hep-ph/0102046

# CP violation effects and high energy neutrinos

A consistent picture is emerging about the neutrino masses and mixing.

Fascinating subject:
Study of $CP$ and $T$ violation effects in $\nu$ flavor transitions:

$$P(\nu_\alpha \to \nu_\beta) \neq P(\overline{\nu}_\alpha) \to P(\overline{\nu}_\beta)$$

$$P(\nu_\alpha \to \nu_\beta) \neq P(\nu_\beta) \to P(\nu_\alpha)$$

Determine the "OPTIMUM STRATEGY" for experimental studies.

- **"LOW ENERGY"**
  - Large effects in the oscillations probabilities.
  - Low Rates (focusing, $\sigma_\nu$), probles with particle ID
- **"HIGH ENERGY"**
  - Small effects in the oscillations probabilities.
  - Very High rates $[\propto E_\nu^3$ (!)]
  - Large matter effects

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24/March/2001
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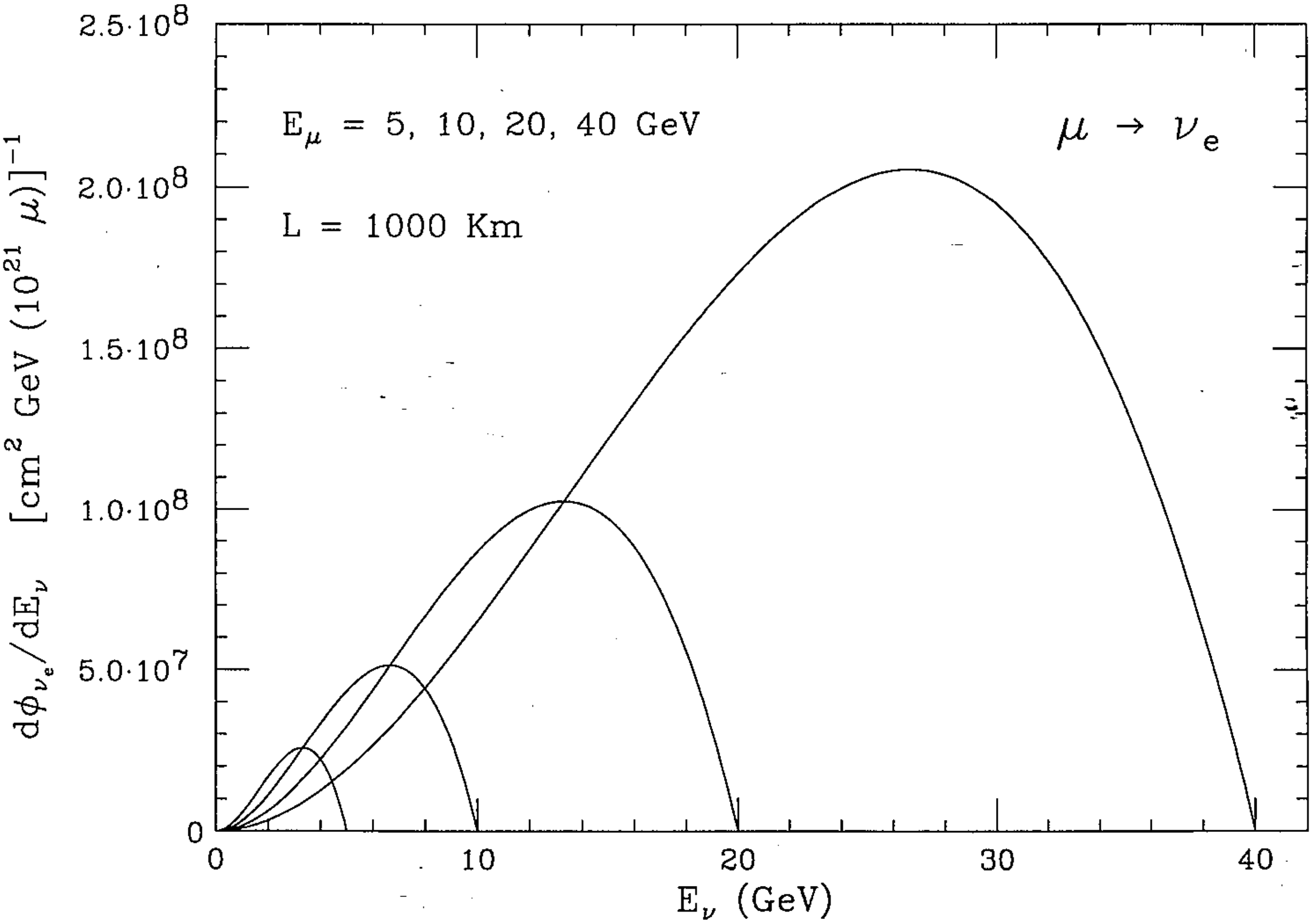
# Neutrino Fluence in a $\nu$–Factory

$$\phi_{\nu_e}(E_\nu) = \frac{12\,N_\mu}{\pi\,L^2}\,\frac{E_\nu^2}{m_\mu^2\,E_\mu}\left(1-\frac{E_\nu}{E_\mu}\right)\,\Theta[E_\mu - E_\nu]$$

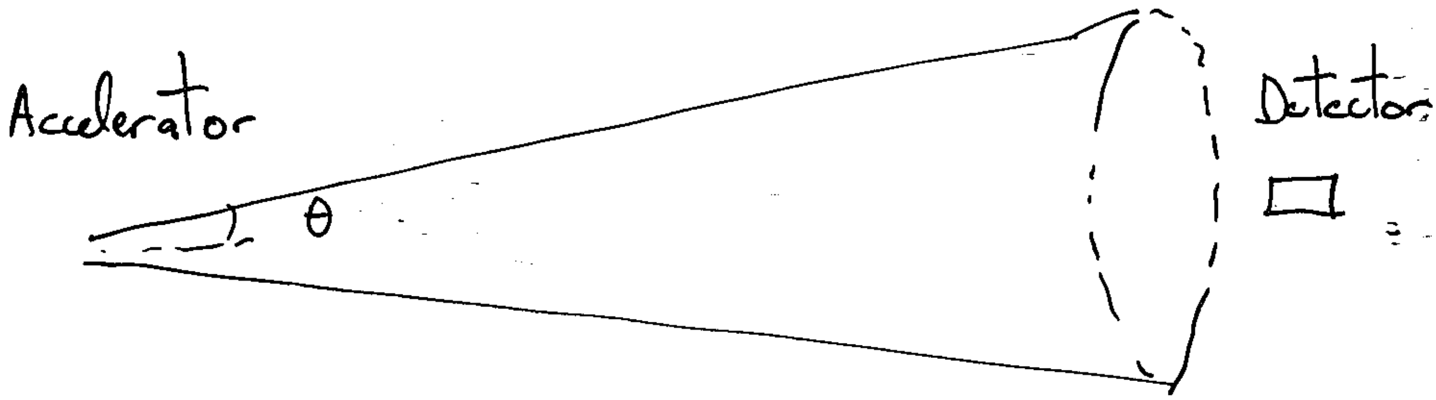



Fixed number of unpolarized $\mu$ decays. ($10^{20}$)

Increasing $E_\mu$ the integrated $\nu$ fluence increases $\propto E_\nu^2$, but the fluence at low energy decreases.

Increase in the flux $\propto E_\mu^2$

**Focusing of $\nu$**




$$\theta \propto \frac{1}{E_\nu} \Rightarrow \Delta\Omega \propto \frac{1}{E_\nu^2}$$

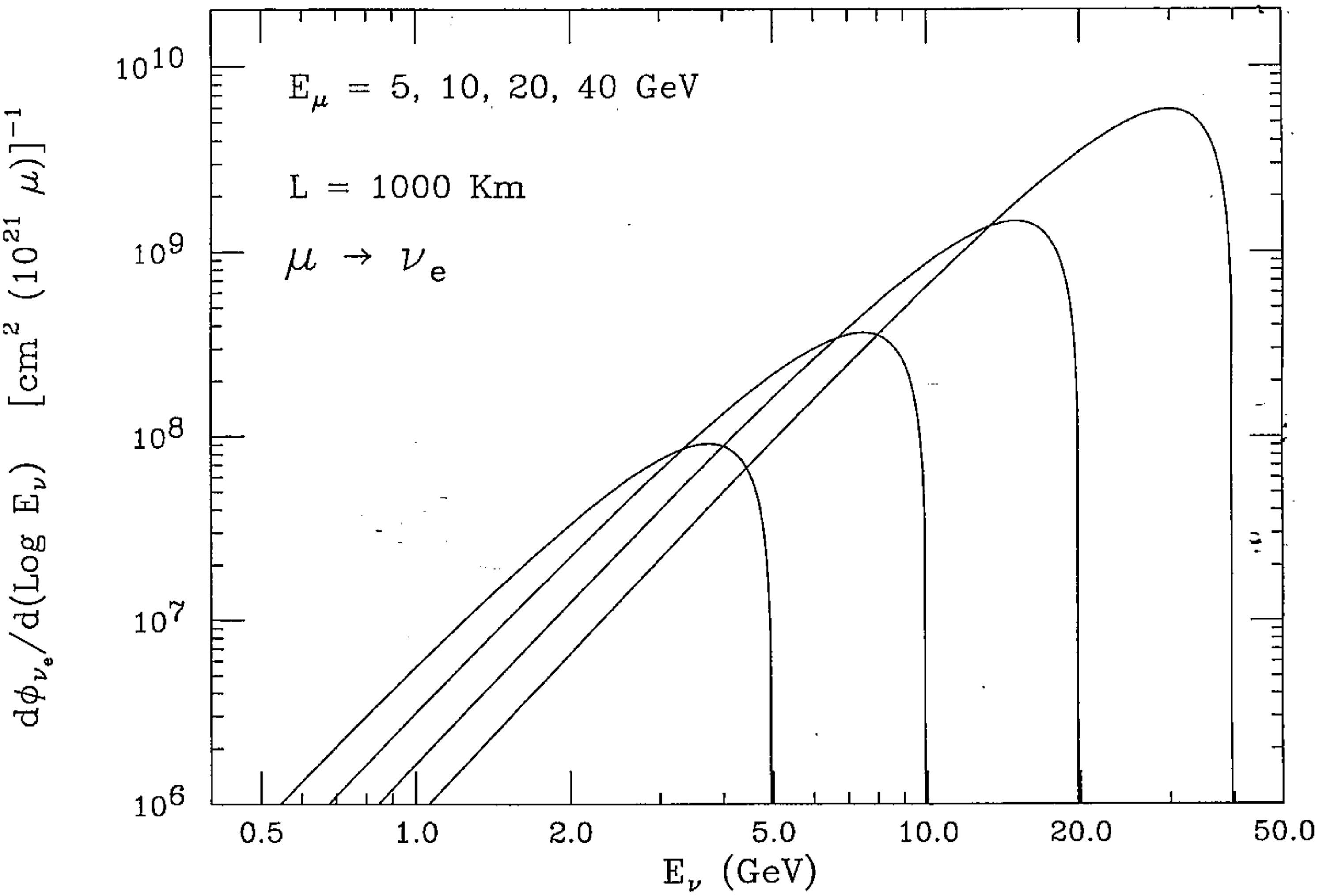
$$\Rightarrow \phi \propto E_\nu^2$$

Cross section

$$\sigma_\nu \propto E_\nu .$$

$$\boxed{\phi_\nu \sigma_\nu \propto E_\nu^3}$$

# Neutrino Fluence in a $\nu$–factory




Electron neutrino fluence in a neutrino factory machine

$$\frac{d\phi_{\nu_e}}{d\log E_\nu}.$$

Fixed $E_\nu$

$$\phi(E_\nu) \propto \frac{1}{E_\mu}$$

# FLUENCE in a NEUTRINO FACTORY

$$\phi_{\nu_e}(E_\nu) = \frac{12\,N_\mu}{\pi\,L^2}\,\frac{E_\nu^2}{m_\mu^2\,E_\mu}\left(1 - \frac{E_\nu}{E_\mu}\right)\,\Theta[E_\mu - E_\nu]$$

# Properties of the OSCILLATION PROBABILITIES

$$P_{\nu_e\to\nu_\mu} \sim E_\nu^{-2}\,L^2$$
$$\Delta P_{\nu_e\to\nu_\mu}(CP) \sim E_\nu^{-3}\,L^3$$
$$\Delta P_{\nu_e\to\nu_\mu}(\text{matter}) \sim E_\nu^{-3}\,L^4$$

# Properties of the RATES

$$\text{Rate} \sim E_\mu^3\,L^{-2}$$
$$\text{Rate}_{\nu_e\to\nu_\mu} \sim E_\mu\,L^0$$
$$\Delta\text{Rate}_{\nu_e\to\nu_\mu}(CP) \sim E_\mu^0\,L$$
$$\Delta\text{Rate}_{\nu_e\to\nu_\mu}(\text{matter}) \sim E_\mu^0\,L^2$$

