E_6 統一模型

Feb. 23-24, 2001

at 宇宙線研 @ 柏

based on

M. Bando & T. K.

Prog. Theor. Phys. 101 ('99) 1313

M. Bando, T. K. & K. Yoshioka

Prog. Theor. Phys. 104 ('00) 211

Phys. Letts. B483 ('00) 163

0.

$\boxed{\text{Why GUT ?}}$

- Anomaly cancellation

  between quarks and leptons

  $\approx$ charge quantization

- Coupling constants unification

  at $\mu \sim 10^{15-16}$ GeV

$\boxed{\text{Why } E_6 \text{ ?}}$

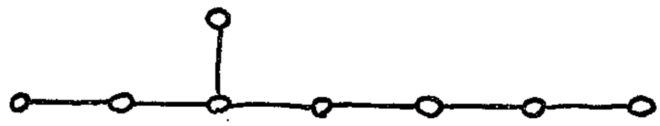
- suggested by String Theory

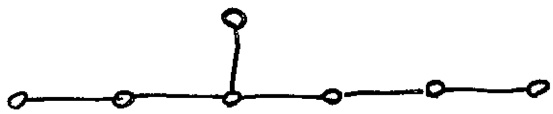
- Maximal group which is safe

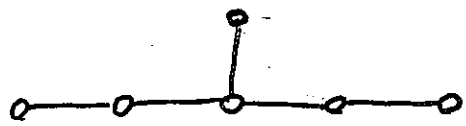
  allows complex reprs.

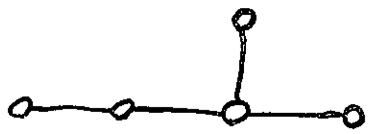
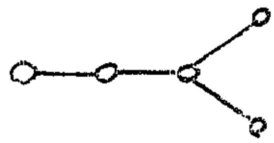
E_6 is natural

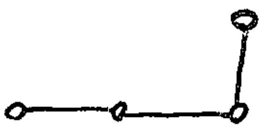
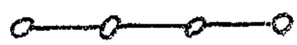
例外群 E series

E_8 

E_7 

E_6 

E_5  =  $SO(10)$

E_4  =  $SU(5)$

E_3  = 

$SU(3) \times SU(2)$

1. Introduction

○ hierarchy in quark/lepton masses

⟶ a possible mechanism

Frogatt-Nielsen mechanism

$$\boxed{y_{ij}\ \Psi_i\ \Psi_j\ H\ \left(\frac{\Theta}{M_P}\right)^{f_i+f_j+h}}$$

$$(\text{anomalous})\ U(1)_X\ :\ \begin{cases} X(\Theta) = -1 \\ X(H) = h \\ X(\Psi_i) = f_i \end{cases}$$

$$y_{ij} \sim y = \mathcal{O}(1)$$

$$\lambda \equiv \frac{\langle\Theta\rangle}{M_P} \sim 0.22$$

⇒ effective Yukawa coupling

$$\boxed{y \times \lambda^{f_i+f_j-h}}$$

hierarchical masses

However,

○ Relations
{ up-type quarks u_i
down-type quarks d_i
charged leptons l_i
neutrinos ν_i }

○ $\exists\ \nu$ masses ⟹ $\boxed{\text{GUT} \geq SO(10)}$

Differences in

1. mass hierarchies $u \leftrightarrow d$

$$\boxed{m_t : m_c : m_u \simeq 1 : \lambda^4 : \lambda^7}$$

$$\updownarrow$$

$$\boxed{m_b : m_s : m_d \simeq 1 : \lambda^2 : \lambda^4}$$

2. $m_t \gg m_b$

$$\boxed{\frac{m_b}{m_t} \sim \lambda^{2-3}}$$

3. Mixing angles

<u>small</u> quark mixing $\boxed{U_{CKM} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ & 1 & \lambda^2 \\ & & 1 \end{pmatrix}}$

<u>large</u> lepton/ν mixing $\boxed{U_{MNS} \overset{?}{\sim} \begin{pmatrix} & & \\ & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}}$

$\nu_\mu \leftrightarrow \nu_\tau$

> Can these differences be explained in such a large GUT framework?

$\longrightarrow$ Yes, <u>possible</u> in E_6!

