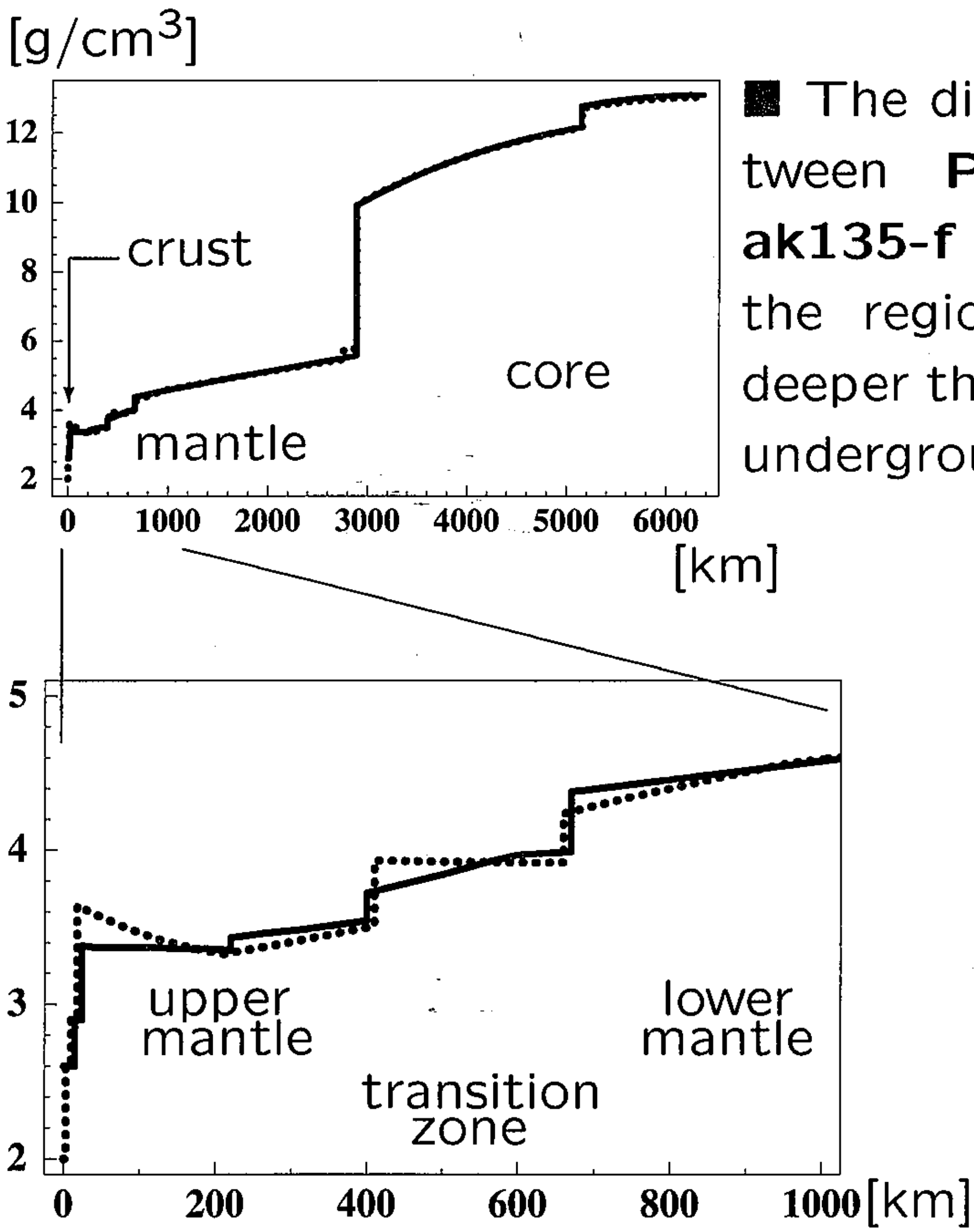


The probability calculated using the constant profile with $\rho_0 = 4.42$[g/cm^3] coincides with that using the full-PREM profile.

▼

■ There is a strong correlation between a_0 and a_1 in the region where $L \lesssim$7,500 km.

Uncertainty of the Earth models

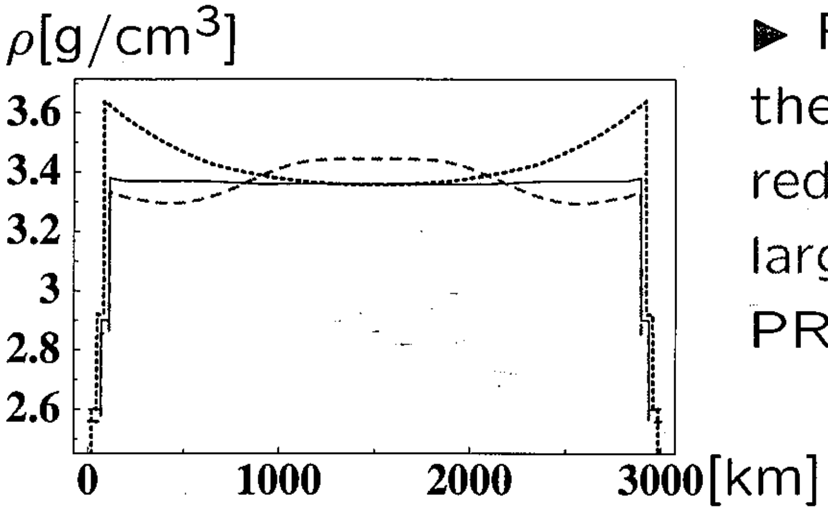


■ The difference between **PREM** and **ak135-f** is small in the region which is deeper than 1000 km underground.