# MIXING MATRIX

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}\, e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}\, e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix}$$

3 Mixing angles $\theta_{jk} \in [0, \frac{\pi}{2}]$

1 phase $\delta$ $\delta \in [-\pi, +\pi]$

↗

U complex $\delta \neq 0, \pi$

T, CP violation effects.

$$H_0 = \frac{1}{2E_\nu} U \begin{bmatrix} m_1^2 & & \\ & m_2^2 & \\ & & m_3^2 \end{bmatrix} U^\dagger$$

# MATTER EFFECTS

## Effective potential

$V = V_{\nu_e} = -V_{\overline{\nu}_e} = \sqrt{2}\, G_F\, n_e$

In the Earth:

$$V^{-1} \simeq 1850 \left(\frac{2.8 \text{ g cm}^{-3}}{\rho}\right) \left(\frac{0.5}{Y_e}\right) \text{ Km.}$$

Effective Hamiltonians for flavor evolution:

$$\mathcal{H}(\nu) = \mathcal{H}_0 + \mathcal{H}_{\rm m}$$
$$\mathcal{H}(\overline{\nu}) = \mathcal{H}_0^* - \mathcal{H}_{\rm m}$$

$$\mathcal{H}_0 = \frac{1}{2E_\nu} U \text{ diag}[m_1^2, m_2^2, m_3^2]\, U^\dagger$$

$$(\mathcal{H}_{\rm m})_{\alpha\beta} = V\, \delta_{\alpha e}\, \delta_{\beta e}$$

Flavor Evolution

$$i \frac{\mathrm{d}}{\mathrm{d}x} \nu_\alpha = \mathcal{H}\, \nu_\alpha$$

Oscillation Probability in vacuum

$$P(\nu_\alpha \to \nu_\beta) =$$

$$\frac{A^{12}_{\alpha\beta}}{2} \left[1 - \cos\left(\frac{\Delta m^2_{12}\, L}{2\, E_\nu}\right)\right] +$$

$$\frac{A^{23}_{\alpha\beta}}{2} \left[1 - \cos\left(\frac{\Delta m^2_{23}\, L}{2\, E_\nu}\right)\right] +$$

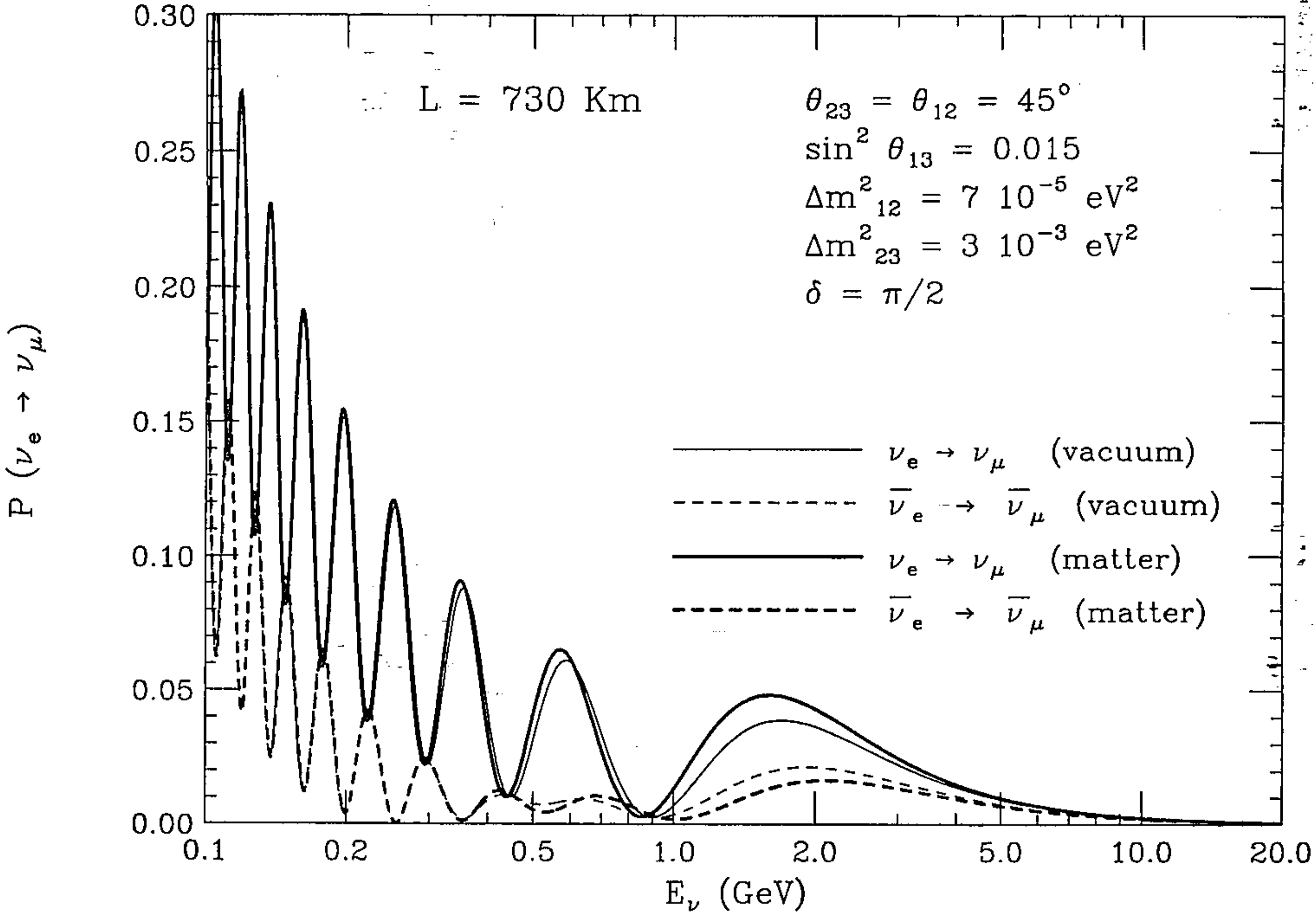
$$\frac{A^{13}_{\alpha\beta}}{2} \left[1 - \cos\left(\frac{\Delta m^2_{13}\, L}{2\, E_\nu}\right)\right] +$$

$$\pm 8\, J\ \sin\left(\frac{\Delta m^2_{12}\, L}{2\, E_\nu}\right)\ \sin\left(\frac{\Delta m^2_{23}\, L}{2\, E_\nu}\right)\ \sin\left(\frac{\Delta m^2_{14}\, L}{2\, E_\nu}\right)$$

$$A^{jk}_{\alpha\beta} = -4\ \mathrm{Re}[U_{\alpha j}\, U^*_{\beta j}\, U^*_{\alpha k}\, U_{\beta k}]$$

$$J = J^{12}_{e\mu} = -\mathrm{Im}[U_{e1}\, U^*_{\mu 2}\, U^*_{e2}\, U_{\mu 2}]$$
$$= c^2_{13}\, s_{13}\, s_{12}\, c_{12}\, s_{23}\, c_{23}\, \sin\delta$$

