Low-Energy Predictions

of

Lopsided Family Charges

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1 Introduction

∘ Large ν_μ-ν_τ mixing by Superkamiokande

⇓

Many models with Family symmetry

mini seesaw

∘ Froggatt-Nielsen (FN) mechanism

broken **U(1)** family symmetry

$$L = h_{ij}\psi_i\psi_j H(\text{ or } H^*)\left(\frac{<\phi>}{M_*}\right)^{Q_{\psi_i}+Q_{\psi_j}},$$
$$\equiv h_{ij}\psi_i\psi_j H(\text{ or } H^*)\epsilon^{Q_{\psi_i}+Q_{\psi_j}}$$

ϕ : family symmetry breaking field

h_{ij} : O(1) constant

Q_{ψ_i} : FN charge for ψ_i

M_* : fundamental (gravitational?) scale

$$\epsilon \equiv \frac{<\phi>}{M_*}$$

∘ Lopsided U(1) charges to $L_i (i = 1-3)$

Large mixing

$\Longrightarrow$

same U(1) family charges A for L_2 and L_3

	I	II
Q_{l_3}	A	A
Q_{l_2}	A	A
Q_{l_1}	$A+1$	A

U(1) charges for lepton doublets

Sato and Yanagida, ⋯ (I)
Hisano Kurosawa and Nomura, ⋯ (II)

To reproduce charged lepton masses

	$\mathbf{10}_1$	$\mathbf{10}_2$	$\mathbf{10}_3$	$\bar{\mathbf{5}}_1$	$\bar{\mathbf{5}}_2$	$\bar{\mathbf{5}}_3$	$\mathbf{1}_1$	$\mathbf{1}_2$	$\mathbf{1}_3$	H
Model I	2	1	0	$A+1$	A	A	c	b	a	0
Model II	3(2?)	1	0	A	A	A	c	b	a	0

Family Charges in SU(5) language.

$\mathbf{10}_i = (Q_i,\ \bar{U}_i,\ \bar{E}_i),\ \bar{\mathbf{5}}_i = (\bar{D}_i,\ L_i)$

∘ Consequences of these models?

cf Vissani, Asaka *et. al*

○ **In GUT ($G \supseteq$ SU(5))**

$$M_E \Longleftrightarrow \bar{5}_i 10_j H \epsilon^{Q_{\bar{5}_i}+Q_{10_j}}$$

$$\Updownarrow$$

$$m_e : m_\mu : m_\tau \simeq \epsilon^{Q_{\bar{5}_1}+Q_{10_1}} : \epsilon^{Q_{\bar{5}_2}+Q_{10_2}} : \epsilon^{Q_{\bar{5}_3}+Q_{10_3}}$$

$$\Downarrow$$

$$\epsilon \simeq 0.1$$

$$Q_{10_i} = \{2,1,0\} \textbf{ and/or } \{\mathbf{3,1,0}\}$$

$$M_U \Longleftrightarrow 10_i 10_j H \epsilon^{Q_{10_i}+Q_{10_j}}$$

$$\Updownarrow$$

$$m_u : m_c : m_t \simeq \epsilon^{Q_{10_1}+Q_{10_1}} : \epsilon^{Q_{10_2}+Q_{10_2}} : \epsilon^{Q_{10_3}+Q_{10_3}}$$

$$\Downarrow$$ ⊕ **No suppression on Top yukawa**

$$\epsilon \simeq 0.1$$

$$Q_{10_i} = \{2,1,0\}$$

2 Mass Matrix

(Base with Charged lepton mass matrix diagonalized)

○ Neutrino Dirac mass matrix

Model I

$$
\begin{aligned}
M_D &= \epsilon^A m_0 \begin{pmatrix} h_{11}\epsilon^{c+1} & h_{12}\epsilon^{b+1} & h_{13}\epsilon^{a+1} \\ h_{21}\epsilon^{c} & h_{22}\epsilon^{b} & h_{23}\epsilon^{a} \\ h_{31}\epsilon^{c} & h_{32}\epsilon^{b} & h_{33}\epsilon^{a} \end{pmatrix} \\
&= \epsilon^A m_0 \begin{pmatrix} h_{11}\epsilon & h_{12}\epsilon & h_{13}\epsilon \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix} \begin{pmatrix} \epsilon^c & 0 & 0 \\ 0 & \epsilon^b & 0 \\ 0 & 0 & \epsilon^a \end{pmatrix}
\end{aligned}
$$

Model II

$$
\begin{aligned}
M_D &= \epsilon^A m_0 \begin{pmatrix} h_{11}\epsilon^{c} & h_{12}\epsilon^{b} & h_{13}\epsilon^{a} \\ h_{21}\epsilon^{c} & h_{22}\epsilon^{b} & h_{23}\epsilon^{a} \\ h_{31}\epsilon^{c} & h_{32}\epsilon^{b} & h_{33}\epsilon^{a} \end{pmatrix} \\
&= \epsilon^A m_0 \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix} \begin{pmatrix} \epsilon^c & 0 & 0 \\ 0 & \epsilon^b & 0 \\ 0 & 0 & \epsilon^a \end{pmatrix}
\end{aligned}
$$

m_0 : **representative of weak scale**

$\circ\nu_R$ Majorana mass matrix

$$
\begin{aligned}
M_{\nu_R} &= M_R \begin{pmatrix} m_{11}\epsilon^{2c} & m_{12}\epsilon^{c+b} & m_{13}\epsilon^{c+a} \\ m_{12}\epsilon^{b+c} & m_{22}\epsilon^{2b} & m_{23}\epsilon^{b+a} \\ m_{13}\epsilon^{a+c} & m_{23}\epsilon^{a+b} & m_{33}\epsilon^{2a} \end{pmatrix} \\
&= M_R \begin{pmatrix} \epsilon^c & 0 & 0 \\ 0 & \epsilon^b & 0 \\ 0 & 0 & \epsilon^a \end{pmatrix} \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{12} & m_{22} & m_{23} \\ m_{13} & m_{23} & m_{33} \end{pmatrix} \begin{pmatrix} \epsilon^c & 0 & 0 \\ 0 & \epsilon^b & 0 \\ 0 & 0 & \epsilon^a \end{pmatrix},
\end{aligned}
$$

M_R : **right-handed neutrino scale**

m_{ij} : **O(1) constant**

○ ν_L Majorana mass matrix

Model I

$$M_{\nu_L} = \frac{\epsilon^{2A} m_0^2}{M_R} \begin{pmatrix} h_{11}\epsilon & h_{12}\epsilon & h_{13}\epsilon \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix} \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{12} & m_{22} & m_{23} \\ m_{13} & m_{23} & m_{33} \end{pmatrix}^{-1} \begin{pmatrix} h_{11}\epsilon & h_{12}\epsilon & h_{13}\epsilon \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix}^T \sim \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix}$$

Model II

$$M_{\nu_L} = \frac{\epsilon^{2A} m_0^2}{M_R} \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix} \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{12} & m_{22} & m_{23} \\ m_{13} & m_{23} & m_{33} \end{pmatrix}^{-1} \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix}^T \sim \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

The FN charges of ν_R's are irrelevant to M_{ν_L}'s!

3 Neutrino Property and Mixings

∘ Procedure

1. Generate h_{ij} and m_{ij}:

$$0.8 \leq |h_{ij}|, |m_{ij}| \leq 1.2$$

2. Select sets of h_{ij} and m_{ij} with constraints

A. CHOOZ limit
$|U_{e3}| < 0.15$

B. Atmospheric neutrino anomaly
$4|U_{\mu 3}|^2(1 - |U_{\mu 3}|^2) > 0.5$

C. Solar neutrino deficit

(a) $10^{-4} < r < 10^{-2}$
$10^{-4} < \tan^2\theta < 5 \times 10^{-3}$
$\longleftarrow$ Small angle solution

(b) $r < 0.1$
$10^{-1} < \tan^2\theta < 10$
$\longleftarrow$ Large angle solution

$\tan^2\theta \equiv |U_{e2}/U_{e1}|^2$
$r \equiv \delta m^2_{\rm solar}/\delta m^2_{\rm atm}$