■ In the upper mantle and transition zone, the difference is large enough to affect the CP sensitivity.

■Yet Another Correlation:
~ *Effect for the CP-violation search* ~

■ If the uncertainty in each point is 5%, then the error of a_1 can become more than 100% without tuning a_0.

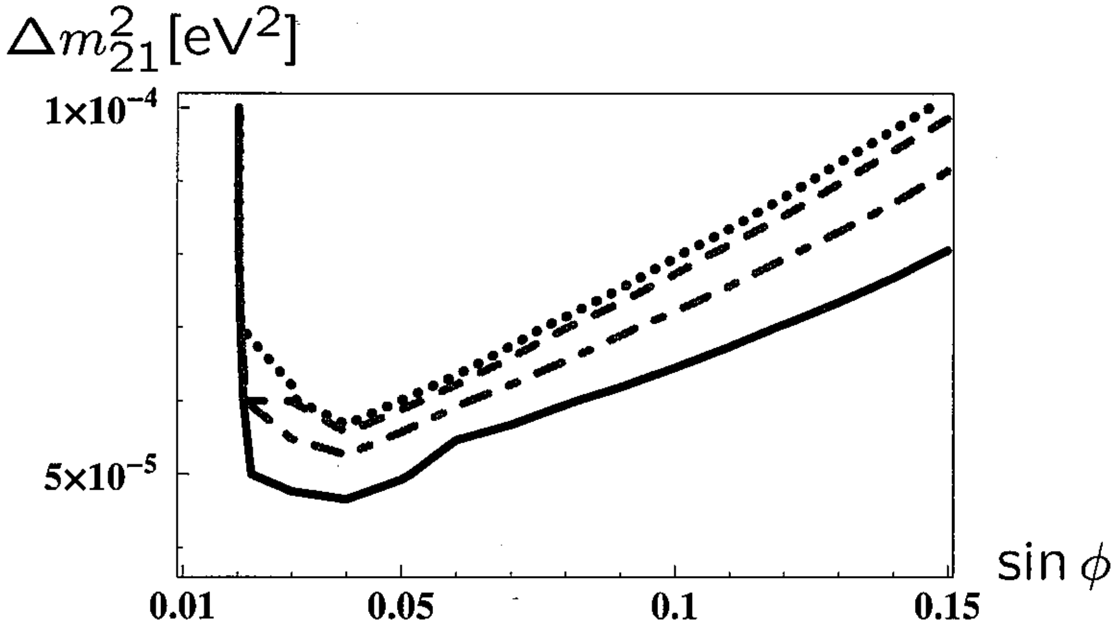


▶ For example, the value of a_1 of the red profile is 2 times larger than that of PREM profile.

Does not really the error affect the CP sensitivity?

▼

■ The uncertainty of a_1 brings $2 \sim 3\%$ additional uncertainty to a_0.



■Definition of the test statistics ~ χ_1^2 *and* χ_3^2 ~

■ Test statistics are the quantities to measure the deviation from the theory with δ_0.

$$\chi_1^2 \equiv \sum_i^{\text{bin}} \frac{\left|N_i^{\text{ex}} - N_i^{\text{th}}(\delta_0)\right|^2}{N_i^{\text{ex}}} + \sum_i^{\text{bin}} \frac{\left|\bar{N}_i^{\text{ex}} - \bar{N}_i^{\text{th}}(\delta_0)\right|^2}{\bar{N}_i^{\text{ex}}}.$$

▶ We propose

$$\chi_3^2 \equiv \sum_i^{\text{bin}} \frac{\left|\bar{N}_i^{\text{th}}(\delta_0) N_i^{\text{ex}} - N_i^{\text{th}}(\delta_0)\bar{N}_i^{\text{ex}}\right|^2}{\bar{N}_i^{\text{th}}(\delta_0)^2 N_i^{\text{ex}} + N_i^{\text{th}}(\delta_0)^2 \bar{N}_i^{\text{ex}}}.$$

■ Hypothesis testing:

When the deviation is larger than the criterion,

$$\chi^2 > \chi^2_{90\%}(\text{d.o.f} = \text{bin}),$$

we can reject the theory with δ_0.

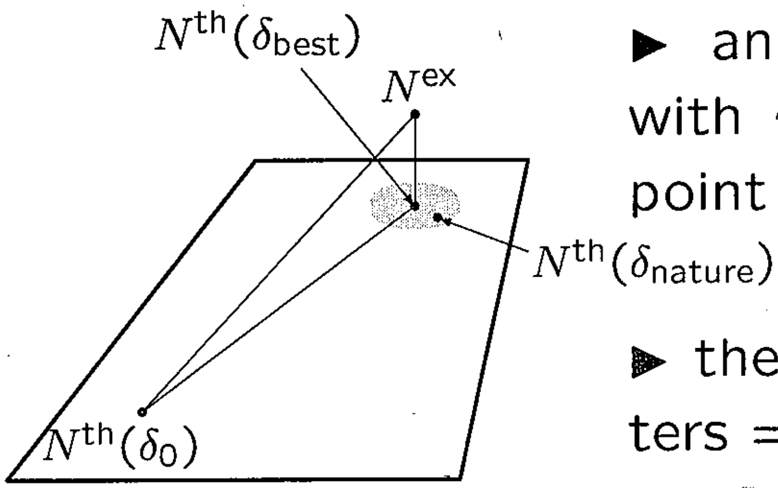
▼

Now, we do not have an experimental sample, N^{ex}. What we can do is the comparison between the theory with δ and that with δ_0.