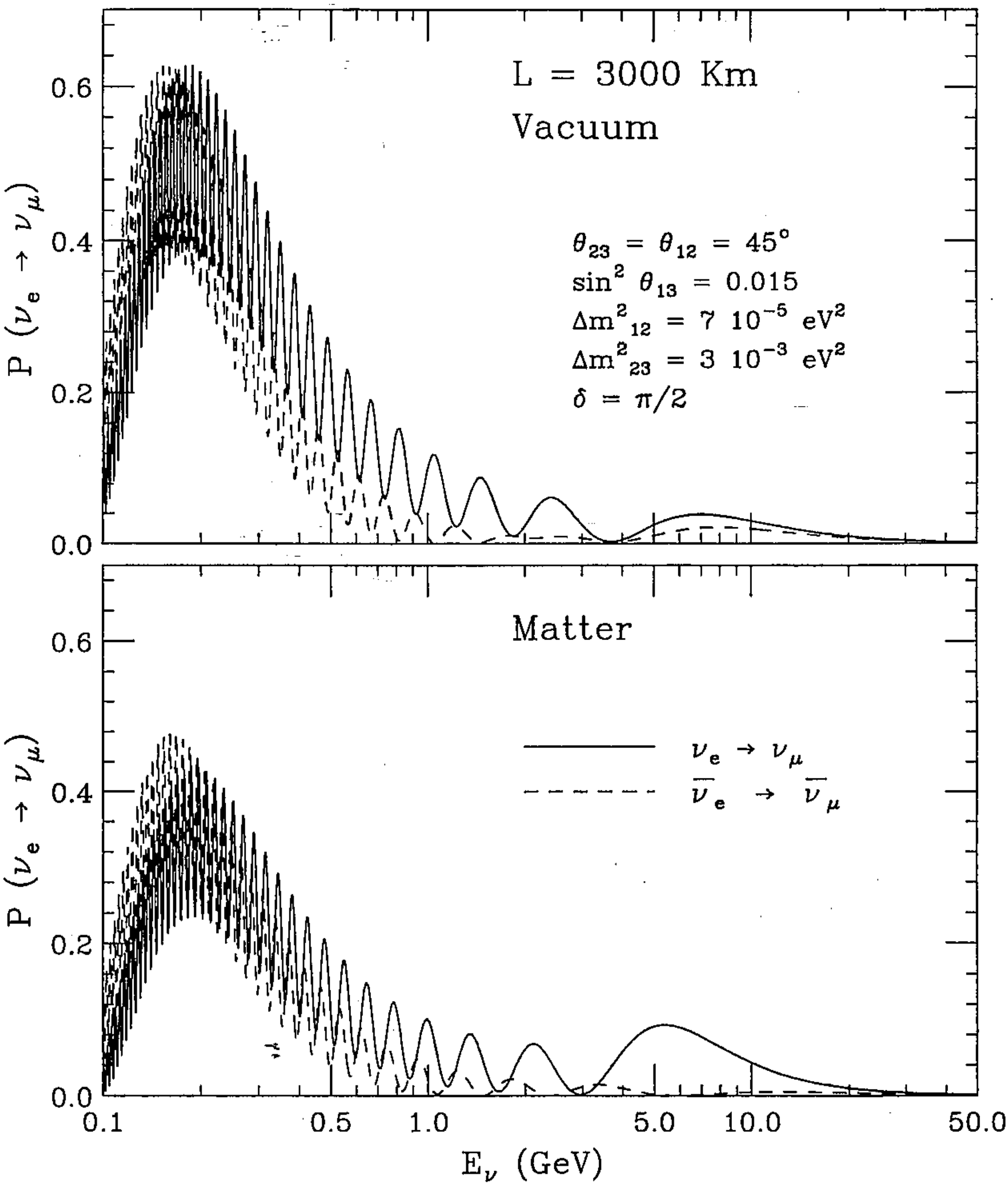
Jarlskog parameter




$P(\nu_e \to \nu_\mu)$, $P(\overline{\nu}_e \to \nu_\mu)$

$L = 730$ Km

L = 3000 km




$P(\nu_e \to \nu_\mu)$, $P(\overline{\nu}_e \to \nu_\mu)$

$L = 3000$ Km

L = 7000 km

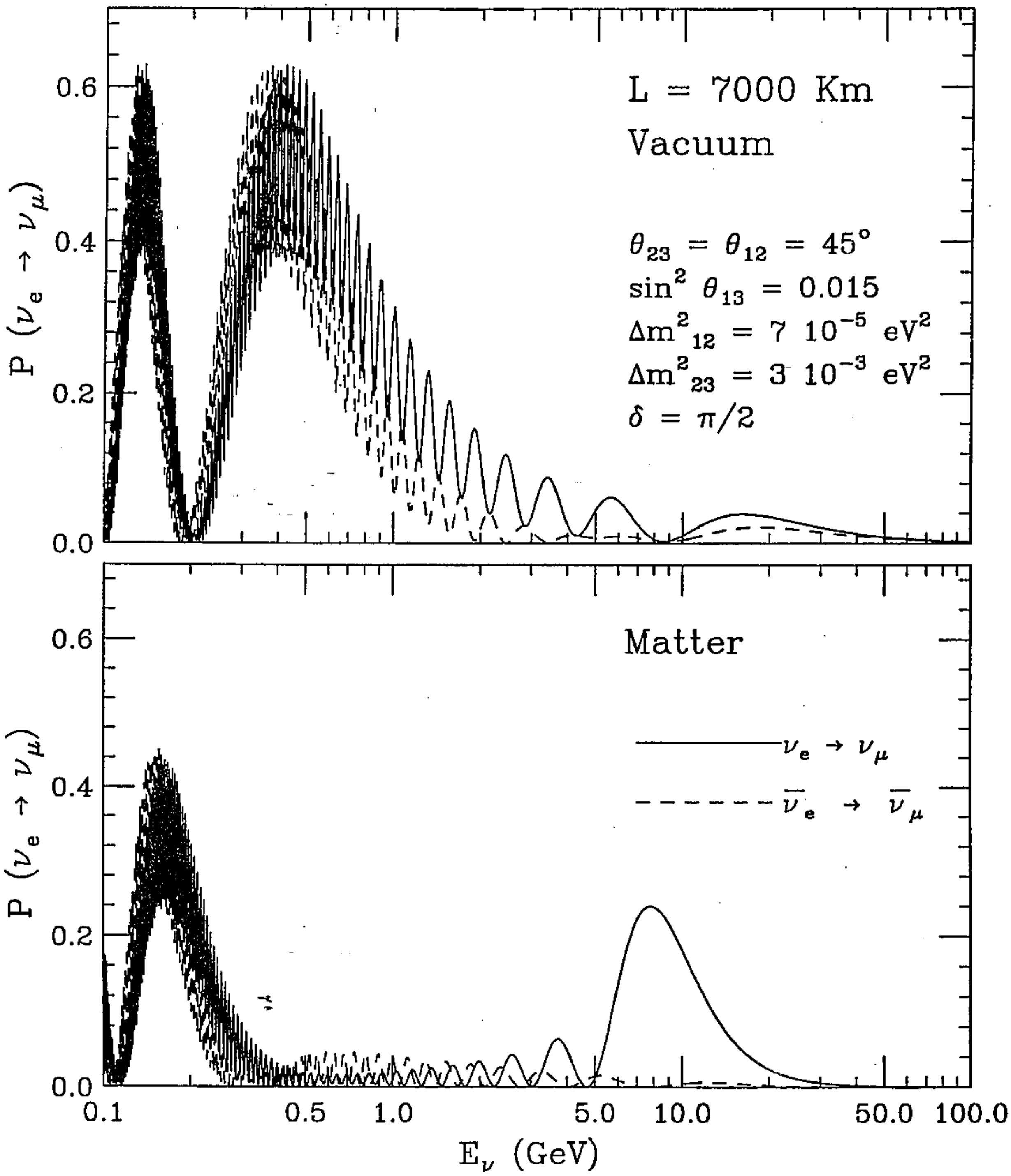



Figure 10: Oscillation probability for the transition $\nu_e \to \nu_\mu$ plotted as a function of $E_\nu$ for a fixed value of the pathlength $L = 7000$ Km. The oscillation parameters are fixed, and are indicated in the figure. The solid (dashed) curves are for $\nu$ ($\overline{\nu}$). The top (bottom) panel gives the probability for propagation in vacuum (matter).

## High energy limit

$$
\begin{aligned}
P(\nu_\alpha \to \nu_\beta) \;=\; & A^{12}_{\alpha\beta}\left(\frac{\Delta m^2_{12}\,L}{4\,E_\nu}\right)^2 + \\
& A^{23}_{\alpha\beta}\left(\frac{\Delta m^2_{22}\,L}{4\,E_\nu}\right)^2 + \\
& A^{13}_{\alpha\beta}\left(\frac{\Delta m^2_{13}\,L}{4\,E_\nu}\right)^2 + \\
& \pm 8\,J\left(\frac{\Delta m^2_{12}\,L}{2\,E_\nu}\right)\left(\frac{\Delta m^2_{23}\,L}{2\,E_\nu}\right)\left(\frac{\Delta m^2_{13}\,L}{2\,E_\nu}\right) + \\
& \text{higher order terms}
\end{aligned}
$$

$$
P(\nu_\alpha \to \nu_\beta) = A_{\alpha\beta}\,\frac{L^2}{E_\nu^2} + B_{\alpha\beta}\frac{L^3}{E_\nu^3} + \ldots
$$

Equal for

$\nu_\alpha \to \nu_\beta$

$\nu_\beta \to \nu_\alpha$

$\overline{\nu_\alpha} \to \overline{\nu_\beta}$

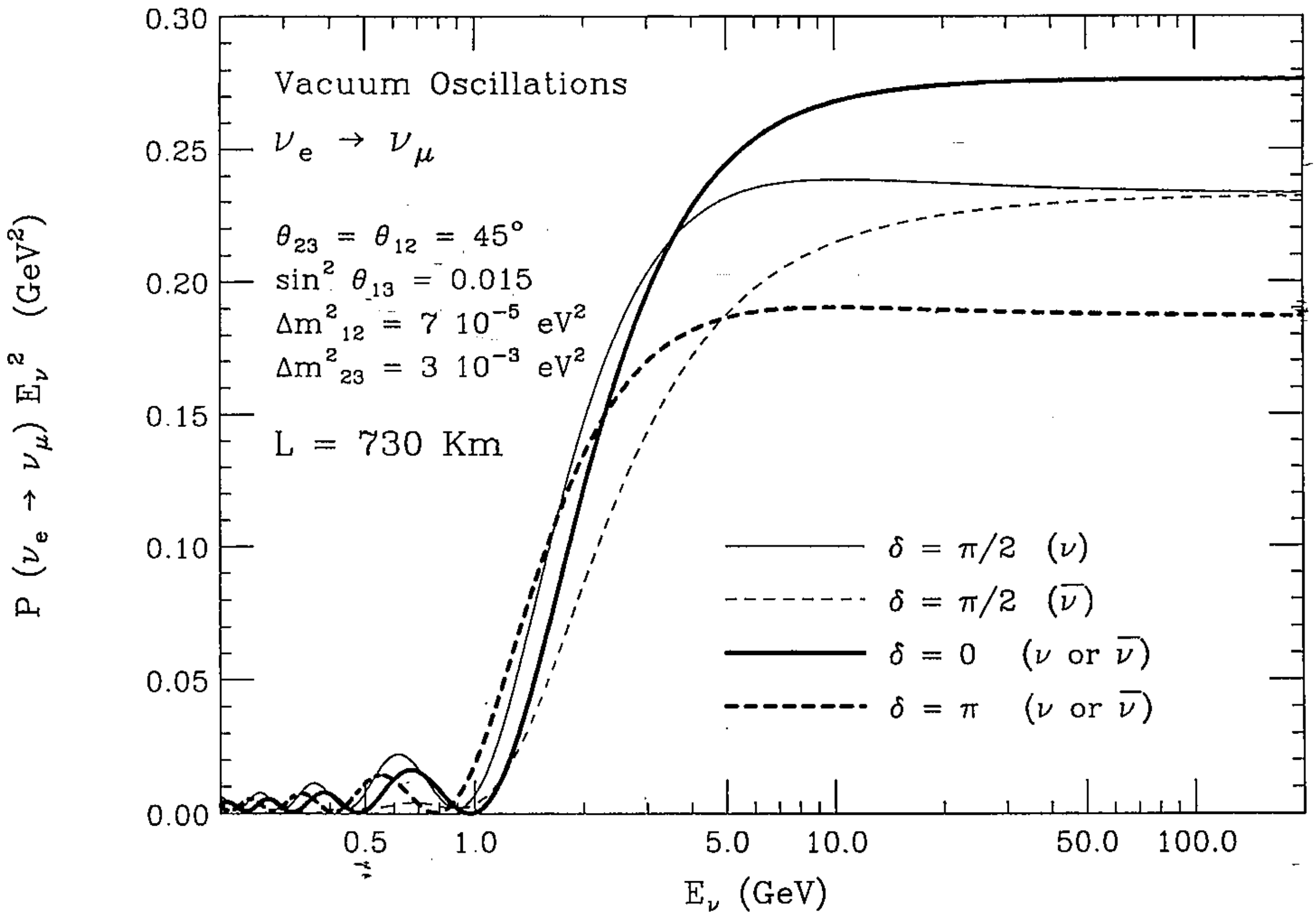
$\overline{\nu_\beta} \to \overline{\nu_\alpha}$

CP, T violations

Product $P(\nu_e \to \nu_\mu)$, $P(\overline{\nu}_e \to \nu_\mu)$

$L = 730$ Km

Dependence on phase $\delta$.



# Oscillation probability in matter

$$P(\nu_\alpha \to \nu_\beta) =$$

$$A_{\alpha\beta}\left(\frac{\Delta m^2_{23}\,L}{4E_\nu}\right)^2 \left[\left(\frac{2}{LV}\right)^2 \sin^2\left(\frac{L\,V}{2}\right)\right] +$$

$$B_{\alpha\beta}\left(\frac{\Delta m^2_{23}\,L}{4E_\nu}\right)^3 \left[\left(\frac{2}{LV}\right)^2 \sin^2\left(\frac{L\,V}{2}\right)\right] +$$

$$C_{\alpha\beta}\left(\frac{\Delta m^2_{23} L}{4\,E_\nu}\right)^3 LV \left\{\frac{48}{(L\,V)^4}\left[\sin^2\left(\frac{LV}{2}\right) - \left(\frac{LV}{4}\right)\sin(LV)\right]\right.$$

$$+ \ldots$$

$$\begin{aligned} A_{\alpha\beta} &= +A_{\beta\alpha} = +A_{\overline{\alpha}\overline{\beta}} = +A_{\overline{\beta}\overline{\alpha}} \\ B_{\alpha\beta} &= -B_{\beta\alpha} = -B_{\overline{\alpha}\overline{\beta}} = +B_{\overline{\beta}\overline{\alpha}} \\ C_{\alpha\beta} &= +C_{\beta\alpha} = -C_{\overline{\alpha}\overline{\beta}} = -C_{\overline{\beta}\overline{\alpha}} \end{aligned}$$

$$A_{\beta\alpha} = (\mathcal{H}_0)_{\alpha\beta}\,(\mathcal{H}_0)^*_{\alpha\beta}\,\varepsilon^{-2}$$

$$B_{\beta\alpha} = \mathrm{Im}[(\mathcal{H}_0)^*_{\alpha\beta}\,(\mathcal{H}_0^2)_{\alpha\beta}]\,\varepsilon^{-3} \quad = 8J.$$

$$C_{\beta\alpha} = \pm\frac{1}{6}\mathrm{Re}\{(\mathcal{H}_0)^*_{\alpha\beta}\,[2\,(\mathcal{H}_0)_{\alpha e}\,(\mathcal{H}_0)_{e\beta} - (\mathcal{H}_0^2)_{\alpha\beta}\,(\delta_{\alpha e}+\delta_{\beta e})]\}$$

$$\epsilon = \frac{\Delta m^2_{23}}{4E_\nu}$$

$(A_{\alpha\beta}, B_{\alpha\beta}, C_{\alpha\beta}$ = adimensional constants$)$

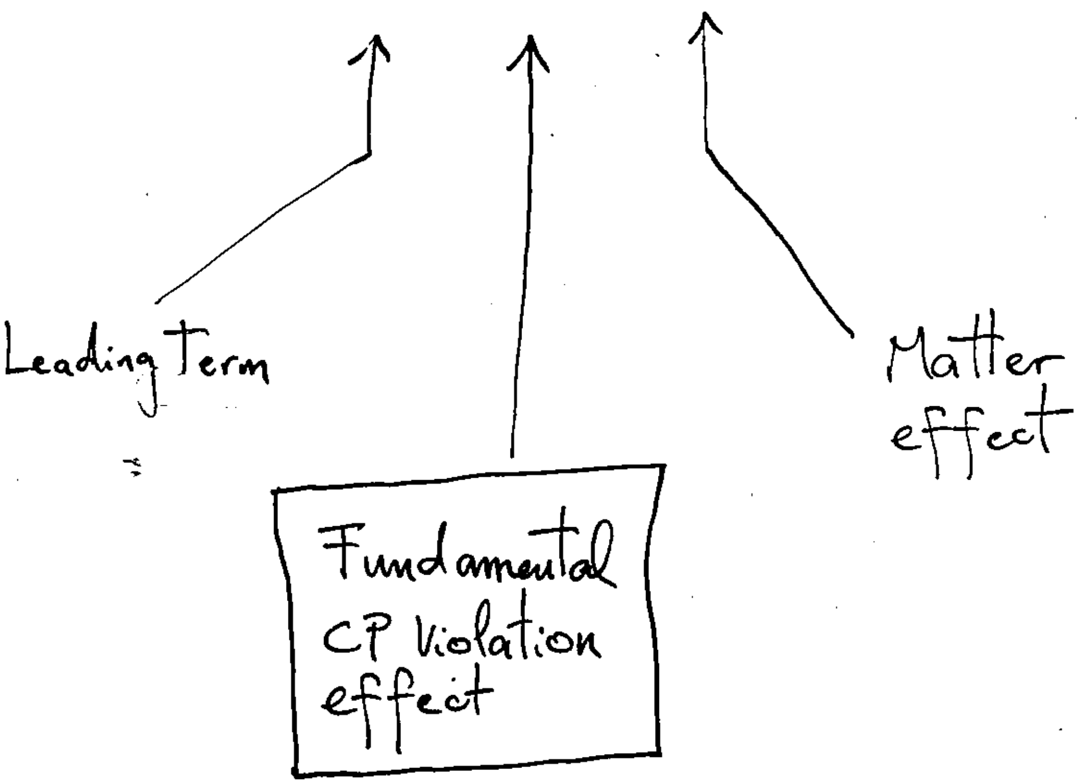
**Developing in series in $VL$:**

$$P(\nu_\alpha \to \nu_\beta) \simeq A_{\alpha\beta} \left(\frac{\Delta m^2_{23}}{4\,E_\nu}\right)^2 L^2 \left[1 - \frac{(V\,L)^2}{12} + \ldots\right]$$
$$+ B_{\alpha\beta} \left(\frac{\Delta m^2_{23}}{4E_\nu}\right)^3 L^3 \left[1 - \frac{(V\,L)^2}{12} + \ldots\right]$$
$$+ C_{\alpha\beta} \left(\frac{\Delta m^2_{23}}{4\,E_\nu}\right)^3 L^4\,V \left[1 - \frac{(V\,L)^2}{15} + \ldots\right];$$

