Updated Analysis of Solar Neutrino Data

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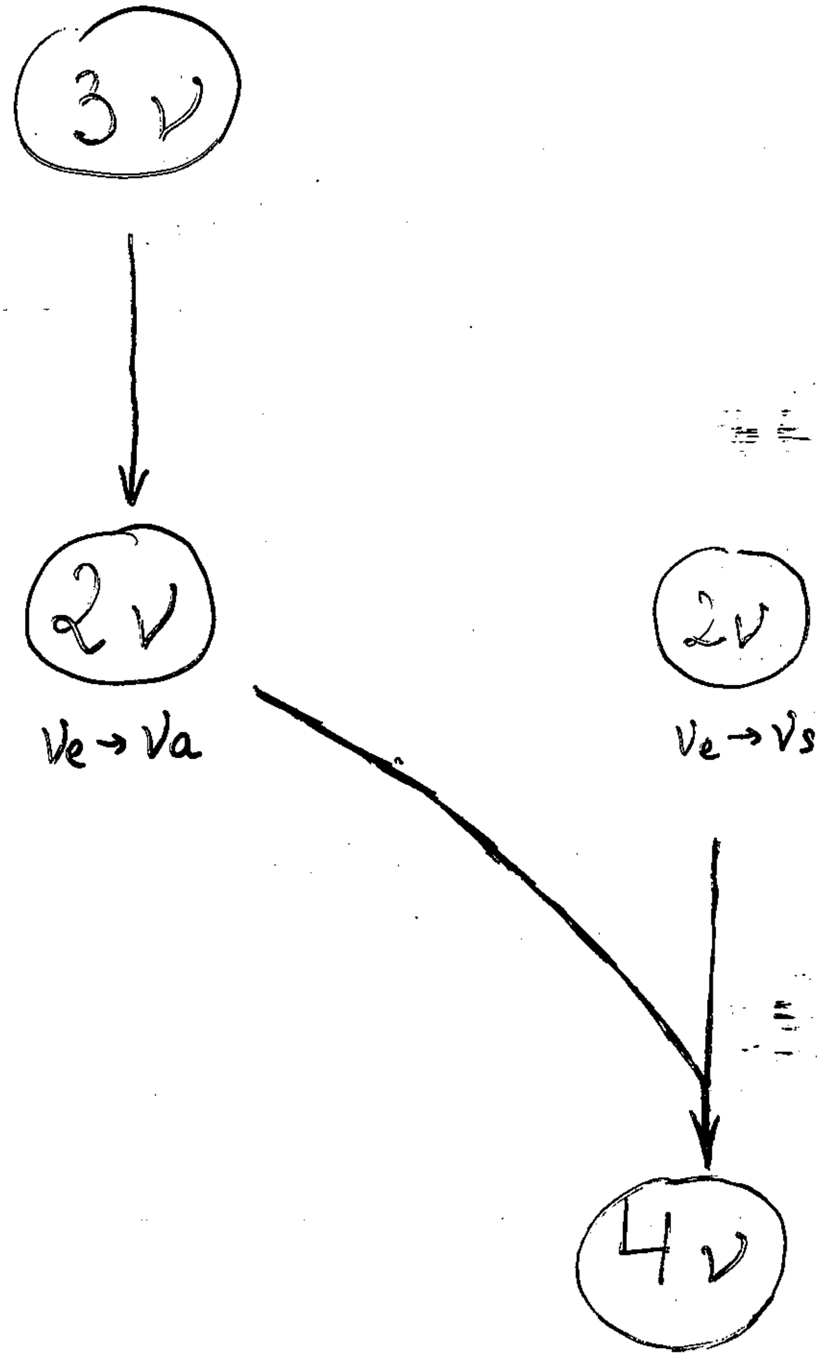
OUTLINE

I. The Solar Neutrino Problem.
II. Two–Neutrino Oscillations.
III. Three–Neutrino Oscillations.
IV. *Unifying* Active and Sterile Oscillations:
Four–neutrino Oscillations.
V. Conclusions.

Work done in colaboration with M.C Gonzalez-Garcia.

M.C Gonzalez-Garcia and C. P-G, NPB Proc. Supl. 91 (2000)
M.C Gonzalez-Garcia, M. Maltoni, C. P-G and J.W.F. Valle, PRD63 (2001)
C. Giunti, M.C Gonzalez-Garcia and C. P-G, PRD62 (2001)

OUTLINE

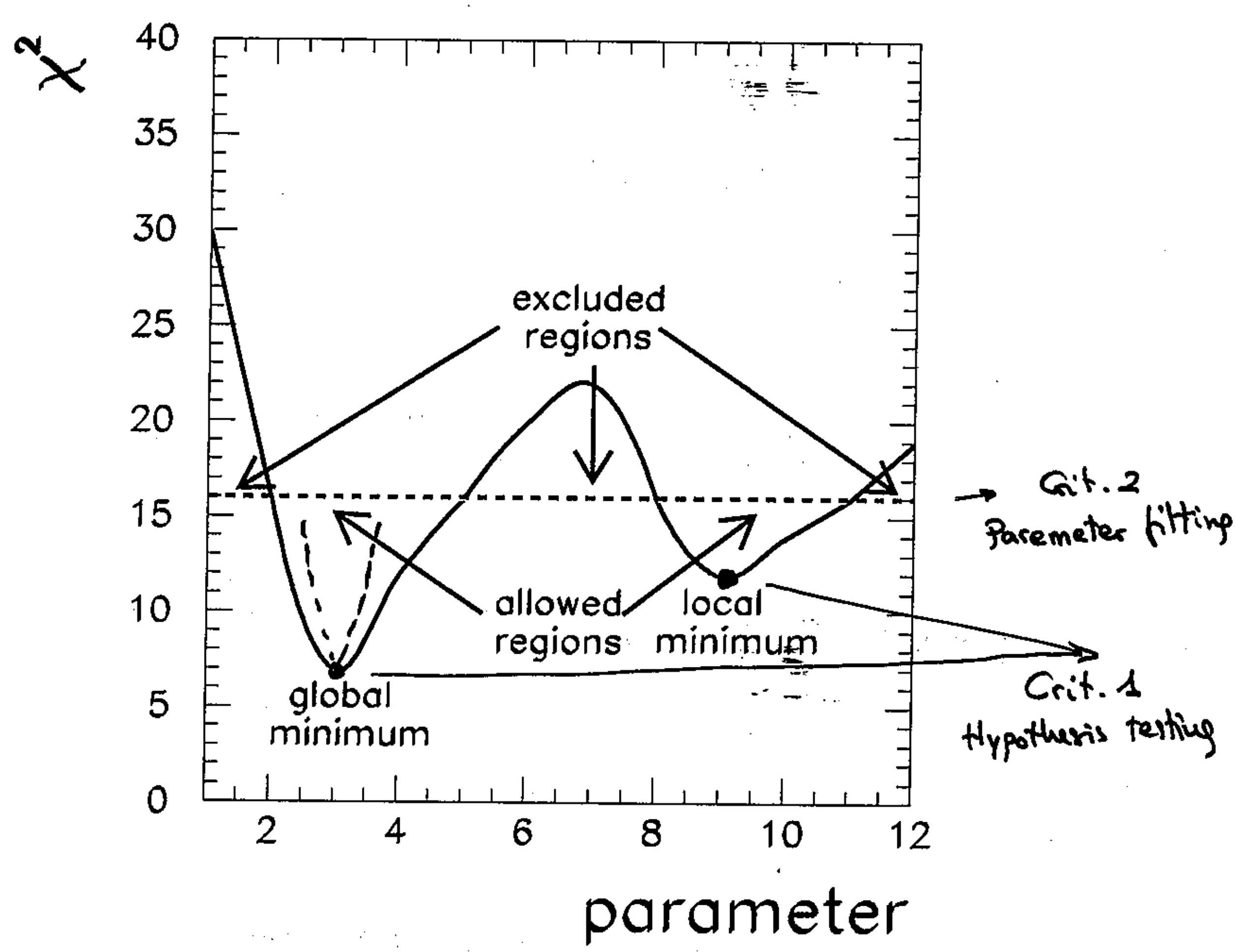


- Principally, Solar neutrino analysis.
- No exotic solutions
- No δ ; Only Δm^2_{ji}, θ_{ij}

Definition of Allowed regions

- For solar neutrinos χ^2 presents several local minima
- allowed regions at a given CL are defined as regions of parameters where

$$\chi^2(\text{param}) < \chi^2_{\text{global minimum}} + \Delta\chi^2(CL, \sharp \text{ of parameters})$$



- The statistical probability of an allowed solution is given by

$$\chi^2_{\text{local minim}}/\text{d.}o.f \quad d.o.f = \sharp \text{ data points} - \sharp \text{ parameters}$$

3ν

Solar : $P_{ee}(\Delta m^2_{21}, \theta_{12}, \theta_{13}, \Delta m^2_{32})$

$\xrightarrow[\Delta m^2_{32} \gg \Delta m^2_{21}]{\text{hierarchy}}$ $P_{ee}(\Delta m^2_{21}, \theta_{12}, \theta_{13})$

Atm : $P_{\alpha\beta}(\Delta m^2_{21}, \theta_{12}, \theta_{13}, \Delta m^2_{32}, \theta_{23})$

$\xrightarrow[\Delta m^2_{32} \gg \Delta m^2_{21}]{}$ $P_{\alpha\beta}(\Delta m^2_{32}, \theta_{23}, \theta_{13})$

Solar :

$$P_{ee} = \cos^4\theta_{13} \cdot P_{ee}^{2\nu}(\Delta m^2_{21}, \theta_{12}; \cos^2\theta_{13} \cdot A) + \sin^4\theta_{13}$$

At present, hierarchy is a very good aprox. !

$$\chi^2_{global} = \chi^2_{solar} + \chi^2_{atm} + \chi^2_{Chooz}$$

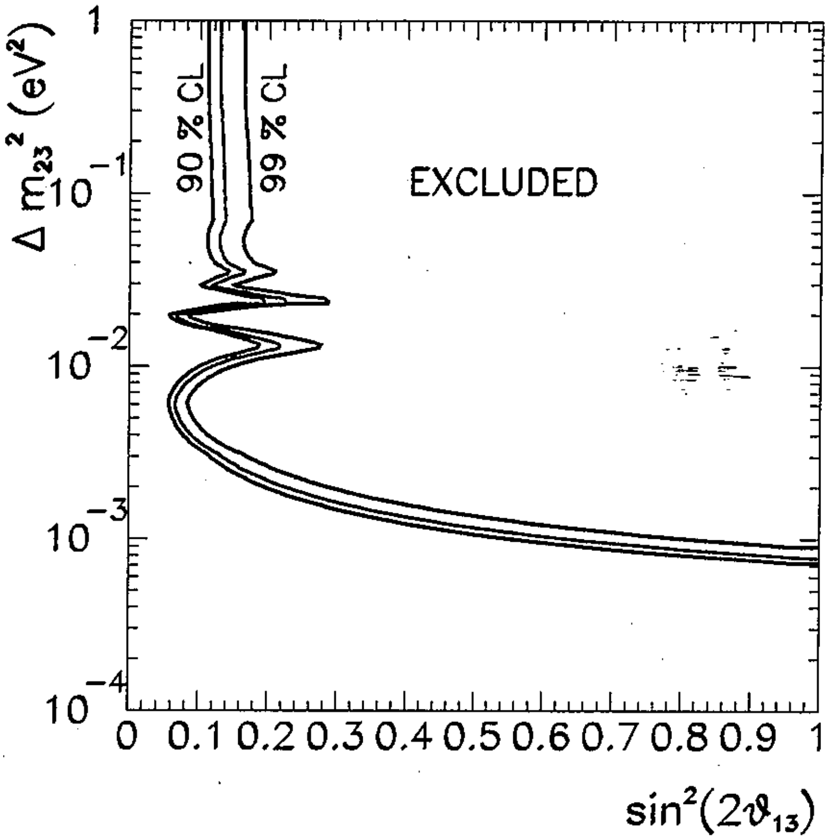
How do you show the analysis:
let's look at the theorems in statistics !

$$\chi^2(a,b,c,d) \longrightarrow \chi^2(a,b,c; d_{min}(a,b,c))$$

If this is a χ^2 $\Longrightarrow$ This remains as a χ^2 but with −1 param.

