

Double Beta Decay and Neutrino Oscillation

H. Nishiura, T. Fukuyama, K. Matsuda, and N. Takeda

西浦宏幸 (a), 福山武志 (b), 松田光市 (b), 武田信弘 (b)

(a)大阪工大短大

(b)立命館大理工

1. Motivation

neutrinoless double beta decay ($\beta\beta_{0\nu}$)

+ neutrino oscillation

+ single β decay



Majorana ν

Possible constraints

on

the MNS lepton mixing matrix

(CP violating phases and mixing angles)

Contents:

- \mathcal{CP} phases in averaged neutrino mass $\langle m_\nu \rangle$ in $(\beta\beta)_{0\nu}$
- Graphical representation of M_{ee} and \mathcal{CP} phases
 - "Complex-mass triangle"
- Constraints on mixing angles from $(\beta\beta)_{0\nu}$.
 - Allowed region for
 - mixing angles for fixed \mathcal{CP} phases:
 - mixing angles S_1^2 vs. S_3^2 (independent of \mathcal{CP} phases)
 - S_3^2 vs. $\langle m_\nu \rangle$ (independent of \mathcal{CP} phases)
 - m_2 vs m_3 for given S_3^2 and $\langle m_\nu \rangle$
(independent of \mathcal{CP} phases)
- Constraints on \mathcal{CP} phases from $(\beta\beta)_{0\nu}$
 - Allowed region for \mathcal{CP} phases β and ρ'
(independent of mixing angles)
- Constraints on \mathcal{CP} phases from $[(\beta\beta)_{0\nu} + \nu \text{ Oscillation}]$
- Averaged neutrino $(m_\nu)_\beta$ mass from β decay
- Constraints on two averaged mass, $\langle m_\nu \rangle$ vs. $(m_\nu)_\beta$
- Constraints on mixing angles
 - from $[(\beta\beta)_{0\nu} + \nu \text{ Oscillation} + \beta \text{ decay}]$
- Constraints on \mathcal{CP} phases from $[(\beta\beta)_{0\nu} + \beta \text{ decay}]$
- Summary

Based on the work in the following papers :

1. "Graphical Representation of CP Violation Effects in Neutrinoless Double Beta Decay",
K. Matsuda, N. Takeda, T. Fukuyama, and H. Nishiura,
hep-ph/0007237.
2. "CP Violation in Lepton Number Violation Process and Neutrino Oscillations",
K. Matsuda, N. Takeda, T. Fukuyama, and H. Nishiura,
Physical Review D62, 093001, 2000. (hep-ph/0003055)
3. "Constraints of Mixing Angles from Lepton Number Violating Processes",
H. Nishiura, K. Matsuda, and T. Fukuyama,
Modern Physics Letters A14, 433-446, 1999. (hep-ph/9809556)
4. "Constraints of Mixing Angles from Neutrino Oscillation Experiments and Neutrinoless Double Beta Decay",
T. Fukuyama, K. Matsuda, and H. Nishiura,
Modern Physics Letters A13, 2279-2288, 1998. (hep-ph/9804262)
5. "CP Violation in Neutrinoless Double Beta Decay and Neutrino Oscillation",
T. Fukuyama, K. Matsuda, and H. Nishiura,
Physical Review D57, 5844-5850, 1998. (hep-ph/9708397)

2. CP phases in the averaged neutrino mass from $(\beta\beta)_{0\nu}$

- averaged neutrino mass:

$$\langle m_\nu \rangle = |M_{ee}|$$

- complex mass M_{ee} :

$$M_{ee} = |U_{e1}|^2 m_1 + |U_{e2}|^2 m_2 + |U_{e3}|^2 m_3$$

$$= |U_{e1}|^2 m_1 + |U_{e2}|^2 m_2 e^{2i\beta} + |U_{e3}|^2 m_3 e^{2i\rho'}$$

Maki-Nakagawa-Sakata lepton mixing matrix U:

$$U = \begin{pmatrix} c_1 c_3 & s_1 c_3 e^{i\beta} & s_3 e^{i(\rho-\phi)} \\ (-s_1 c_2 - c_1 s_2 s_3 e^{i\phi}) e^{-i\beta} & c_1 c_2 - s_1 s_2 s_3 e^{i\phi} & s_2 c_3 e^{i(\rho-\beta)} \\ (s_1 s_2 - c_1 c_2 s_3 e^{i\phi}) e^{-i\rho} & (-c_1 s_2 - s_1 c_2 s_3 e^{i\phi}) e^{-i(\rho-\beta)} & c_2 c_3 \end{pmatrix}$$

standard representation

$$\equiv \begin{pmatrix} |U_{e1}| & |U_{e2}| e^{i\beta} & |U_{e3}| e^{i\rho'} \\ * & * & * \\ * & * & * \end{pmatrix} \quad \rho' \equiv \rho - \phi$$