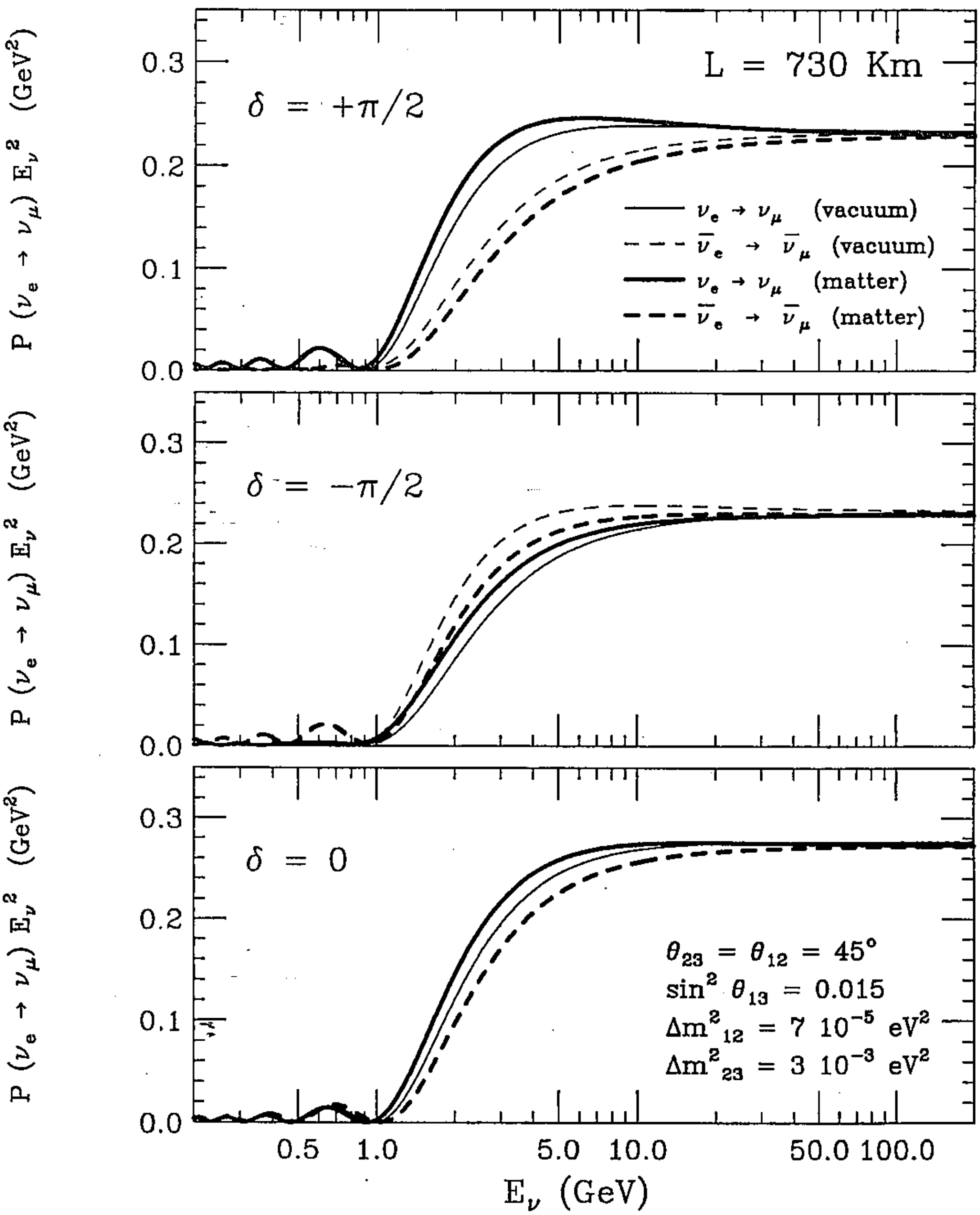
The constants $A_{\alpha\beta}$ and $B_{\alpha\beta}$ are **IDENTICAL** to the vacuum case.

4 transitions related by
$CP$ and $T$ transoformations:

$$P(\nu_\alpha \to \nu_\beta) = \frac{A}{E_\nu^2} + \frac{B}{E_\nu^3} + \frac{C}{E_\nu^3} + \ldots$$
$$P(\nu_\beta \to \nu_\alpha) = \frac{A}{E_\nu^2} - \frac{B}{E_\nu^3} + \frac{C}{E_\nu^3} + \ldots$$
$$P(\overline{\nu}_\alpha \to \overline{\nu}_\beta) = \frac{A}{E_\nu^2} - \frac{B}{E_\nu^3} - \frac{C}{E_\nu^3} + \ldots$$
$$P(\overline{\nu}_\beta \to \overline{\nu}_\alpha) = \frac{A}{E_\nu^2} + \frac{B}{E_\nu^3} - \frac{C}{E_\nu^3} + \ldots$$

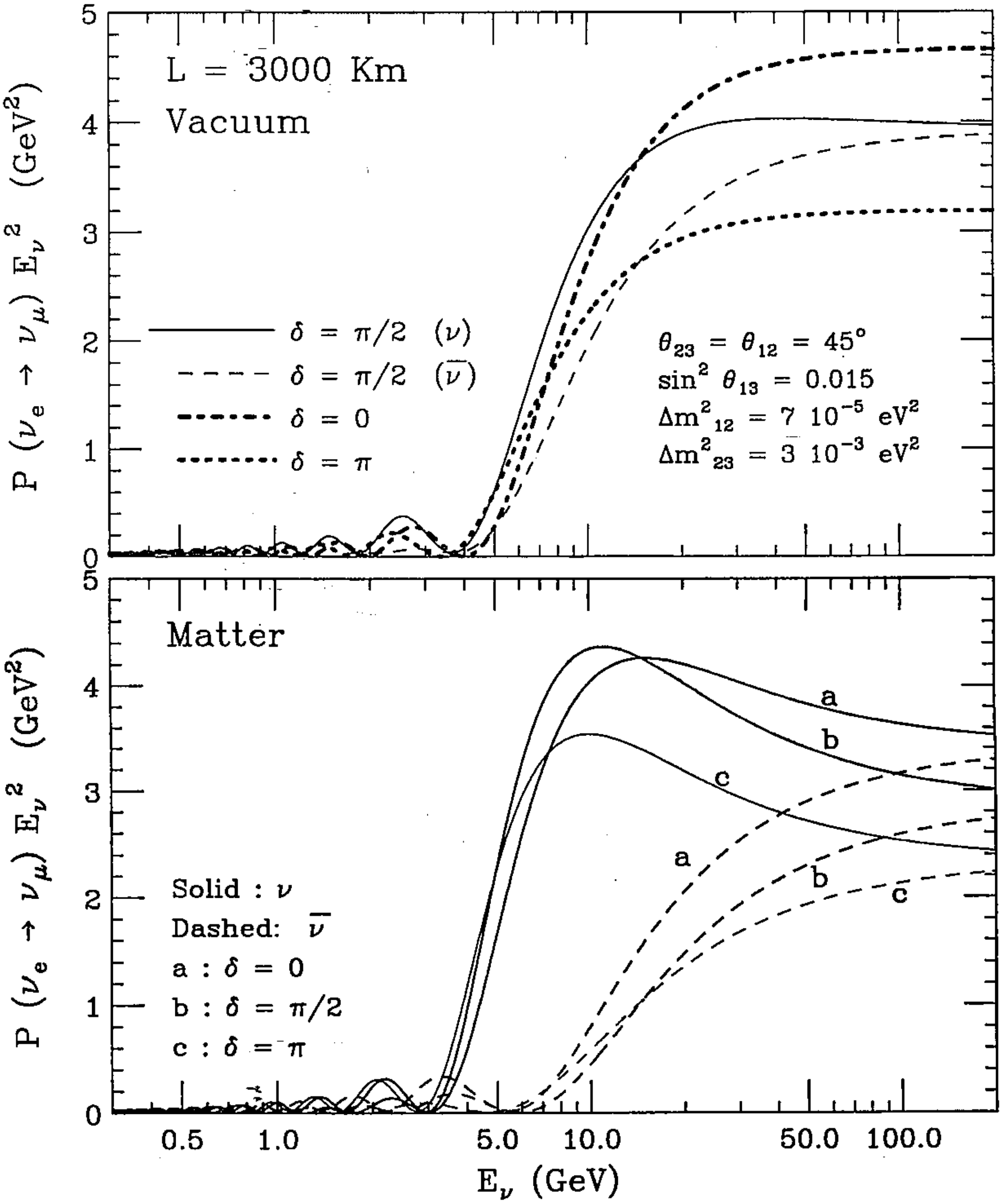


# Inclusion of Matter




$\Delta m^2_{23} > 0$ $\quad P_\nu$ enhanced

$P_{\bar{\nu}}$ suppressed.



L = 3000 Km
Vacuum
δ = π/2 (ν)
δ = π/2 ($\bar{\nu}$)
δ = 0
δ = π
$\theta_{23} = \theta_{12} = 45°$
$\sin^2 \theta_{13} = 0.015$
$\Delta m^2_{12} = 7\ 10^{-5}\ eV^2$
$\Delta m^2_{23} = 3\ 10^{-3}\ eV^2$
Matter
Solid : ν
Dashed: $\bar{\nu}$
a : δ = 0
b : δ = π/2
c : δ = π
a
b
c
$P(\nu_e \to \nu_\mu)\ E_\nu^2\ (GeV^2)$
$E_\nu$ (GeV)
0.5
1.0
5.0
10.0
50.0
100.0

Difference between $\nu$, $\bar{\nu}$

## High $V E_\nu$ limit of Matter effects

Behaviour of the effective squared masses and mixing parameters in the limit of very large matter effects ($|\Delta m^2_{23}|/(2VE_\nu) \to 0$).

- Case A corresponds to neutrinos for $\Delta m^2_{23}$ positive or to anti–neutrinos for $\Delta m^2_{23}$ negative. In this case one has:

$$(\Delta m^2_{12})^{\rm m} \to \Delta m^2_{23},$$

$$(\Delta m^2_{23})^{\rm m} \to 2VE_\nu,$$

$$\sin^2\theta^{\rm m}_{13} \to 1, \qquad \cos^2\theta^{\rm m}_{13} \to \sin^2\theta_{13}\left(\frac{\Delta m^2_{23}}{2E_\nu V}\right)^2, \qquad \theta_3 \to 90^\circ$$

$$\sin^2\theta^{\rm m}_{12} \to \sim 1, \qquad \cos^2\theta^{\rm m}_{12} \to \underline{\underline{\text{const}}} \simeq \sin^2\theta_{12}\,\frac{\Delta m^2_{12}}{\Delta m^2_{23}}. \qquad \theta_2 \to \sim 90^\circ$$