2. Model & E-symmetry

Characters

	Name (E_6 repn.)	$U(1)_X$	R-parity
Quarks/Leptons	$\Psi_i(27)$ $(i=1,2,3)$	f_i	odd
Higgs	$H(27)$, $\overline{H}(27^*)$	$\pm h = 0$	even
	$\Phi(27)$, $\overline{\Phi}(27^*)$	$\pm x$	
Singlet	$\Theta(1)$	-1	even
Adjoint Higgs	$\phi(78)$	-2	even

Yukawa coupling

$$W_Y(H) = y_{ij}\ \Psi_i(27)\,\Psi_j(27)\,H(27)\left(\frac{\Theta}{M_P}\right)^{f_i+f_j+h}$$

$$W_Y(\Phi) = y'_{ij}\ \Psi_i(27)\,\Psi_j(27)\,\Phi(27)\left(\frac{\Theta}{M_P}\right)^{f_i+f_j+x}$$

$$y \sim y' \sim \mathcal{O}(1)$$

'Higher dimensional' interactions

$$M_P^{-1}\ \Psi_i(27)\,\Psi_j(27)\,\overline{H}(27^*)\,\overline{H}(27^*)\left(\frac{\Theta}{M_P}\right)^{f_i+f_j-2h}$$

$$M_P^{-1}\ \Psi_i(27)\,\Psi_j(27)\,\overline{H}(27^*)\,\overline{\Phi}(27^*)\left(\frac{\Theta}{M_P}\right)^{f_i+f_j-h-x}$$

$$M_P^{-1}\ \Psi_i(27)\,\Psi_j(27)\,\overline{\Phi}(27^*)\,\overline{\Phi}(27^*)\left(\frac{\Theta}{M_P}\right)^{f_i+f_j-2x}$$

	E_6	$U(1)$	R-parity
Ψ_1	27	3	$-$
Ψ_2	27	2	$-$
Ψ_3	27	0	$-$
$H, \bar{H}$	$27, \overline{27}$	0	$+$
$\Phi, \bar{\Phi}$	$27, \overline{27}$	∓ 4	$+$
ϕ	78	-2	$+$
Θ	1	-1	$+$

$\Phi(78)$ "dressing"

$$W_\phi = \sum_{i,j} S_{ij} \frac{1}{M_P} \Psi_i(27)\Psi_j(27)\Big(\phi(78)H(27)\Big)_{27}\left(\frac{\Theta}{M_P}\right)^{f_i+f_j-2}$$

$$+ \sum_{ij} a_{ij}\frac{1}{M_P}\Big(\phi(78)\Psi_i(27)\Big)_{27}\Psi_j(27)H(27)\left(\frac{\Theta}{M_P}\right)^{f_i+f_j-2}$$

$$\langle\phi(78, 8_R)\rangle = \lambda^2 M_P \begin{pmatrix} \omega+\chi_R & & \\ & -\omega+\chi_R & \\ & & -2\chi_R \end{pmatrix}$$

$$\langle\phi(78, 8_L)\rangle = \lambda^2 M_P \begin{pmatrix} \chi_L & & \\ & \chi_L & \\ & & -2\chi_L \end{pmatrix}$$

$$78 = 8_c + 8_L + 8_R$$

$$+ (3,3,3) + (3^*,3^*,3^*)$$

Mass matrices

$$M_u = \begin{pmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & 1 \end{pmatrix} y_{33} v \sin\beta$$

$$M_d = \begin{pmatrix} Y^d_{11}\cos\theta & Y^d_{12}\cos\theta & (Y_{13} - (\omega + 3\chi_R)\tilde{S}_{13})\sin\theta \\ Y^d_{21}\cos\theta & Y^d_{22}\cos\theta & (Y_{23} - (\omega + 3\chi_R)\tilde{S}_{23})\sin\theta \\ -3\chi_L\tilde{S}_{13}\cos\theta & -3\chi_L\tilde{S}_{23}\cos\theta & \sin\theta \end{pmatrix} ;$$