∘ To determine ϵ in model I, calculate charged lepton masses;

$$M_l \propto \begin{pmatrix} l_{11}\epsilon^3 & l_{12}\epsilon^2 & l_{13}\epsilon^1 \\ l_{21}\epsilon^2 & l_{22}\epsilon^1 & l_{23}\epsilon^0 \\ l_{31}\epsilon^2 & l_{32}\epsilon^1 & l_{33}\epsilon^0 \end{pmatrix},$$

$$14 < \frac{m_\tau}{m_\mu} < 20$$

$$180 < \frac{m_\mu}{m_e} < 240$$

$$\Downarrow$$

$$0.05 < \epsilon < 0.1$$

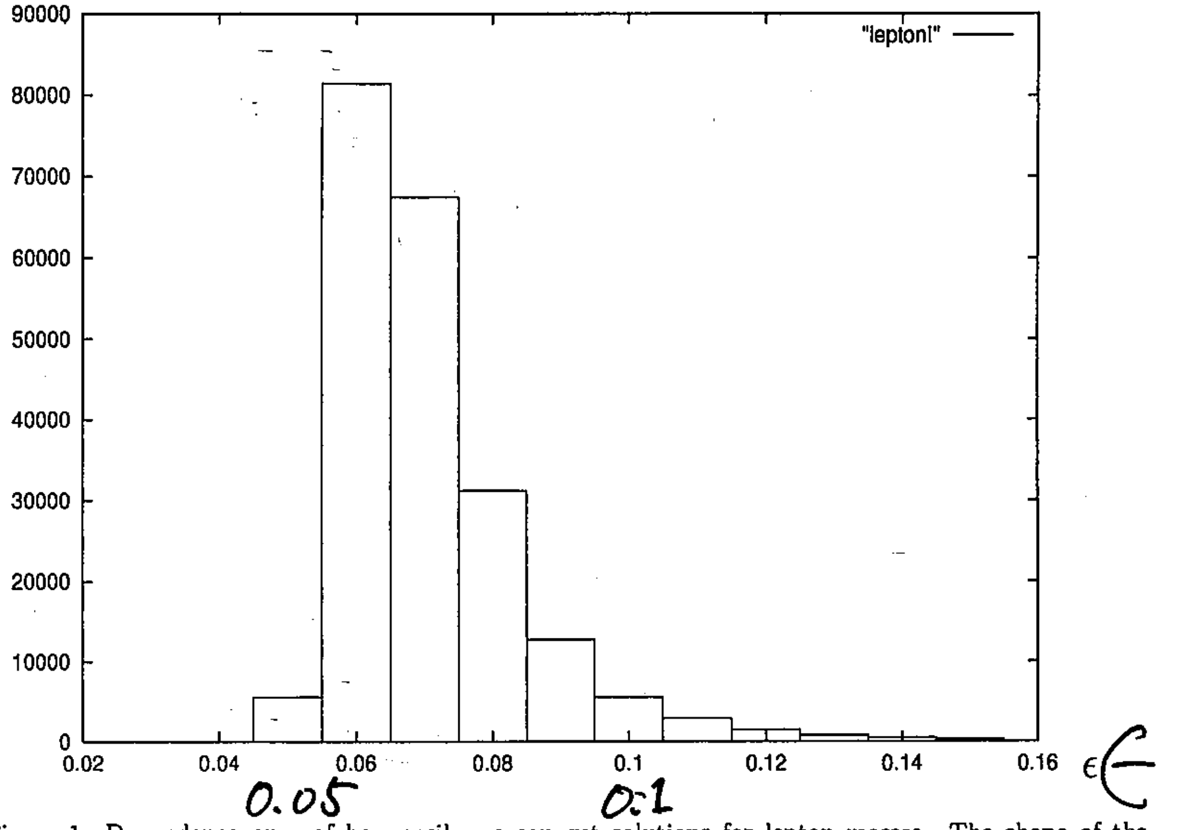

Figure 1: Dependence on ϵ of how easily we can get solutions for lepton masses. The shape of the dependence does not change with the tightness of the selection.

○ Model I **Vissani**

10% sets of coefficients satisfy the constraints independently of ϵ.

○Model II

1% sets can satisfy the criterion.

$\longleftarrow |U_{e3}| < 0.15$

	small mixing	large mixing
$I(\epsilon = 0.05)$	81818	20025
$I(\epsilon = 0.06)$	64353	28837
$I(\epsilon = 0.07)$	51867	38638
$I(\epsilon = 0.08)$	42436	49616
$I(\epsilon = 0.09)$	34714	61220
$I(\epsilon = 0.1)$	29330	72372
II	6	9703

Table 1: Sample of how many sets can satisfy the criterion. Here, "small(large) mixing" implies that the set satisfies the criterion for the small(large) mixing angle solution to the solar neutrino problem.

3.1 Solar neutrino parameters

○Model II

r **and** $\tan^2\theta$

Uniformly distributed in LMA region

○ Model I

◇ **Both small and large mixing angles**

◇ **Disfavored region at right top end of LMA**

$\Longleftarrow$

Majorana mass matrix after diagonalizing the dominant 2 by 2 part

$$\begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & \delta & 0 \\ \epsilon & 0 & 1 \end{pmatrix},$$

δ^2 corresponds to r

$$\tan\theta \simeq \epsilon/\delta \text{ for } \epsilon < \delta.$$

$$r \times \tan^2\theta \simeq \epsilon^2$$

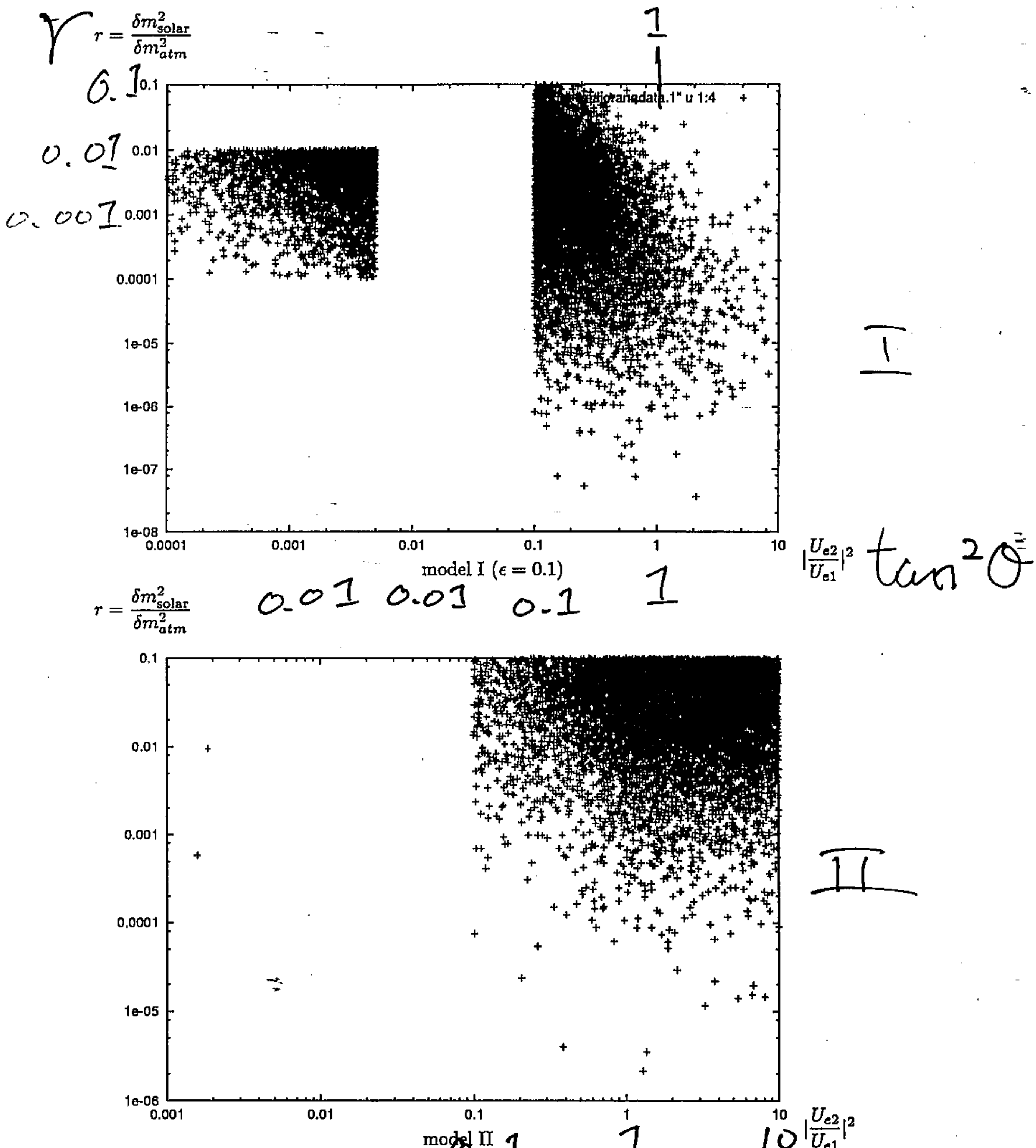

Figure 2: Relations between the mixing angles and mass ratio. The shape of the dependence does not change much with the tightness of the selection.

