# THE BRANE WORLD

*Imamura, Watari, T. Y.*

# Neutrino Mass

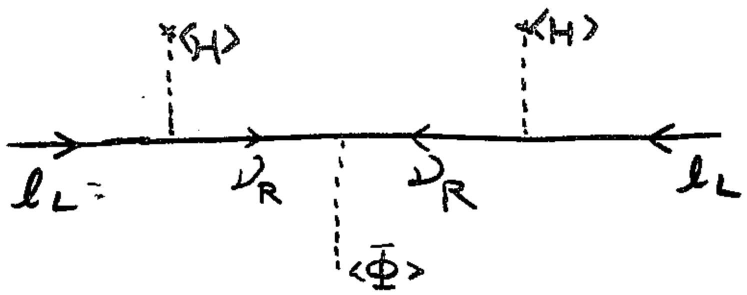
$$\mathcal{L} \sim \frac{1}{M_R}\, l_L l_L \langle H\rangle^2$$

$$l_L = \begin{pmatrix} e \\ \nu \end{pmatrix}_L$$

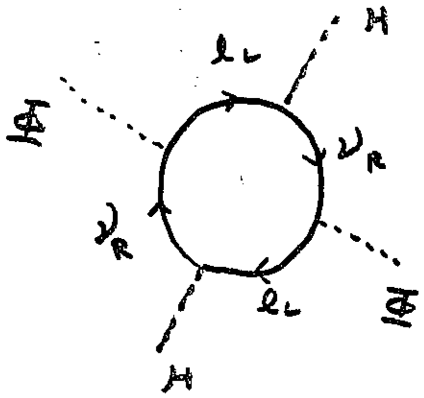
THE SEESAW MASS

$$m_\nu \sim \frac{1}{M_R} \langle H\rangle^2$$

$$m_\nu \simeq 0.1\ \mathrm{eV} \quad \rightarrow \quad M_R \simeq 10^{15}\ \mathrm{GeV}$$




$$\langle \Phi \rangle \simeq M_R \simeq 10^{15}\ \mathrm{GeV}$$




$$\simeq \frac{1}{16\pi^2} \mathcal{H}^\dagger \mathcal{H}\, \Phi^\dagger \Phi$$

$$\Downarrow$$

$$\frac{|\langle \Phi \rangle|^2}{16\pi^2} \mathcal{H}^\dagger \mathcal{H}$$

$$m_{\mathcal{H}} \sim 10^{14}\,\mathrm{GeV} \gg 10^{2}\,\mathrm{GeV}$$

We need some cancellation mechanism !!

SUSY !!

$$\begin{cases} \mathcal{H}(x,\theta) = \mathcal{H}(x) + \psi_H(x)\,\theta \\ \text{chiral multiplet} \\ \bar{\mathcal{H}}(x,\theta) = \bar{\mathcal{H}}(x) + \psi_{\bar{H}}(x)\,\theta \end{cases}$$

Higgs mass term

$$W = m_H \cdot \bar{H} H \qquad \leftarrow \text{SUSY invariant}$$

$$\text{WHY} \quad m_H \ll M_{Pl} \; ?$$

SUSY is not sufficient.

*R* symmetry

$$\theta \to e^{i\alpha}\theta$$

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$$H(x,\theta) = H(x) + \psi_H(x)\,\theta$$

$$\longrightarrow H(x) + \psi_H(x)\, e^{-i\alpha} \cdot e^{i\alpha}\theta$$

$R$ :

$$\begin{cases} H(x) \longrightarrow H(x) \\ \psi_H(x) \longrightarrow e^{-i\alpha}\, \psi_H(x) \end{cases}$$

$$\begin{cases} \bar{H}(x) \longrightarrow \bar{H}(x) \\ \psi_{\bar{H}}(x) \longrightarrow \psi_{\bar{H}}(x)\, e^{-i\alpha} \end{cases}$$

Mass term is forbidden :

$$\psi_H \psi_{\bar{H}} \longrightarrow e^{-i2\alpha}\, \psi_H \psi_{\bar{H}} .$$

$\psi_H, \psi_{\bar{H}}$ are massless.

$\underset{\text{SUSY}}{\hookrightarrow}$ $H, \bar{H}$ are massless

$$W = m_H\, H(x,\theta)\, \bar{H}(x,\theta)$$

$$m_H = 0 .$$

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BUT.