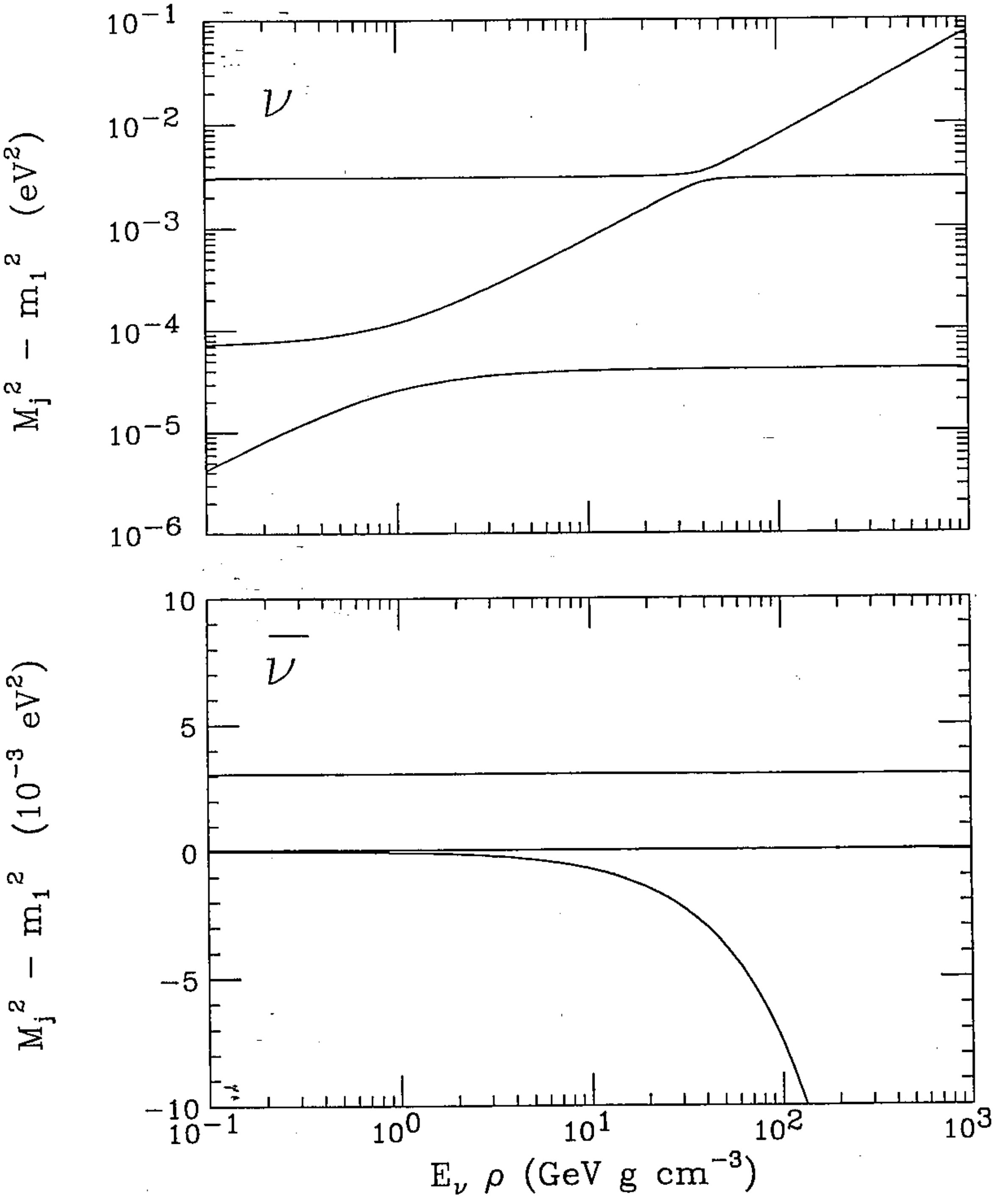
- Case B corresponds to neutrinos for $\Delta m^2_{23}$ negative or to anti–neutrinos for $\Delta m^2_{23}$ positive. In this case one has:

$$(\Delta m^2_{12})^{\rm m} \to 2VE_\nu,$$

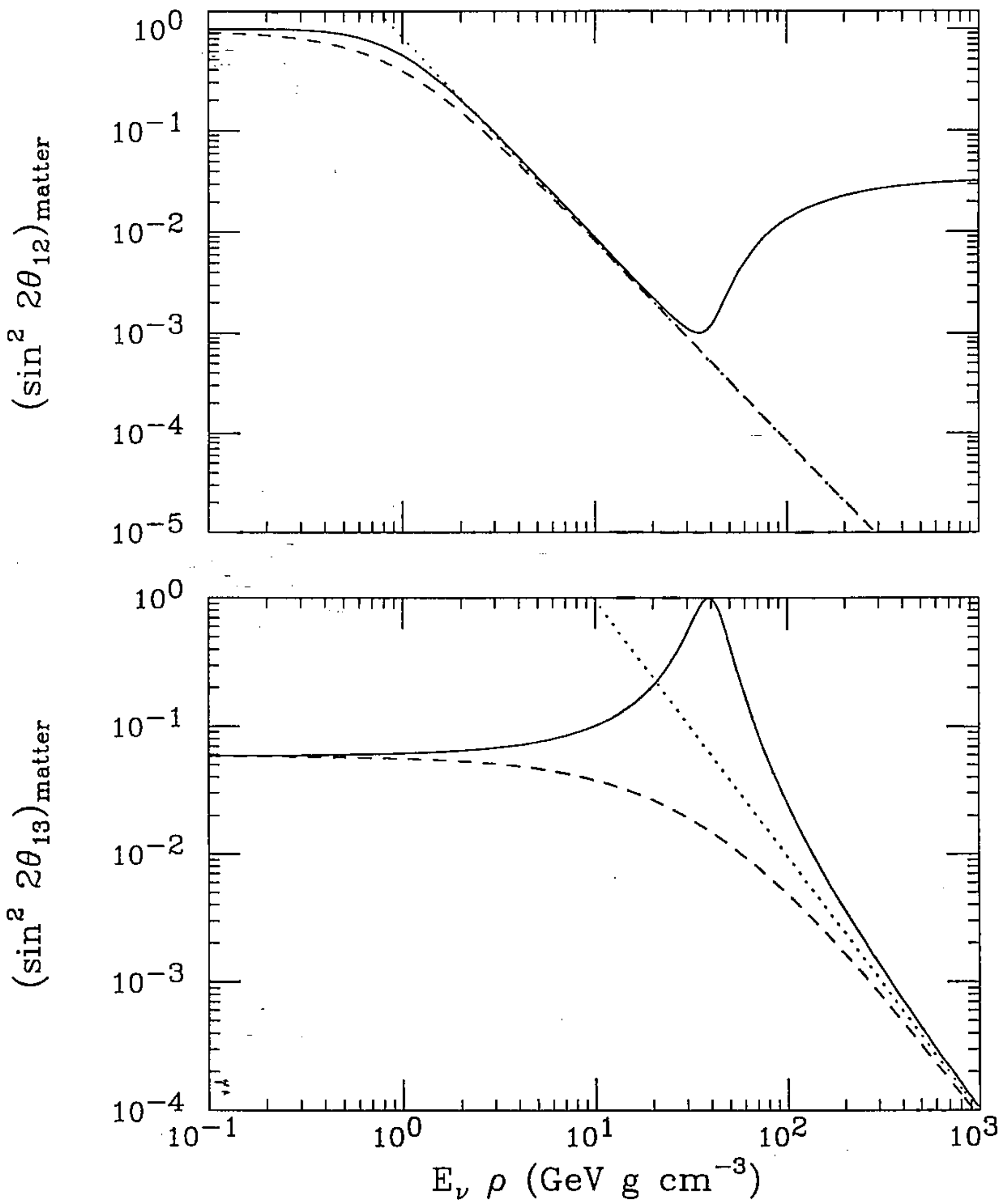
$$(\Delta m^2_{23})^{\rm m} \to \Delta m^2_{23},$$

$$\sin^2\theta^{\rm m}_{13} \to \sin^2\theta_{13}\left(\frac{\Delta m^2_{23}}{2E_\nu V}\right)^2, \qquad \cos^2\theta^{\rm m}_{13} \to 1, \qquad \theta_{13} \to 0$$

$$\sin^2\theta^{\rm m}_{12} \to \sin^2\theta_{12}\left(\frac{\Delta m^2_{12}}{2E_\nu V}\right)^2, \qquad \cos^2\theta^{\rm m}_{12} \to 1. \qquad \theta_2 \to 0$$

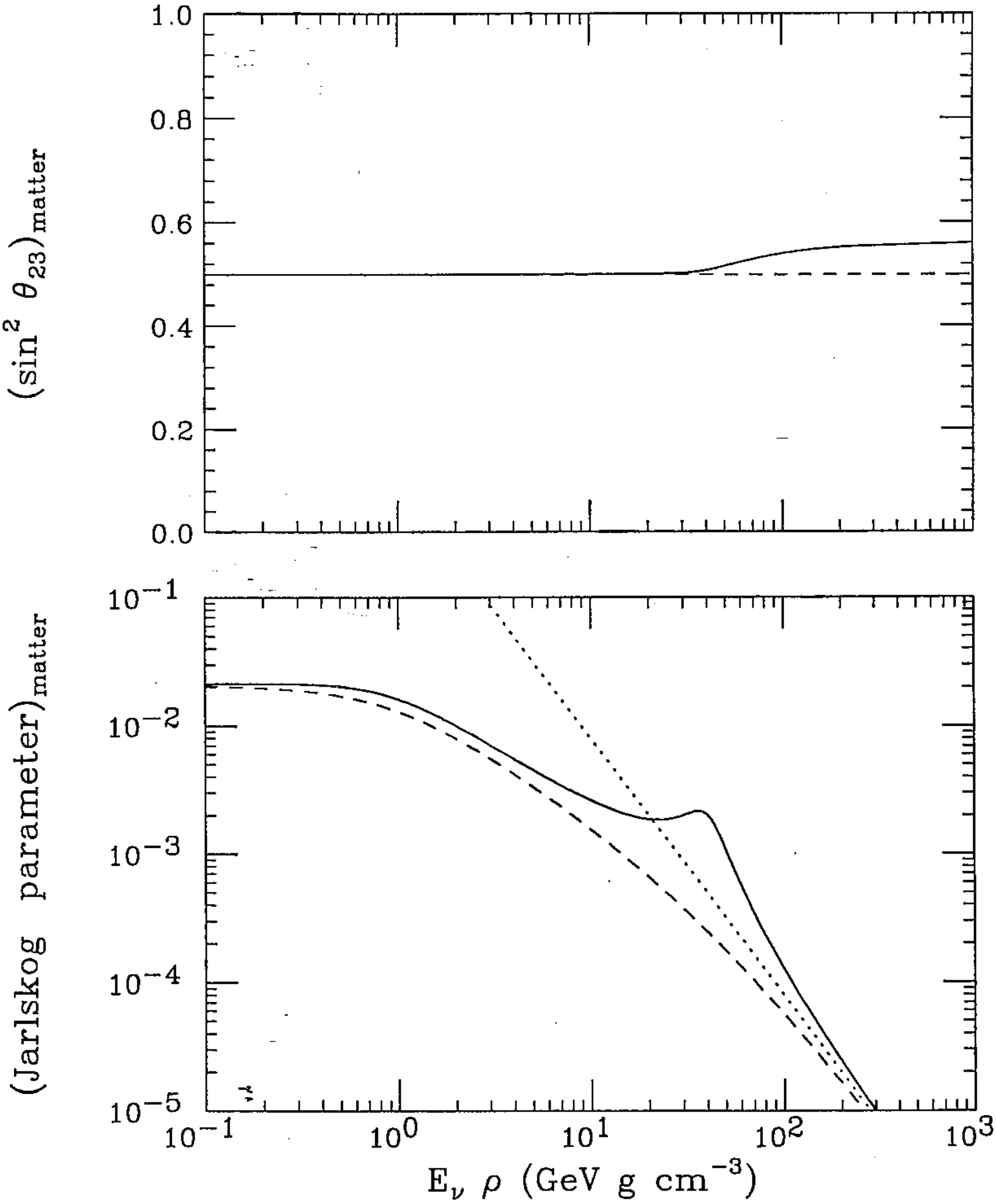



Effective squared mass eigenvalues in matter
$m_1^2 = 0$, $m_2^2 = 7 \times 10^{-5}$ eV$^2$, $m_3^2 = 3 \times 10^{-3}$ eV$^2$;
$\theta_{12} = 40°$, $\theta_{23} = 45°$, $\theta_{13} = 7°$, $\delta = 45°$.




$\sin^2 2\theta_{12}^{\rm m}$, $\sin^2 2\theta_{13}^{\rm m}$

The dotted lines in the top panel is $\sin^2 2\theta_{12}\,(\Delta m_{12}^2/(2E_\nu V))^2$. The dotted line in the bottom panel is $\sin^2 2\theta_{13}\,(\Delta m_{13}^2/(2E_\nu V))^2$.




$\sin^2\theta_{23}^{\rm m}$, , $J = (c_{13}^{\rm m})^2\, s_{13}^{\rm m}\, s_{12}^{\rm m}\, c_{12}^{\rm m}\, s_{23}^{\rm m}\, c_{23}^{\rm m}\, \sin\delta_{\rm m}$.