3.2 U_{e3}

○Model II

Larger U_{e3}

$\Longleftarrow$ No symmetry

○ Model I

Automatically small $\sim \epsilon$

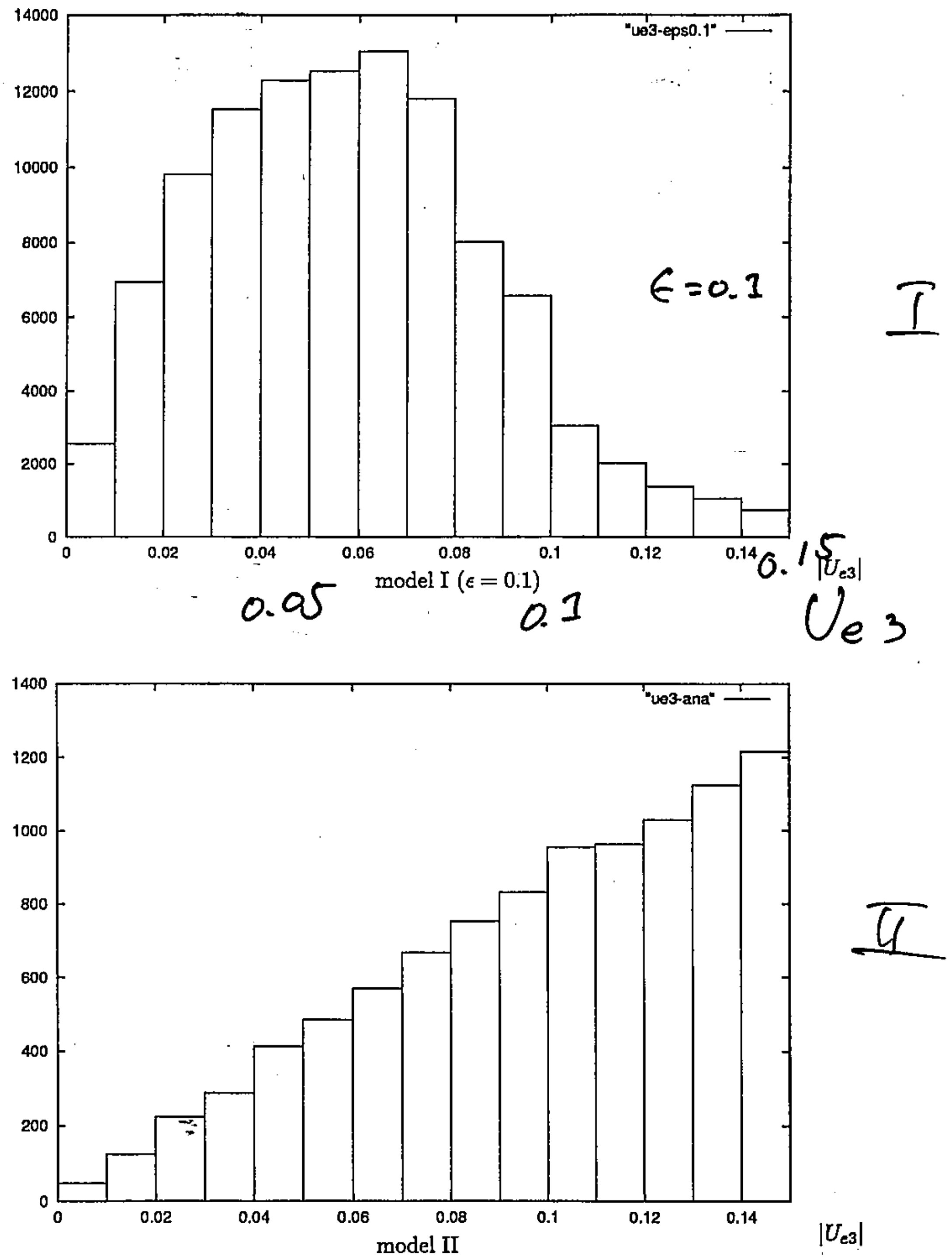

Figure 3: Distributions of $|U_{e3}|$.

3.3 Jarlskog Parameter
~ CP violation ~

○ 3.3.1 Jarlskog parameter

$$
\begin{aligned}
J &\equiv |\mathrm{Im}(U^*_{\alpha i}U^*_{\beta j}U_{\alpha j}U_{\beta i})| \\
&= |\mathrm{Im}(U^*_{e3}U^*_{\mu 2}U_{e2}U_{\mu 3})| \\
&= \frac{1}{4}|\sin\theta_{13}\cos^2\theta_{13}\sin 2\theta_{23}\sin 2\theta_{12}\sin\delta|.
\end{aligned}
$$

$\sin\theta_{13}$: U_{e3}

θ_{23} : θ_{atm}

θ_{12} : θ_{solar}

δ : **CP violating phase**

○ Model I

$J \propto \epsilon$

$$
J \sim \underset{}{0.25} \times \underset{\epsilon}{U_{e3}} \times \underset{O(1)}{\text{other contributions(mixings)}}
$$

$$
\simeq 0.1\epsilon.
$$

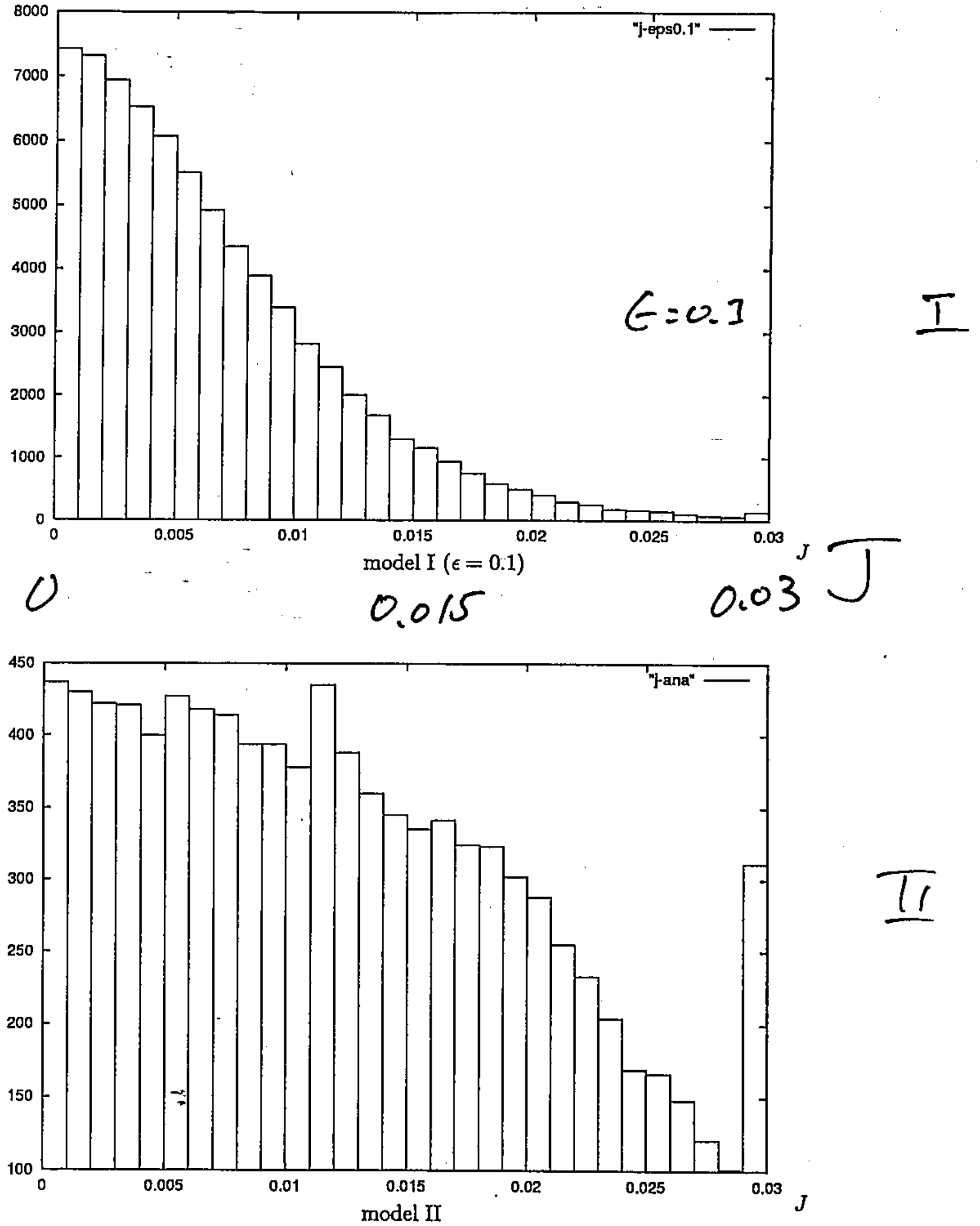

Figure 4: Distributions of J. The number of J in the right bin means that of $J > 0.029$.