β, ρ are Majorana CP violating phases appeared in MNS matrix.

neutrino mass with CP phases:

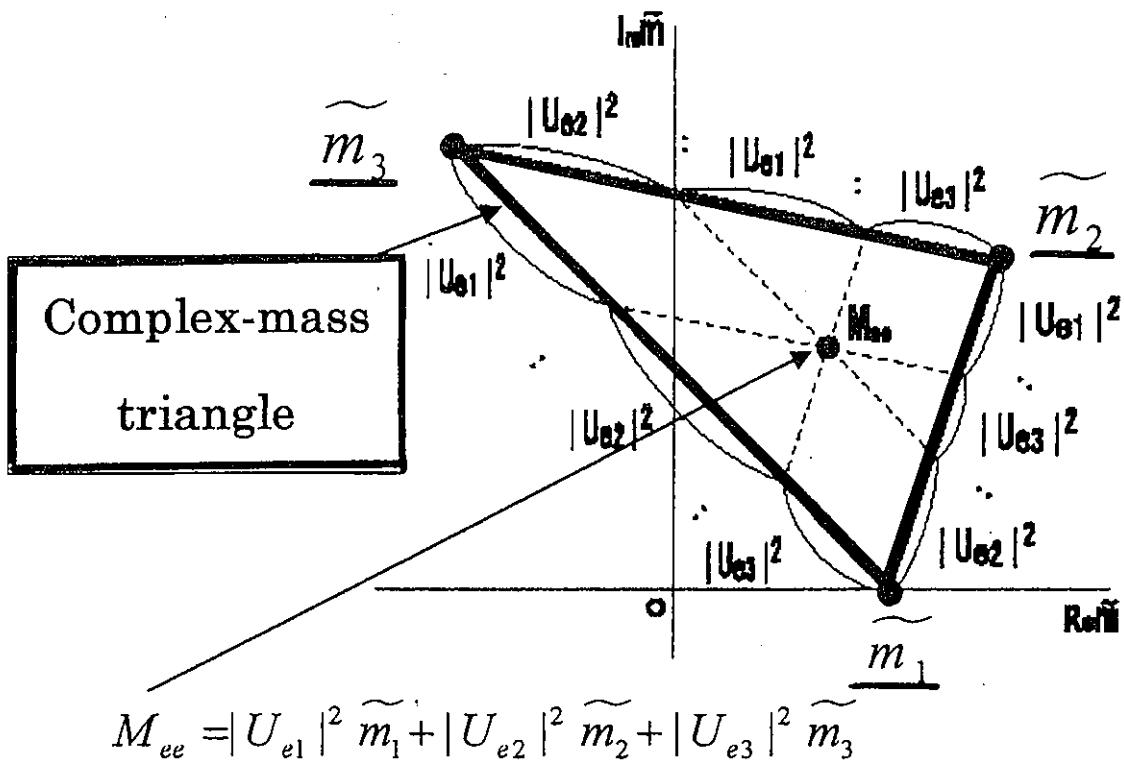
$$\left\{ \begin{array}{l} \tilde{m}_1 \equiv m_1 \\ \tilde{m}_2 \equiv m_2 e^{2i\beta} \\ \tilde{m}_3 \equiv m_3 e^{2i\rho'} \end{array} \right.$$

Then, we have

$$M_{ee} = |U_{e1}|^2 \tilde{m}_1 + |U_{e2}|^2 \tilde{m}_2 + |U_{e3}|^2 \tilde{m}_3$$

3. Graphical representation of M_{ee} and CP phases

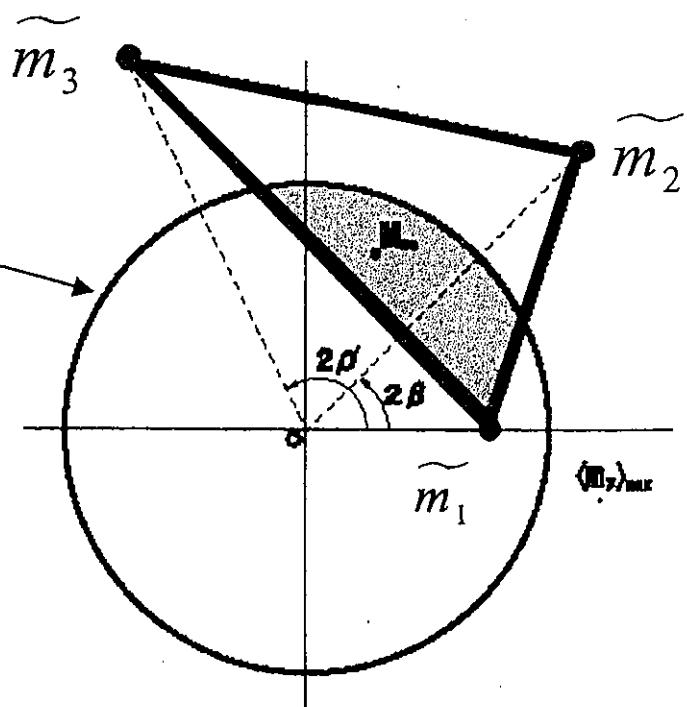
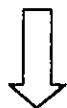
- Complex-mass triangle in the complex mass plane:



- Allowed region for M_{ee} :

$(\beta\beta)_{0\nu}$ experimental upper bound $\langle m_\nu \rangle_{\max}$ for $\langle m_\nu \rangle$

$$\langle m_\nu \rangle = |M_{ee}| < \langle m_\nu \rangle_{\max}$$



The allowed position of M_{ee} is in the intersection (shaded area) of the inside of the complex mass triangle and the inside of the circle of radius $\langle m_\nu \rangle_{\max}$ around the origin.

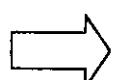
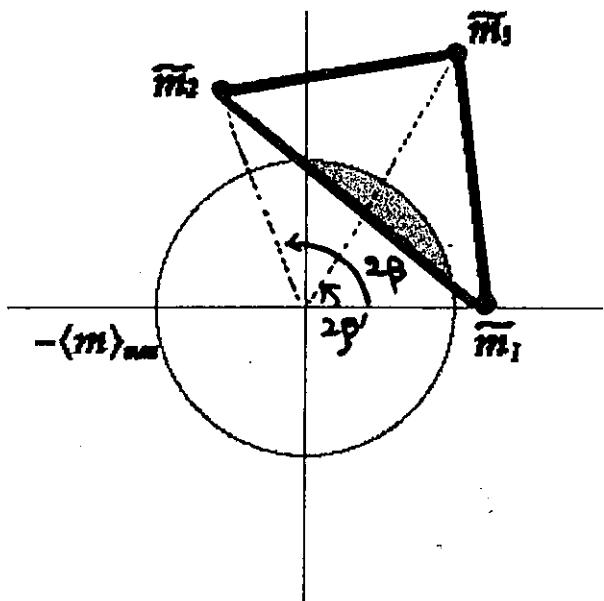
4. Constraints on mixing angles from $(\beta\beta)_{0\nu}$

- Allowed region for mixing angles for fixed CP violating phases:

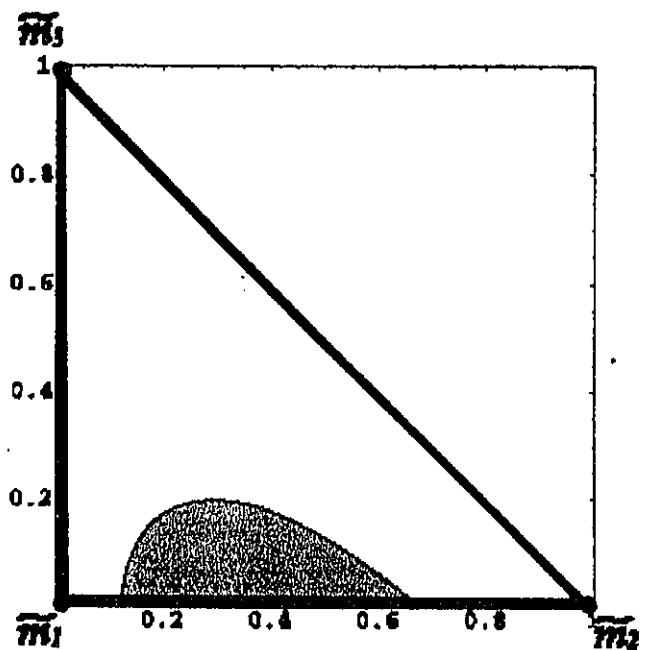
The complex-mass triangle is deformed to the isosceles right triangle retaining the ratios of the heights from the vertices \tilde{m}_i