$\Lambda_{cos} \simeq 0$ :

$$\cancel{\text{SUSY}} \simeq \cancel{R}$$

$$m_H \simeq \text{SUSY-Breaking scale}$$

$$\simeq 100\,\text{GeV} \sim 1\,\text{TeV}$$

$$\underset{(2)}{W} = \cancel{m_H}\, \underset{(0)}{H}\, \underset{(0)}{\bar{H}}$$

$$\mathcal{L} = \int \underset{(-2)}{d^2\theta}\; \underset{(2)}{W}$$

R charge

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*Inconsistent with GUT!*

$$\begin{cases} H(5) = \begin{pmatrix} H_c \\ H_f \end{pmatrix} & : \quad Q_R = 0 \\ \\ \bar{H}(5^*) = \begin{pmatrix} \bar{H}_c \\ \bar{H}_f \end{pmatrix} & : \quad Q_R = 0 \end{cases}$$

$$\underset{(2)}{W} = m_c \underset{(0)}{H_c} \underset{(0)}{\bar{H}_c} + m_H \underset{(0)}{H_f} \underset{(0)}{\bar{H}_f}$$

$$m_c \sim m_H \sim R\text{-breaking scale}$$
$$\sim \cancel{\text{SUSY}} \text{ scale}$$

But we need

$$m_c \gtrsim 10^{16}\,\text{GeV}. \leftarrow \text{proton lifetime}.$$

SOLUTION :

T. Y. ('85)

$$\begin{pmatrix} H_c \\ H_f \end{pmatrix} \quad + \quad \bar{\xi}_c$$
$$(0) \qquad\qquad (2)$$

$$\begin{pmatrix} \bar{H}_c \\ \bar{H}_f \end{pmatrix} \quad + \quad \xi_c$$
$$(0) \qquad\qquad (2)$$

$$W = M_c \underset{(0)}{H_c} \underset{(2)}{\bar{\xi}_c} + \bar{M}_c \underset{(0)}{\bar{H}_c} \underset{(2)}{\xi_c}$$

$H_f$ , $\bar{H}_f$ : massless

How to introduce $\xi_c$ and $\bar{\xi}_c$ ?

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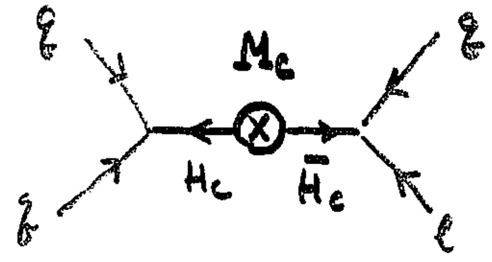
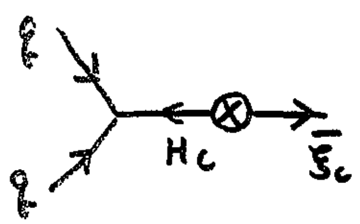
# D= 5 OPERATOR

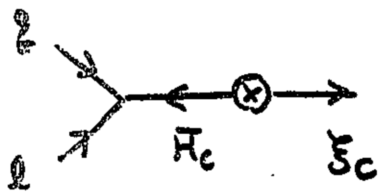
Sakai, T.Y.
Weinberg ('82)




$\tau(p \to K\nu) \lesssim 10^{31}$ years ×

Now Suppressed !!




THIS RESULT IS GENERIC.

" D=6 Proton Decay "

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# $SU(5)_{GUT} \times U(3)_H$

T. Y.

Hyperqarks $\quad Q^{i}{}_{\alpha}\ (5^*, 3) \quad \left(\begin{array}{l}\alpha = 1-3 \\ i = 1-5\end{array}\right.$

$\bar{Q}_i{}^{\alpha}\ (5, 3^*)$

$$\langle Q \rangle = \begin{pmatrix} v & & & & \\ & v & & & \\ & & v & 0 & 0 \end{pmatrix}$$

$$\langle \bar{Q} \rangle = \begin{pmatrix} v & & \\ & v & \\ & & v \\ 0 & & \\ 0 & & \end{pmatrix}$$

$$SU(5)_{GUT} \times U(3)_H$$

$$\longrightarrow SU(3)_C \times SU(2)_L \times U(1)_Y$$

## Gauge couplings

$$SU(3)_C \subset SU(3)_{GUT} \times SU(3)_H$$

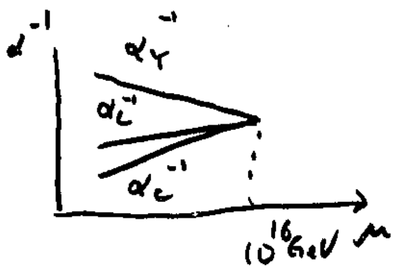
$$U(1)_Y \subset U(1)_{GUT} \times U(1)_H$$

$$SU(2)_L \subset SU(5)_{GUT}$$

$$\alpha_C \simeq \frac{\alpha_{GUT}}{1 + \alpha_{GUT}/\alpha_{3H}}$$

$$\alpha_Y \simeq \frac{\alpha_{GUT}}{1 + \frac{1}{15}\alpha_{GUT}/\alpha_{1H}}$$

$$\alpha_2 = \alpha_{GUT}$$




"Unification"

$$\alpha_{1H} \sim \alpha_{3H} \gg \alpha_{GUT}$$

$$\alpha_C \simeq \alpha_L \simeq \alpha_Y$$

THE HYPERCOLOR $U(3)_H$ IS IN STRONG COUPLING REGION!