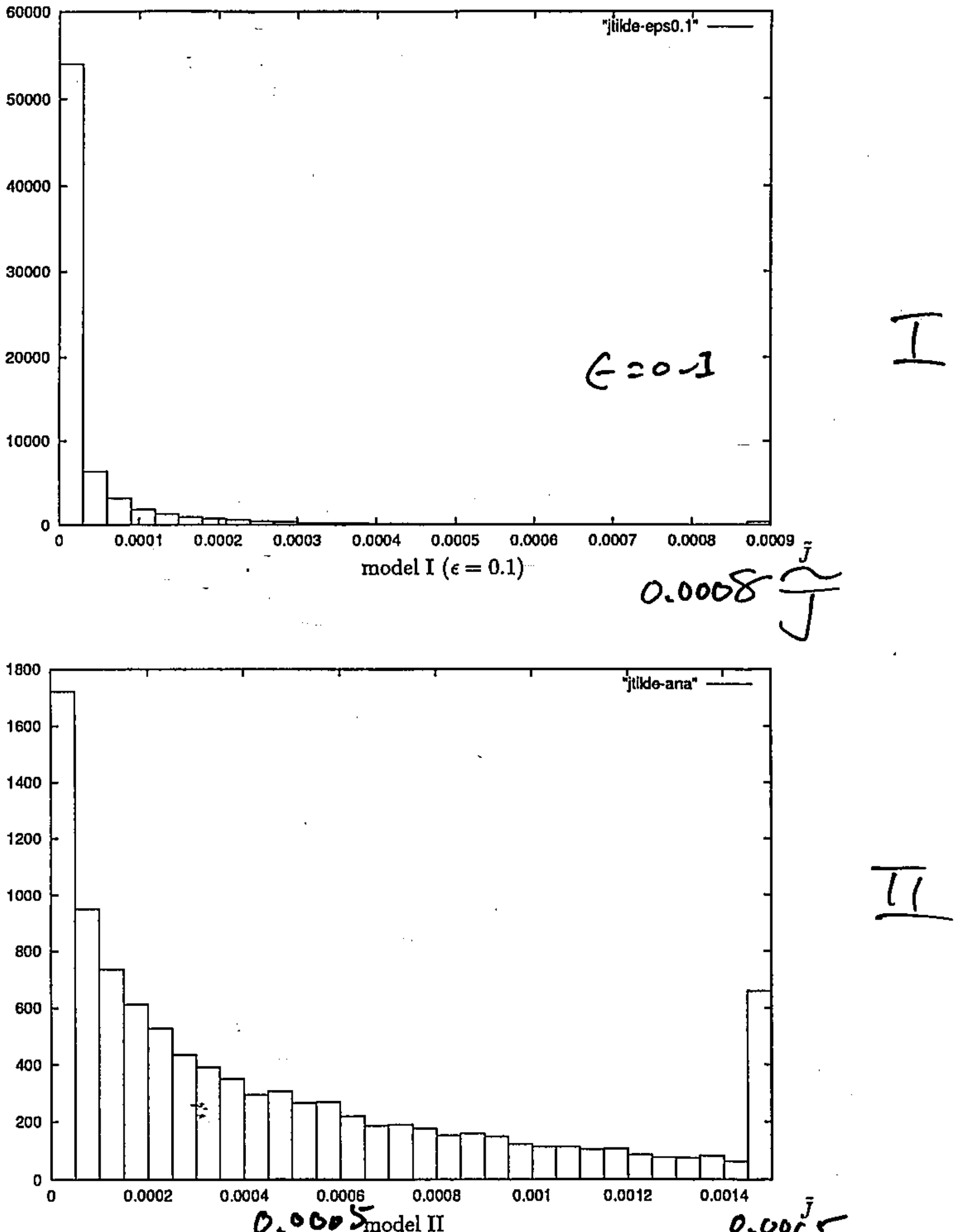

Figure 5: Distributions of $\tilde{J} \equiv |J \times r|$. The number of $\tilde{J}$ in the right bin means that of $\tilde{J} > 0.0009$ for the model I and $\tilde{J} > 0.00145$ for the model II.

∘ Model II

$$J \sim \underset{}{0.25} \times \underset{0.15}{U_{e3}} \times \underset{O(1)}{\text{other contributions}}$$

$$\sim 0.01.$$

Slightly larger than model I.

◦ 3.3.2 Jarlskog parameter $\times \frac{\delta m^2_{\rm solar}}{\delta m^2_{\rm atm}}$

Measurable quantity for CP violation in the realistic neutrino oscillation experiment

$$\tilde{J} \equiv |J \times \frac{\delta m^2_{\rm solar}}{\delta m^2_{\rm atm}}|.$$

◦ Model I

$$\begin{array}{llllll} \tilde{J} \sim & 0.5 \times & U_{e3} & \times \frac{\delta m^2_{\rm solar}}{\delta m^2_{\rm atm}} \tan\theta_{12} & \times & \text{other contribution} \\ & & \sim \epsilon & \epsilon^2/\tan\theta_{12} & & O(1) \end{array}$$

$$\simeq 0.1 \times \epsilon^3.$$

◦ Model II

$$J \times \frac{\delta m^2_{\rm solar}}{\delta m^2_{\rm atm}} \sim 0.01 \times (0.1 - 0.01) \sim (0.001 - 0.0001)$$

CP violation observed $\longrightarrow$ Model II

3.4 $2\nu0\beta$ :: $m_{ee} \equiv |\Sigma_i U_{ei}^2 m_i|$

$\delta m_{\rm atm}^2 = 3 \times 10^{-3}$ eV^2

∘ Model I

$$m_{ee} \sim \sqrt{3 \times 10^{-3}}\epsilon^2$$
$$\simeq 0.05 \times \epsilon^2 \text{ eV}.$$

∘ Model II

Naively

$$\sqrt{3 \times 10^{-3}} \sim 0.05 \text{ eV}.$$

However

result :: lower by one order

$2\beta0\nu$ decay observed → Model II

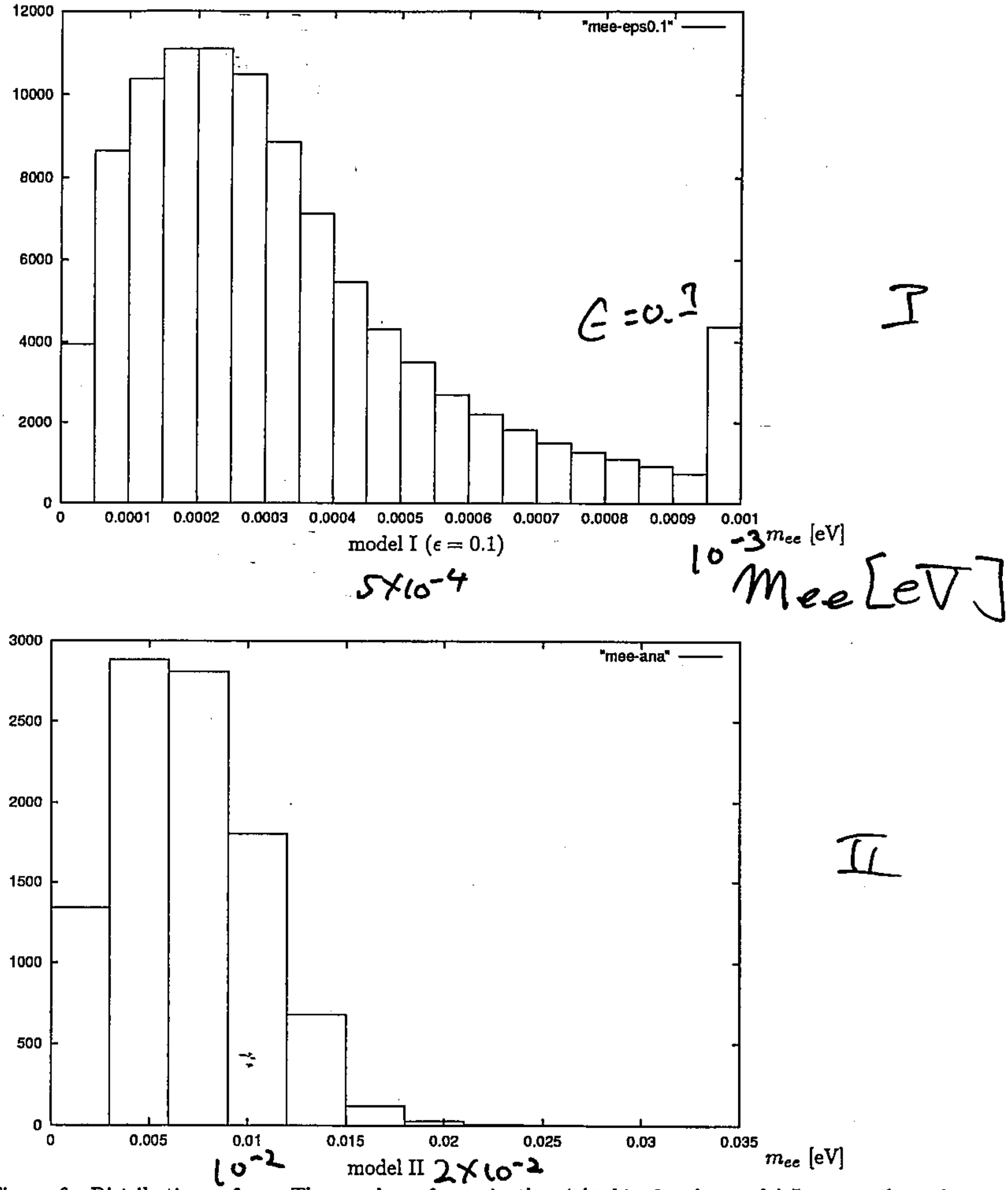

Figure 6: Distributions of m_{ee}.The number of m_{ee} in the right bin for the model I means that of $m_{ee} > 0.00095$ eV.

4 Lepton-Flavor Violation

Borzumati and Maiero
Hisano *et. al.*

(Dirac) Neutrino Yukawa couplings

$$W = f_\nu^{ji} L_j \bar{N}_i H_u$$

$$\Downarrow$$

LFV in the charged-lepton

even if universal scalar mass(m_0^2) at GUT scale(M_G)

$$\mu \frac{d(m_{\tilde{L}}^2)_{ij}}{d\mu} = \left(\mu \frac{d(m_{\tilde{L}}^2)_{ij}}{d\mu}\right)_{\mathrm{MSSM}} + \frac{1}{16\pi^2}\left[m_{\tilde{L}}^2 f_\nu^\dagger f_\nu + f_\nu^\dagger f_\nu m_{\tilde{L}}^2 + 2(f_\nu^\dagger m_{\tilde{\nu}}^2 f_\nu + \tilde{m}_{H_u}^2 f_\nu^\dagger f_\nu + A_\nu^\dagger A_\nu)\right]_{ij}$$

SUSY breaking	$m_{\tilde{L}}^2$	**scalor lepton doublet**
	$m_{\tilde{\nu}}^2$	**right-handed sneutrino**
	$\tilde{m}_{H_u}^2$	**doublet Higgs**

$$U^{Dirac^T} f_\nu^{ij} V^{Dirac^*} = \mathrm{diag}(f_{\nu 1}, f_{\nu 2}, f_{\nu 3})$$

Approximately (a_0:: **universal A term**)

$$(\Delta m^2_{\tilde{L}})_{ij} \simeq -\frac{(6+a_0^2)m_0^2}{16\pi^2}(f_\nu^\dagger f_\nu)_{ij}\log\frac{M_G}{M_R}$$

$$\simeq -\frac{(6+a_0^2)m_0^2}{16\pi^2}U_{ik}^{Dirac}U_{jk}^{Dirac*}|f_{\nu k}|^2\log\frac{M_G}{M_R}$$