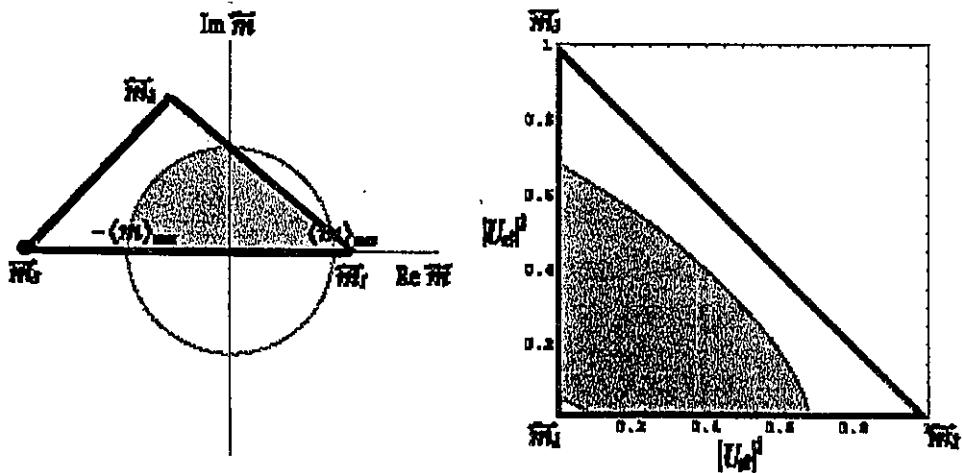
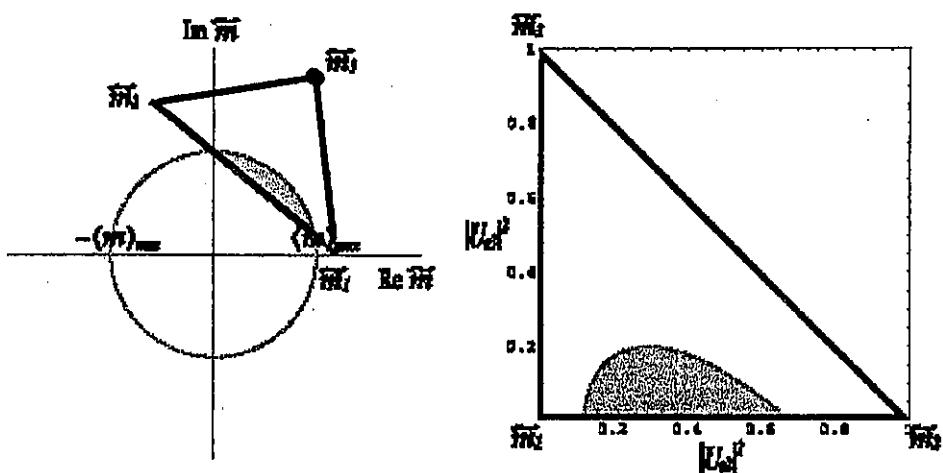
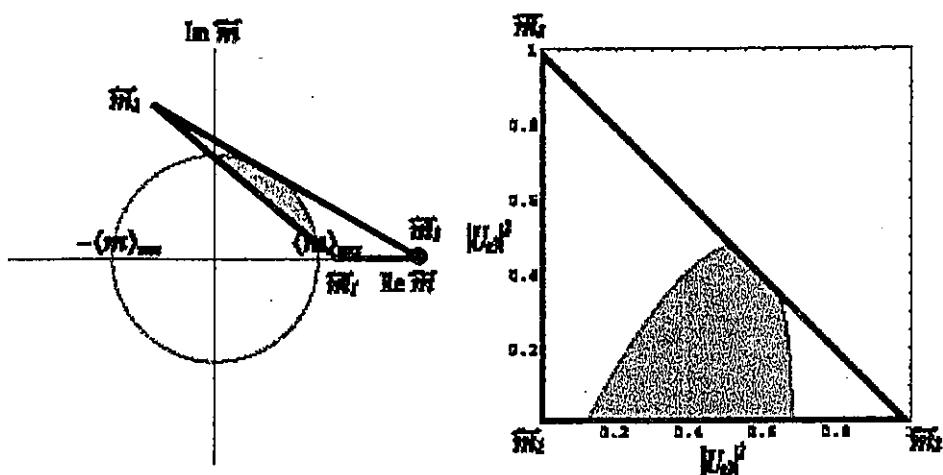
Allowed region of $|U_{e2}|^2$ and $|U_{e3}|^2$ for fixed CP violating phases β and ρ'



$$|U_{e3}|^2$$



Constraints for some cases:



- Allowed region for mixing angles S_1^2 vs S_3^2 independent of CP violating phases:

Note that the mixing angle θ_2 does not contribute to $(\beta\beta)_{0\nu}$.

$$\begin{aligned} \langle m_\nu \rangle^2 &= |c_1^2 c_3^2 m_1 + s_1^2 c_3^2 m_2 e^{2i\beta} + s_3^2 m_3 e^{2i\rho'}|^2 \\ &= (c_1^2 c_3^2 m_1 + s_1^2 c_3^2 m_2 \cos 2\beta + s_3^2 m_3 \cos 2\rho')^2 + (s_1^2 c_3^2 m_2 \sin 2\beta + s_3^2 m_3 \sin 2\rho')^2 \end{aligned}$$