(What we can control is *the power of test*.)

■Nature value and the best-fit value

Why d.o.f.=f?



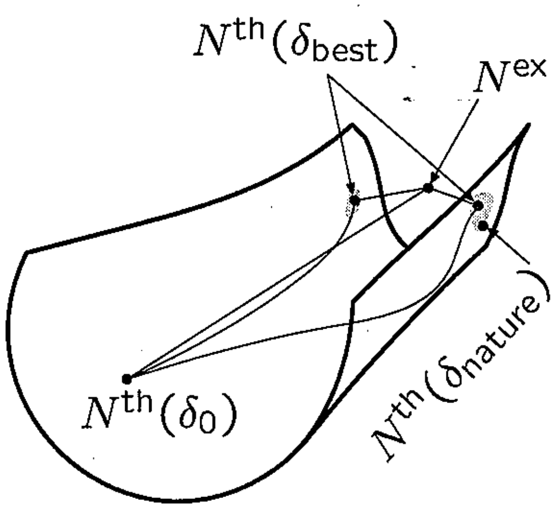
▶ an experimental result with n-bin method = the point in n-dim space.

▶ the space for f parameters = f-dim plane

▶ the best-fit value = the nearest point from N^{ex} on the plane

However, the parameters are entangled each other. Not so plain.

Why d.o.f.=n?



▶ The allowed regions can distribute separately each other.

▶ If so, we should use the distance between $N^{th}(\delta_0)$ and the nearest allowed region in the test.

▶ It follows from the fact $N^{ex} \simeq N^{th}(\delta_{best})$ that we make the test in n-dim (= with n bin method).

■Numerical calculations:
~ *procedure to calculate the necessary data size* ~

■The figures indicate the necessary data size (muon number × detector mass) to distinguish between the theory with δ and that with δ_0 at 90% confidence level.

►Concretely,

$$\chi^2_{90\%} < \chi^2 \equiv \min_{\delta_0 \in \{0,\pi\};\{x'\}} \sum_i^{\text{bin}} \frac{|N_i(\delta, x) - N_i(\delta_0, x')|^2}{N_i(\delta, x)}$$

$$\equiv N_\mu M_{\text{det}} \min_{\delta_0 \in \{0,\pi\};\{x'\}} \sum_i^{\text{bin}} X_i$$

$$N_\mu M_{\text{det}} > \chi^2_{90\%} \Big/ \min \sum_i^{\text{bin}} X_i$$

In each E_μ and L, this value is ploted.

The contour "1" means if we have 10^{21} muons and 100 kt detector, then we can observe the difference between "δ" and "δ_0" at 90% C.L..

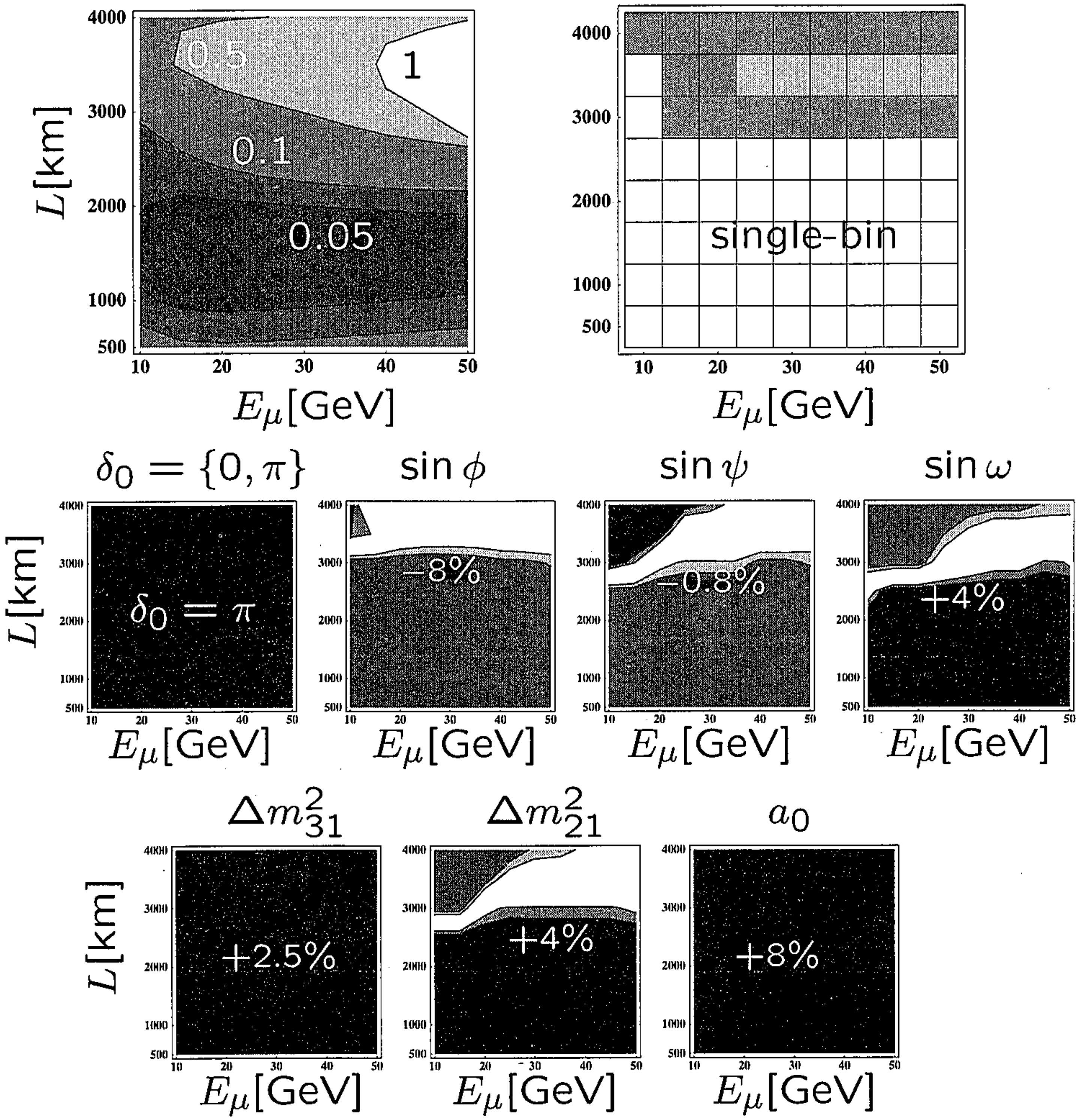
■Numerical calculations ~ *optimum set-up* ~