Note U_{ij}^{Dirac} **is relevant.**

$$\tau \to \mu\gamma \Leftrightarrow (\Delta m^2_{\tilde{L}})_{32}$$

$$\mu \to e\gamma \Leftrightarrow (\Delta m^2_{\tilde{L}})_{21}$$

∘ **Right-handed neutrino family charge**
$(a, b, c) = (0, 0, 0)$ **and** $(0, 1, 2)$**(cf Asaka et,al)**

∘ **Lepton doublet family charge**

Sato, Tobe, Yanagida

$A = 0 \Leftrightarrow$ **large** $\tan\beta$

$\Downarrow$

Too large LFV

We consider

$A = 1 \Leftrightarrow$ **small** $\tan\beta$

4.1 Model I with $(a, b, c) = (0, 1, 2)$

$$m_D = m_3\epsilon^A \begin{pmatrix} \bar{C}_3\epsilon^3 & \bar{C}_2\epsilon^2 & \bar{C}_1\epsilon^1 \\ \bar{B}_3\epsilon^2 & \bar{B}_2\epsilon & \bar{B}_1 \\ \bar{A}_3\epsilon^2 & \bar{A}_2\epsilon & \bar{A}_1 \end{pmatrix}$$

$$U^{Dirac} \simeq \begin{pmatrix} 1 & -\frac{\epsilon X}{\sqrt{1+|\bar{B}_1/\bar{A}_1|^2}} & \frac{\epsilon\bar{C}_1^*/\bar{A}_1^*}{\sqrt{1+|\bar{B}_1/\bar{A}_1|^2}} \\ \frac{\epsilon\bar{A}_2\bar{C}_1}{\bar{A}_1\bar{B}_2-\bar{B}_1\bar{A}_2} & \frac{1}{\sqrt{1+|\bar{B}_1/\bar{A}_1|^2}} & \frac{\bar{B}_1^*/\bar{A}_1^*}{\sqrt{1+|\bar{B}_1/\bar{A}_1|^2}} \\ -\frac{\epsilon\bar{B}_2\bar{C}_1}{\bar{A}_1\bar{B}_2-\bar{B}_1\bar{A}_2} & -\frac{\bar{B}_1/\bar{A}_1}{\sqrt{1+|\bar{B}_1/\bar{A}_1|^2}} & \frac{1}{\sqrt{1+|\bar{B}_1/\bar{A}_1|^2}} \end{pmatrix}$$

$$X = \bar{C}_1^*(\bar{A}_1\bar{A}_2^* + \bar{B}_1\bar{B}_2^*)/\bar{A}_1(\bar{A}_1^*\bar{B}_2^* - \bar{A}_2^*\bar{B}_1^*)$$

$$f_{\nu 1} : f_{\nu 2} : f_{\nu 3} \simeq \epsilon^2 : \epsilon : 1$$
$$f_{\nu 3} \propto \epsilon^A$$

$\circ(\Delta m^2_{\tilde{L}})_{32}$ $\quad \tau \longrightarrow \mu\gamma$

$$(\Delta m^2_{\tilde{L}})_{32} \simeq -\frac{(6+a_0^2)m_0^2}{16\pi^2} U_{33}^{Dirac} U_{23}^{Dirac*} |f_{\nu 3}|^2 \log\frac{M_G}{M_R}.$$

$\circ(\Delta m^2_{\tilde{L}})_{21}$ $\quad \mu \longrightarrow e\gamma$

$$(\Delta m^2_{\tilde{L}})_{21} \simeq -\frac{(6+a_0^2)m_0^2}{16\pi^2} U_{23}^{Dirac} U_{13}^{Dirac*} |f_{\nu 3}|^2 \log\frac{M_G}{M_R}.$$

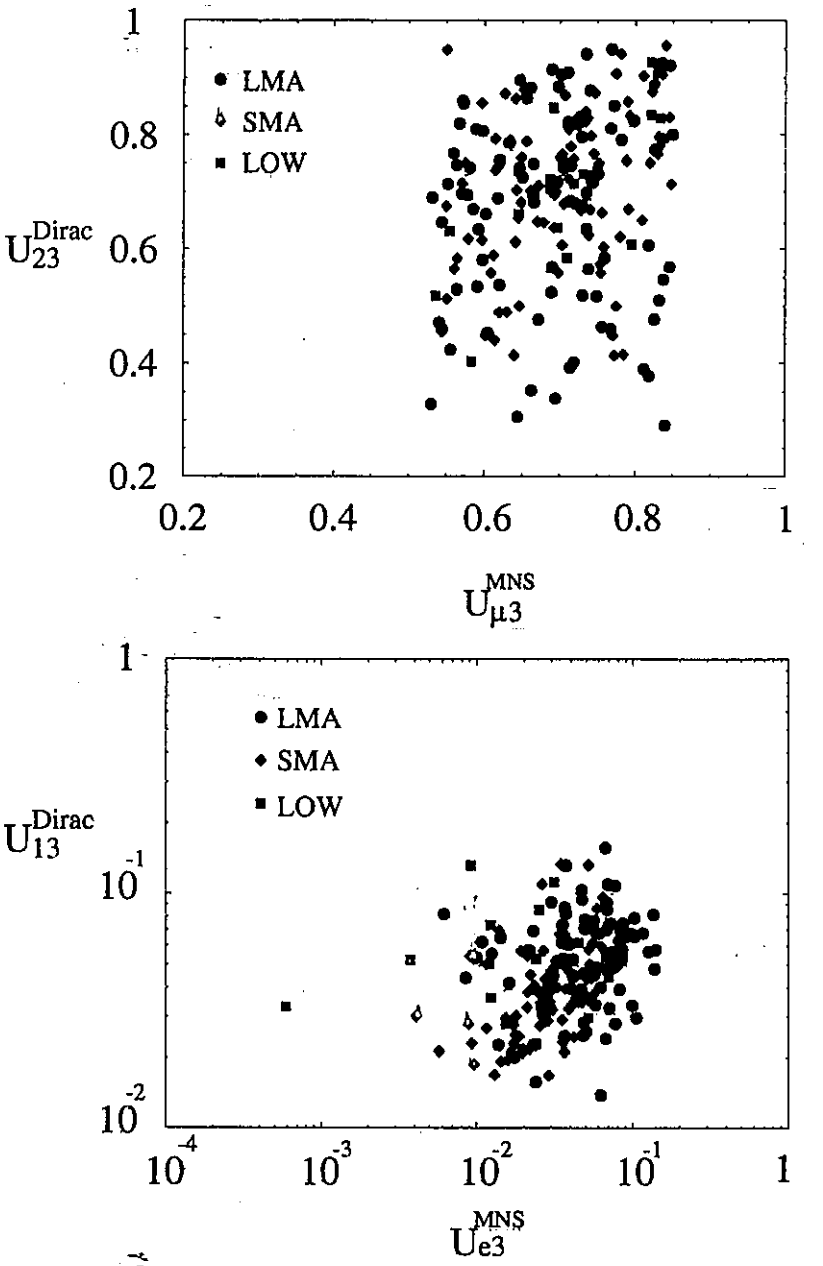

Figure 5: U^{MNS} versus U^{Dirac} in Model I with $(a, b, c) = (0, 1, 2)$.

∘ **Branching ratio for the** $\tau \to \mu\gamma$ ($\mu \to e\gamma$)
$\propto |(\Delta m^2_{\tilde{L}})_{32}|^2$ ($|(\Delta m^2_{\tilde{L}})_{21}|^2$)

$$\frac{\mathrm{Br}(\mu \to e\gamma)}{\mathrm{Br}(\tau \to \mu\gamma)} \sim \left|\frac{(\Delta m^2_{\tilde{L}})_{21}}{(\Delta m^2_{\tilde{L}})_{32}}\right|^2 \simeq \left|\frac{U^{Dirac}_{13}}{U^{Dirac}_{33}}\right|^2 \sim \epsilon^2 \left|\frac{\bar{C}_1}{\bar{A}_1}\right|^2$$

A-independent prediction of Model I with $(a, b, c) = (0, 1, 2)$!!

cf. **Current limits**

$$\mathrm{Br}(\mu \to e\gamma) < 1.2 \times 10^{-11}$$
$$\mathrm{Br}(\tau \to \mu\gamma) < 1.1 \times 10^{-6}$$

$\Longrightarrow$ $\mu \to e\gamma$ **is much more sensitive**

∘ **Solar neutrino parameter independent**

$A = 0$ :: **almost excluded**

$A = 1$ :: **will soon be measured**

In near future

$$\mathrm{Br}(\mu \to e\gamma) \sim 10^{-14} \text{ PSI}$$
$$\mathrm{R}(\mu \to e \text{ in Al}) \sim 10^{-16} \text{ MECO}$$
$$\mathrm{R}(\mu \to e \text{ in Ti}) \sim 10^{-18} \text{ PRISM}$$

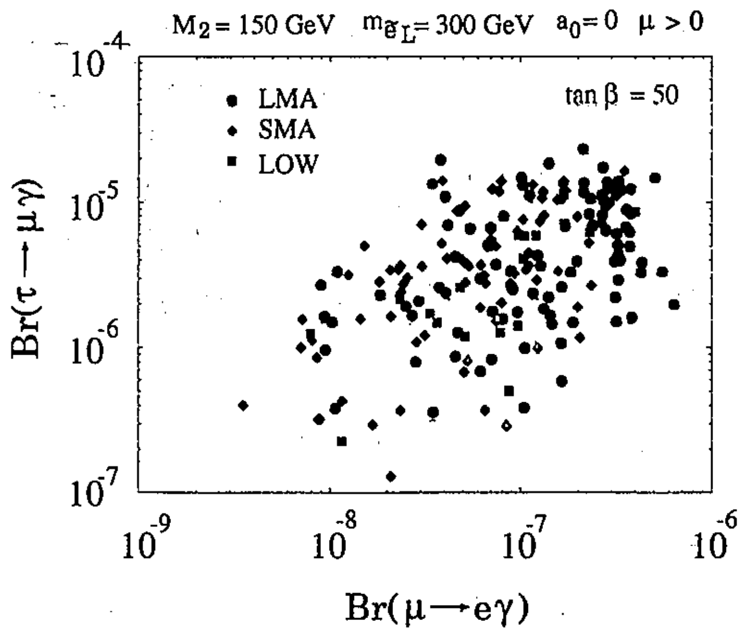

Figure 6: Br($\mu \to e\gamma$) versus Br($\tau \to \mu\gamma$) in Model I with $(a, b, c, \tau) = (0, 1, 2, 0)$. Here we take the left-handed slepton mass to be 300 GeV, the Wino mass to be 150 GeV, and $\epsilon = 0.07$.