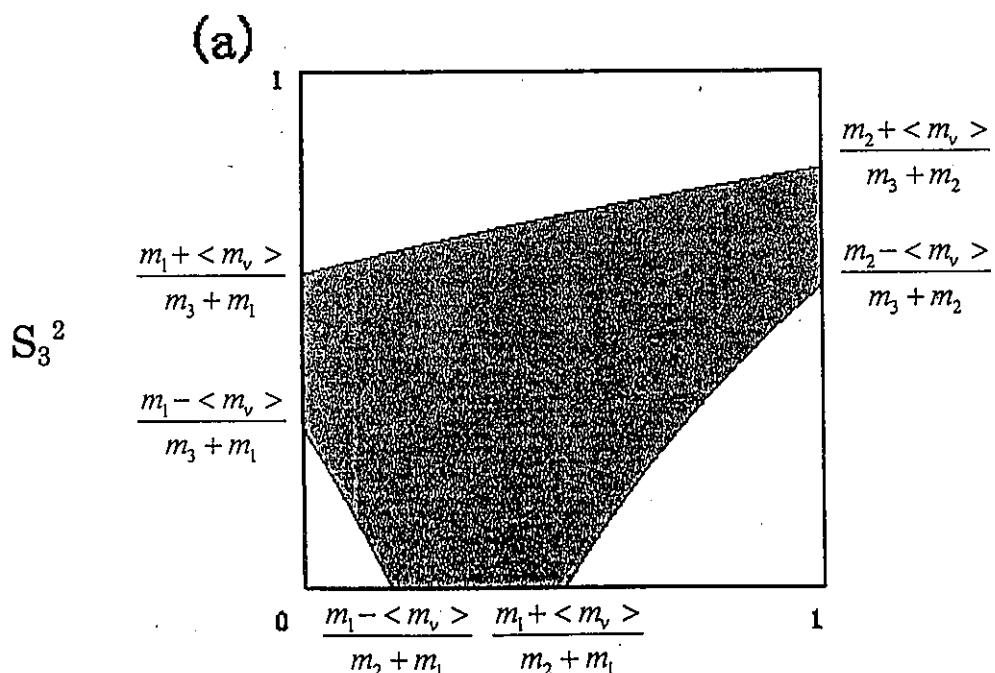


$$-1 \leq \cos 2\beta \leq 1, \quad -1 \leq \cos 2\rho' \leq 1$$

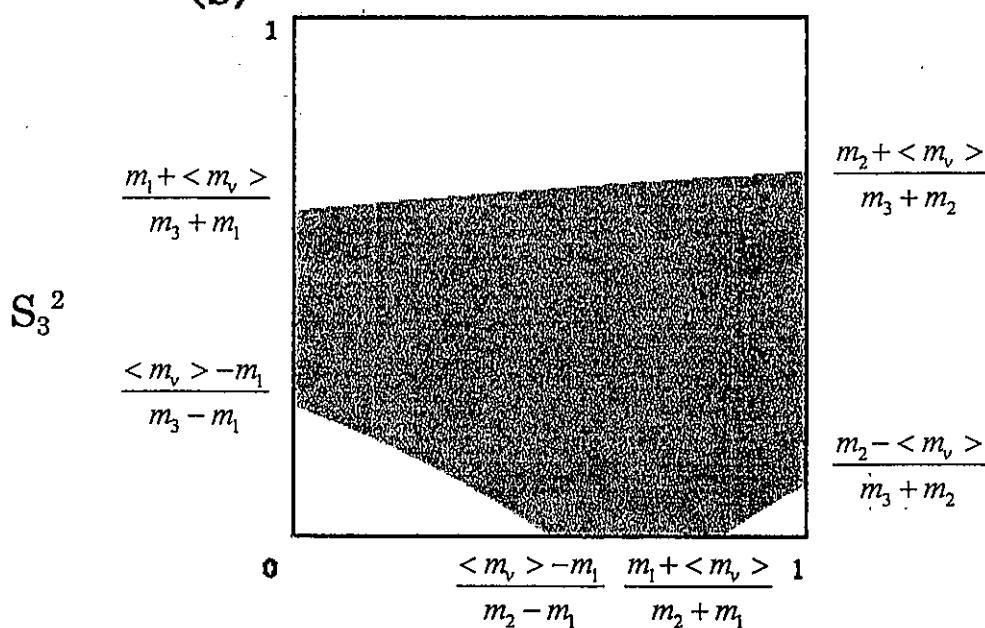
$$\begin{aligned} (\langle m_\nu \rangle - s_3^2 m_3)^2 - (c_1^2 c_3^2 m_1 + s_1^2 c_3^2 m_2)^2 &\leq 0 \\ (\langle m_\nu \rangle + s_3^2 m_3)^2 - (c_1^2 c_3^2 m_1 - s_1^2 c_3^2 m_2)^2 &\geq 0 \\ (\langle m_\nu \rangle - s_1^2 c_3^2 m_2)^2 - (c_1^2 c_3^2 m_1 + s_3^2 m_3)^2 &\leq 0 \\ (\langle m_\nu \rangle + s_1^2 c_3^2 m_2)^2 - (c_1^2 c_3^2 m_1 - s_3^2 m_3)^2 &\geq 0 \end{aligned}$$