Including CHOOZ results:

$$R = 1.01 \pm 2.8\,\%(\text{stat}) \pm 2.7\,\%(\text{syst})$$

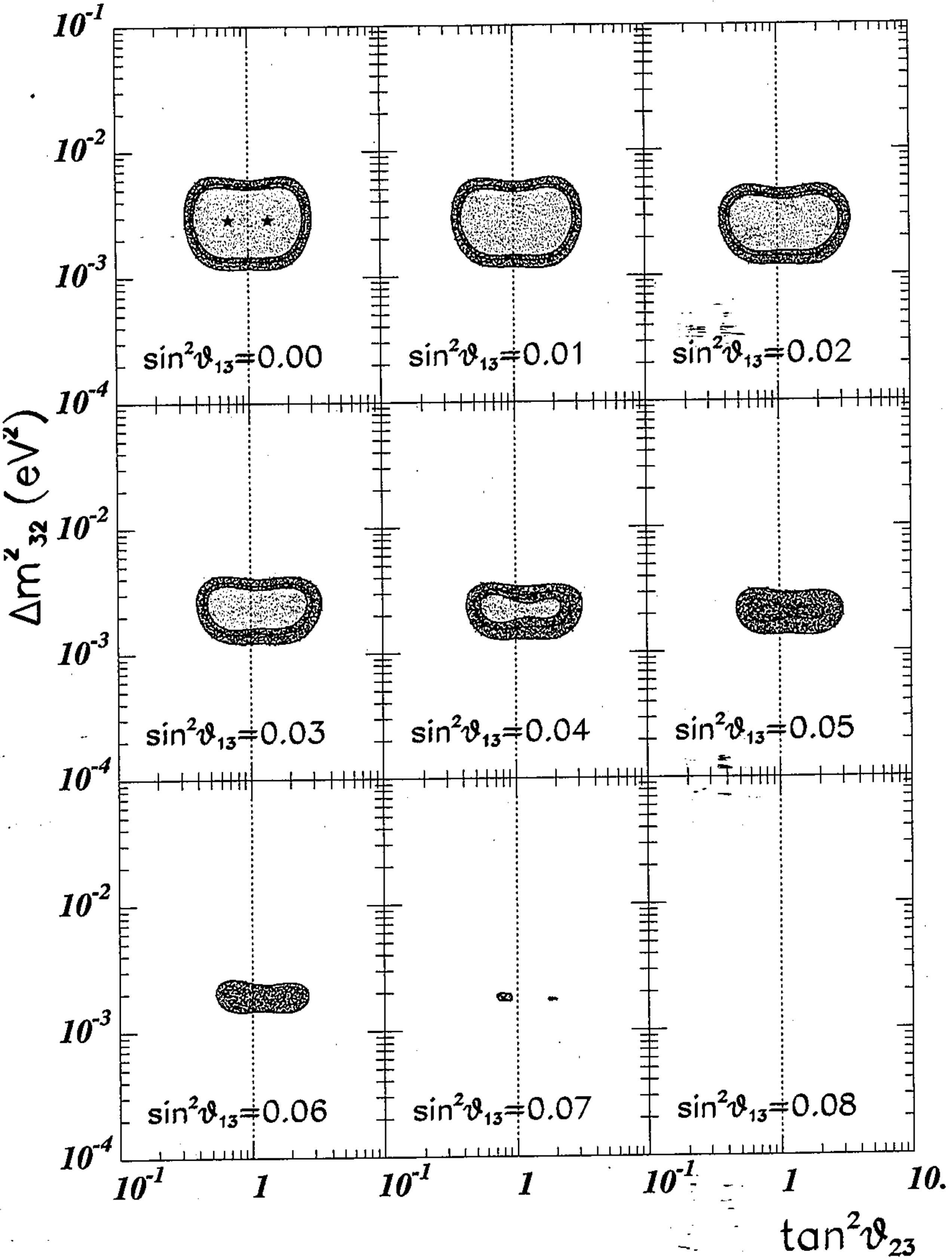


- Two–neutrino and Four–neutrino scenario:

$$P_{ee}^{CHOOZ} = 1 - \sin^2(2\theta_{12})\sin^2(\frac{\Delta m_{12}^2 L}{4E_\nu})$$

- Three–neutrino scenario:

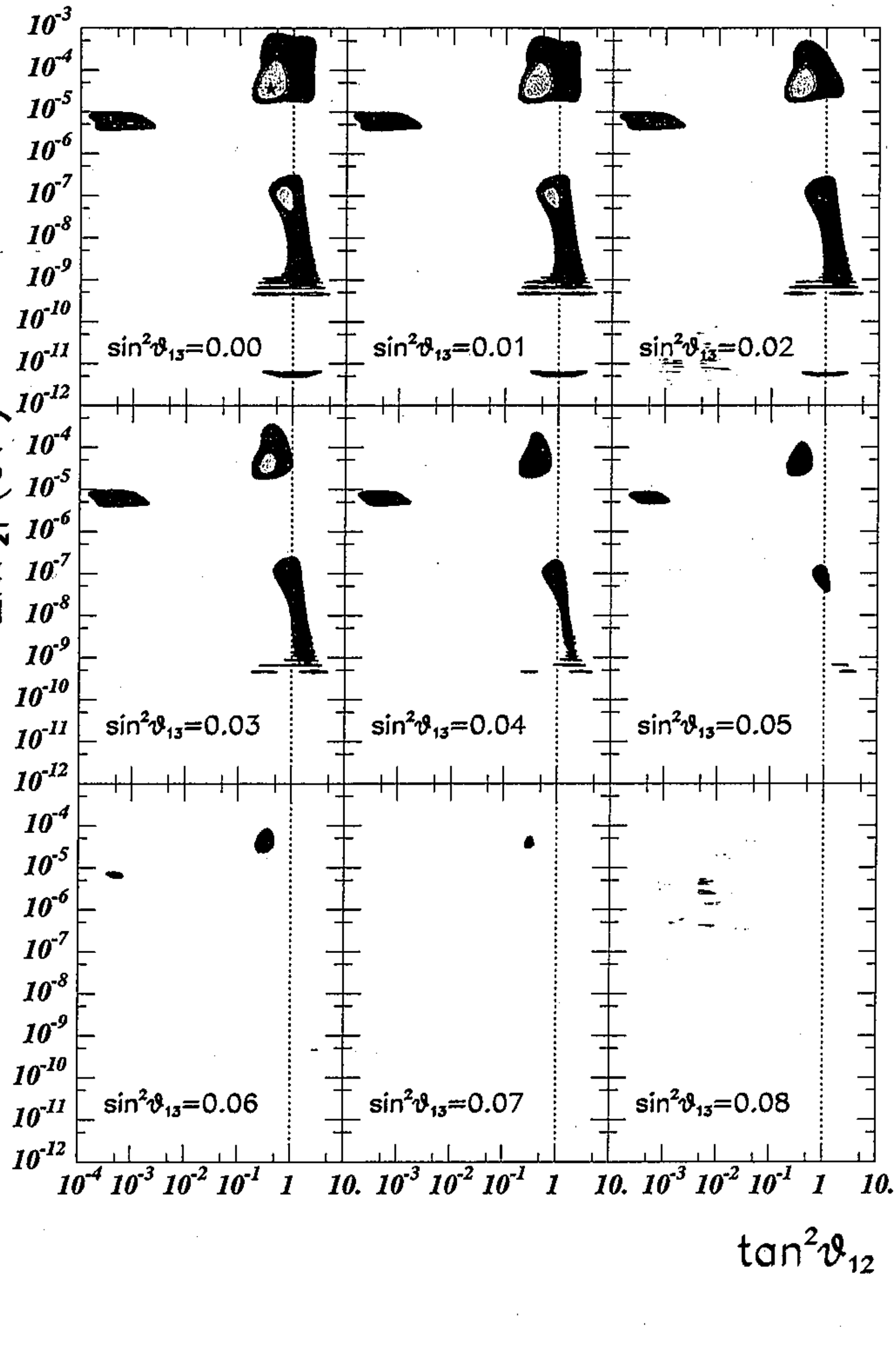
$$\begin{aligned} P_{ee}^{CHOOZ} &= 1 - \cos^4\theta_{13}\sin^2(2\theta_{12})\sin^2\left(\frac{\Delta m_{21}^2 L}{4E_\nu}\right) \\ &-\sin^2(2\theta_{13})\cos^2\theta_{12}\sin^2\left(\frac{\Delta m_{31}^2 L}{4E_\nu}\right) \\ &-\sin^2(2\theta_{13})\sin^2\theta_{12}\sin^2\left(\frac{\Delta m_{32}^2 L}{4E_\nu}\right) \end{aligned}$$



Δm^2_{32} (eV^2)
10^{-1}
10^{-2}
10^{-3}
10^{-4}
$\sin^2\vartheta_{13}$=0.00
$\sin^2\vartheta_{13}$=0.01
$\sin^2\vartheta_{13}$=0.02
$\sin^2\vartheta_{13}$=0.03
$\sin^2\vartheta_{13}$=0.04
$\sin^2\vartheta_{13}$=0.05
$\sin^2\vartheta_{13}$=0.06
$\sin^2\vartheta_{13}$=0.07
$\sin^2\vartheta_{13}$=0.08
10^{-1}
1
10.
$\tan^2\vartheta_{23}$

Solutions for Three–neutrino Oscillations

Solar+Atmospheric+CHOOZ data.



II.Two–Neutrino Oscillations.

– The ν_e survival amplitude after propagation from the Sun to the detector at the Earth:

$$A(\nu_e \to \nu_e) = A_{Sun}(\nu_e \to \nu_1) \times A_{\rm vac}(\nu_1 \to \nu_1) \times A_{Earth}(\nu_1 \to \nu_e) + A_{Sun}(\nu_e \to \nu_2) \times A_{\rm vac}(\nu_2 \to \nu_2) \times A_{Earth}(\nu_2 \to \nu_e)$$

– Where

$$|A_{Sun}(\nu_e \to \nu_1)|^2 = P_{e1}^{Sun} = 1 - |A_{Sun}(\nu_e \to \nu_2)|^2$$
$$|A_{Earth}(\nu_1 \to \nu_e)|^2 = P_{1e}^{Earth} = 1 - |A_{Earth}(\nu_2 \to \nu_e)|^2$$
$$A_{vac}(\nu_i \to \nu_i) = \exp\left(-i\, m_i^2\, (L - R_{Sun})/2E\right)$$

– So in general:

$$P_{ee} = P_{e1}^{Sun} P_{1e}^{Earth} + (1 - P_{e1}^{Sun})(1 - P_{1e}^{Earth}) + 2\sqrt{P_{e1}^{Sun}(1 - P_{e1}^{Sun}) P_{1e}^{Earth}(1 - P_{1e}^{Earth})} \cos\left(\frac{\Delta m^2 L}{2E} + \delta\right)$$

– When matter doesn't matter

$P_{e1}^{Sun} = P_{1e}^{Earth} = \cos^2\theta \Rightarrow$ Vacuum Oscillations (VO)

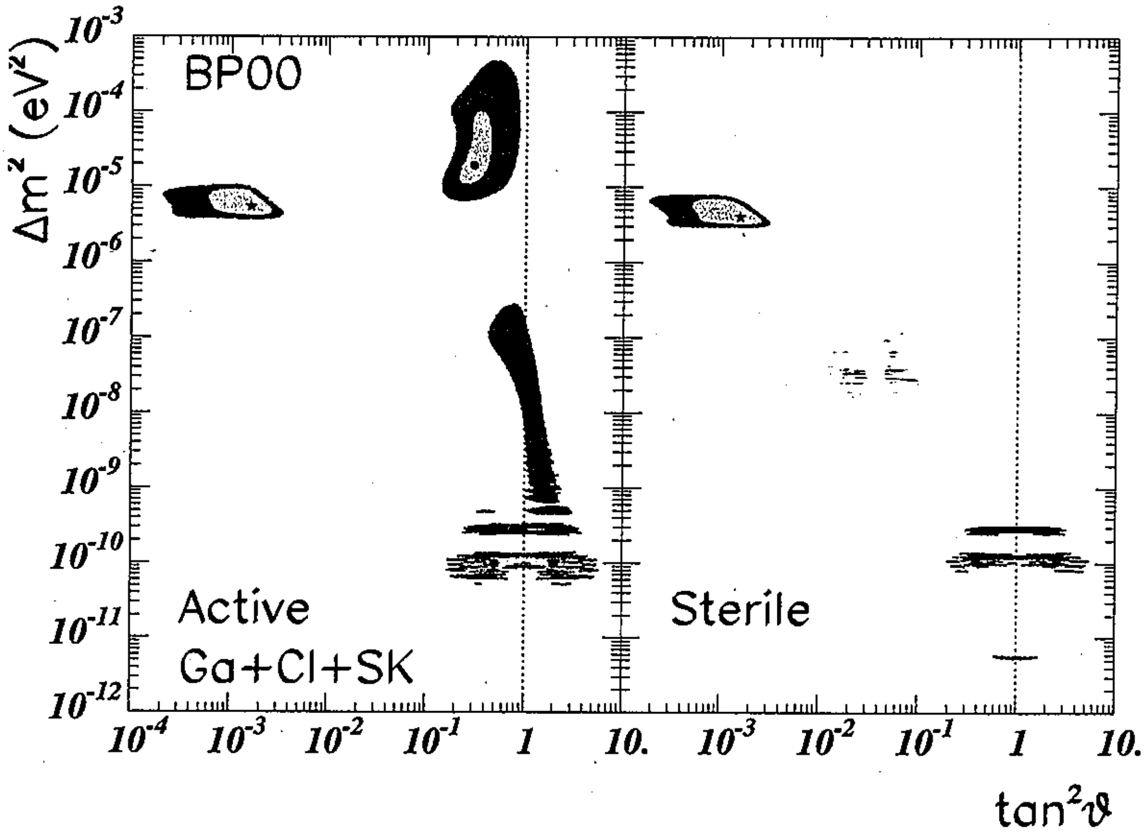
For $\Delta m^2 \gtrsim 10^{-8}$ eV$^2 \Rightarrow$ VO averaged, $P_{ee} = 1 - \frac{1}{2}\sin^2 2\theta$