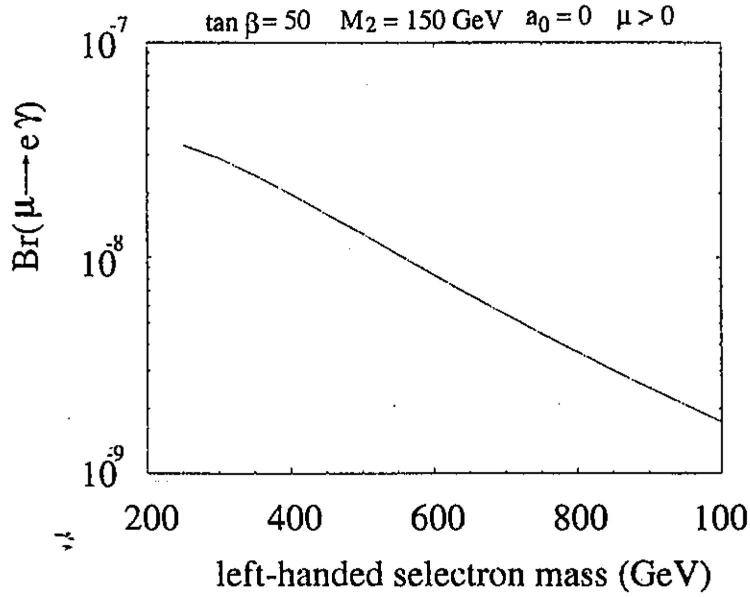

Figure 7: Br($\mu \to e\gamma$) as a function of the left-handed selectron mass in Model I with $(a, b, c) = (0, 1, 2, 0)$.

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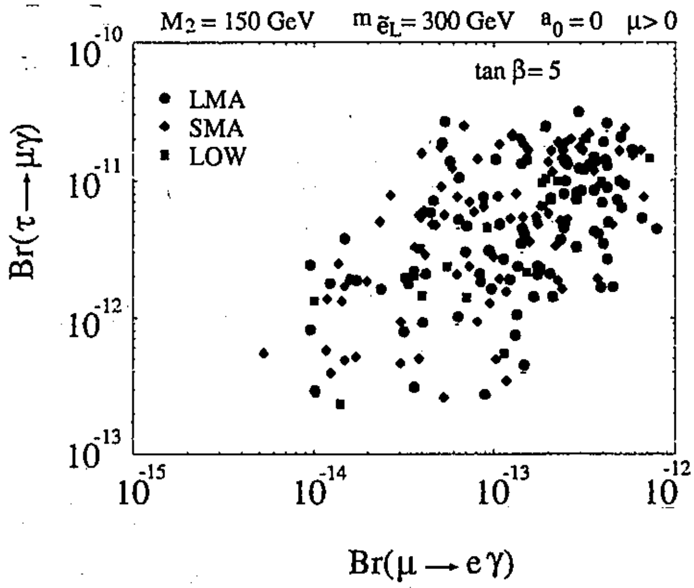

Figure 8: $\mathrm{Br}(\mu \to e\gamma)$ versus $\mathrm{Br}(\tau \to \mu\gamma)$ in Model I with $(a, b, c, \tau) = (0, 1, 2, 1)$. Here we take the left-handed slepton mass to be 300 GeV, the Wino mass to be 150 GeV, and $\epsilon = 0.07$.

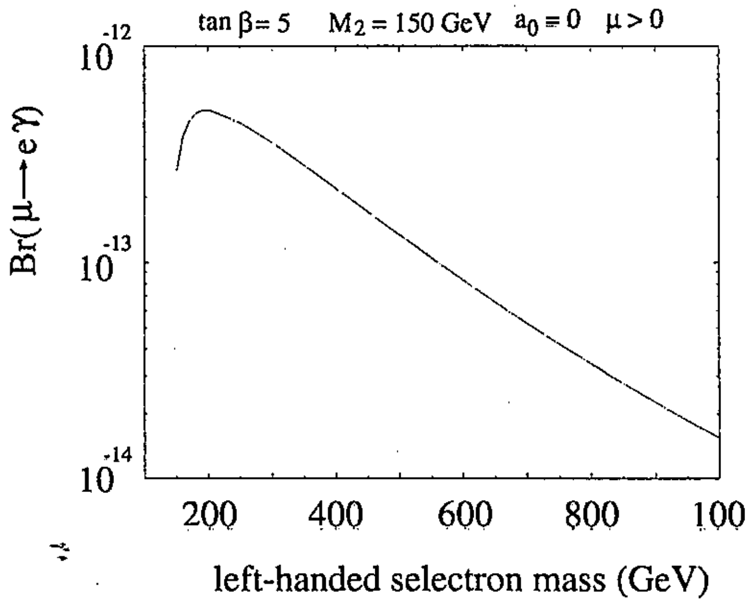

Figure 9: $\mathrm{Br}(\mu \to e\gamma)$ as a function of the left-handed selectron mass in Model I with $(a, b, c, \tau) = (0, 1, 2, 1)$.

$$\frac{\mathrm{Br}(\mu \to eee)}{\mathrm{Br}(\mu \to e\gamma)} \simeq 6 \times 10^{-3},$$

$$\frac{\mathrm{R}(\mu \to e \text{ in Ti (Al)})}{\mathrm{Br}(\mu \to e\gamma)} \simeq 5\ (3) \times 10^{-3}.$$

$\diamond$ LFV serach :: Significant probe to neutrino models.

4.2 Model I with $(a,b,c) = (0,0,0)$

$$m_{\nu D} = m_3 \epsilon^A \begin{pmatrix} \bar{C}_3\epsilon & \bar{C}_2 & \bar{C}_1 \\ \bar{B}_3\epsilon & \bar{B}_2 & \bar{B}_1 \\ \bar{A}_3\epsilon & \bar{A}_2 & \bar{A}_1 \end{pmatrix}.$$

$$U^{Dirac} \simeq \begin{pmatrix} 1 & \epsilon\frac{X_-}{\sqrt{N_-}} & \epsilon\frac{X_+}{\sqrt{N_+}} \\ \epsilon Y_0 & \frac{Y_-}{\sqrt{N_-}} & \frac{Y_+}{\sqrt{N_+}} \\ \epsilon Z_0 & \frac{Z}{\sqrt{N_-}} & \frac{Z}{\sqrt{N_+}} \end{pmatrix},$$

$$f_{\nu 1} : f_{\nu 2} : f_{\nu 3} \simeq \epsilon : \epsilon - 1 : 1$$
$$f_{\nu 3} \propto \epsilon^A$$

○ $(\Delta m^2_{\tilde{L}})_{32}$ $\qquad \tau \longrightarrow \mu\gamma$

$$(\Delta m^2_{\tilde{L}})_{32} \simeq -\frac{(6+a_0^2)m_0^2}{16\pi^2}\left[U_{33}^{Dirac}U_{23}^{Dirac*}|f_{\nu 3}|^2 + U_{32}^{Dirac}U_{22}^{Dirac*}|f_{\nu 2}|^2\right]\log\frac{M_G}{M_R},$$
$$\sim -\frac{(6+a_0^2)m_0^2}{16\pi^2}U_{33}^{Dirac}U_{23}^{Dirac*}\left(|f_{\nu 3}|^2 - |f_{\nu 2}|^2\right)\log\frac{M_G}{M_R}.$$