# OSCILLATIONS PROBABILITIES as POWER SERIES

$$i\frac{\mathrm{d}}{\mathrm{d}x}\nu_\alpha = \mathcal{H}_0\,\nu_\alpha = \left[\lambda\mathbf{1} + \frac{1}{2E_\nu}U\ \mathrm{diag}[m_1^2, m_2^2, m_3^2]\ U^\dagger\right]\ \nu_\alpha$$

$$S(\nu_\alpha \to \nu_\beta) \equiv S_{\beta\alpha} = \exp[-i\mathcal{H}_0\,L]_{\beta\alpha}.$$

$$S_{\beta\alpha} = \delta_{\beta\alpha} + (-i\,L)(\mathcal{H}_0)_{\beta\alpha} + \frac{1}{2!}(-i\,L)^2(\mathcal{H}_0^2)_{\beta\alpha} + \ldots$$

$$P(\nu_\alpha \to \nu_\beta) = |S_{\beta\alpha}|^2$$

$$\mathcal{H}_0 = \frac{\Delta m_{23}^2}{4E_\nu}\ U\ \mathrm{diag}[-(1+2x_{12}), -1, 1]\ U^\dagger = \frac{\Delta m_{23}^2}{4E_\nu}\ \hat{h}$$

$$y = \frac{\Delta m_{23}^2\,L}{4\,E_\nu}.$$

$$\boxed{P_{\alpha\to\beta}(L, E_\nu) = \sum_{n=2}^{\infty} c_n^{\alpha\to\beta}\ y^n}$$

$L/E_\nu$

$$P_{\beta\to\alpha} = [\hat{h}_{\alpha\beta}\,\hat{h}^*_{\alpha\beta}]\ y^2 + \mathrm{Im}[\hat{h}^*_{\alpha\beta}\ (\hat{h}^2)_{\alpha\beta}]\ y^3 + \ldots$$

$$\begin{cases} c_n^{\alpha\to\beta} = +c_n^{\beta\to\alpha} = +c_n^{\overline{\alpha}\to\overline{\beta}} = +c_n^{\overline{\beta}\to\overline{\alpha}} & \text{for } n \text{ even,} \\ c_n^{\alpha\to\beta} = -c_n^{\beta\to\alpha} = -c_n^{\overline{\alpha}\to\overline{\beta}} = +c_n^{\overline{\beta}\to\overline{\alpha}} & \text{for } n \text{ odd,} \end{cases}$$

# INCLUSION of MATTER

$$\begin{aligned}\mathcal{H}_\nu\, L &= \mathcal{H}_0\, L + \hat{p}_e\, V\, L = \hat{h}\, y + \hat{p}_e\, z \\ \mathcal{H}_{\overline{\nu}}\, L &= \mathcal{H}_0^*\, L - \hat{p}_e\, V\, L = \hat{h}^*\, y - \hat{p}_e\, z\end{aligned}$$

$$z = V\, L$$

$$\boxed{P_{\alpha\to\beta}(L, E_\nu) = P_{\alpha\to\beta}(y, z) = \sum_{n=2}^{\infty} \sum_{m=0}^{\infty} c_{n,m}^{\alpha\to\beta}\, y^n\, z^m}$$

$$\begin{cases} c_{n,m}^{\alpha\to\beta} = +c_{n,m}^{\beta\to\alpha} = +c_{n,m}^{\overline{\alpha}\to\overline{\beta}} = +c_{n,m}^{\overline{\beta}\to\overline{\alpha}} & \text{for } n \text{ even and } m \text{ even,} \\ c_{n,m}^{\alpha\to\beta} = +c_{n,m}^{\beta\to\alpha} = -c_{n,m}^{\overline{\alpha}\to\overline{\beta}} = -c_{n,m}^{\overline{\beta}\to\overline{\alpha}} & \text{for } n \text{ even and } m \text{ odd} \\ c_{n,m}^{\alpha\to\beta} = -c_{n,m}^{\beta\to\alpha} = -c_{n,m}^{\overline{\alpha}\to\overline{\beta}} = +c_{n,m}^{\overline{\beta}\to\overline{\alpha}} & \text{for } n \text{ odd and } m \text{ even} \\ c_{n,m}^{\alpha\to\beta} = -c_{n,m}^{\beta\to\alpha} = +c_{n,m}^{\overline{\alpha}\to\overline{\beta}} = -c_{n,m}^{\overline{\beta}\to\overline{\alpha}} & \text{for } n \text{ odd and } m \text{ odd} \end{cases}$$

$$c^{\alpha\to\beta}_{2,m(\text{odd})} = 0$$

$$c^{\alpha\to\beta}_{2,m(\text{even})} = [\hat{h}_{\alpha\beta}\,\hat{t}^*_{\alpha\beta}]\; d_{2,m}$$

$$d_{2,m(\text{even})} = i^m \sum_{k=0}^{m} \frac{(-1)^k}{(k+1)!\,(m+1-k)!} = \frac{2}{(m+2)!}\, i^m$$

$$c^{\alpha\to\beta}_{3,m(\text{even})} = \text{Im}[\hat{h}^*_{\alpha\beta}\; (\hat{h}^2)_{\alpha\beta}]\; d_{3,m}$$

$$c^{\alpha\to\beta}_{3,m(\text{odd})} = \frac{1}{6}\text{Re}\{\hat{h}^*_{\alpha\beta}\;[2\,\hat{h}_{\alpha e}\,\hat{h}_{e\beta} - (\hat{h}^2)_{\alpha\beta}\,(\delta_{\alpha e} + \delta_{\beta e})]\}\; d_{3,m}$$

$$d_{3,m(\text{even})} = 2\,i^m \sum_{k=0}^{m} \frac{(-1)^k}{(k+1)!\,(m+2-k)!} = \frac{2}{(m+2)!}\, i^m$$

$$d_{3,m(\text{odd})} = 12\,i^{m-1} \sum_{k=0}^{m} \frac{(-1)^k}{(k+1)!\,(m+2-k)!} = \frac{12(m+1)}{(m+3)!}\, i^{m-1}$$

$(d_{2,0} = d_{3,0} = d_{3,1} = 1)$.

---

$$\sum_{m=0}^{\infty} d_{2,m}\, z^m \doteq \sum_{m(\text{even})=0}^{\infty} d_{2,m}\, z^m = \left(\frac{2}{z}\right)^2 \sin^2\left(\frac{z}{2}\right)$$

$$\sum_{m(\text{odd})=1}^{\infty} d_{2,m}\, z^m = \frac{48}{z^3}\;\left[\sin^2\left(\frac{z}{2}\right) - \frac{z}{4}\sin(z)\right]$$

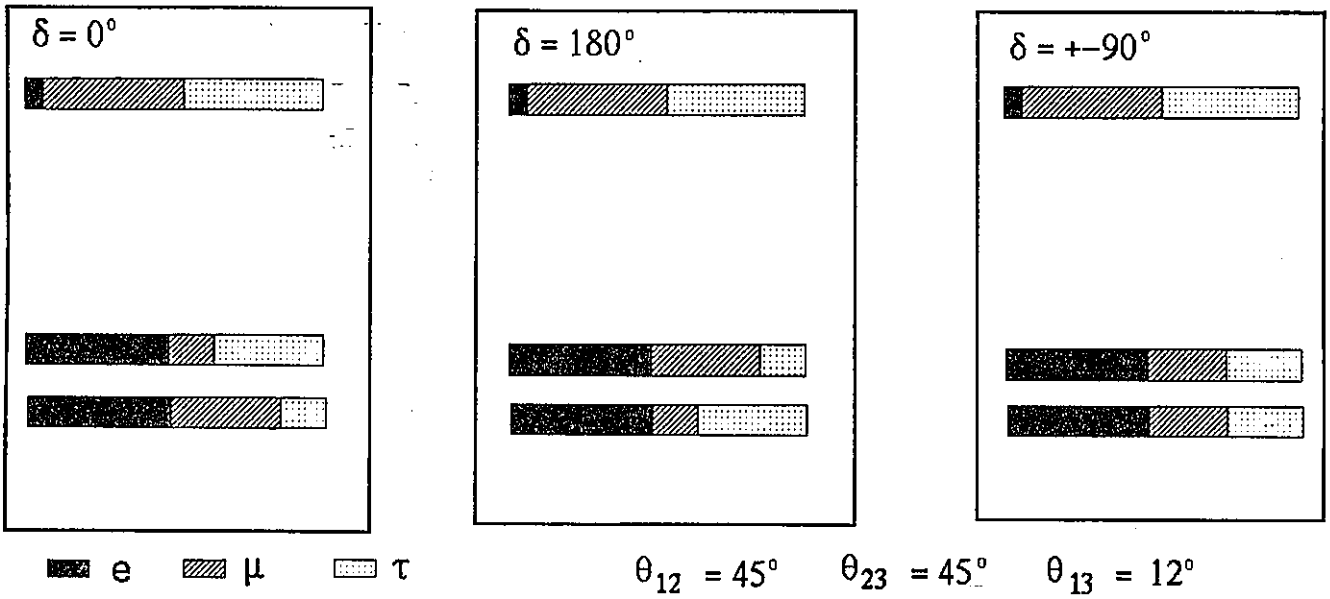
Geometrical meaning of the three mixing angles in the determination of the "overlaps" $|\langle\nu_\alpha|\nu_j\rangle|^2$:

1. The angle $\theta_{13}$ determines how much electron flavor is in the state $|\nu_3\rangle$, and how much is shared between $|\nu_1\rangle$ and $|\nu_2\rangle$ (fractions $\sin^2\theta_{13}$ and $\cos^2\theta_{13}$ respectively).

2. The angle $\theta_{23}$ describes how the non–electron content of the state $|\nu_3\rangle$ is shared between $\nu_\mu$ and $\nu_\tau$ (fractions $\sin^2\theta_{23}$ and $\cos^2\theta_{23}$).

3. The angle $\theta_{12}$ describes how the electron flavor not taken by the $|\nu_3\rangle$ is "shared" between the $|\nu_1\rangle$ and the $|\nu_2\rangle$ (fractions $\cos^2\theta_{12}$ and $\sin^2\theta_{12}$).

**WHAT IS the "GEOMETRICAL MEANING of $\delta$ ?**




Role of the phase $\delta$ in determining the flavor overlaps $|\langle \nu_\alpha | \nu_j \rangle|^2$.
$\theta_{12} = \theta_{23} = 45°$, $\theta_{13} = 12°$,
$\delta$ is 0, 180° and $\pm 90°$.

Fixing $\theta_{12}$, $\theta_{13}$, $\theta_{23}$ $\quad |\langle \nu_{\mu,\tau} | \nu_{1,2} \rangle|^2$ are not fixed. depend on $\delta$

$\delta = 0$ $\quad |\langle \nu_\mu | \nu_1 \rangle|^2$ maximum

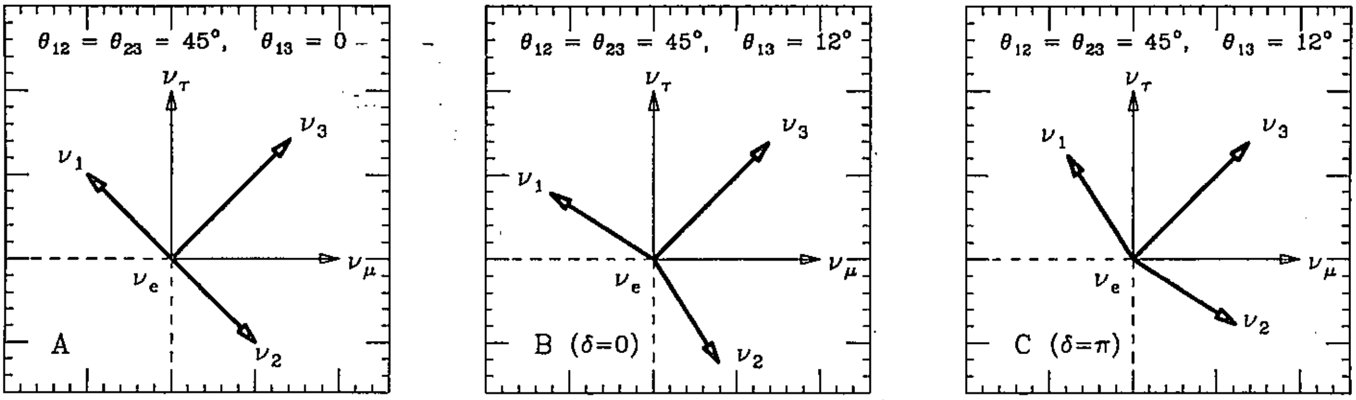
$|\delta| = \pi$ $\quad |\langle \nu_\mu | \nu_1 \rangle|^2$ minimum.

## BIMAXIMAL MIXING

$$U = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ +\frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Include small $\theta_{13}$: First order:

$$U = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & s_{13}\, e^{-i\delta} \\ -\frac{1}{2}(1 + s_{13}\, e^{i\delta}) & \frac{1}{2}(1 - s_{13}\, e^{i\delta}) & \frac{1}{\sqrt{2}} \\ +\frac{1}{2}(1 - s_{13}\, e^{i\delta}) & -\frac{1}{2}(1 + s_{13}\, e^{i\delta}) & \frac{1}{\sqrt{2}} \end{pmatrix}$$




## Special case: REAL matrix $\delta = 0$ or $\pi$.

Geometrical relation between the flavor and mass eigenvectors in the case of a real mixing matrix ($\delta = 0$ or $\pi$). The figure shows the projections in the ($\nu_\mu$,$\nu_\tau$) plane of the mass eigenvectors. The $\nu_e$ vector is coming out of the plane of the figure. The parameters of the mixing matrix are indicated in the plots.