Introduce A pair of $Q^6_\alpha$ and $\bar{Q}^\alpha_6$.

$$\begin{array}{c} Q^6 \\ 5^*\left\{\begin{array}{c} Q^1 \\ Q^2 \\ Q^3 \\ Q^4 \\ Q^5 \end{array}\right. \end{array} \qquad\qquad \begin{array}{c} \bar{Q}^6 \\ \left.\begin{array}{c} \bar{Q}^1 \\ \bar{Q}^2 \\ \bar{Q}^3 \\ \bar{Q}^4 \\ \bar{Q}^5 \end{array}\right\} 5 \end{array}$$

$$\langle Q \rangle = \begin{pmatrix} v & & & \\ & v & & 0 \\ & & v & \end{pmatrix} \; ; \; \langle \bar{Q} \rangle = \begin{pmatrix} v & & \\ & v & \\ & & v \\ & 0 & \end{pmatrix}$$

Global Symmetry : $SU(6)_L \times SU(6)_R$

$$\Downarrow$$

$$SU(3)_c \times SU(3)_L \times SU(3)_R$$

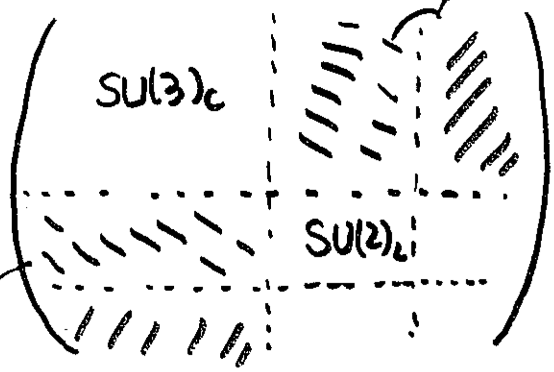
Gauge Symmetry :

$$SU(5)_{GUT} \times U(3)_H$$

$$\longrightarrow SU(3)_c \times SU(2)_L \times U(1)_Y$$

Nambu - Goldstone multiplets

X BOSON に (multiplet)




(multiplet) X Boson に 吸収

[illegible] が massless として残る。

color (3 + 3*)

$\xi_c$ $\bar{\xi}_c$

$H_c$ $\bar{H}_c$ と mass をくむ.

$H_f + \bar{H}_f$ が massless として残る。

But more N-G multiplets appear, since $G = SU(6)_L \times SU(6)_R$.

$$\mathcal{N}=1 \overbrace{\phantom{x}}^{\text{SUSY}} \text{HYPER-COLOR} \quad SU(3)$$

$$Q^{i}_{\alpha} \qquad \bar{Q}^{\alpha}_{i} \qquad \alpha = 1,2,3$$

$$i = 1 - N$$

$$\mathcal{L} = \int d^4\theta \, \mathcal{H}$$

$$\mathcal{H} = Q^{\dagger} e^{V} Q + \bar{Q}^{\dagger} e^{-V} Q$$

Global Symmetry : $U(N)_L \times U(N)_R$

$\mathcal{N}=2$ SUSY HYPER-COLOR $SU(3)$

$$Q^{i}_{\alpha} \ , \ \bar{Q}^{\alpha}_{i} \qquad X^{\beta}_{\alpha}$$

$$\mathcal{L} = \int d^4\theta \, \mathcal{H} + \int d^2\theta \, W + h.c$$

$$W = g \, Q_{\alpha} X^{\alpha}_{\beta} \bar{Q}^{\beta}$$

Global Symmetry : $U(N)_{L+R}$ !!

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# $N=2$ SUSY $U(3)_H$ THEORY

Izawa, T.Y.

$$W = Q^i_\alpha X^\alpha_\beta Q_i^\beta + Q^i_\alpha X_0 Q_i^\alpha$$
$$-3V^2 X_0$$

FI - F-term

consistent with $N=2$ SUSY.