$$\times\, y_{33} v \cos\beta$$

$$M_e^T = \begin{pmatrix} Y^e_{11}\cos\theta & Y^e_{12}\cos\theta & (Y'_{13} - (\omega + 3\chi_R)\tilde{S}_{13})\sin\theta \\ Y^e_{21}\cos\theta & Y^e_{22}\cos\theta & (Y'_{23} - (\omega + 3\chi_R)\tilde{S}_{23})\sin\theta \\ -3\chi_L\tilde{S}_{13}\cos\theta & -3\chi_L\tilde{S}_{23}\cos\theta & \sin\theta \end{pmatrix}$$

$$\times\, y_{33} v \cos\beta$$

E_6 27

$$E_6 \supset SO(10) \supset SU(5)$$

$$27 = 16 + 10 + 1$$

$$= \overbrace{\left(10 + \underset{\begin{pmatrix} d^c \\ e \\ -\nu \end{pmatrix}}{\underline{\underline{5}}^*} + \underset{\nu^c}{1}\right)} + \overbrace{\left(5 + \underset{\begin{pmatrix} D^c \\ E \\ -N \end{pmatrix}}{\underline{\underline{5}}^*}\right)} + \underset{S}{1}$$

Two 5^*'s : $\Phi(16, 5^*)$ & $\Phi(10, 5^*)$

two 1 s : $\Phi(16, 1)$ & $\Phi(1, 1)$

while 10 & 5 are unique.

$$
\begin{array}{lcccccc}
 & \multicolumn{3}{c}{\overbrace{\qquad\qquad\qquad\qquad\qquad}^{16}} & \multicolumn{2}{c}{\overbrace{\qquad\qquad\quad}^{10}} & 1 \\
\Psi_3(27) & 10_3 & 5^* & 1 & 5 & 5^* & 1 \\
\Psi_2(27) & 10_2 & 5^* & 1 & 5 & 5^* & 1 \\
\Psi_1(27) & 10_1 & 5^* & 1 & 5 & 5^* & 1
\end{array}
$$

$$H(16,1) \qquad \Phi(1,1)$$

$$\Psi_i(27)\,\Psi_j(27)\,H(27)$$

$$\Phi(27)$$

$$\supset \quad 16\cdot 16\cdot 10$$

$$10\cdot 10\cdot 1$$

3. Up quark mass matrix

$$y_{ij}\, \bar{\Psi}_i(27)\, \bar{\Psi}_j(27)\, H(27) \left(\frac{\Theta}{M_P}\right)^{f_i+f_j+h}$$

$$\downarrow \qquad \lambda \equiv \frac{\langle \Theta \rangle}{M_P} \sim 0.22$$

$$y^{\rm eff}_{ij}\, \Psi_i(16,10)\, \Psi_j(16,10)\, H(10,5)$$

$$y^{\rm eff}_{ij} \sim y \;\lambda^{f_i+f_j+h}$$

$$\boxed{M_u = y\, v_u \times \begin{array}{c c} & \begin{array}{ccc} 10_1 & 10_2 & 10_3 \end{array} \\ \begin{array}{c} 10_1 \\ 10_2 \\ 10_3 \end{array} & \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \end{array}}$$