– When matter does matter ($10^{-8} \lesssim \Delta m^2 \lesssim 10^{-3}$) : Matter effects in the Sun (MSW) and in the Earth

Solutions for $\nu_e \longrightarrow \nu_{active}$ or $\nu_e \longrightarrow \nu_{sterile}$

Updated with Super–Kamiokande 1258 days, BP00 v2

Allowed regions from Rates (90,95, 99, 99.7 % CL):



- Why differences for oscillations into active or sterile?
  Main effect is different contribution to event rates in SuperK

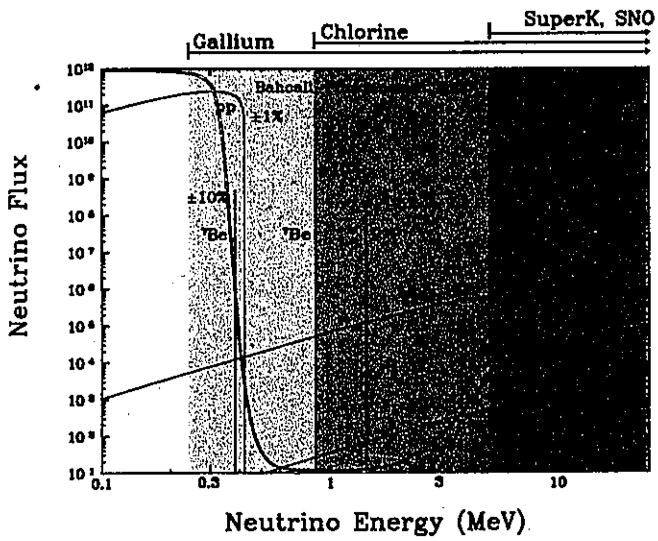
$$\nu_{\mu(\tau)} + e \longrightarrow \nu_{\mu(\tau)} + e \longrightarrow \text{NC events in SuperK}$$
$$\nu_s + e \not\rightarrow \nu_s + e \longrightarrow \text{no NC events in SuperK}$$

Also slightly different survival probabilities in the sun

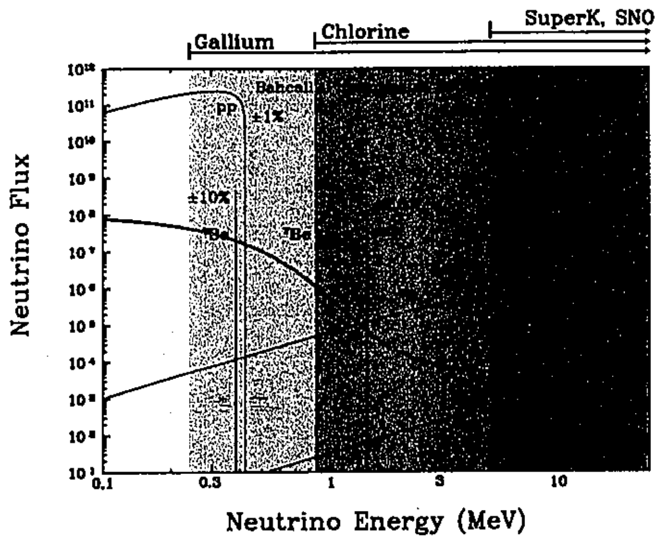
		Active				Sterile
		SMA	LMA	LOW	VAC	SMA
Rates	Δm^2/eV^2	5.5×10^{-6}	1.9×10^{-5}	9.2×10^{-8}	9.7×10^{-11}	4.1×10^{-6}
	$\tan^2\theta$	1.6×10^{-3}	0.30	0.67	0.50 (2.0)	1.5×10^{-3}
	Prob (%)	56 %	8 %	0.5 %	2 %	14%

Energy Dependence of P_{ee}^{Day} for Different Solutions

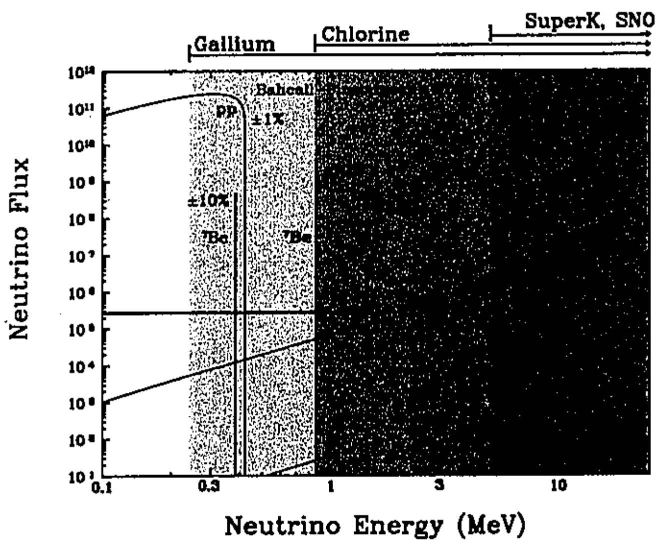
SMA



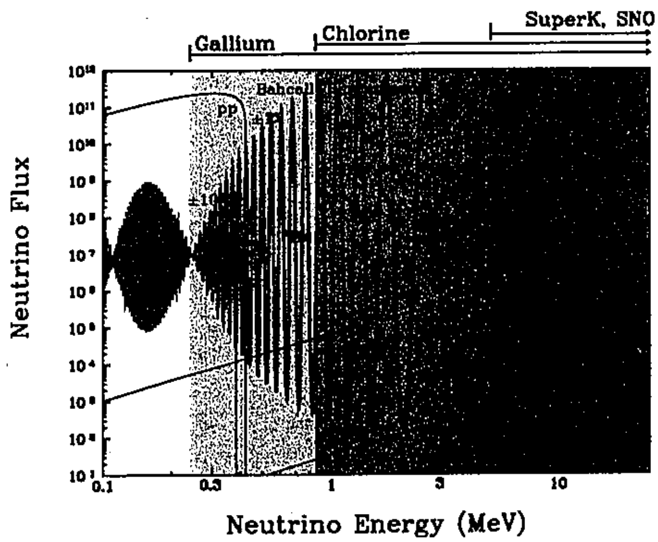
LMA



LOW



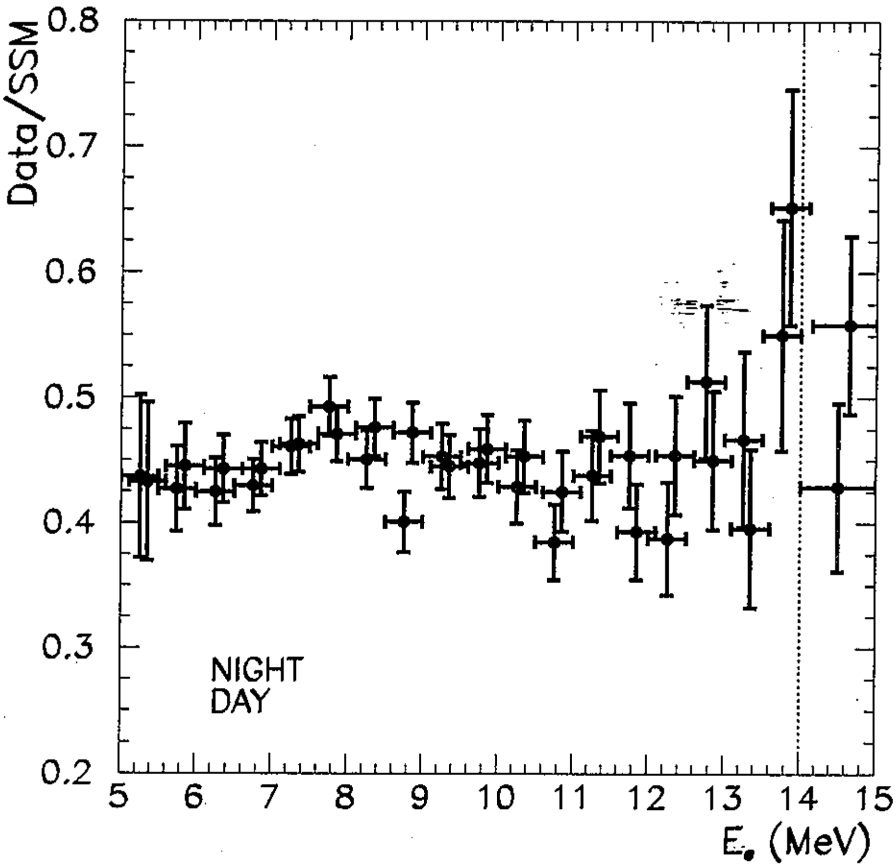
VAC



Other Super–Kamiokande Measurements

Super–Kamiokande 1258 Days

- Recoil Day–Night Electron Energy Spectrum



→ Spectrum compatible with flat $\chi^2_{flat} = 33/(37dof)$

→ Few more events at N than D $2\frac{N-D}{N+D} = 0.033 \pm 0.022 \pm 0.013\ (1.3\sigma)$

Solutions for $\nu_e \rightarrow \nu_{active}$ or $\nu_e \rightarrow \nu_{sterile}$

Updated with Super–Kamiokande 1258 days, BP00 v2

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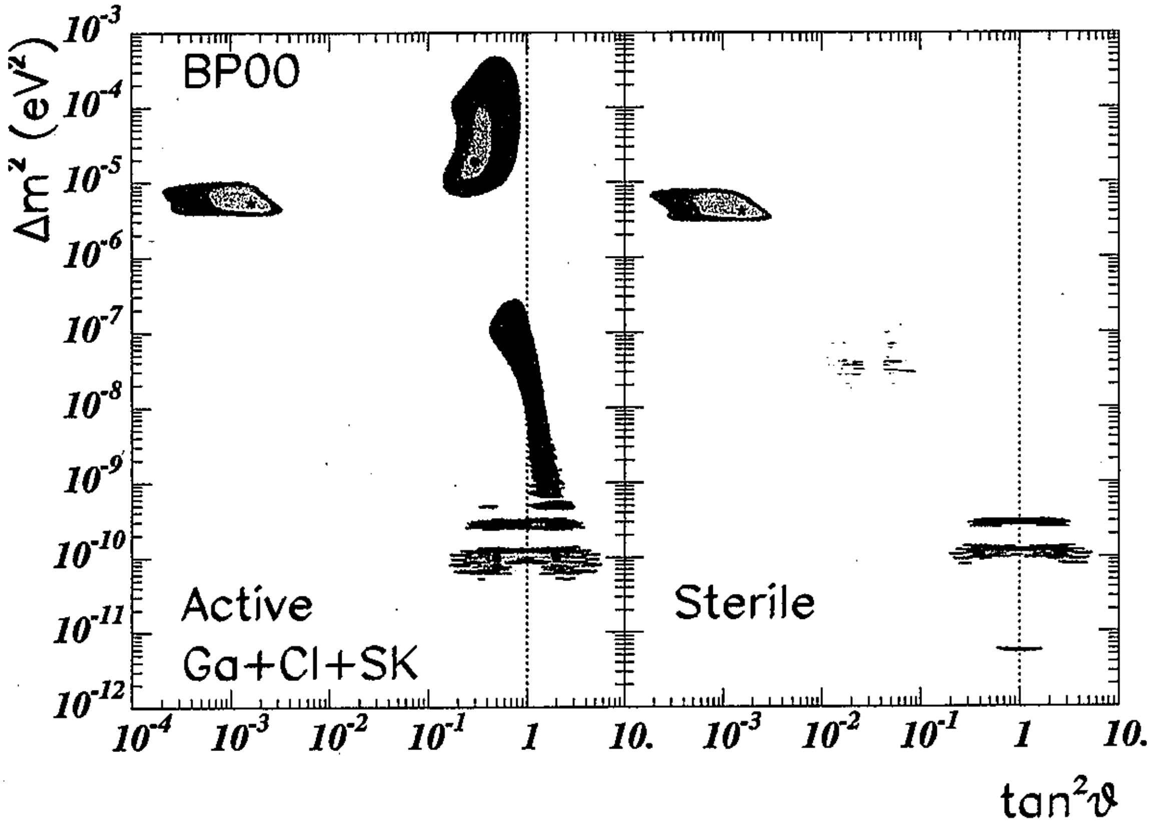