Reference values:

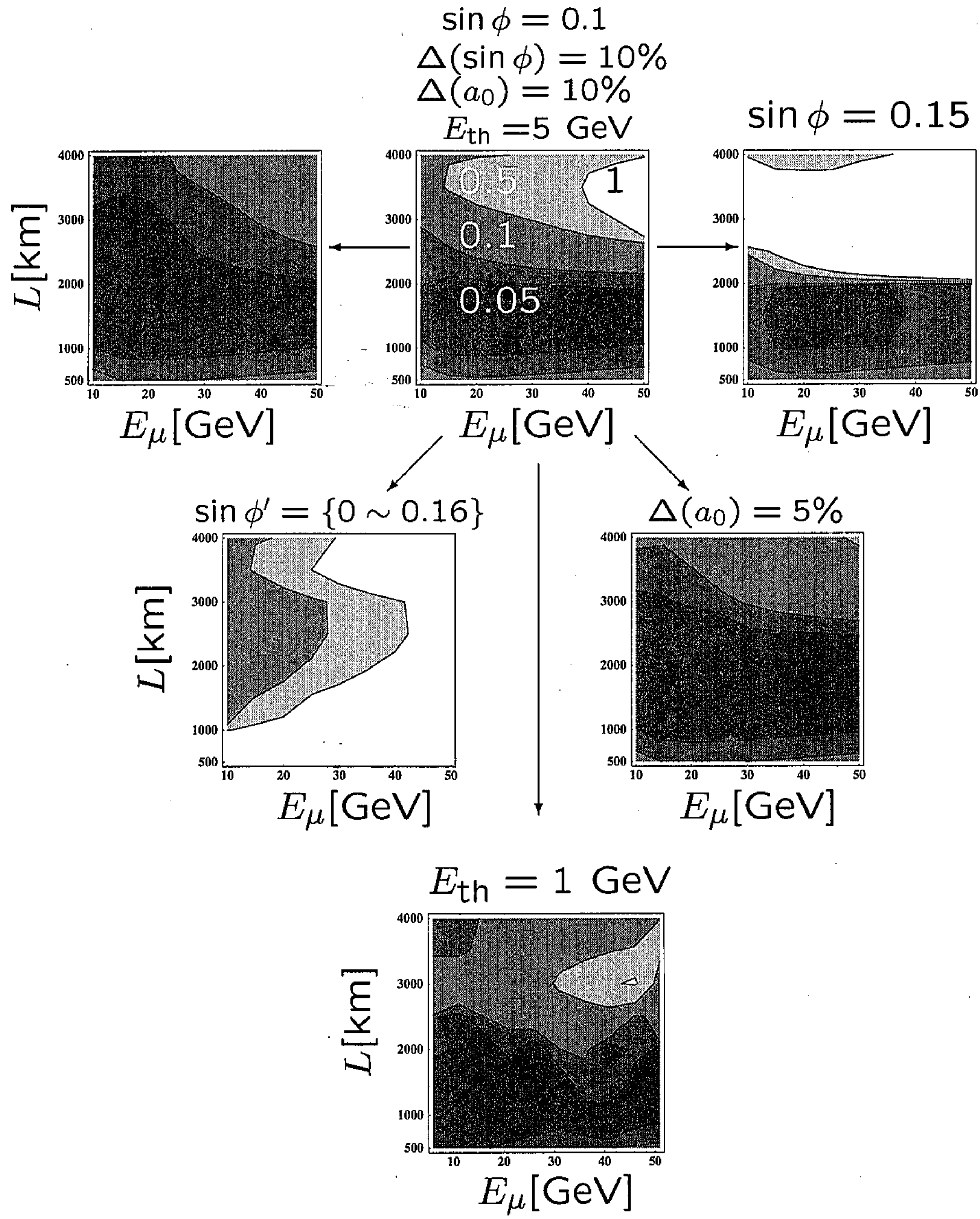
$$\sin\phi = 0.1 \pm 10\%,\ \sin\psi = 1/\sqrt{2} \pm 1\%,\ \sin\omega = 1/2 \pm 5\%,$$

$$\Delta m^2_{31} = 2.5\times 10^{-3} \pm 3\%, \quad \Delta m^2_{21} = 5\times 10^{-5} \pm 5\%$$

$N(\delta)$:full-PREM, $\quad N(\delta_0)$:const.±10%.