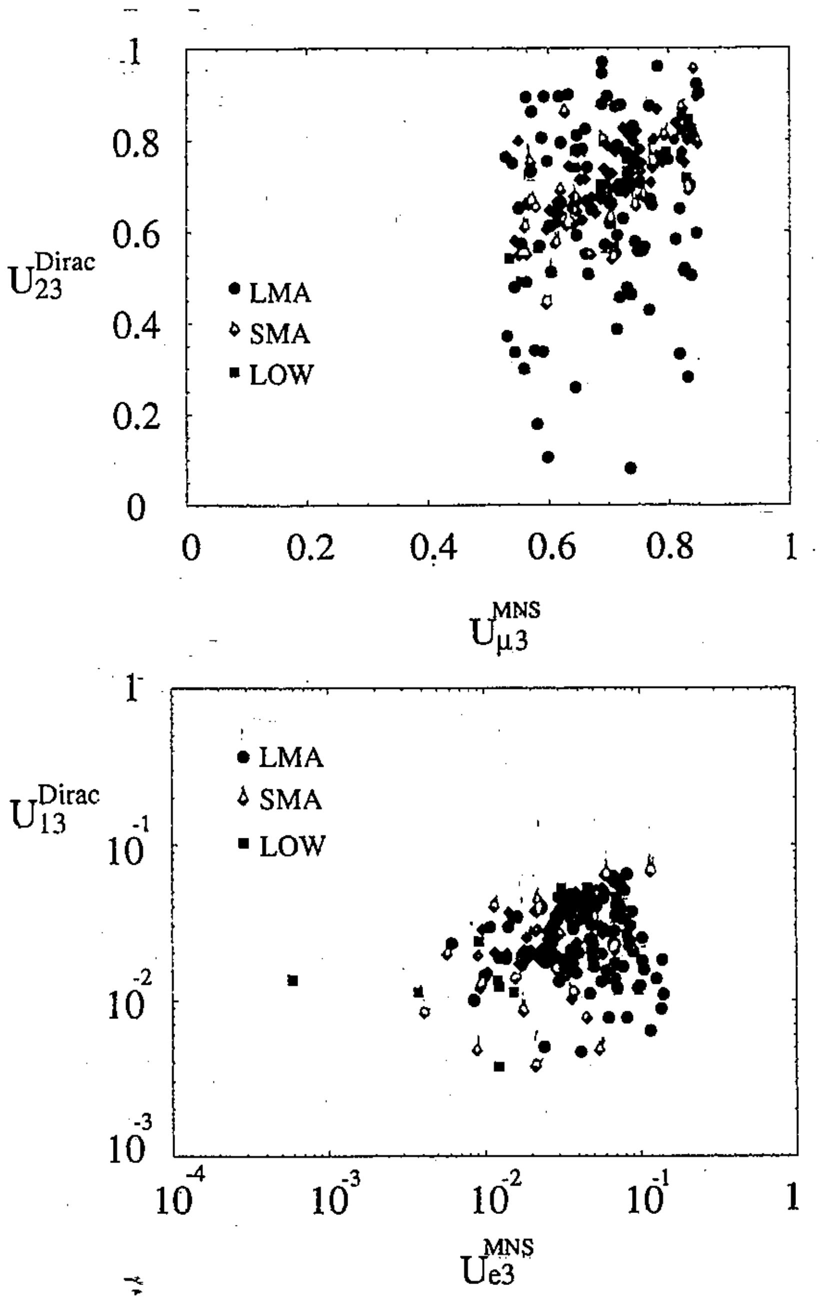

Figure 10: U^{MNS} versus U^{Dirac} in Model I with $(a, b, c) = (0, 0, 0)$.

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∘$(\Delta m^2_{\tilde{L}})_{21}$ $\mu \longrightarrow e\gamma$

$$
\begin{aligned}
(\Delta m^2_{\tilde{L}})_{21} &\simeq -\frac{(6+a_0^2)m_0^2}{16\pi^2}\left[U_{23}^{Dirac}U_{13}^{Dirac*}|f_{\nu 3}|^2\right.\\
&\qquad \left.+U_{22}^{Dirac}U_{12}^{Dirac*}|f_{\nu 2}|^2\right]\log\frac{M_G}{M_R},\\
&= -\frac{(6+a_0^2)m_0^2}{16\pi^2}\left[U_{23}^{Dirac}U_{13}^{Dirac*}\left(|f_{\nu 3}|^2-|f_{\nu 2}|^2\right)\right.\\
&\qquad \left.-U_{21}^{Dirac}U_{11}^{Dirac*}|f_{\nu 2}|^2\right]\log\frac{M_G}{M_R}.
\end{aligned}
$$

∘ **SMA and LOW solutions**

$f_{\nu 3}$ **dominant**

$\longrightarrow$ **similar result with previous one**

$$
\frac{\mathrm{Br}(\mu \to e\gamma)}{\mathrm{Br}(\tau \to \mu\gamma)} \propto \epsilon^2
$$

∘ **LMA solutions**

Both $f_{\nu 2}$ and $f_{\nu 3}$ can be large

U_{12}^{Dirac} **large**

$\longrightarrow$ **Br is widely distributed**

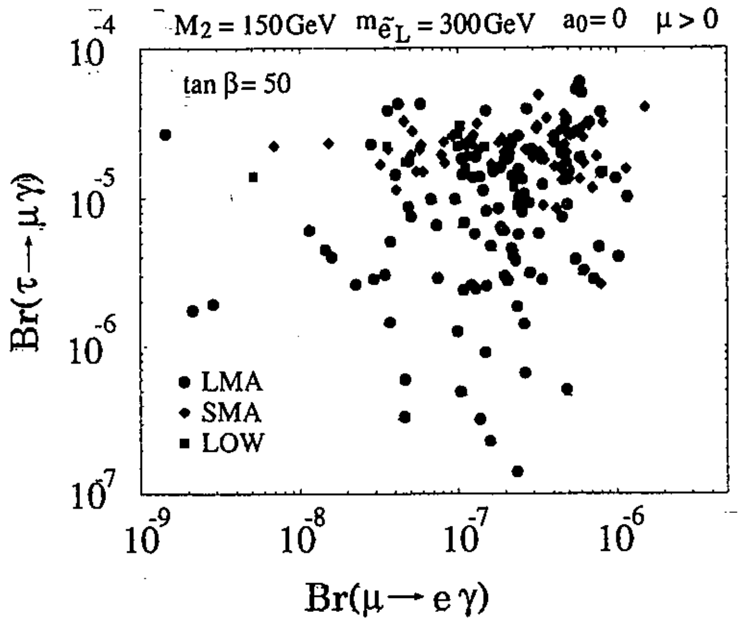

Figure 11: Br($\mu \to e\gamma$) versus Br($\tau \to \mu\gamma$) in Model I with $(a, b, c, \tau) = (0, 0, 0, 0)$. Here we take the left-handed slepton mass to be 300 GeV, the Wino mass to be 150 GeV, and $\epsilon = 0.07$.

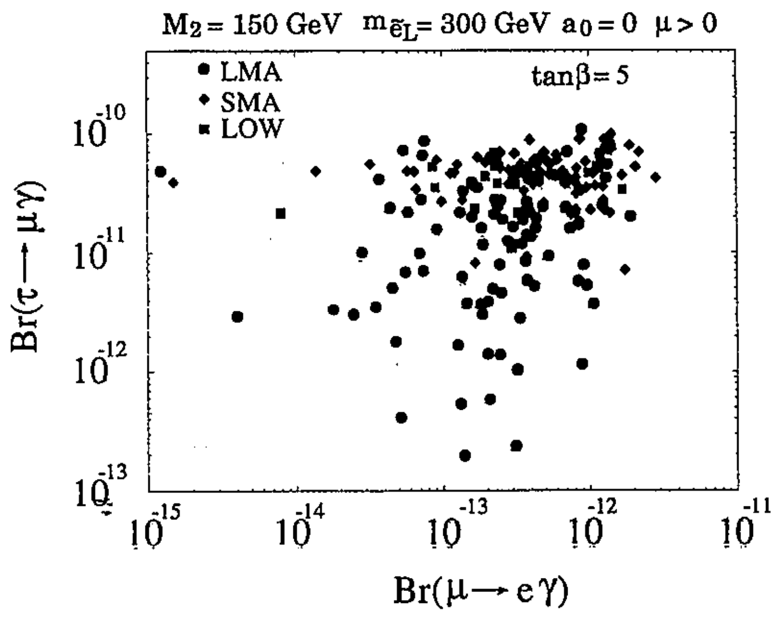

Figure 12: Same as Fig.11 except for Model I with $(a, b, c, \tau) = (0, 0, 0, 1)$.

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4.3 Model II with $(a,b,c)=(0,1,2)$

$$m_{\nu D} = m_3\epsilon^A \begin{pmatrix} \bar{C}_3\epsilon^2 & \bar{C}_2\epsilon^2 & \bar{C}_1\epsilon^2 \\ \bar{B}_3\epsilon & \bar{B}_2\epsilon & \bar{B}_1\epsilon \\ \bar{A}_3 & \bar{A}_2 & \bar{A}_1 \end{pmatrix}.$$

$$U_{ij}^{Dirac} \sim O(1)$$

$$f_{\nu 1} : f_{\nu 2} : f_{\nu 3} \simeq \epsilon^2 : \epsilon : 1$$
$$f_{\nu 3} \propto \epsilon^A$$

○ $(\Delta m^2_{\tilde{L}})_{32}$ $\quad\quad$ $\tau \longrightarrow \mu\gamma$

$$(\Delta m^2_{\tilde{L}})_{32} \simeq -\frac{(6+a_0^2)m_0^2}{16\pi^2} U_{33}^{Dirac} U_{23}^{Dirac*} |f_{\nu 3}|^2 \log\frac{M_G}{M_R},$$

○ $(\Delta m^2_{\tilde{L}})_{21}$ $\quad\quad$ $\mu \longrightarrow e\gamma$

$$(\Delta m^2_{\tilde{L}})_{21} \simeq -\frac{(6+a_0^2)m_0^2}{16\pi^2} U_{23}^{Dirac} U_{13}^{Dirac*} |f_{\nu 3}|^2 \log\frac{M_G}{M_R}.$$

$$\frac{\mathrm{Br}(\mu \to e\gamma)}{\mathrm{Br}(\tau \to \mu\gamma)} \propto \left|\frac{(\Delta m^2_{\tilde{L}})_{21}}{(\Delta m^2_{\tilde{L}})_{32}}\right|^2 \propto \left|\frac{U_{13}^{Dirac}}{U_{33}^{Dirac}}\right|^2 \sim O(1)$$