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	$\tan^2\theta$	1.6×10^{-3}	0.30	0.67	0.50 (2.0)	1.5×10^{-3}
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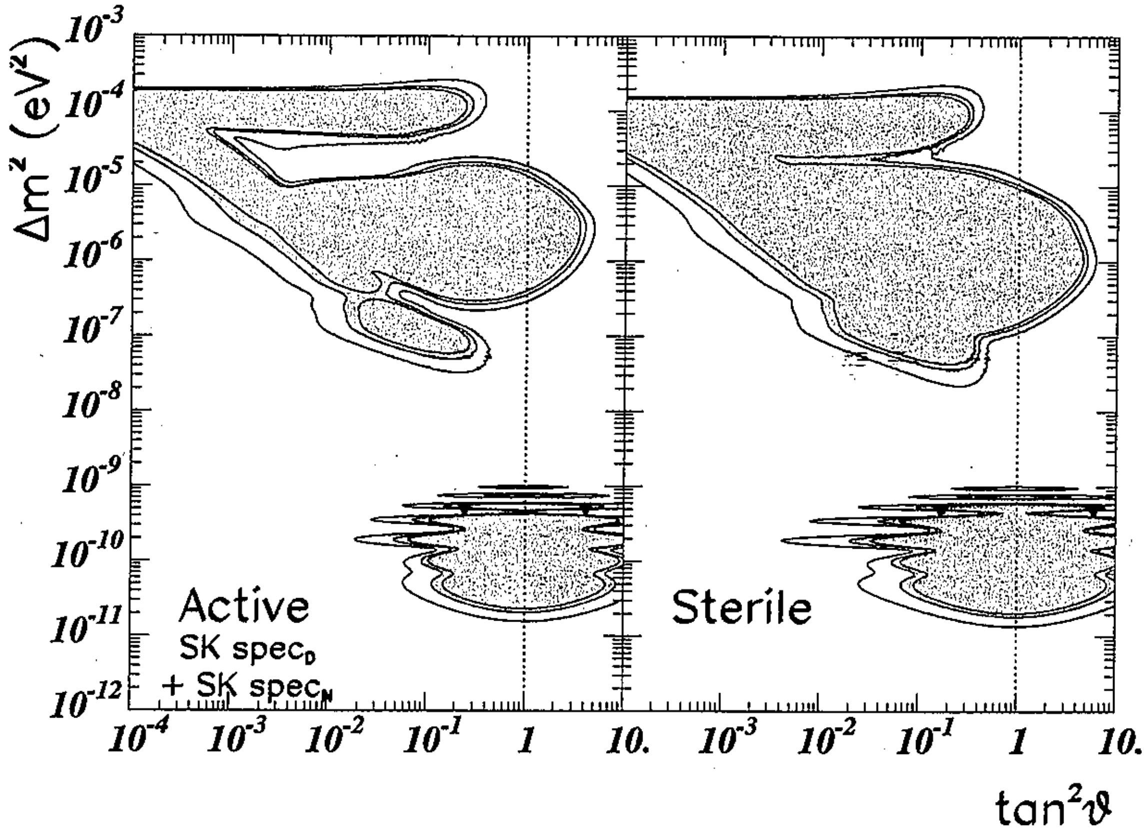
Solutions for $\nu_e \rightarrow \nu_{active}$ or $\nu_e \rightarrow \nu_{sterile}$

Updated with Super–Kamiokande 1258 days, BP00 v2

Excluded from Sp_D and Sp_N:



∗ Total Rates:

$$\chi^2_R = \sum_{i,j=1,3} (R_i^{th} - R_i^{exp})\sigma_{ij}^{-2}(R_j^{th} - R_j^{exp})$$

∗ Day and Night Recoil Electron Energy Spectra: 2×19 bins

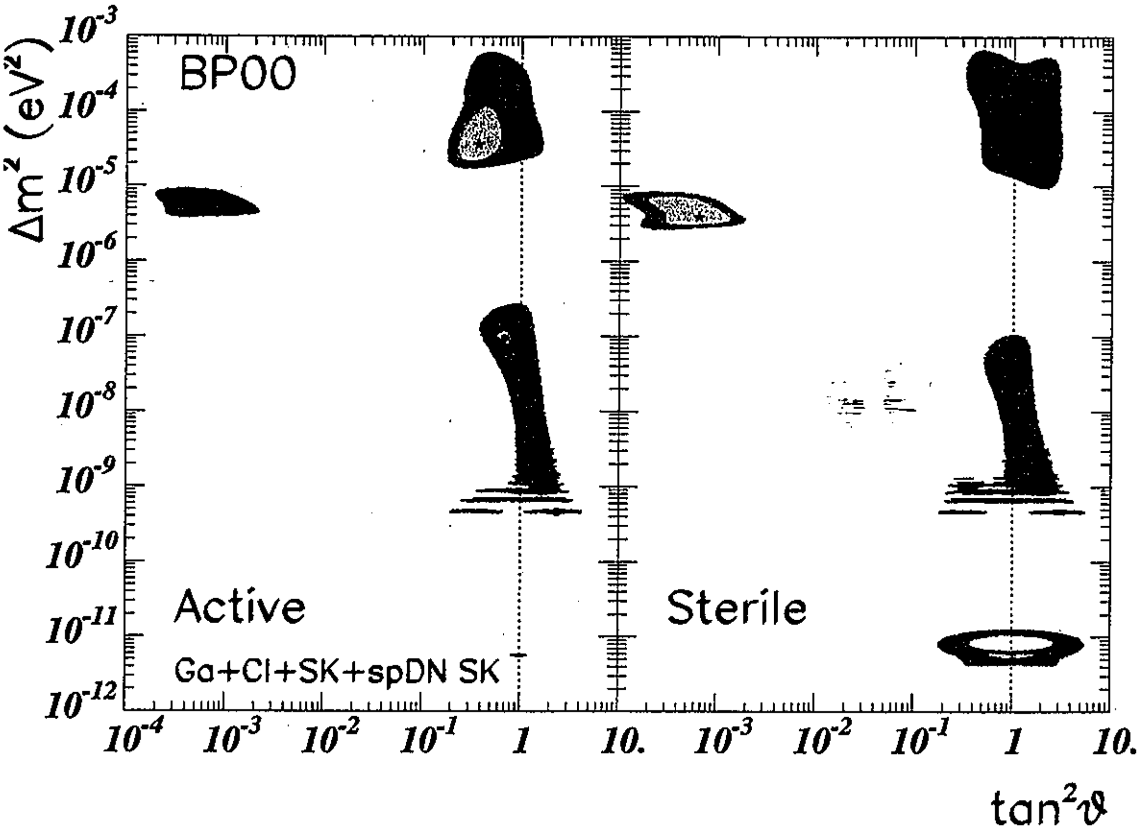
$$\chi^2_{SpD/SpN} = \sum_{i,j=1,38} (\alpha_{sp} R_i^{th} - R_i^{exp})\sigma_{ij}^{-2}(\alpha_{sp} R_j^{th} - R_j^{exp})$$

In Global Analysis we use 3 Rates + SK Day and Night spectra:
A total of 3+38-1=40 independent data points

Solutions for $\nu_e \longrightarrow \nu_{active}$ or $\nu_e \longrightarrow \nu_{sterile}$

Updated with Super–Kamiokande 1258 days, BP00 v2

Allowed regions from Global Analysis (Solar+CHOOZ data):



		Active				Sterile
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		Active				Sterile
		SMA	LMA	LOW	QVO	SMA
Global	$\Delta m^2/\mathrm{eV}^2$	5.2×10^{-6}	3.7×10^{-5}	1.0×10^{-7}	4.6×10^{-16}	3.9×10^{-6}
	$\tan^2\theta$	7.5×10^{-4}	0.37	0.67	2.37	6.5×10^{-4}
	Prob (%)	30 %	61 %	42 %	35 %	30%

Allowed regions: How to discriminate them?

- Solar neutrinos oscillate ?

- Scenario : 3, 4 ?

- Regime : LMA, LOW, SMA, VAC, JS² ?

⇓

Details in natural sources of neutrinos !

- Solar interior

- ...

(b) Oscillations in the Earth Matter :

$$P_{ee}^{Night} = P_{ee}^{Day} - (1 - 2P_c)\cos(2\theta_{m,0}) f_{reg}$$

with

$$f_{reg} = P_{2e}^{Earth} - \sin^2\theta$$

In the case of crossing the distance L inside a constant V_e:

$$f_{reg} = \frac{V_e}{\frac{\Delta m^2}{4E}} \sin^2 2\theta_m \sin^2 \frac{\pi L}{l_m} = \frac{V_e}{\frac{\Delta m^2}{4E}} \sin^2 2\theta_m \sin^2 \frac{\phi}{2}$$

with the oscillation lenght in matter l_m

$$l_m = \frac{\pi}{\frac{\Delta m^2}{4E}} \frac{\sin 2\theta_m}{\sin 2\theta}$$

- As a result:

– f_{reg} is ALWAYS positive.
– $P_{ee}^{Night} - P_{ee}^{Day}$ positive or negative depending on P_c.
The adiabatic case, $P_c = 0$, implies maximal $P_{ee}^{Night} > P_{ee}^{Day}$.

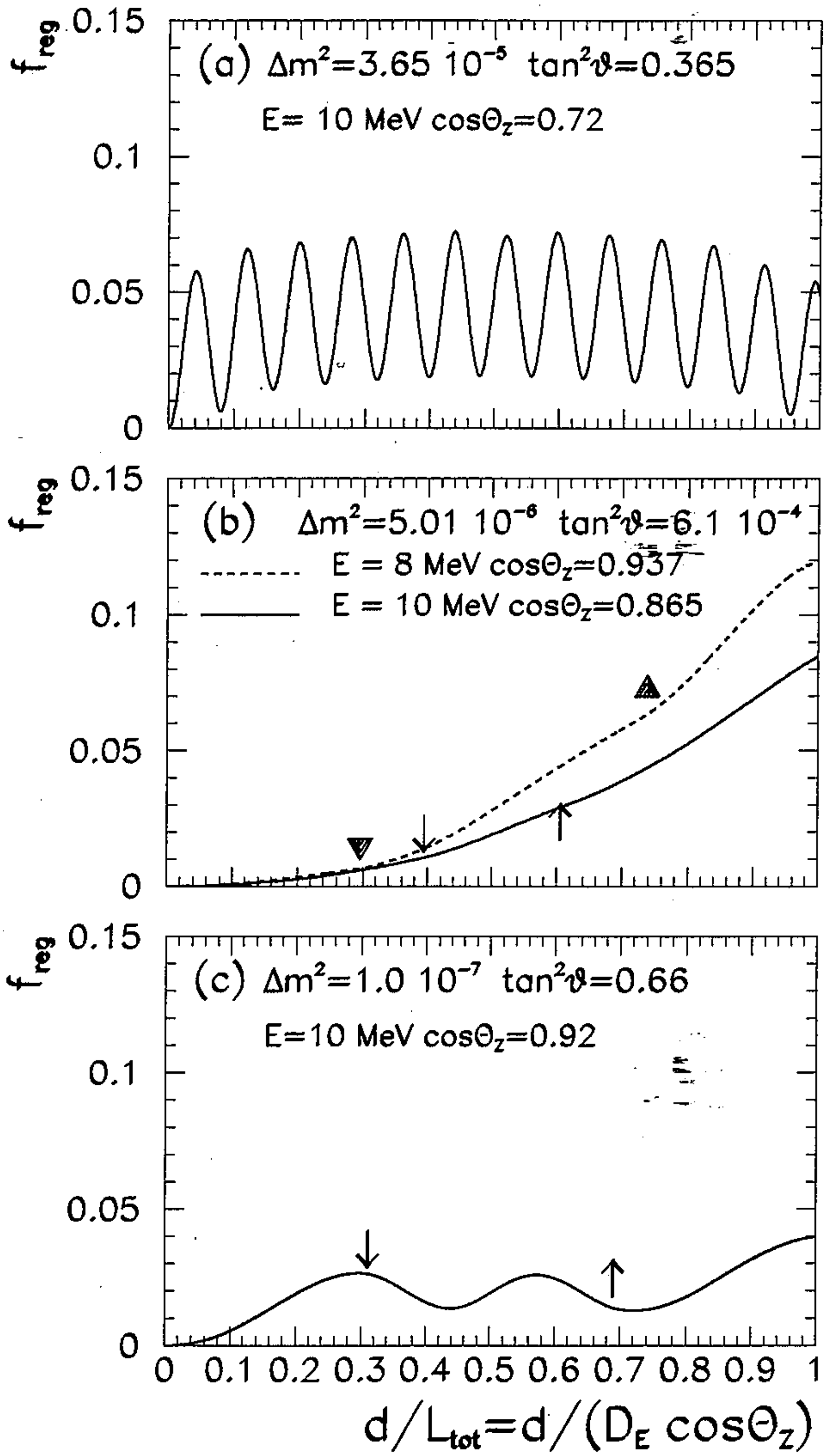
- Matter effects in the Earth are maximal when

$$\Delta m^2 \cos(2\theta) \sim 2EV_e$$

Since $\langle V_{e,Earth} \rangle = 2 - 5$ g cm^{-3} and $E \sim 10$ MeV.
Effect is most important for $\Delta m^2 \cos(2\theta) \sim 10^{-6} - 10^{-7}$ eV^2