Hereafter we assume that $m_1 \leq m_2 \leq m_3$

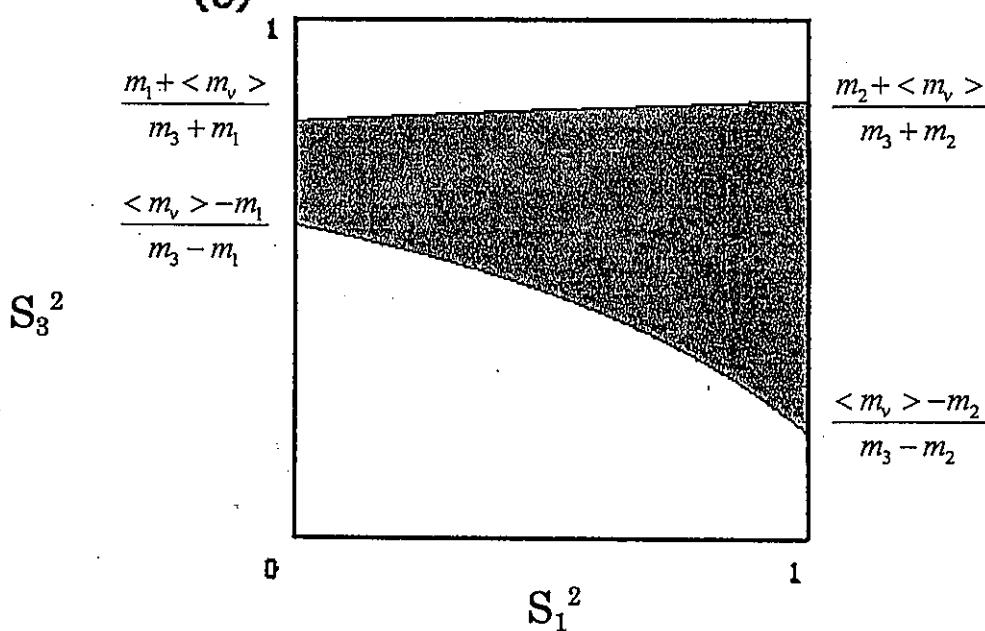
$$(a) \langle m_\nu \rangle \leq m_1 \quad (b) m_1 \leq \langle m_\nu \rangle \leq m_2 \quad (c) m_2 \leq \langle m_\nu \rangle \leq m_3$$



(b)



(c)



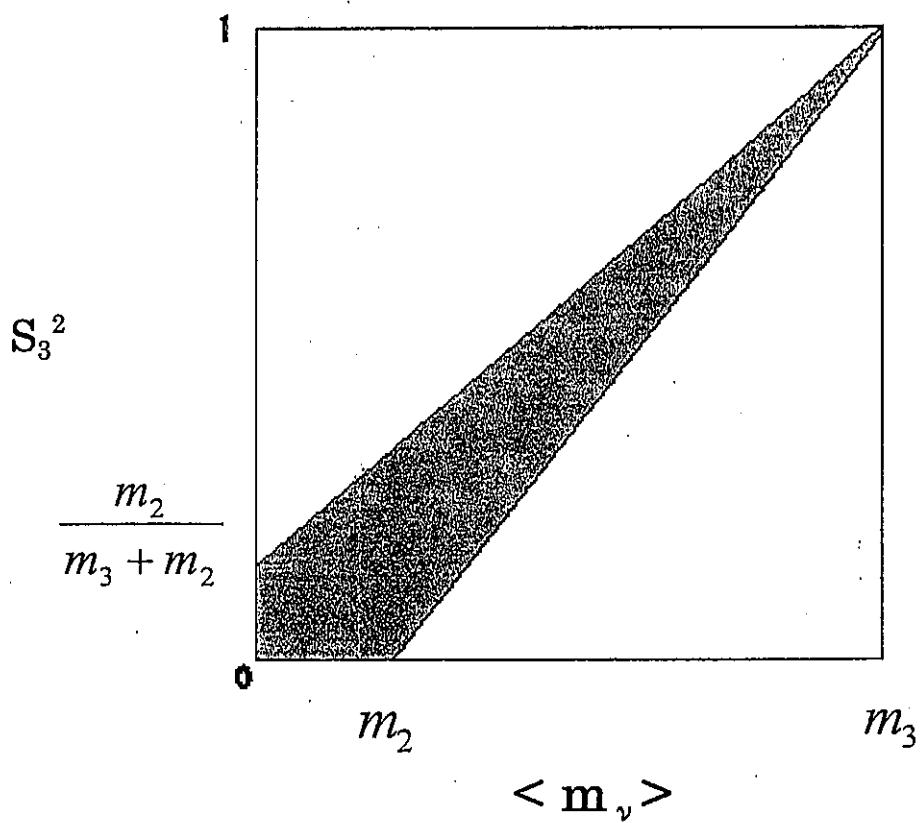
● Allowed region for S_3^2 :