We have a unique vacuum

$$\langle Q \rangle = \begin{pmatrix} v & & \\ & v & \\ & & 0 \end{pmatrix} ; \quad \langle \bar{Q} \rangle = \begin{pmatrix} v & & \\ & v & \\ & & 0 \end{pmatrix}$$

Global Symmetry: $U(6) \to SU(3) \times U(3)$

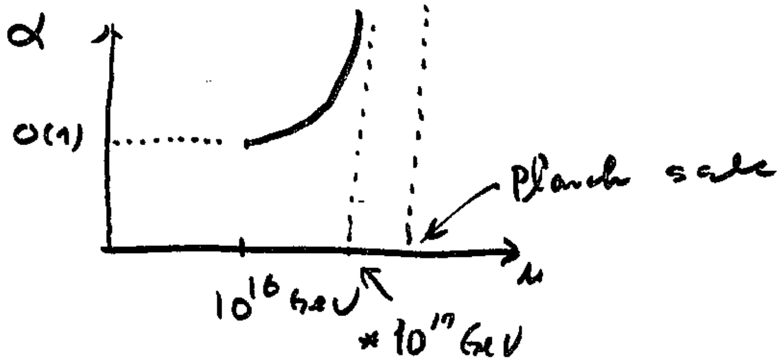
: Gauge symmetry :

$$SU(5)_{GUT} \times U(3)_H$$
$$\to SU(3)_c \times SU(2)_L \times U(1)_Y$$

We solved the problem, but new problems arise.

① $\alpha_H \gg \alpha_{GUT}$

$\alpha_{1H}$ blows up below $M_{pl}$.




② Why $\mathcal{N}=2$ Theory for the hypercolor sector?

① → Cut-off Scale $M_* \sim 10^{17}$ GeV.

The Brane World.

solves the problems.

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$M_* \ll M_{pl}$ suggests

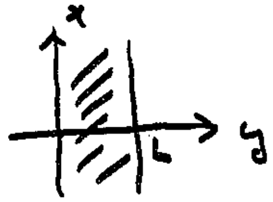
a higher dimensional theory.

Witten ('96)

$$\mathcal{L} = M_*^3 \int d^4x\, dy\, \sqrt{-g^{(5)}}\, R_{(5)}$$

$$g^{(5)} = \left( \begin{array}{c|c} g^{(4)} & \\ \hline & 1 \end{array} \right)$$

$$= \underset{\underset{M_{pl}^2}{\parallel}}{M_*^3 L} \int d^4x \sqrt{-g}\, R$$




$$M_* L = (M_{pl}/M_*)^2$$

If $L \gg M_*^{-1}$ $\rightarrow$ $M_{pl} \gg M_*$

The Planck scale is only an effective scale.

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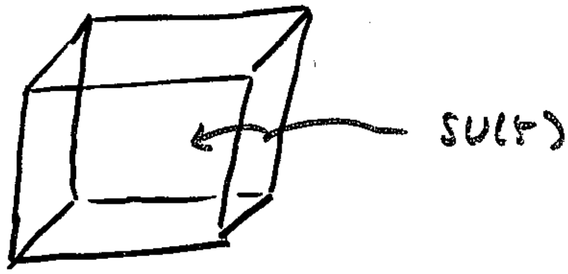
Bulk SU(5) GUT.




$$\mathcal{L} = \frac{M_*}{g_0^2} \int d^4x\, dy\ \overline{W}_\alpha W^\alpha$$

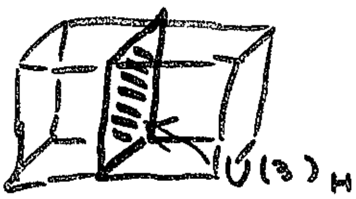
SU(5) gauge multiplet

$$= \frac{M_* L}{g_0^2} \int d^4x\ \overline{W}_\alpha W^\alpha$$

$$\frac{M_* L}{g_0^2} = \frac{1}{g_{GUT}^2}$$

$$\therefore\ g_{GUT}^2 = \frac{g_0^2}{M_* L} \ll 1 \qquad \text{if } M_* L \sim \text{large}$$

$g_{GUT}^2 \ll g_H^2 \longrightarrow$ HYPER-COLOR SECTOR on 3-brane

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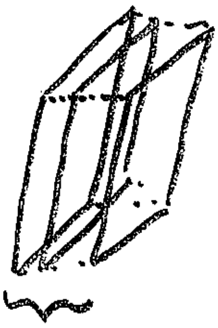
String Theory D-brane



N枚.