For large $E_\nu$ the effective masses and mixing parameters in vacuum and in matter are very different from the vacuum case, (and very different for $\nu$ and $\overline{\nu}$)
However the oscillation probability are very similar to the vacuum case.

Why ? Cancellation effect:

**Certain combinations of $\Delta m^2_{jk}$ and mixing parameters are independent from the matter effects:**

$$\begin{aligned}\mathcal{F} &= J\,\Delta m^2_{12}\,\Delta m^2_{23}\,\Delta m^2_{13}\\ &= c^2_{13}\,s_{13}\,c_{12}\,s_{12}\,c_{23}\,s_{23}\,\sin\delta\,\Delta m^2_{12}\,\Delta m^2_{23}\,\Delta m^2_{13}\end{aligned}$$

$$\mathcal{F}_{\mathrm{mat},\nu} = \mathcal{F}_{\mathrm{mat},\overline{\nu}} = \mathcal{F}_{\mathrm{vacuum}} \quad *$$

$$\mathcal{G}_{\alpha\beta} = A^{12}_{\alpha\beta}\,(\Delta m^2_{12})^2 + A^{13}_{\alpha\beta}\,(\Delta m^2_{13})^2 + A^{23}_{\alpha\beta}\,(\Delta m^2_{23})^2$$

where $A^{jk}_{\alpha\beta} = -4\,\mathrm{Re}[U_{\alpha j}\,U^*_{\beta j}\,U^*_{\alpha k}\,U_{\beta k}]$

$$(\mathcal{G}_{\alpha\beta})_{\mathrm{mat},\nu} = (\mathcal{G}_{\alpha\beta})_{\mathrm{mat},\overline{\nu}} = (\mathcal{G}_{\alpha\beta})_{\mathrm{vacuum}}$$

* Harrison Scott
Phys. Rev. Lett. B476 (2000)

## The leading term in the oscillation probability and $|\delta|$

$$P(\nu_\alpha \to \nu_\beta) = A_{\alpha\beta} \left(\frac{\Delta m^2 L}{4E_\nu}\right)^2$$

$$A_{\alpha\beta} = A^{12}_{\alpha\beta} \times x^2_{12} + A^{13}_{\alpha\beta} \times (1 + x_{12})^2 + A^{23}_{\alpha\beta} \times 1$$

$$A^{jk}_{\alpha\beta} = -4\mathrm{Re}[U_{\alpha j} U^*_{\alpha k} U^*_{\beta j} U_{\beta k}]$$

$$x_{12} = \frac{\Delta m^2_{12}}{\Delta m^2_{23}}$$

$$A^{12}_{e\mu} = 4\, s^2_{12}\, c^2_{12}\, c^2_{13}\, (c^2_{23} - s^2_{13}\, s^2_{23}) + (c^2_{12} - s^2_{12})\, s_{13}\, s_{23}\, c_{23}\, \cos\delta$$
$$A^{13}_{e\mu} = 4 s^2_{13}\, c^2_{13}\, s^2_{23}\, c^2_{12} + 4\, s_{12}\, c_{12}\, s_{13}\, c^2_{13}\, s_{23}\, c_{23}\, \cos\delta$$
$$A^{23}_{e\mu} = 4\, s^2_{13}\, c^2_{13}\, s^2_{23}\, s^2_{12} - 4\, s_{12}\, c_{12}\, s_{13}\, c^2_{13}\, s_{23}\, c_{23}\, \cos\delta$$

"QUASI–BIMAXIMAL MIXING"

$$A_{e\mu} = \frac{1}{2}(1 - s^2_{13})\, x^2_{12} + 2\, s^2_{13} + s_{13}\, \cos\delta\, [(1 + x_{12})^2 - 1]$$

Three main contributions:

1. "solar contribution" ($\propto x^2_{12} \propto (\Delta m^2_{12})^2$)
2. "$\theta_{13}$ contribution" ($\propto s^2_{13}$)
3. "mass splitting effect" ($\propto s_{13}\, x_{12} \cos\delta$)

## Measurement of $\theta_{13}$

$$E_{\text{res}} \simeq \frac{|\Delta m^2_{23}|}{2V} \cos 2\theta_{13}$$
$$\simeq 14.1 \left(\frac{|\Delta m^2_{23}|}{3 \times 10^{-3}\ \text{eV}^2}\right) \left(\frac{2.8\ \text{g cm}^{-3}}{\rho}\right) \text{GeV}$$

$$P_{\nu_e \leftrightarrow \nu_\mu} = \frac{s^2 \sin^2 \theta_{23}}{s^2 + c^2 (1 \mp E/E_{\text{res}})^2} \times$$
$$\times \sin^2 \left[\frac{\Delta m^2 L}{4 E_\nu} \sqrt{s^2 + c^2 (1 \mp E/E_{\text{res}})}\right]$$

(shorthand notation $s = \sin 2\theta_{13}$, $c = \cos 2\theta_{13}$).

**MAXIMUM of PROBABILITY**

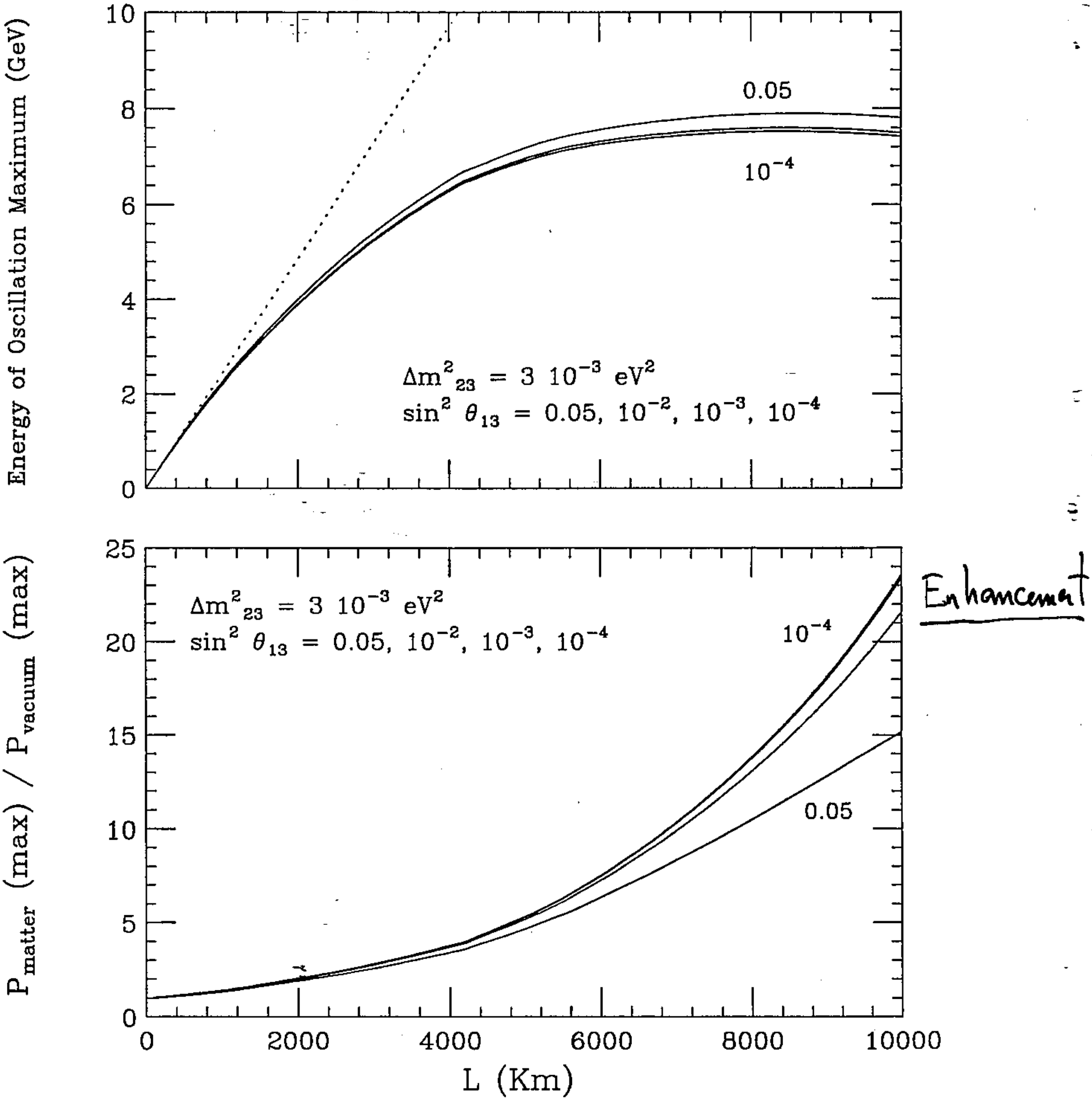
Enhancement Region

$$E_{\text{peak}} \simeq \frac{|\Delta m^2_{23}|\, L}{2\pi + 2VL}$$

$$\frac{P_{\text{peak}}}{P_{\text{vacuum}}} \simeq \left(1 - \frac{VL}{VL + \pi}\right)^{-2} \simeq 1 + \frac{2}{\pi} VL + \frac{2}{\pi} (VL)^2 + \ldots$$

$$\lambda_{\text{res}} = \frac{\lambda_{\text{vacuum}}(E_{\text{res}})}{\sin 2\theta_{13}}$$

E: maximum of Probability



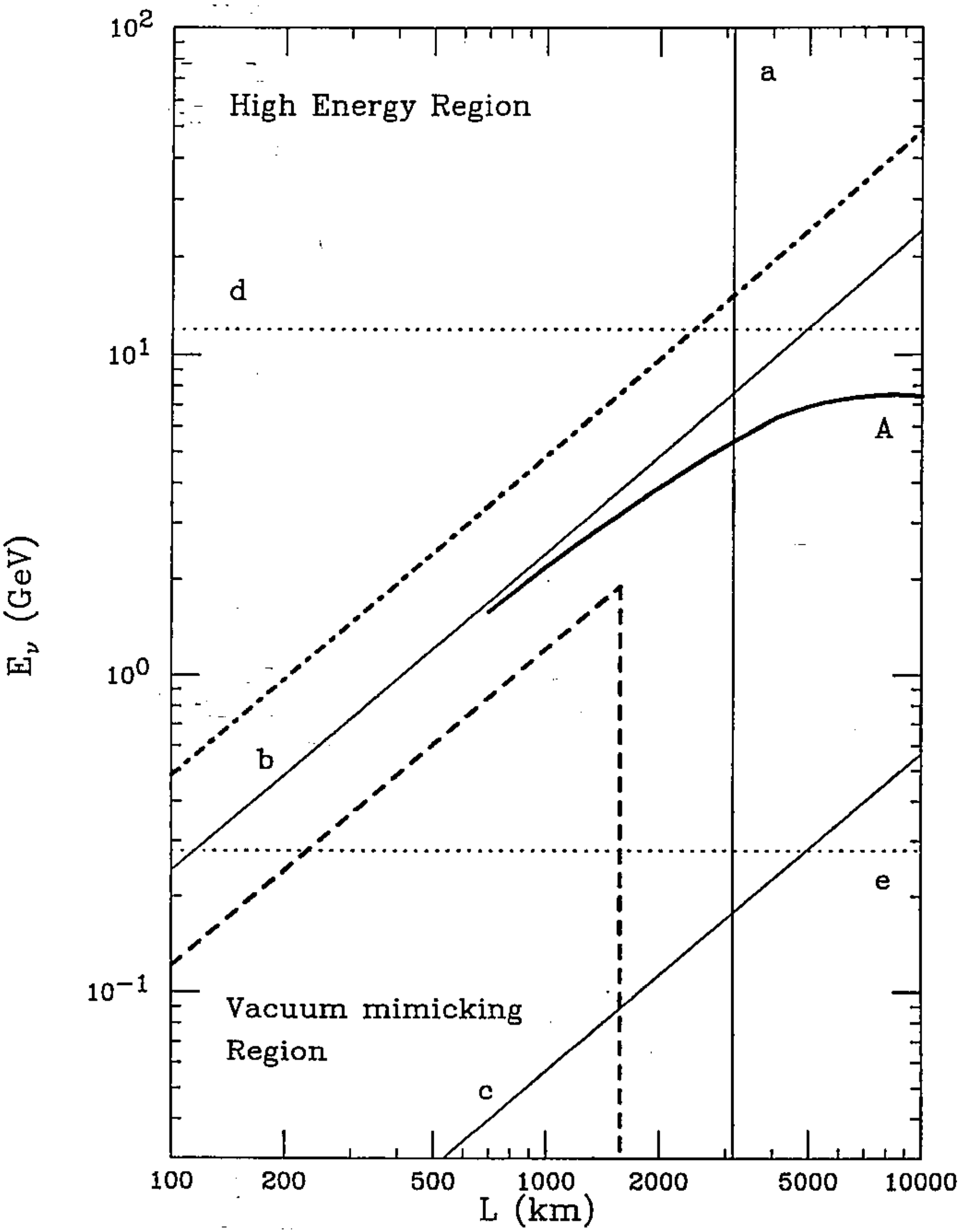



Figure 15: In this figure are indicated some interesting regions in the space ($L$,$E_\nu$) of the oscillation probability for $\nu_e \leftrightarrow \nu_\mu$ and $\nu_e \leftrightarrow \nu_\tau$ transitions. The region above the dot–dashed line is the one where the high energy expansion is valid. In the region delimited by the dashed line the oscillation probabilities in matter and in vacuum are to a good approximation equal. The line labeled $A$ shows the $\nu$ energy where the oscillation probability has the largest matter induced enhancement. The line is drawn only if the maximum enhancement is larger than 20%. The line labeled with $a$ shows the relation $L = 2\,V^{-1}$ for the Earth's crust ($\rho = 2.8$ g cm$^{-3}$). The lines $b$ and $c$ show the relations $E_\nu = |\Delta m^2_{23}|\,L/(2\pi)$ and $E_\nu = \Delta m^2_{12}\,L/(2\pi)$, that is the highest energy where the oscillation probability has a maximum. Line $d$ indicates the approximate energy for which $\theta_{13}$ passes through a resonance, and line $e$ shows the energy above which matter effects modify significantly the mixing parameters.