$$v_u \equiv \langle H(10,5) \rangle$$

by choosing

$$\boxed{\begin{aligned} f_1 &= 0 \\ f_2 &= 2 \\ f_3 &= 3 \end{aligned}}$$

$$\underline{h = 0}$$

Most important information for GUT

masses and mixing angles

of **Quarks** and **Leptons**

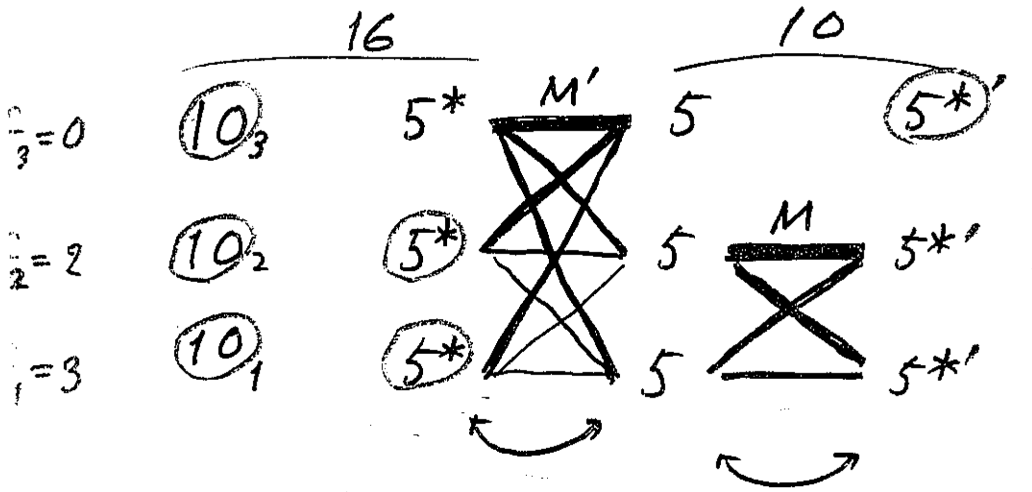
U_{CKM} 相互関係 U_{MNS}

ν masses $\longleftrightarrow$ information of

GUT scale masses

4. $\left(\begin{array}{l}\overset{5^*}{\text{down quarks}} \\ \text{charged leptons}\end{array}\right)$ mass matrix

123′ model



$$\langle \underline{H(16,1)} \rangle \equiv M' \ll \langle \underline{\Phi(1,1)} \rangle \equiv M$$

- Φ … $x = -4$ を assign

$$\Psi_1,\ \Psi_1,\ \Phi \quad \text{はない.} \qquad \left(\frac{\langle H \rangle}{M_p}\right)$$

$$0 \quad 0 \quad -4 \qquad\qquad -1$$

$\neq$

— SUSY zero

- Higgs 5^* also mixes

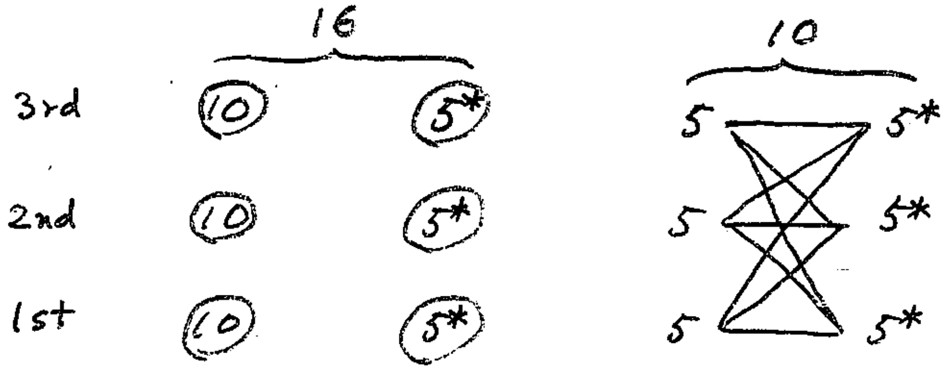
$$\boxed{H(5^*) = H(10, 5^*)\cos\theta + H(16, 5^*)\sin\theta}$$

couples to $(16,10) \times (16, 5^*)$ $\qquad$ $(16,10) \times (10, 5^*)$

4. 5* Family Structure

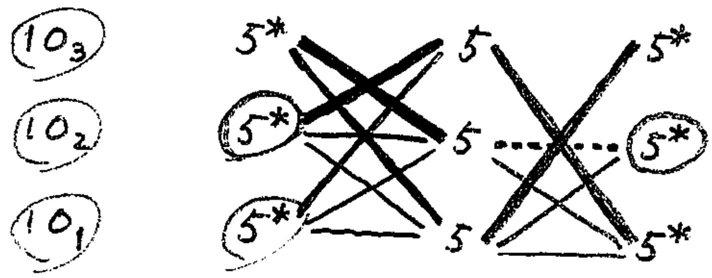
Three typical structures

1) Parallel family structure



—— $= \langle \Phi(1,1) \rangle$

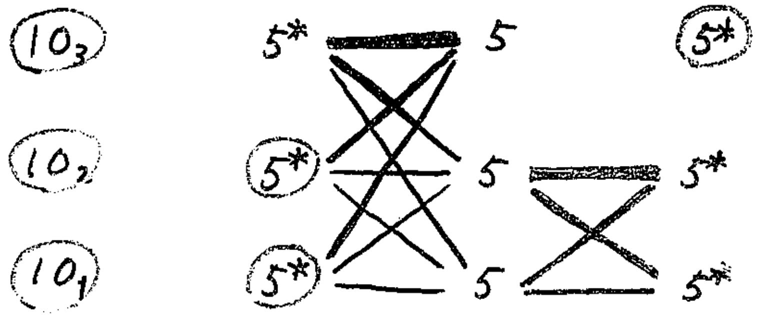
2) Nonparallel family str.