U(N) Gauge Theory

3枚の D-3 brane

Bulk SU(5)GUT と より高次元

時空の D-brane (5枚)

10次元 String Theory

(type II B)

D7−D3 brane

5枚の D7 ⊕ 3枚の D3 branes.

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# The type IIB String Theory

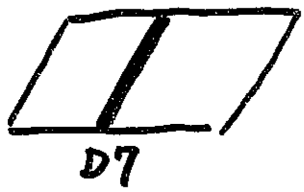
## with D7-D3 Brane.

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| D7 | | o | o | o | o | o | o | o | | |
| D3 | | o | o | o | | | | | | |

SUSY charges $Q^i$

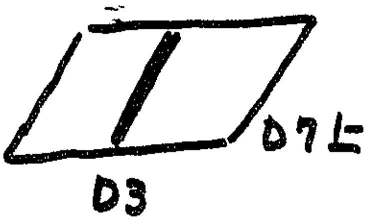
$Q^i$ (1 — 32) $\underline{32}$

D7 半分に落ちる $\underline{16}$




$N=4$ SUSY

D3 さらに半分に落ちる $\underline{8}$




$N=2$ SUSY

D3 Brane 上の $U(3)_H$ は $N=2$ SUSY をもつ

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D7 上の $SU(5)_{GUT}$ Gauge 場

$\mathcal{N}=4$ SUSY

$$A_\mu \qquad \begin{matrix} \chi_1 \\ \chi_2 \\ \chi_3 \\ \chi_4 \end{matrix} \qquad \begin{matrix} \Sigma \\ \Sigma' \\ \Sigma'' \end{matrix}$$

fermions　　　scalar bosons

$$SO(1,9) \simeq SO(1,3) \times SO(6)$$

16 の super charge $Q^i$ は

4つの $Q_\alpha^i$ に分解される

$$Q_\alpha^i = (\underline{2}, \underline{4}) \quad \text{complex}$$

$SO(1,3)$ の spinor　　　$SO(6)$ の $\underline{4}$

$$A_\mu \xrightarrow{Q^i} \begin{matrix} \chi_1 \\ \chi_2 \\ \chi_3 \\ \chi_4 \end{matrix}$$

残したいのは $(A_\mu, \chi_1)$ の対

$\mathcal{N}=1$ SUSY

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Orbifold とする. $SO(6) \simeq SU(4)$ の subgroup

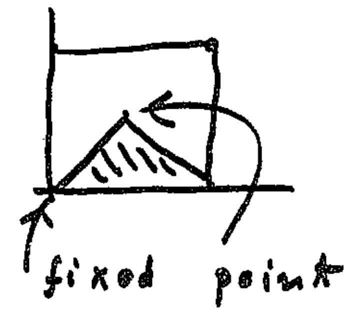
$$\begin{pmatrix} 1 & & & \\ & i & & \\ & & i & \\ & & & -1 \end{pmatrix}$$

$$\begin{pmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & & & \\ & i & & \\ & & i & \\ & & & -1 \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{pmatrix}$$

$\theta_1$ のみが残る.

$T^6/Z_4$ orbifold を仮定する.

$T^2$




Two fixed point

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & & & \\ & i & & \\ & & i & \\ & & & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$$

$x_2, x_3, x_4$ は fixed point で 0 になる.

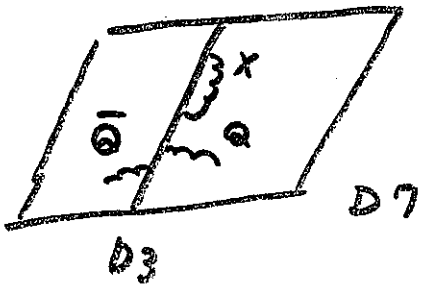
zero mode として残るのは $x_1$ のみ. $x_1$ のみ生きのこる.

$\rightarrow$ ($A_\mu$ $x_1$) $\mathcal{N}=1$ SUSY. 21

Fixed Point 上に quark, lepton ($5^* + 10$)
Higgs chiral multiplet を おく。

- R-symmetry は 8-9 plane の回転。
- FI term の origin は B-field.
- $Q^\alpha_i$: $\bar{Q}^i_\alpha$ は D3−D7 string

  $X^\alpha_\beta$, $X_0$ は D3−D3 string




But Fixed Point 上の $5^* + 10$ は、今の所
理解できていない。 fixed point の 特殊性によるか?

5枚の D7-Branes.

$$U(5) = SU(5)_{GUT} \times U(1)$$

B-L と考える.

Anomaly Cancellation

↳ $\nu_R$ is required !!!

Neutrinos are massive.

Seesaw mechanism

T.Y. (79)

G.M. R.S.

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