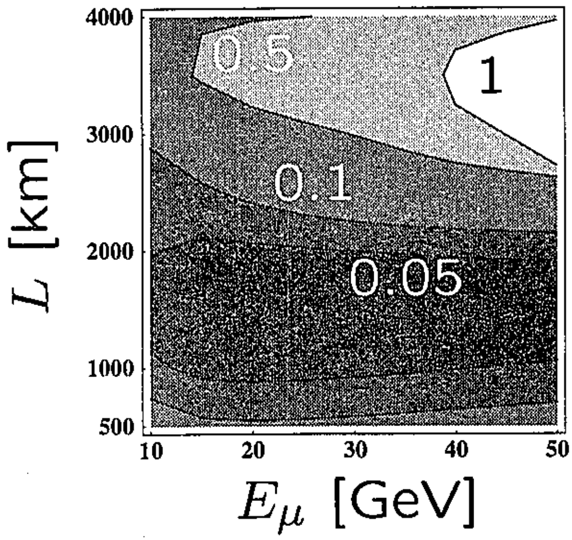
■Numerical calculations:
~ options for some parameter sets ~



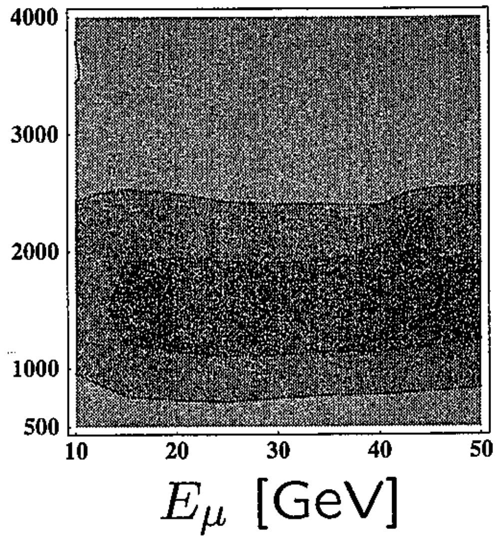
■Numerical results:
~ *Comparison between* χ_1^2 *and* χ_3^2 ~

■ In $\sin\phi = 0.1$,

χ_3^2



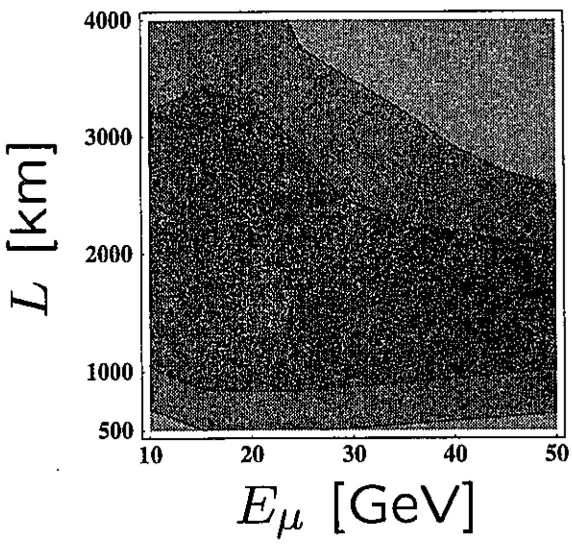
χ_1^2



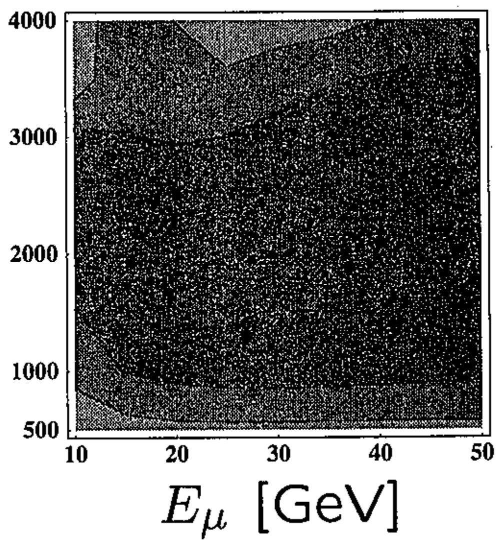
▶ The difference is slight.

■ In $\sin\phi = 0.05$,

χ_3^2



χ_1^2



▶ In high energy region, χ_1^2 is advantageous.