Earth matter effects generally represented by the day–night asymmetry

$$A_{DN} = 2\frac{D-N}{D+N}$$



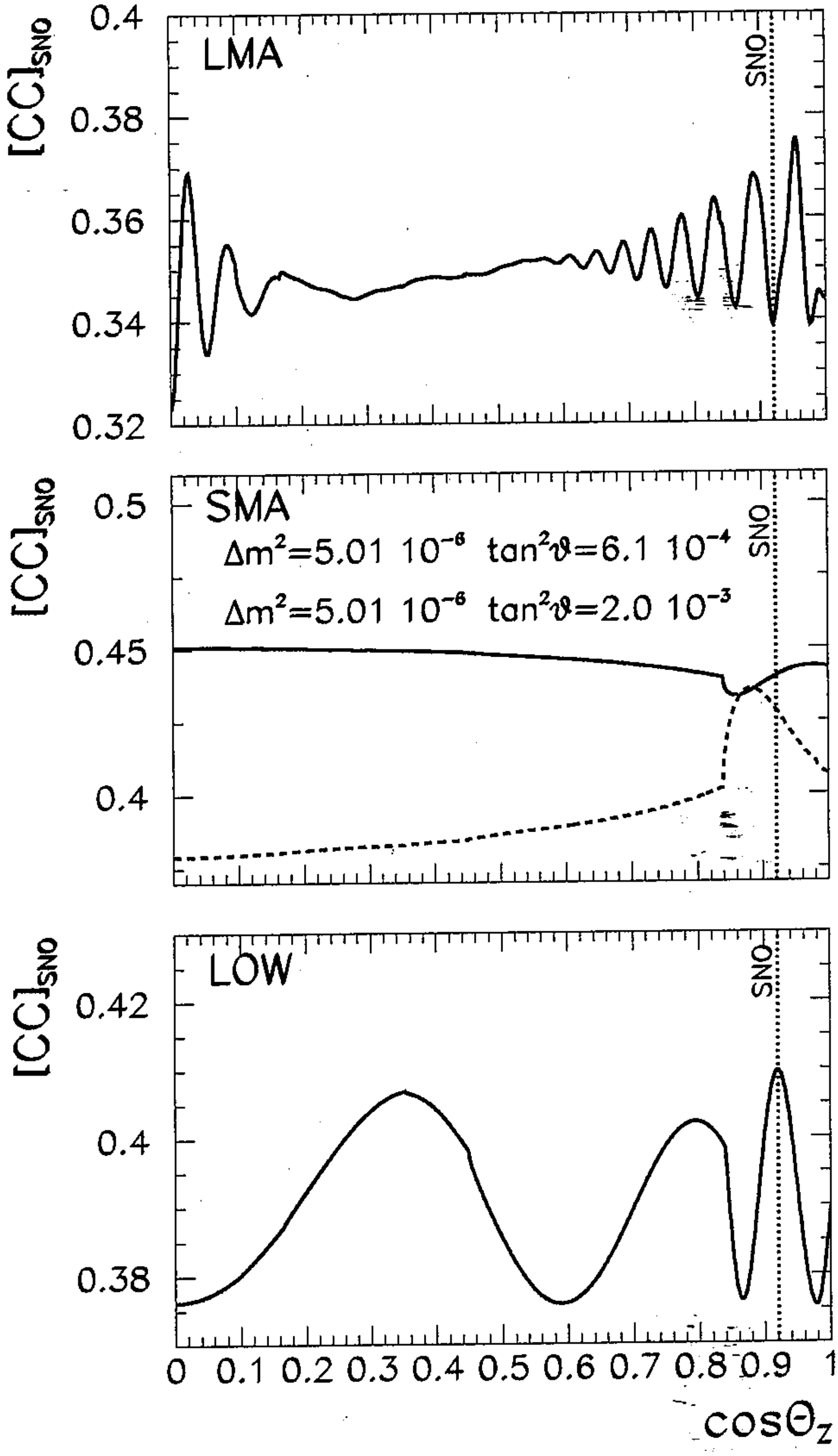
$$f_{reg} = \frac{V_e}{\frac{\Delta m^2}{4E}} \sin^2 2\theta_m \sin^2 \frac{\pi L}{l_m} \qquad l_m = \frac{\pi}{\frac{\Delta m^2}{4E}} \frac{\sin 2\theta_m}{\sin 2\theta}$$

$$\sin(2\theta_m) = \frac{\Delta m^2 \sin(2\theta)}{\sqrt{(\Delta m^2 \cos(2\theta) - A)^2 + (\Delta m^2 \sin(2\theta))^2}}$$

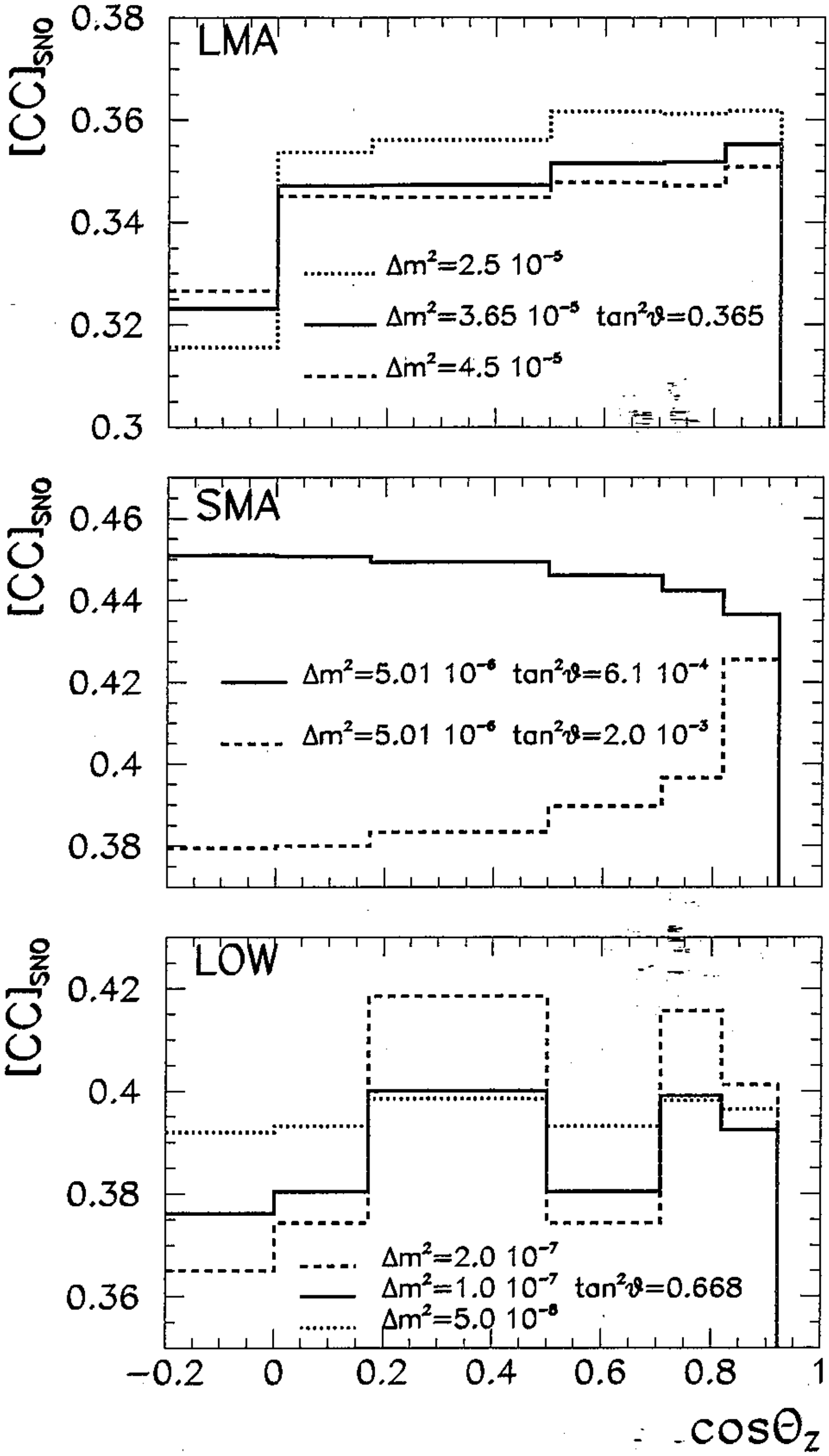
IV.Predictions for SK and SNO.

- Zenith Angle Distributions at SNO.

M.C. Gonzalez-Garcia, C.P-G and A.Yu. Smirnov, PRD 63 (2001)

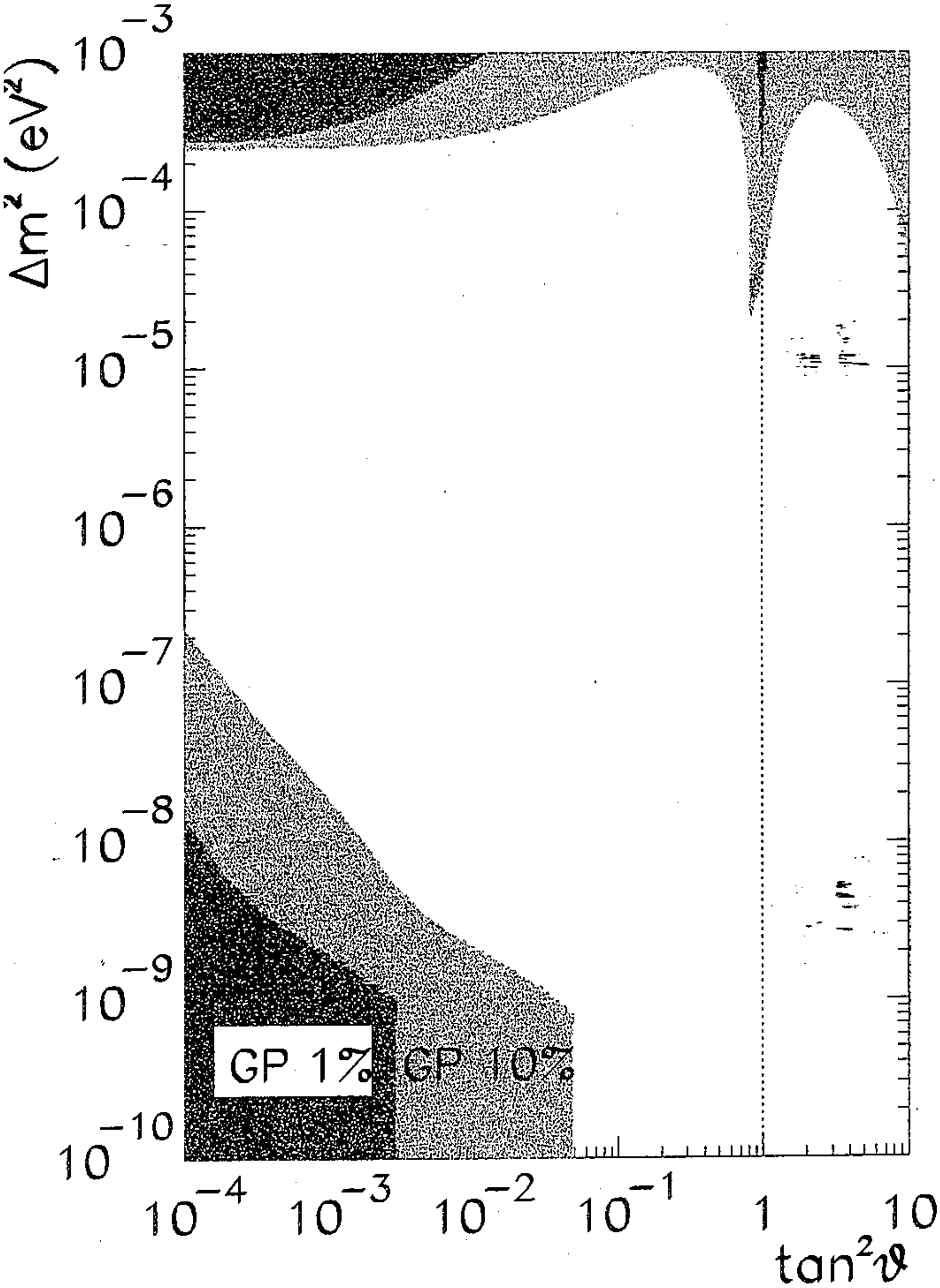


• Optimum Binning for Zenith Angle Distributions at SNO.