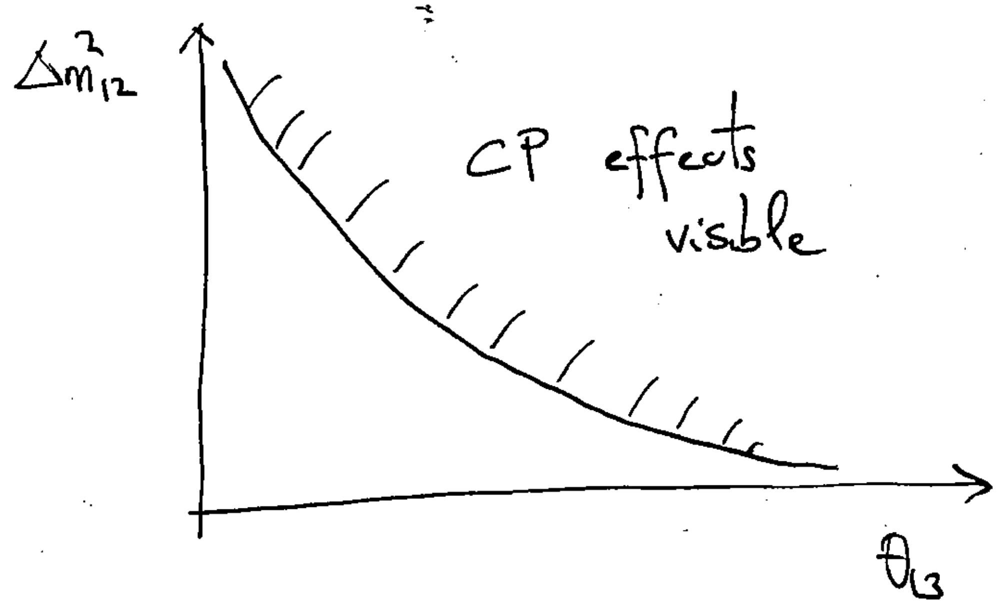
# Perspectives for CP Violation studies in the $\nu$-sector

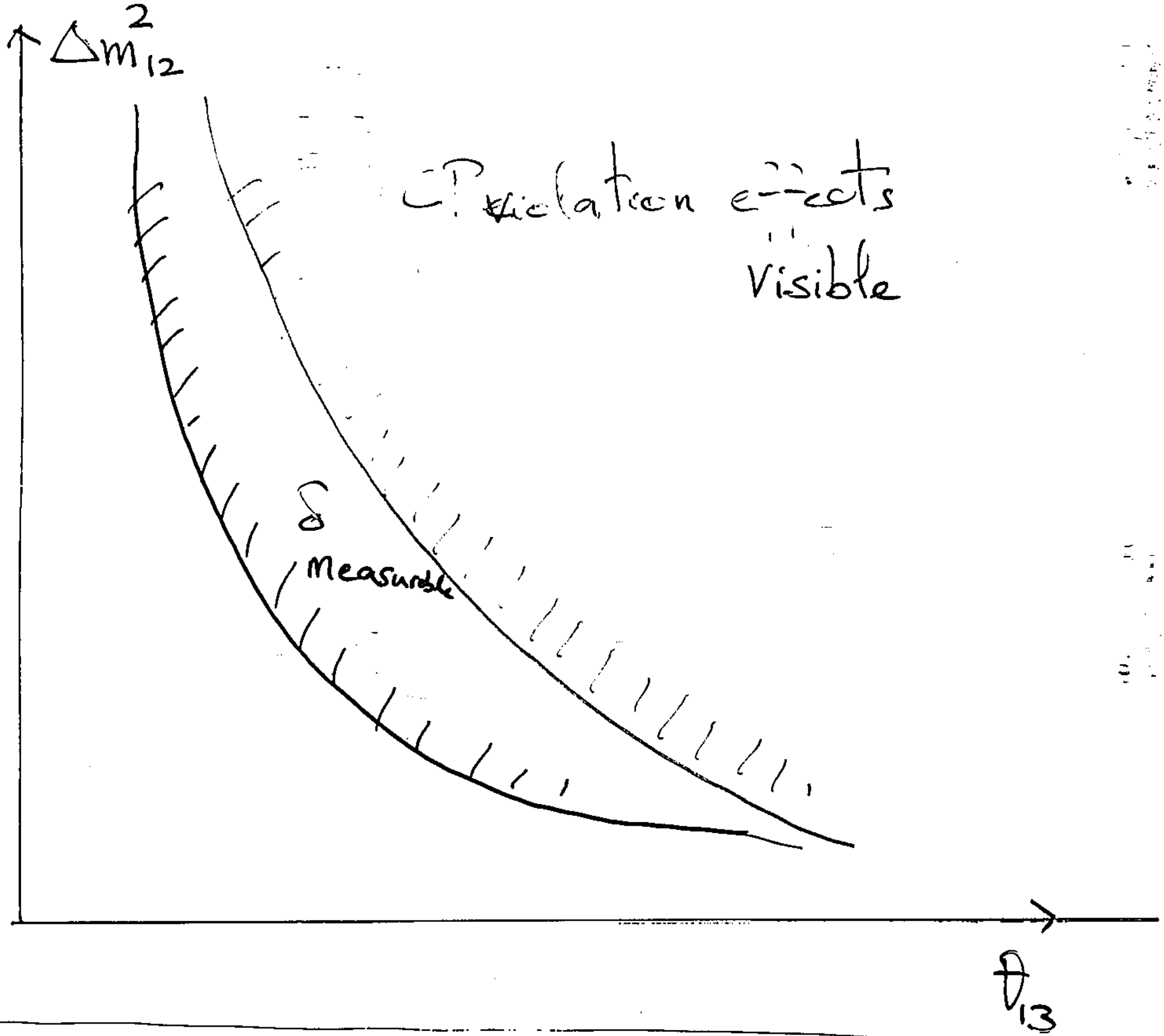
Depend crucially on two parameters.

$\Delta m^2_{12}$ : the "solar" $\Delta m^2$

$\theta_{13}$ : $\sin^2\theta_{13} = \left| \langle \nu_e | \nu_3 \rangle \right|^2$

$$|\nu_3\rangle \simeq \frac{1}{\sqrt{2}}|\nu_\mu\rangle + \frac{1}{\sqrt{2}}|\nu_\tau\rangle + \underbrace{s_{13}|\nu_e\rangle}$$

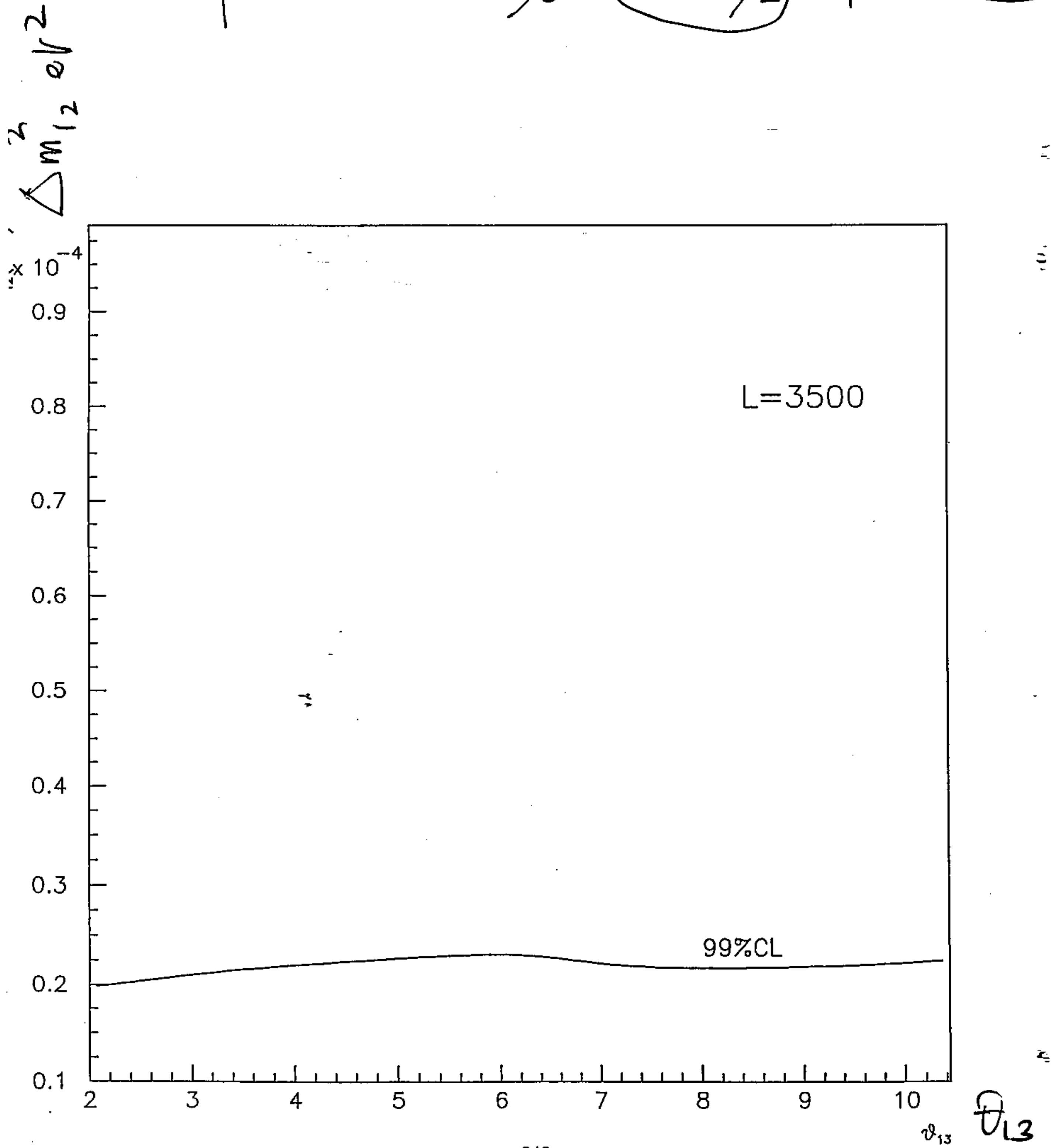





It is easier to measure |δ| than to measure CP violation effects.

Fig 22 of [Cervera et al.]
"Golden Measurements at a $\nu$-factory"
hep-ph/0002183

Lower limit on $\Delta m^2_{12}$
to separate at 99% $\delta = \pi/2$ from $\delta = 0$



## CONCLUSIONS

- A simple formula has been developed to describe the Oscillation probabilities for $E_\nu$ large:

$$P_{\nu_\alpha \to \nu_\beta} = \frac{A\, L^2}{E_\nu^2} \pm \frac{B_{CP}\, L^3}{E_\nu^3} \pm \frac{C_{\rm matter}\, L^4}{E_\nu^3}$$

- Conceptual difference between the measurement of $|\delta|$, and the measurement of $CP$ violation effects. Measuring $|\delta|$ determines the SIZE of $CP$ violation effects but not their SIGN.

- It is important to develop a real "UNDERSTANDING" of the measurable effects that allow to study $\delta$ and the $CP$ violation effects.

- What is the "OPTIMUM STRATEGY" for the study of $CP$ violation in the $\nu$ sector ?

Probably a "COMPROMISE" between the "LOW ENERGY" and "HIGH ENERGY" options.