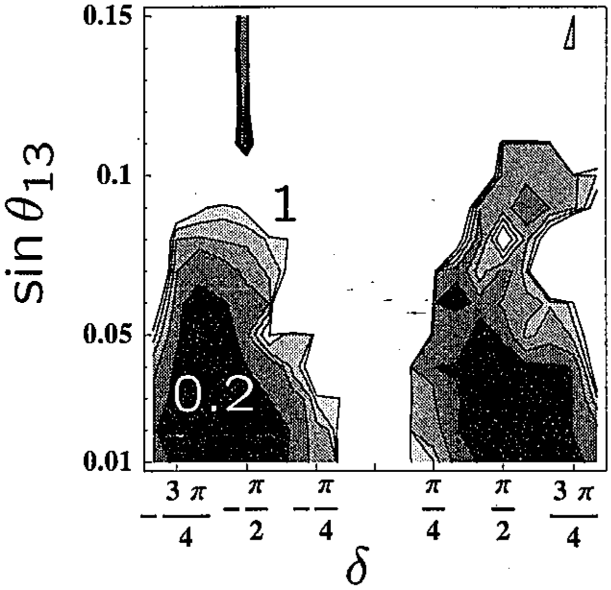
▶ In low energy region...

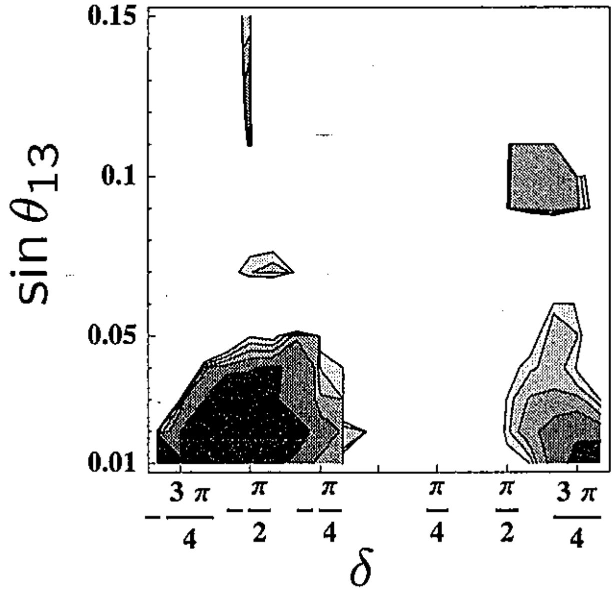
■Numerical results:
Comparison "low and short" with "high and long"

$\Delta(a_0) = 10\%$, $\Delta(\sin\phi) = 10\%$

■ In long baseline and high energy option,

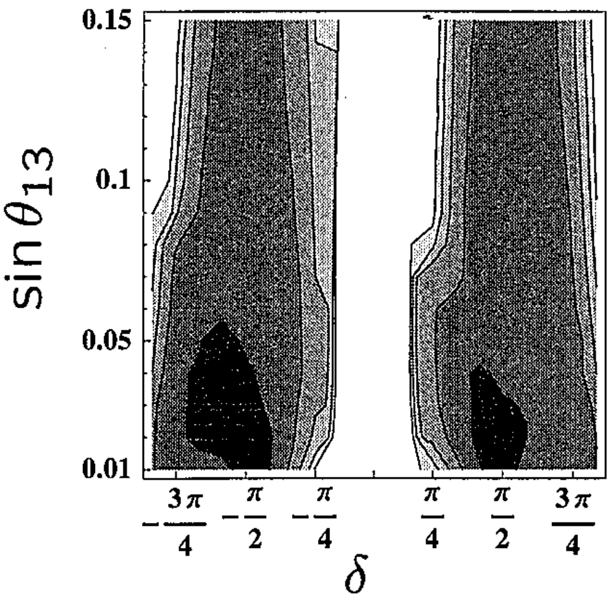
χ_1^2 :(3,000 km, 50 GeV) χ_3^2 :(3,000 km, 50 GeV)

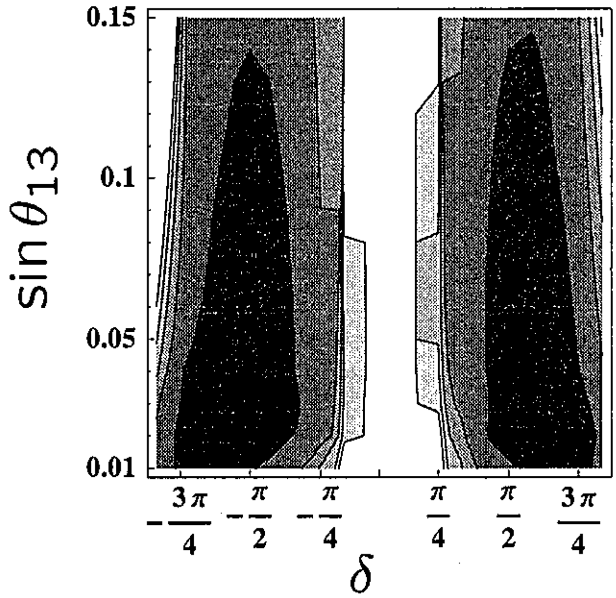




■ In short baseline and low energy option,

χ_1^2 :(1,000 km, 10 GeV) χ_3^2 :(1,000 km, 10 GeV)





■Conclusion

■ Through the correlation between a_0 and a_1 the uncertainty of profile gives an extra uncertainty to a_0.

▼

■ We consider the short baseline option with the test statistic which is mainly sensitive to $\sin\delta$.

■ We re-consider the method of the hypothesis testing.

▼

■ If $|U_{e3}|$ is as large as the value just under the current bound,

then the matter effect is enhanced.