—— $= \langle \Phi^{x=-2}(16,1) \rangle$ $<$ —— $= \langle \Phi'^{x=-3}(1,1) \rangle$

---- を消すために $\Phi'(351)$

3) E-twisted str. 123' model



—— $= \langle H(16,1) \rangle \ll$ —— $= \langle \Phi^{x=-4}(1,1) \rangle$

$$M_d^T = y v_d \times \begin{array}{c|ccc} & 5^*_1 & 5^*_2 & 5^{*\prime}_3 \\ \hline 10_1 & & & \\ 10_2 & & & \\ 10_3 & & & \end{array}\!\!\begin{pmatrix} \lambda^6 \cos\theta & \lambda^5 \cos\theta & \lambda^3 \sin\theta \\ \lambda^5 \cos\theta & \lambda^4 \cos\theta & \lambda^2 \sin\theta \\ \lambda^3 \cos\theta & \lambda^2 \cos\theta & \sin\theta \end{pmatrix}$$

$$\lambda^{f_i + f_j} \times (\cos\theta \text{ or } \sin\theta)$$

$\boxed{\sin\theta \sim \lambda^2}$ にとる

$$\longrightarrow \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & 1 & 1 \end{pmatrix} \times y v_d \lambda^2$$

V_{MNS}^{e3}

$$\frac{m_t}{m_b} \sim \lambda^2$$

if $v_t \sim v_d$

Since $M_d^T \propto M_\ell$

$$\boxed{U_d M_d^T U_\ell^\dagger = \text{diag.}}$$

hierarchical

$$\begin{pmatrix} 1 & \lambda & \lambda \\ \lambda & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \lambda & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$U_{CKM} = U_u U_d^\dagger = \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \quad \text{OK}$$

$$U_{MNS} = U_\ell U_\nu^\dagger$$

$$\text{if } U_\nu \sim \mathbf{1} \qquad U_{MNS} = \begin{pmatrix} 1 & & \\ & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

U_ℓ のみで bimaximal は無理

if bimaximal $\longrightarrow$ U_ν から large 1-2 mixing

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \\ & & 1 \end{pmatrix}$$

が必要

6. Neutrino mass

- **Dirac mass M_D** $\quad \nu \nu^c, \ N S$

$$\Psi(27)\,\Psi(27)\,H(27)$$

$$\supset \langle H(10,5)\rangle \times \left(\begin{array}{ccc|ccc} \multicolumn{3}{c|}{16} & 10 & & 1 \\ 10 & 5^* & 1 & 5 & 5^* & 1 \\ u & \nu & \nu^c & & N & S \end{array} \right)$$

- **Right-handed neutrino mass**

$M_R \quad 6\times 6 \qquad (\nu_1^c\ \nu_2^c\ \nu_3^c\ \ S_1\ S_2\ S_3)$

$$\frac{1}{M_p}\,\Psi_i(27)\,\Psi_j(27)\begin{pmatrix}\overline{\Phi}(27)\\ \overline{H}(27)\end{pmatrix}\begin{pmatrix}\overline{\Phi}(27)\\ \overline{H}(27)\end{pmatrix}\left(\frac{\Theta}{M_P}\right)^{f_i+f_j+\cdots}$$