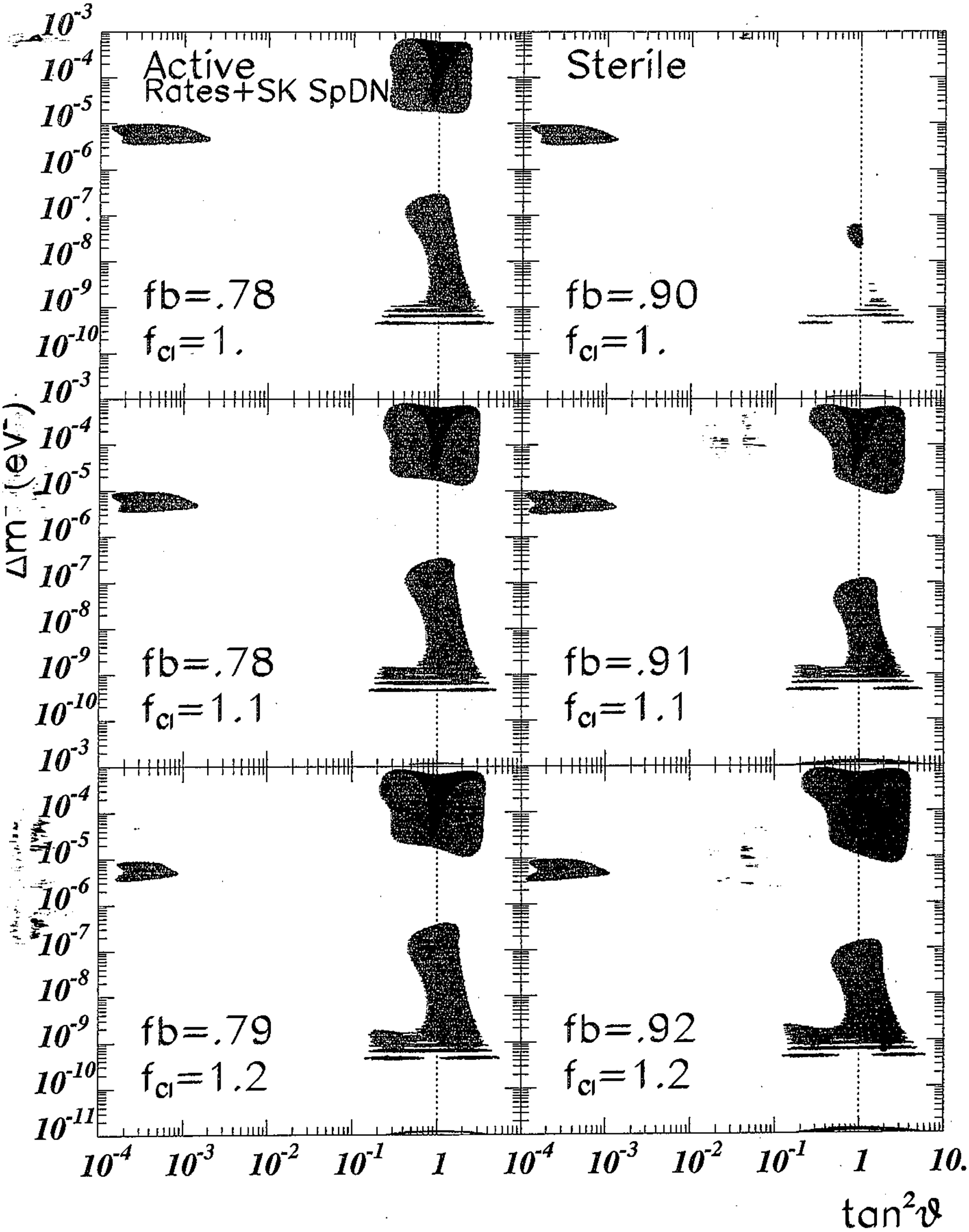


$P_{GP} = 1 - \frac{1}{2}\sin^2 2\theta$

$$\frac{|P_{GP} - P_{ee}|}{P_{GP}} < 10\% \ (1\%)$$



Berezinsky, V., Gonzalez-Garcia and C.P-G, hep-ph/0105294



V. Berezinsky, M.C. Gonzalez-Garcia and C.P-G, hep-ph/0105294

IV. *Unifying* Active and Sterile Oscillations: Four-neutrino Oscillations.

- To fit solar, atmospheric and LSND

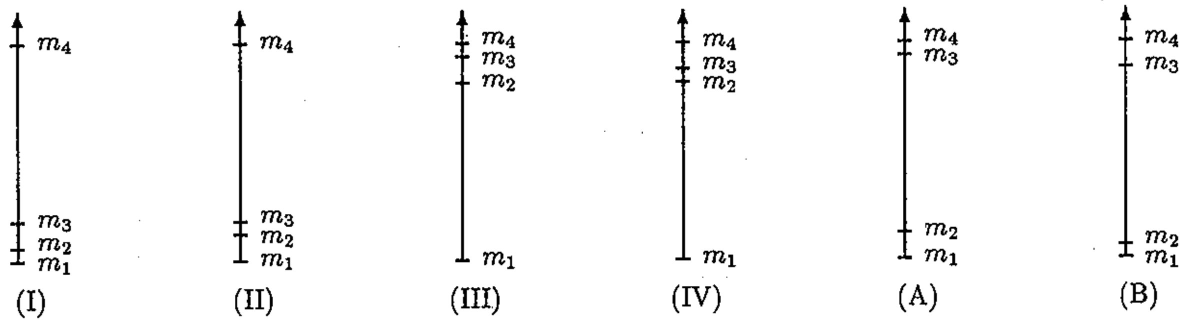
  $\rightarrow 3\ \Delta m^2 : \Delta m^2_{LSND} \ggg \Delta m^2_{atm} \gg \Delta m^2_{sun}$

  $\rightarrow 4\ \nu's$

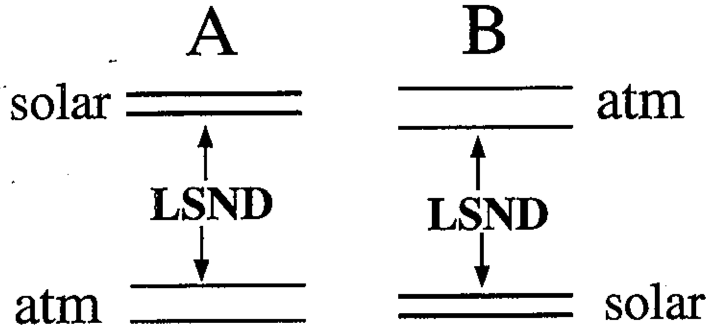
- But LEP data implies only 3 neutrino flavours

  $\rightarrow$ 4th ν must be sterile

- U: 6 mixing angles and 3 CP phases
- Possible mass spectra:



- Limits from Accelerator and Reactor: $\rightarrow$ A or B favoured



In B for example:

Atm: $\nu_\mu \rightarrow \nu_+$

Solar: $\nu_e \rightarrow \nu_-$

$$\nu_- \simeq c_\chi \nu_s + s_\chi \nu_\tau$$

$$\nu_+ \simeq -s_\chi \nu_s + c_\chi \nu_\tau$$

χ function of angles in U

- Open Question: is ν_s part of Atm or Solar Oscillation?

  * In 2-ν oscillations we have seen the limiting cases:

  $$c_\chi = 0 \rightarrow \begin{cases} \text{Atm: } \nu_\mu \rightarrow \nu_s \\ \text{Solar: } \nu_e \rightarrow \nu_\tau \end{cases}$$

  $$c_\chi = 1 \rightarrow \begin{cases} \text{Atm: } \nu_\mu \rightarrow \nu_\tau \\ \text{Solar: } \nu_e \rightarrow \nu_s \end{cases}$$

  * But intermediate cases $0 < s_\chi < 1$ also possible

Bugey : Δm^2_{LSND} $P_{ee} \gtrsim 0.99$

$\Downarrow$

$$c_{14}^2 s_{13}^2 + s_{14}^2 \leq 10^{-2} \quad \rightarrow \quad \theta_{13} \to 0,\ \theta_{14} \to 0$$

CDHS , CCFR

$$\Delta m^2_{LSND} \Rightarrow s_{23}^2 \lesssim 0.2$$

$$U = U_{24} U_{23} U_{14} U_{13} U_{34} U_{12} \simeq U_{24} U_{23} U_{34} U_{12}$$

Solar : U_{34} is not appearing $\quad \nu_e \to c_{23} c_{24} \nu_s + \sqrt{1 - c_{23}^2 c_{24}^2}\, \nu_a$

$$U = \underbrace{U_{24} U_{23}}_{c_{24} c_{23}} \cdot U_{12} \qquad A \equiv A_{cc} + c_{23}^2 c_{24}^2 A_{NC}$$

Atm : hierarchy $\quad U = U_{24} U_{23} U_{34}$

enter independently

$$s_{23} s_{24} \nu_s + c_{23} \nu_\mu - s_{23} s_{34} \nu_\tau$$

$\downarrow$

$$s_{24} \nu_s + c_{24} \nu_\tau$$

but ! only U_{24}, U_{23} enter in the dynamics !

$$A \equiv (s_{24}^2 - s_{23}^2 c_{24}^2) A_{NC}$$

U_{34} only appear in the last step : χ^2 !

$0 \leq \theta_{12} < \pi/2$

$0 \leq \theta_{23} < \pi/2$

$0 \leq \theta_{24} < \pi/2$

$-\pi/2 \leq \theta_{34} < \pi/2 \quad \xrightarrow{\theta_{23} \geq 0} \quad 0 \leq \theta_{34} < \pi/2$

- The mixing matrix $U = U_{24}U_{23}U_{34}U_{12}$

$$\begin{pmatrix} c_{12} & s_{12} & 0 & 0 \\ -s_{12}c_{23}c_{24} & c_{12}c_{23}c_{24} & s_{23}c_{24}c_{34} - s_{24}s_{34} & s_{23}c_{24}s_{34} + s_{24}c_{34} \\ s_{12}s_{23} & -c_{12}s_{23} & c_{23}c_{34} & c_{23}s_{34} \\ s_{12}c_{23}s_{24} & -c_{12}c_{23}s_{24} & -s_{23}s_{24}c_{34} + c_{24}s_{34} & -s_{23}s_{24}s_{34} + c_{24}c_{34} \end{pmatrix}$$

- <u>For solar neutrinos:</u>

  * $c_{23}^2 c_{24}^2 = 0 \longrightarrow$ pure $\nu_e \longrightarrow \nu_{active}$ oscillations with mixing θ_{12}
  * $c_{23}^2 c_{24}^2 = 1 \longrightarrow$ pure $\nu_e \longrightarrow \nu_{sterile}$ oscillations with mixing θ_{12}
  * Intermediate cases $0 < c_{23}^2 c_{24}^2 = |U_{s1}|^2 + |U_{s2}|^2 < 1$ also possible: $\nu_e \longrightarrow \nu_X$ with mixing θ_{12}

$$\nu_X = c_{23}^2 c_{24}^2 \nu_s + \sqrt{1 - c_{23}^2 c_{24}^2}\,\nu_a.$$

– In this general case of simultaneous $\nu_e \longrightarrow \nu_s$ and $\nu_e \longrightarrow \nu_a$:

$$P_{ee}^{4\nu} = P_{ee}^{2\nu}$$

$$P_{es}^{4\nu} = c_{23}^2 c_{24}^2 (1 - P_{ee}^{2\nu})$$

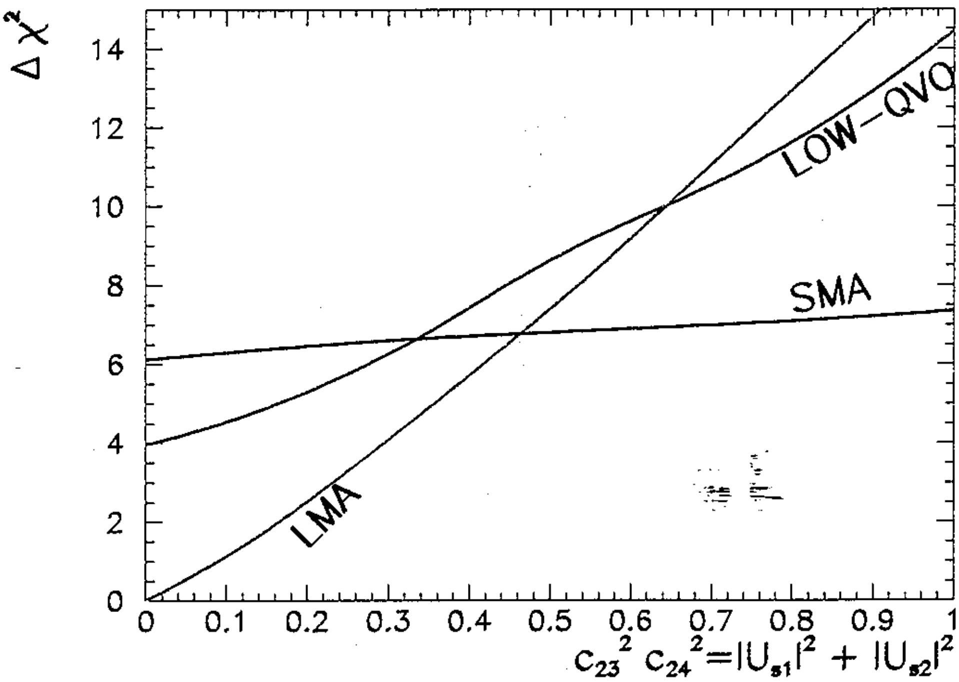
$$P_{ea}^{4\nu} = (1 - c_{23}^2 c_{24}^2)(1 - P_{ee}^{2\nu})$$

but $P_{ee}^{2\nu}$ is computed for a matter potential

$$A \equiv A_{CC} + c_{23}^2 c_{24}^2 \, A_{NC}$$

– Analysis with the 3 parameters: Δm_{12}^2, θ_{12}, and $c_{23}^2 c_{24}^2$

Solutions for Four-neutrino Oscillations



- LMA, LOW-QVO can have a subdominant $\nu_e \rightarrow \nu_s$ component
- SMA allowed at 95% CL for all mixtures active-sterile