- **No suppression on U_{13}^{Dirac}**
  $\longrightarrow$ larger Br$(\mu \to e\gamma)$

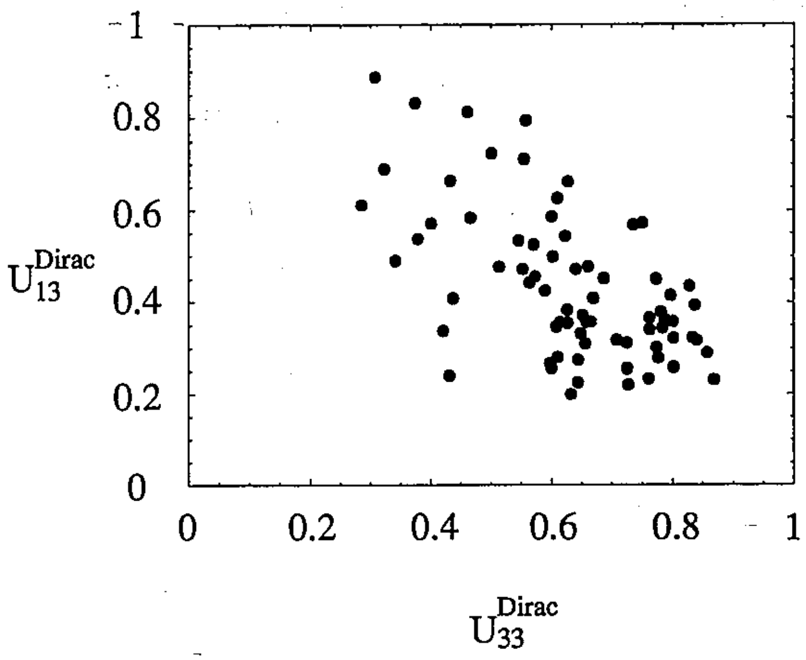

Figure 13: U_{13}^{Dirac} versus U_{33}^{Dirac} in Model II with $(a, b, c) = (0, 1, 2)$.

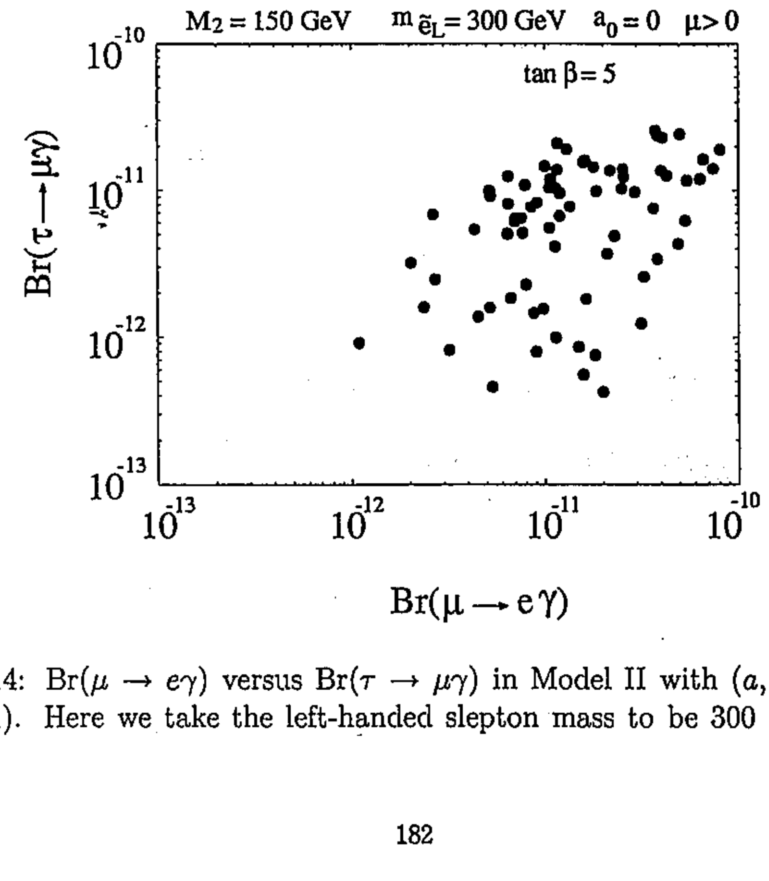

Figure 14: Br($\mu \to e\gamma$) versus Br($\tau \to \mu\gamma$) in Model II with $(a, b, c, \tau) = (0, 1, 2, 1)$. Here we take the left-handed slepton mass to be 300 GeV, the

4.4 Model II with $(a,b,c) = (0,0,0)$

$$m_{\nu D} = m_3 \epsilon^A \begin{pmatrix} \bar{C}_3 & \bar{C}_2 & \bar{C}_1 \\ \bar{B}_3 & \bar{B}_2 & \bar{B}_1 \\ \bar{A}_3 & \bar{A}_2 & \bar{A}_1 \end{pmatrix}.$$

$$U_{ij}^{Dirac} \sim O(1)$$

- **Again very large Br**

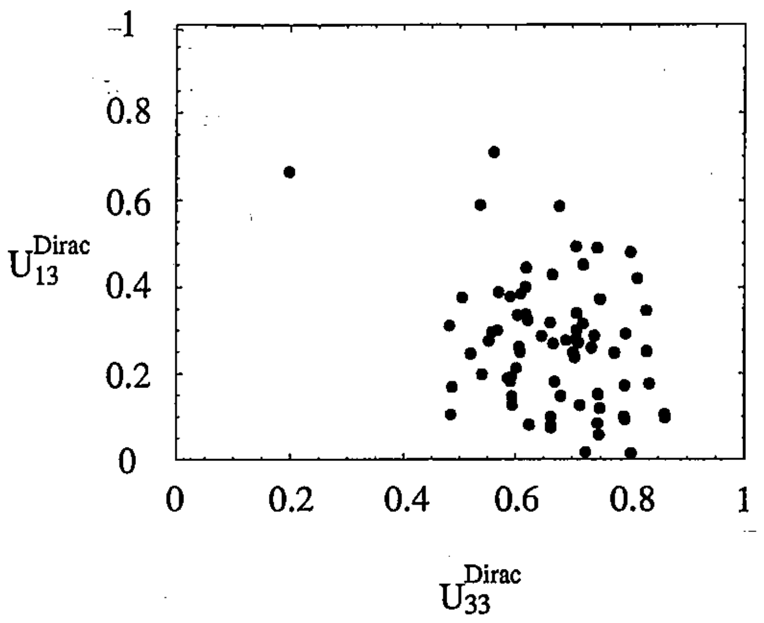

Figure 15: U_{13}^{Dirac} versus U_{33}^{Dirac} in Model II with $(a,b,c) = (0,0,0)$.

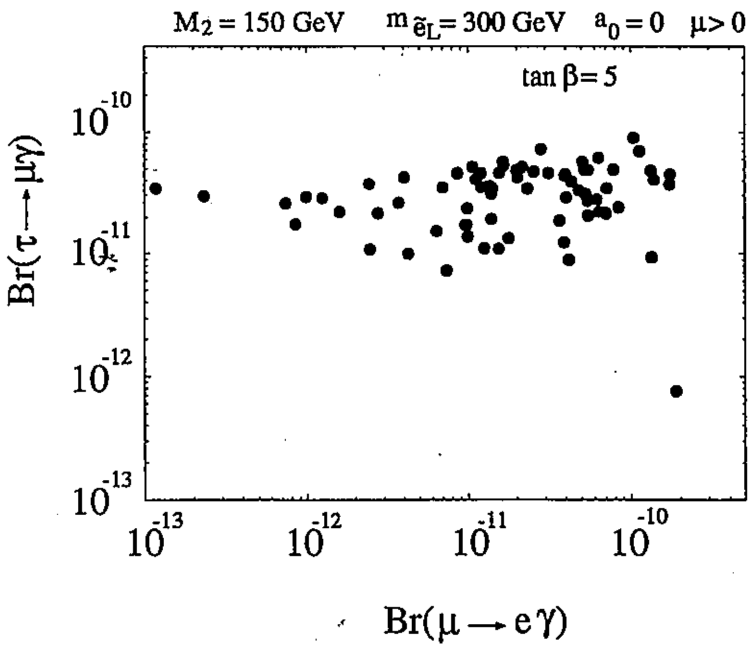

Figure 16: Br($\mu \to e\gamma$) versus Br($\tau \to \mu\gamma$) in Model II with $(a,b,c,\tau) = (0,0,0,1)$. Here we take the left-handed slepton mass to be 300 GeV, the Wino mass to be 150 GeV, and $\epsilon = 0.07$.

5 Conclusion and Summary

∘ Two kinds of FN U(1) charges

Model I (001) and Model II(000)

Very different and distinguishable

∘ Solar parameters

Small mixing → Model I

Large mixing → Both still distinguishable!?

∘ U_{e3}

In both model it cannot be too small

$$U_{e3} \sim 0.01 - 0.1$$

Observable in near future?

∘ CP violation

Too small in Model I

Observable in Model II

∘ **$2\nu0\beta$**

$$m_{ee} \sim 0.05 \times \epsilon^2 \text{ eV Model I}$$

→ **too small**

$$m_{ee} \sim 0.01 \times \text{ eV Model II}$$

→ **observable in near future**

∘ **Lepton-Flavor violation**

Close to current limit on Br($\mu \to e\gamma$)

$$\Uparrow$$

Large U_{e3} ($\to U_{13}^{Dirac}$)