- **light neutrino mass matrix**

$$M_\nu \equiv M_D\, M_R^{-1}\, M_D \quad = \quad \left(\lambda^{f_i+f_j}\times \alpha^{0,-1,-2}\right)$$

$$\alpha \equiv \frac{\langle \Phi(1,1) \rangle \lambda^{-x}}{\langle H(16,1) \rangle} \equiv \frac{M_1}{M_2}$$

in 123′ model

$$M_\nu = \frac{\tilde{m}_t^2 M_p}{M_2^2} \times \begin{array}{c} \\ \nu_1 \\ \nu_2 \\ N_3 \end{array} \begin{array}{c} \begin{array}{ccc} \nu_1 & \nu_2 & N_3 \end{array} \\ \left(\begin{array}{cc} \alpha^{-2} \begin{pmatrix} \lambda^6 & \lambda^5 \\ \lambda^5 & \lambda^4 \end{pmatrix} & \alpha^{-1} \begin{pmatrix} \lambda^3 \\ \lambda^2 \end{pmatrix} \\ \alpha^{-1} \begin{pmatrix} \lambda^3 & \lambda^2 \end{pmatrix} & 1 \end{array} \right) \end{array}$$

atm

$$\Delta m_{23}^2 \sim 10^{-3}\,\mathrm{eV}^2 \sim m_3^2$$

$$\Big\} \; \lambda^3 \longrightarrow \frac{m_3}{m_2} \sim \lambda^2$$

sol.

$$\Delta m_{12}^2 \sim 10^{-5}\,\mathrm{eV}^2 \sim m_2^2$$

$$\alpha \sim \lambda$$

We finally obtain

$$\boxed{M_\nu = \frac{\tilde{m}_t^2 M_P}{M_2^2} \times \begin{array}{c|ccc} & \nu_1 & \nu_2 & N_3 \\ \hline \nu_1 & \alpha^{-2}\lambda^6 & \alpha^{-2}\lambda^5 & \alpha^{-1}\lambda^3 \\ \nu_2 & \alpha^{-2}\lambda^5 & \alpha^{-2}\lambda^4 & \alpha^{-1}\lambda^2 \\ N_3 & \alpha^{-1}\lambda^3 & \alpha^{-1}\lambda^2 & 1 \end{array}}$$

As far as $\alpha = \frac{M_1}{M_2} > \lambda^2$,

this M_ν is hierarchical $\Rightarrow$ $U_\nu \sim \mathbb{1}$

i.e. consistent with large 2-3 mixing in U_{MNS}

<u>Numerology</u>

Suppose

$$\boxed{\begin{aligned} M_2 &\equiv M_\Phi \lambda^4 \sim 5\times 10^{18}\,\text{GeV} \times (2\times 10^{-3}) \\ &= 10^{16}\,\text{GeV} \\ M_1 &\equiv M_H = \alpha M_2 \sim \alpha \times 10^{16}\,\text{GeV} \end{aligned}}$$

i.e.,

$$\begin{cases} \langle \overline{\Phi}^{x=4}(S) \rangle = M_\Phi \sim 5\times 10^{18}\,\text{GeV} \\ \langle \overline{H}(\nu^c) \rangle = M_H \sim \alpha \times 10^{16}\,\text{GeV} \end{cases}$$

Take $\alpha \sim \lambda \quad (> \lambda^2)$

$M_P \sim 10^{19}\,\text{GeV}$

$\tilde{m}_t \sim \frac{1}{3} m_t \sim 60\,\text{GeV}$

then

$$M_\nu = \underbrace{\left(\frac{\tilde{m}_t^2 M_P}{M_2^2}\right)}_{\sim 4\times 10^{-2}\,\mathrm{eV}} \times \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda^1 \\ \lambda^2 & \lambda^1 & 1 \end{pmatrix}$$

eigenvalues

$$m_{\nu_3} \sim 4\times 10^{-2}\ \mathrm{eV}$$

$$m_{\nu_2} \sim \lambda^2 m_{\nu_3} \sim 2\times 10^{-3}\ \mathrm{eV}$$

$$m_{\nu_1} \sim \lambda^4 m_{\nu_3} \sim 1\times 10^{-4}\ \mathrm{eV}$$

Alternative

$$\begin{array}{ccc} \Phi^{x=-4} & \longrightarrow & \Phi^{x=-5} \\ & & \downarrow \\ & & \bar{\Phi}^{x=+5} \end{array}$$

$$\begin{cases} \langle \bar{\Phi}^{x=+5}(\nu^c)\rangle = M_\Phi \sim 5\times 10^{19}\,\mathrm{GeV} & (16,1) \\ \qquad\qquad\qquad\qquad\qquad\quad \vee & \\ \langle \bar{H}\ (S)\rangle = M_H \sim 10^{16}\,\mathrm{GeV} & (1,1) \end{cases}$$