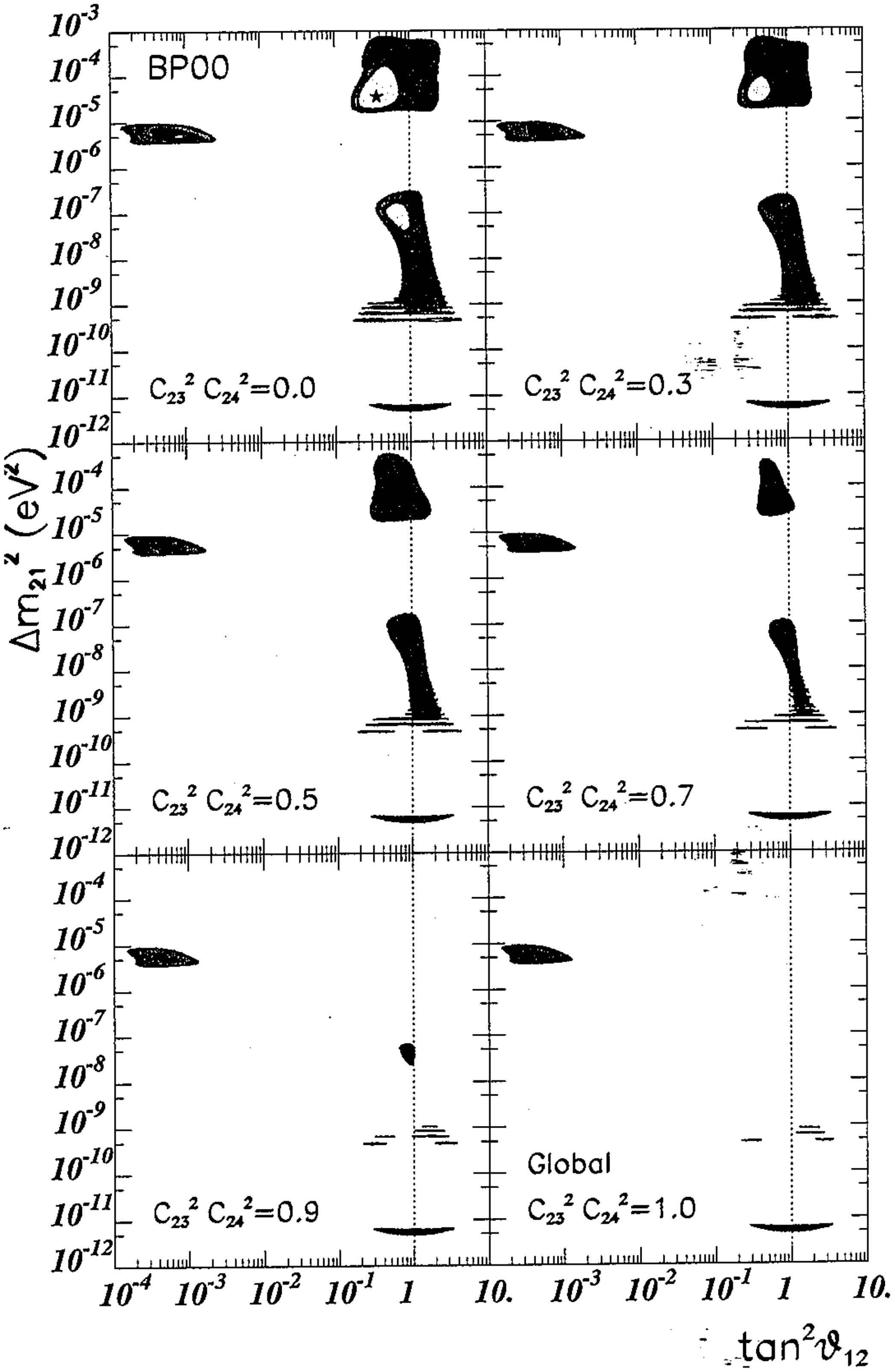
Three dimensional disappearing for $c_{23}^2 c_{24}^2$

% CL	SMA	LMA	LOW-QVO
90	< 0.08	< 0.44	< 0.30
99	all	< 0.67	< 0.77

- Impact of atmospheric data:

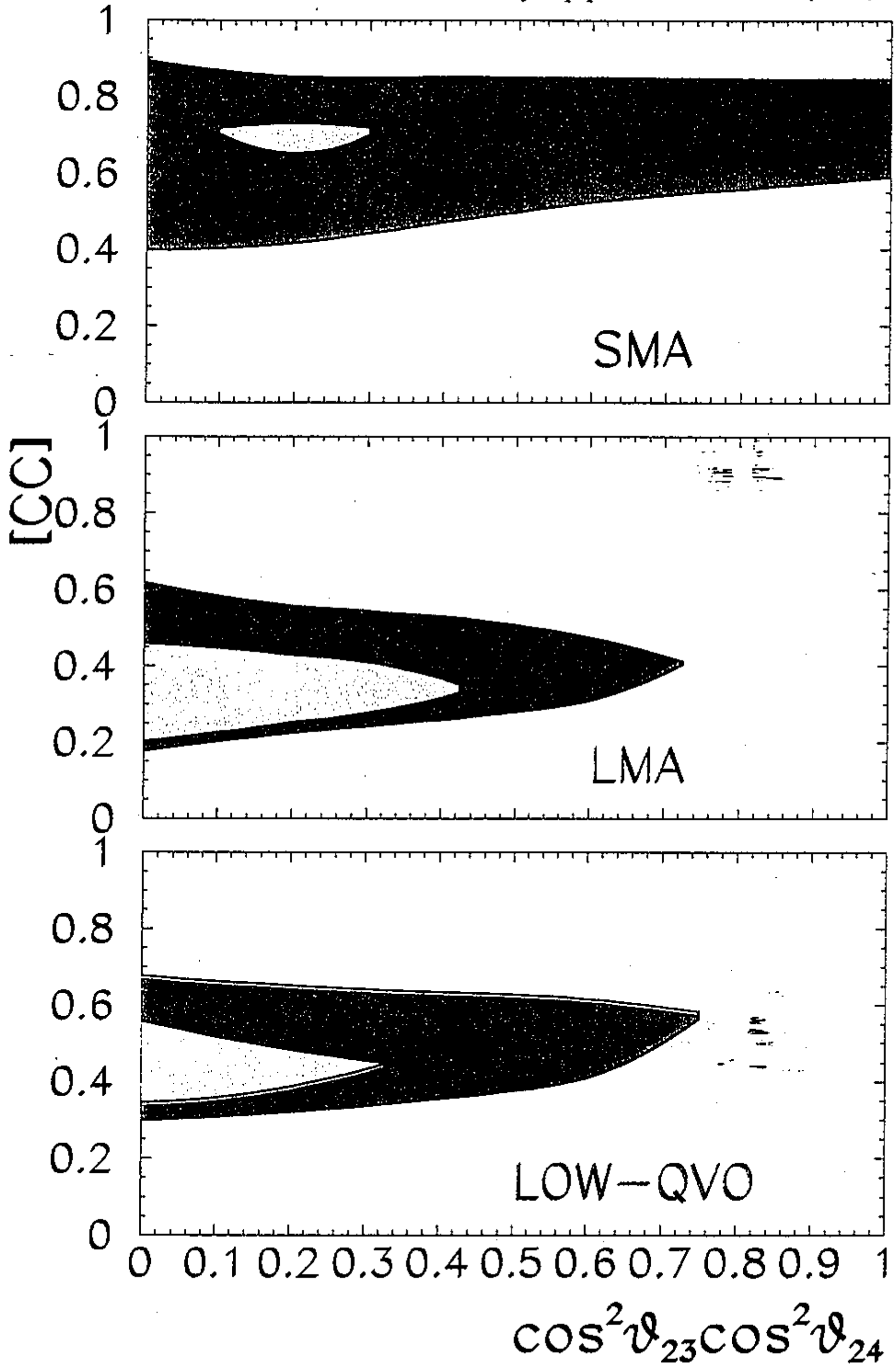
Solutions for Four-neutrino Oscillations

Solar+CHOOZ data.



What will we learn from SNO?

M.C G-G C. Peña-Garay hep-ph/0011245 PRD63 (2001)



$$[CC] \sim \langle P_{\nu_e \to \nu_e} \rangle$$

– Dependence of [CC] on $c_{23}^2 c_{24}^2$ only via modification of the matter potential ⇒ weak limits

– Dependence in figure due to variation of size of allowed regions

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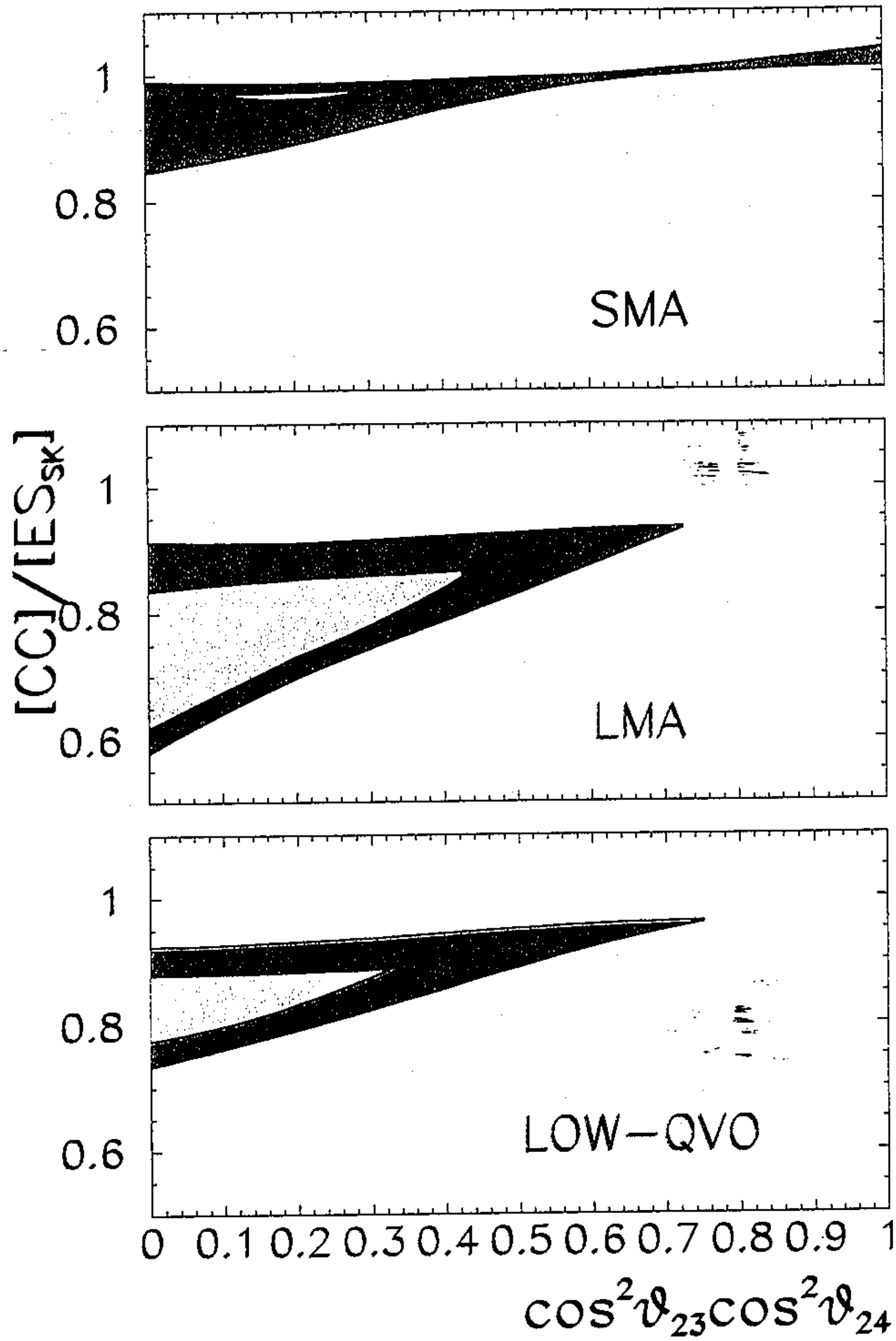

$$\frac{[\mathrm{CC}]}{[\mathrm{ES}]_{\mathrm{SK}}} \sim \frac{6\langle P_{\nu_e\to\nu_e}\rangle_{\mathrm{SNO}}}{1+5\langle P_{\nu_e\to\nu_e}\rangle_{\mathrm{SK}} - c_{23}^2 c_{24}^2(1-\langle P_{\nu_e\to\nu_e}\rangle_{\mathrm{SK}})}$$