▶ The short baseline with χ_3^2 is advantageous.

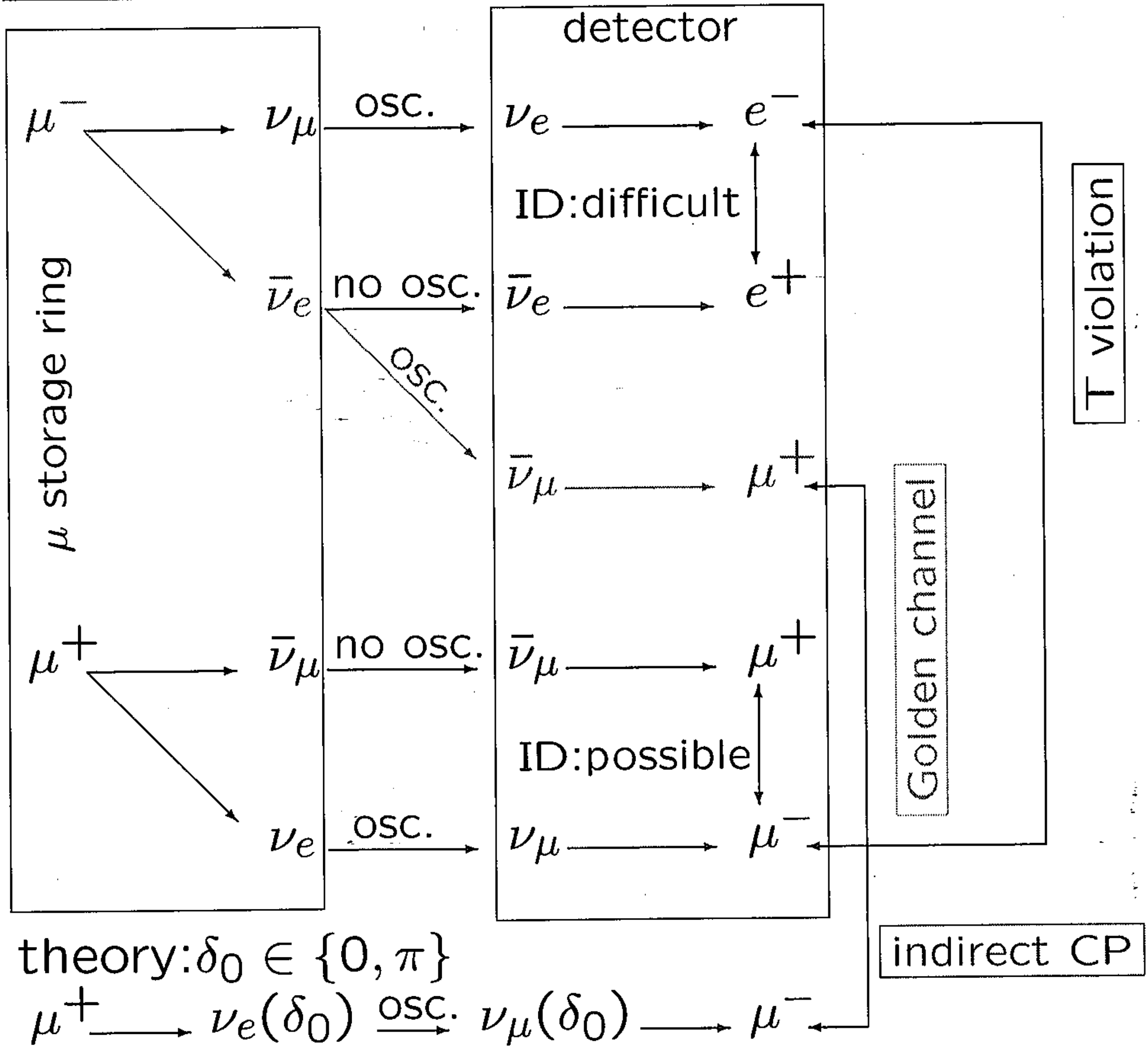
■ If $|U_{e3}|$ is small,

then the matter effect is suppressed.

▶ The long baseline with χ_1^2 is advantageous.

■CP violation search with a neutrino factory: ~ *Golden channel* ~

Neutrino factory strategy



■When we search for the CP-violation effect, the estimation of the matter effect is important since the matter effect make the fake CP signal.

$$P_{\nu_e \to \nu_\mu} \xrightarrow{\delta \to -\delta,\ a \to -a} P_{\bar{\nu}_e \to \bar{\nu}_\mu}$$

■Oscillation probability:

∼ *Exact and approximated solution* ∼

■In 3-generation scheme, the oscillation probability can be exactly expressed as

$$\begin{cases} P_{\nu_e\to\nu_\mu} = A\cos\delta + B\sin\delta + C \\ P_{\bar\nu_e\to\bar\nu_\mu} = \bar A\cos\delta + \bar B\sin\delta + \bar C, \end{cases}$$

this can be approximated at $\mathcal{O}\{(\Delta m^2_{21}/\Delta m^2_{31})^2\}$ as

$$\begin{aligned}
P_{\nu_e\to\nu_\mu} \simeq\ & 4\frac{|\Delta m^2_{31}|^2}{(\lambda_+-\lambda_-)^2}|U_{e3}|^2|U_{\mu 3}|^2\sin^2\frac{\lambda_+-\lambda_-}{4E}L \\
& +4\frac{\Delta m^2_{21}(\Delta m^2_{31})^2}{\lambda_-\lambda_+(\lambda_+-\lambda_-)}\mathrm{Re}[U_{e1}U^*_{\mu 1}U^*_{e3}U_{\mu 3}]\sin^2\frac{\lambda_-}{4E}L \\
& -4\frac{\Delta m^2_{21}(\Delta m^2_{31})^2}{\lambda_-\lambda_+(\lambda_+-\lambda_-)}\mathrm{Re}[U_{e1}U^*_{\mu 1}U^*_{e3}U_{\mu 3}]\sin^2\frac{\lambda_+}{4E}L \\
& +4\frac{\Delta m^2_{21}\Delta m^2_{31}}{(\lambda_+-\lambda_-)^2}\Bigg\{2\mathrm{Re}[U_{e2}U^*_{\mu 2}U^*_{e3}U_{\mu 3}] \\
& \qquad +\frac{\Delta m^2_{31}(\lambda_++\lambda_-)}{\lambda_+\lambda_-}\mathrm{Re}[U_{e1}U^*_{\mu 1}U^*_{e3}U_{\mu 3}]\Bigg\}\sin^2\frac{\lambda_+-\lambda_-}{4E}L \\
& +8\frac{\Delta m_{21}(\Delta m^2_{31})^2}{\lambda_+\lambda_-(\lambda_+-\lambda_-)}\mathrm{Im}(U_{e1}U^*_{\mu 1}U^*_{e3}U_{\mu 3})\sin\frac{\lambda_-}{4E}L\sin\frac{\lambda_+}{4E}L\sin\frac{\lambda_+-\lambda_-}{4E}L \\
& +4\frac{(\Delta m^2_{21})^2\Delta m^2_{31}}{\lambda_-\lambda_+(\lambda_+-\lambda_-)}\left(\frac{\Delta m^2_{31}}{\lambda_-}|U_{e1}|^2|U_{\mu 1}|^2+\mathrm{Re}[U_{e1}U^*_{\mu 1}U^*_{e2}U_{\mu 2}]\right)\sin^2\frac{\lambda_-}{4E}L \\
& -4\frac{(\Delta m^2_{21})^2\Delta m^2_{31}}{\lambda_-\lambda_+(\lambda_+-\lambda_-)}\left(\frac{\Delta m^2_{31}}{\lambda_+}|U_{e1}|^2|U_{\mu 1}|^2+\mathrm{Re}[U_{e1}U^*_{\mu 1}U^*_{e2}U_{\mu 2}]\right)\sin^2\frac{\lambda_+}{4E}L \\
& +4\frac{(\Delta m^2_{21})^2}{(\lambda_+-\lambda_-)^2}\Bigg\{|U_{e2}|^2|U_{\mu 2}|^2+\frac{(\Delta m^2_{31})^2}{\lambda_-\lambda_+}|U_{e1}|^2|U_{\mu 1}|^2 \\
& \qquad +\frac{\Delta m^2_{31}(\lambda_++\lambda_-)}{\lambda_-\lambda_+}\mathrm{Re}[U_{e1}U^*_{\mu 1}U^*_{e2}U_{\mu 2}]\Bigg\}\sin^2\frac{\lambda_+-\lambda_-}{4E}L,
\end{aligned}$$

where

$$\lambda_\mp \equiv \frac{1}{2}\left(\Delta m^2_{31}+a\mp\sqrt{(\Delta m^2_{31}-a)^2+4\Delta m^2_{31}a|U_{e3}|^2}\right).$$

追記

私の報告に対するコメントに関してコメントします。

■ 物質効果の誤差が5%の場合に (E_μ, L) =(50 GeV, 3,000 km) が最適な設定であるということは consensus がとれている。

▶ その通りです。私の主張のスタート地点は、物質効果の誤差が密度分布の効果及びその誤差を考慮することによりこれまで考えられてきた見積もりより大きいのではないだろうか、ということです。そのことが事実であるならば、最適領域は「低く」「短い」方向に引っ張られることは χ_1^2 でも同様であると思います。勿論、このことを主張するためには χ_1^2 についても物質密度分布の効果を調べる必要があります。これは、今後行ない報告したいと思います。

■ χ_3^2 は高いエネルギー領域での情報を捨てるように組んであるので、低いエネルギーの領域が最適となるのは当然である。

▶ その通りです。χ_3^2 の (E_μ, L) 依存性の絵 (11 頁) と、私の「test statistic は $\delta = 0$ (または π) を検定できる量であれば、どう組んでもよい」という発言から、統計量が先にありその統計量から実験設定を決めているように理解されているかと思いますが、本来の意図は上で述べたように物質効果の誤差をよりシリアスに考えた結果、実験設定を再考し、その上でより効率のよい検定を目指して統計量も再考したということです (3 頁, 「Shortly speaking...」参照)。つまり、χ_3^2 は低いエネルギーでの使用を前提としており、その領域では効率のよい量であろうということです (14 頁参照)。

また、統計量の作り方ですが、振動確率はエネルギー依存性の異なる様々な項、様々な情報、の複雑な組み合わせであるので、全てのエネルギー領域で最も効率のよい検定方法を作ることは難しく、各領域においてどの項をメインに取り出すかということで、効率のよしあしがあるのだと思います。