Then $\begin{pmatrix} M_1^2 & M_1 M_2 \\ M_1 M_2 & M_2^2 \end{pmatrix} = \begin{pmatrix} M_\Phi^2 \lambda^{10} & M_H M_\Phi \lambda^5 \\ M_H M_\Phi \lambda^5 & M_H^2 \end{pmatrix}$

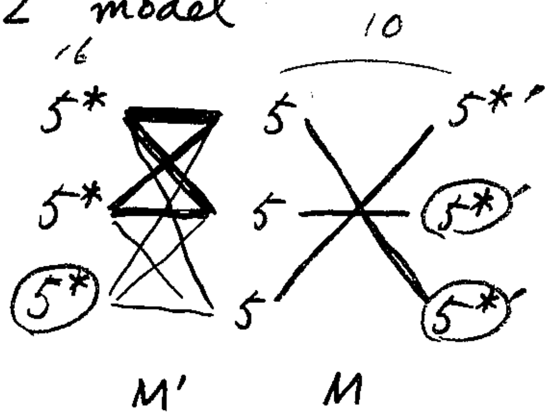
$$M_2 = M_H \sim 10^{16}\,\mathrm{GeV}$$

$$M_1 = M_\Phi \lambda^5 \sim 5\times 10^{18}\,\mathrm{GeV} \times (2\times 10^{-3}) \times \lambda$$

$$= \lambda \times 10^{16}\,\mathrm{GeV} \implies \alpha = \lambda\ !$$

Example of bimaximal mixing

1′ 1 2′ model



$$\underline{\lambda^3 M' > M > \lambda^6 M'}$$

$$M_d^T \sim \begin{array}{c} \\ 10_1 \\ 10_2 \\ 10_3 \end{array} \begin{array}{ccc} 5^{*\prime}_1 & 5^{*}_1 & 5^{*\prime}_2 \\ \end{array}$$

$$M_d^T \sim \begin{array}{cc} & \begin{array}{ccc} 5^{*\prime}_1 & 5^{*}_1 & 5^{*\prime}_2 \end{array} \\ \begin{array}{c} 10_1 \\ 10_2 \\ 10_3 \end{array} & \begin{pmatrix} \lambda^6 \sin\theta & \lambda^6 \cos\theta & \lambda^5 \sin\theta \\ \lambda^5 \sin\theta & \lambda^5 \cos\theta & \lambda^4 \sin\theta \\ \lambda^3 \sin\theta & \lambda^3 \cos\theta & \lambda^2 \sin\theta \end{pmatrix} \end{array}$$

$$\xrightarrow[\sin\theta \sim \lambda]{} \doteqdot \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \underbrace{\lambda}_{V_{MNS}^{e3}} & 1 & 1 \end{pmatrix} \times \underbrace{\lambda^3}_{\frac{m_b}{m_t}}$$

$$
M_\nu \sim \begin{array}{c|ccc} & N_1 & \nu_1 & N_2 \\ \hline N_1 & \lambda^6 & \alpha^{-1}\lambda^6 & \lambda^5 \\ \nu_1 & \alpha^{-1}\lambda^6 & \alpha^{-2}\lambda^6 & \alpha^{-1}\lambda^5 \\ N_2 & \lambda^5 & \lambda^5 & \lambda^4 \end{array}
$$

$$
\xrightarrow[\alpha\sim 1]{} \begin{pmatrix} \lambda^2 & \lambda^2 & \lambda \\ \lambda^2 & \lambda^2 & \lambda \\ \lambda & \lambda & 1 \end{pmatrix} \times \lambda^4
$$

$$
U_\nu \sim \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \lambda \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \lambda \\ \lambda & \lambda & 1 \end{pmatrix}
$$

$$\sin\theta \sim \lambda^{@}$$

$$\alpha \sim \lambda^{\bullet}$$

これらは Φ, H / $\bar{\Phi}, \bar{H}$ の $U(1)_X$ charge assignment

で，potential の minimum 条件で決まっている．