– Explicit dependence on $c_{23}^2 c_{24}^2$

⇒ Better sensitivity but still large errors bars

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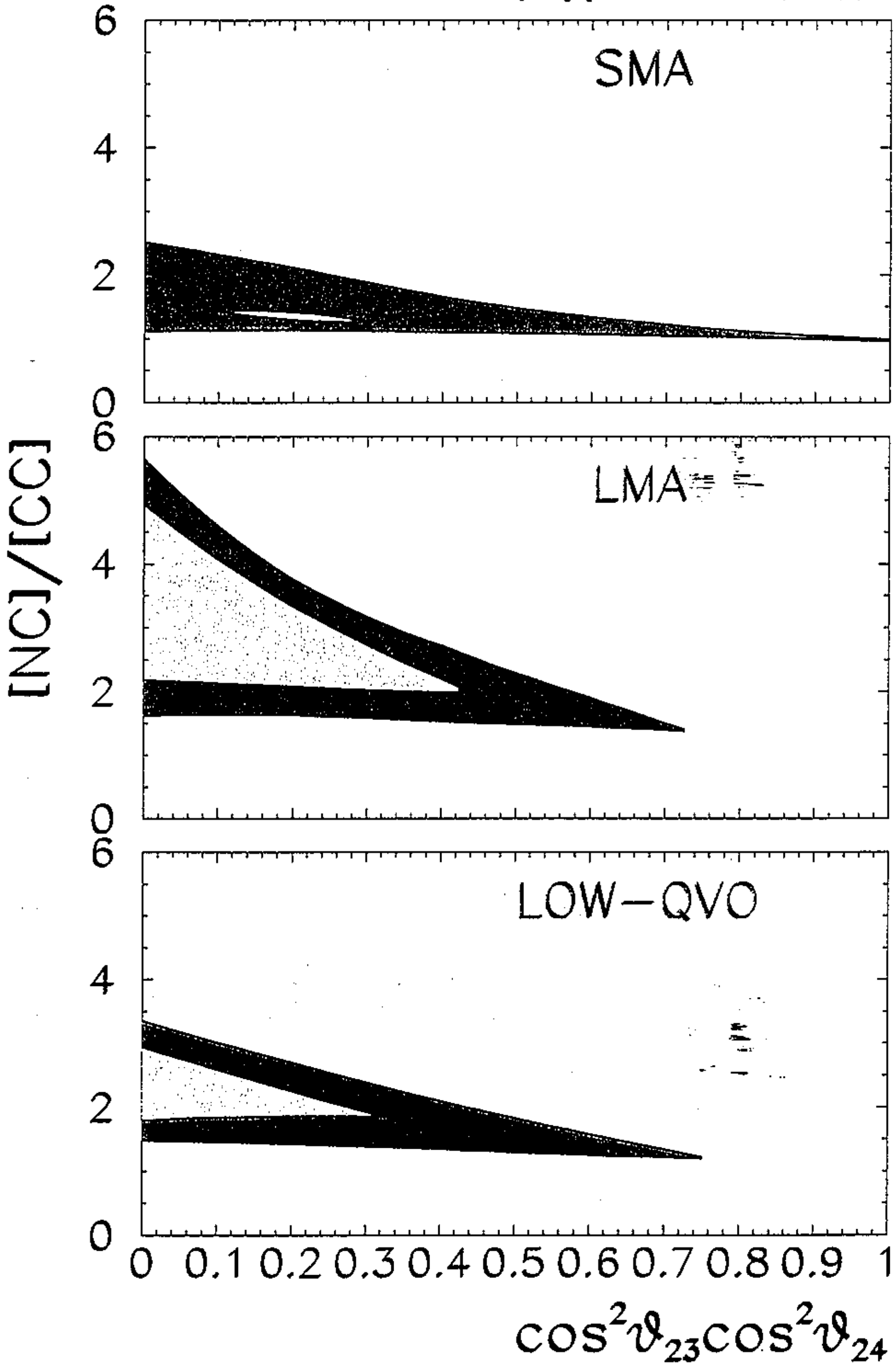


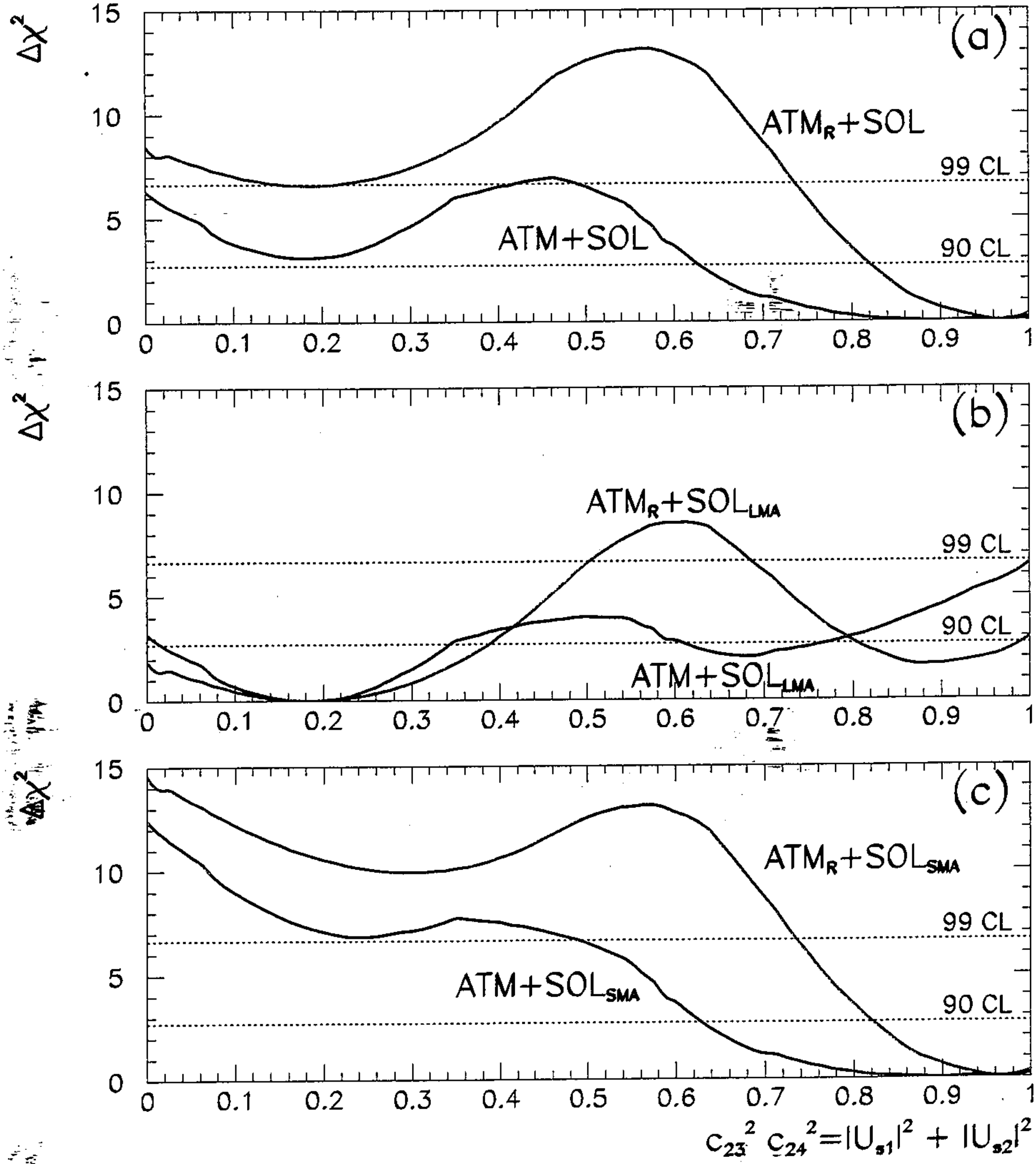
$$\frac{[\mathrm{NC}]}{[\mathrm{CC}]} \sim \frac{1 - c_{23}^2 c_{24}^2 (1 - \langle P_{\nu_e \to \nu_e} \rangle)}{\langle P_{\nu_e \to \nu_e} \rangle}$$

– Explicit dependence on $c_{23}^2 c_{24}^2$

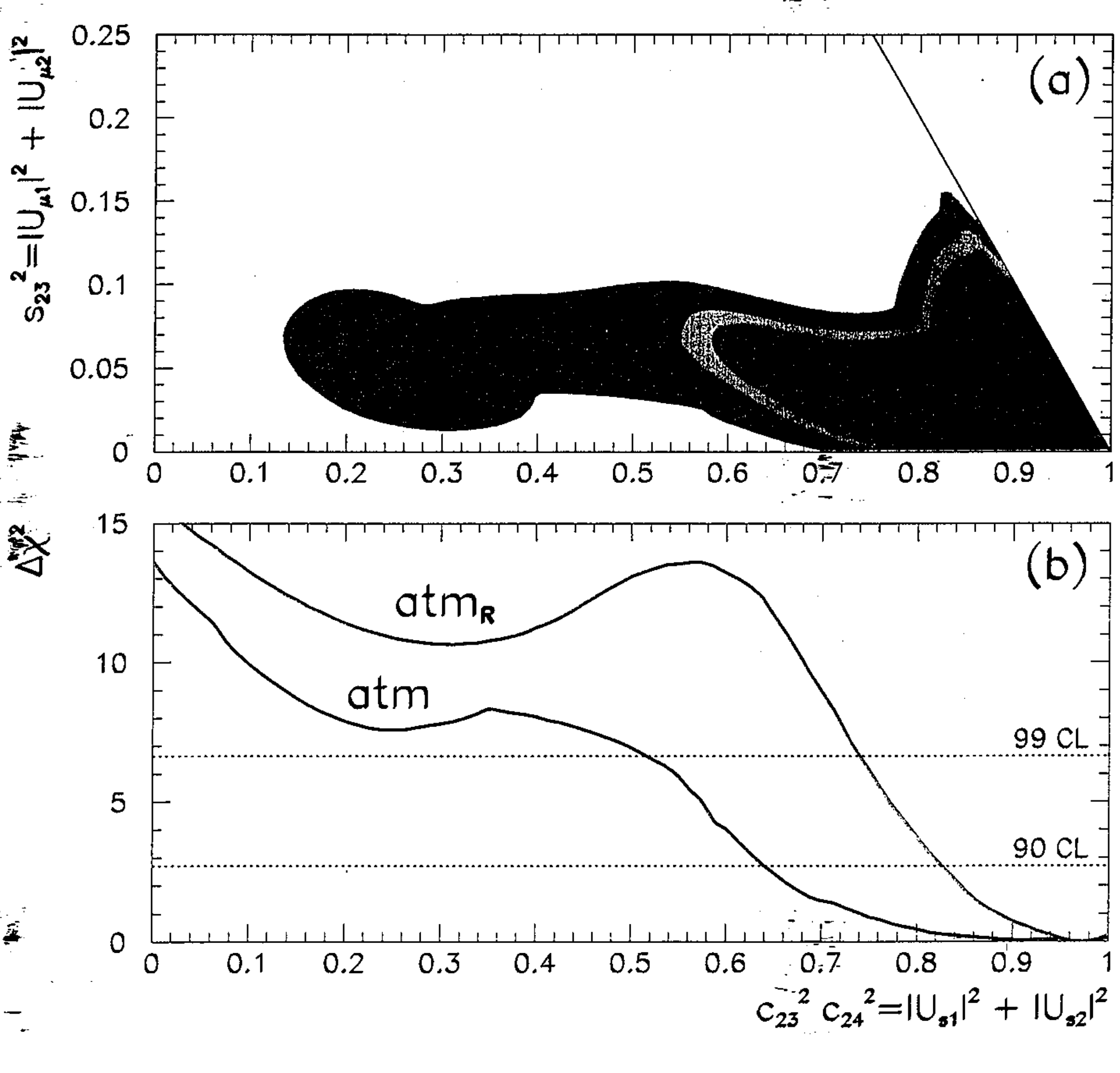
– Best sensitivity:

for instance $\frac{[\mathrm{NC}]}{[\mathrm{CC}]} = 1 \Rightarrow c_{23}^2 c_{24}^2 > 0.44$ at 99%CL

M.C. Gonzalez-Garcia, M. Maltoni and C.P-G, hep-ph/0105269



M.C. Gonzalez-Garcia, H. Maltoni and C.P.G , hep-ph/0105269



V.Summary.

- Solar neutrino data is explained in terms of neutrino oscillations.

- Two–Neutrino Oscillations:
  - ⇒ Several regimes remain allowed : LMA, SMA, LOW-QVO.
  - ⇒ Global fit is not a superposition of partial analysis.
  - ⇒ Sterile worse than Active scenario.
  - ⇒ Small impact from SNO CC rate on the solutions.

- Three–Neutrino Oscillations:
  - ⇒ Full agreement between CHOOZ result and solar and atmospheric data.
  - ⇒ Values of θ_{13} around zero preferred, althought the stronger bound comes from CHOOZ result.

- Four–Neutrino Oscillations:
  - ⇒ Unified picture of Active and Sterile scenarios.
  - ⇒ Pure Sterile oscillations are allowed at 95 % CL when compared with pure Active oscillations.