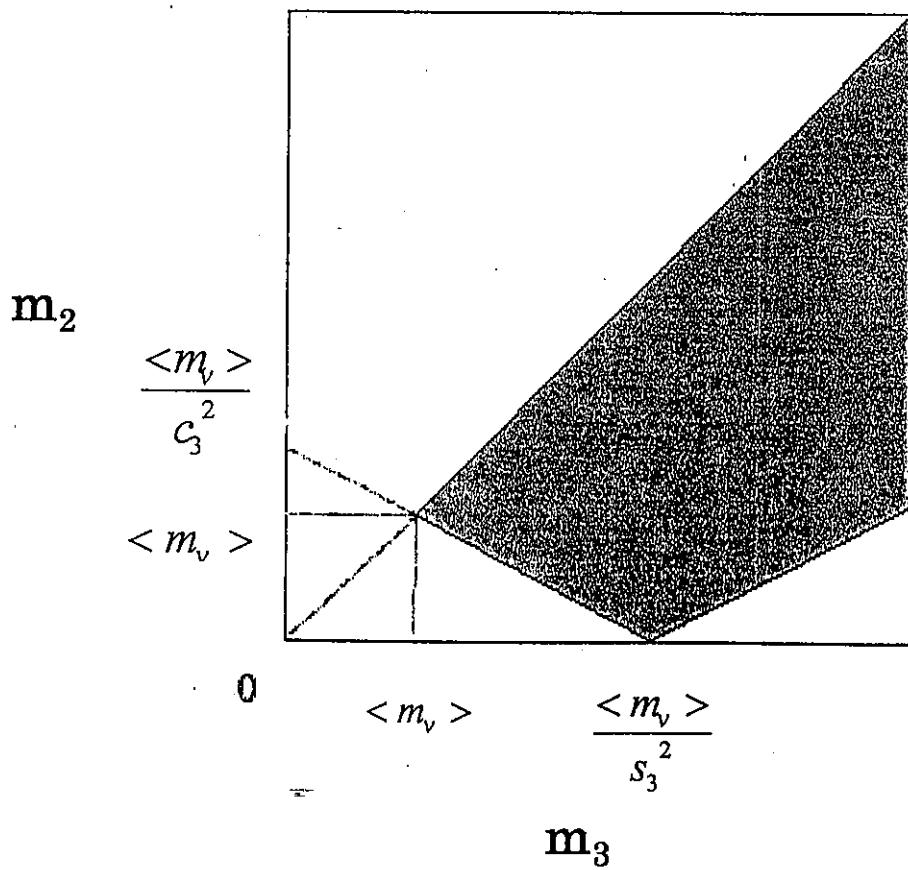
$$\frac{\langle m_\nu \rangle - m_2}{m_3 - m_2} \leq S_3^2 \leq \frac{m_2 + \langle m_\nu \rangle}{m_3 + m_2} \quad \text{for } m_2 \leq \langle m_\nu \rangle$$

$$0 \leq S_3^2 \leq \frac{m_2 + \langle m_\nu \rangle}{m_3 + m_2} \quad \text{for } \langle m_\nu \rangle \leq m_2$$

- Allowed region for S_3^2 vs. $\langle m_\nu \rangle$:



- Allowed region for m_2 vs. m_3 for given S_3^2 and $\langle m_\nu \rangle$:

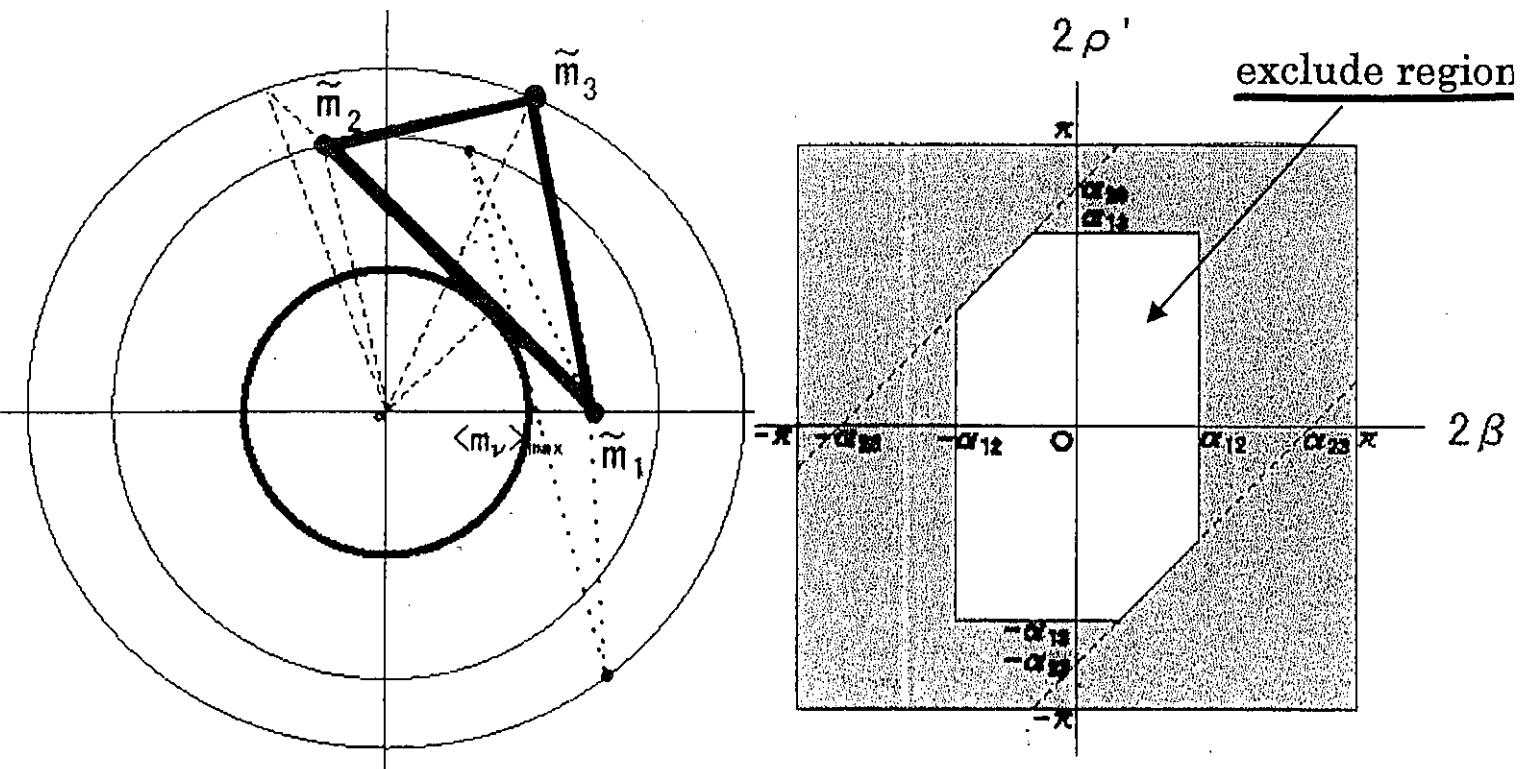
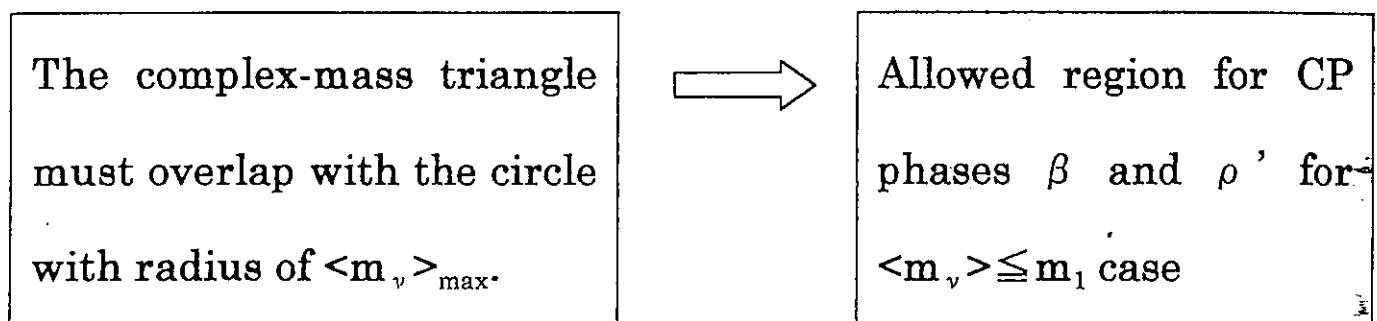


5. Constraints on CP violating phases from $(\beta\beta)_{0\nu}$

- Allowed region for CP violating phases β and ρ' independent of mixing angles:

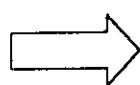
$m_1 \leq \langle m_\nu \rangle$ case: No restriction for CP violating phases

$\langle m_\nu \rangle \leq m_1$ case: Restriction for CP violating phases



$$\alpha_{ij} \equiv \cos^{-1}\left(\frac{\langle m_\nu \rangle_{\max}}{m_i}\right) + \cos^{-1}\left(\frac{\langle m_\nu \rangle_{\max}}{m_j}\right), \quad \alpha_{12} = \cos^{-1}\left(\frac{\langle m_\nu \rangle_{\max}}{m_1}\right) + \cos^{-1}\left(\frac{\langle m_\nu \rangle_{\max}}{m_2}\right)$$

$$\alpha_{23} - \alpha_{12} = \cos^{-1}\left(\frac{\langle m_\nu \rangle_{\max}}{m_3}\right) - \cos^{-1}\left(\frac{\langle m_\nu \rangle_{\max}}{m_1}\right)$$



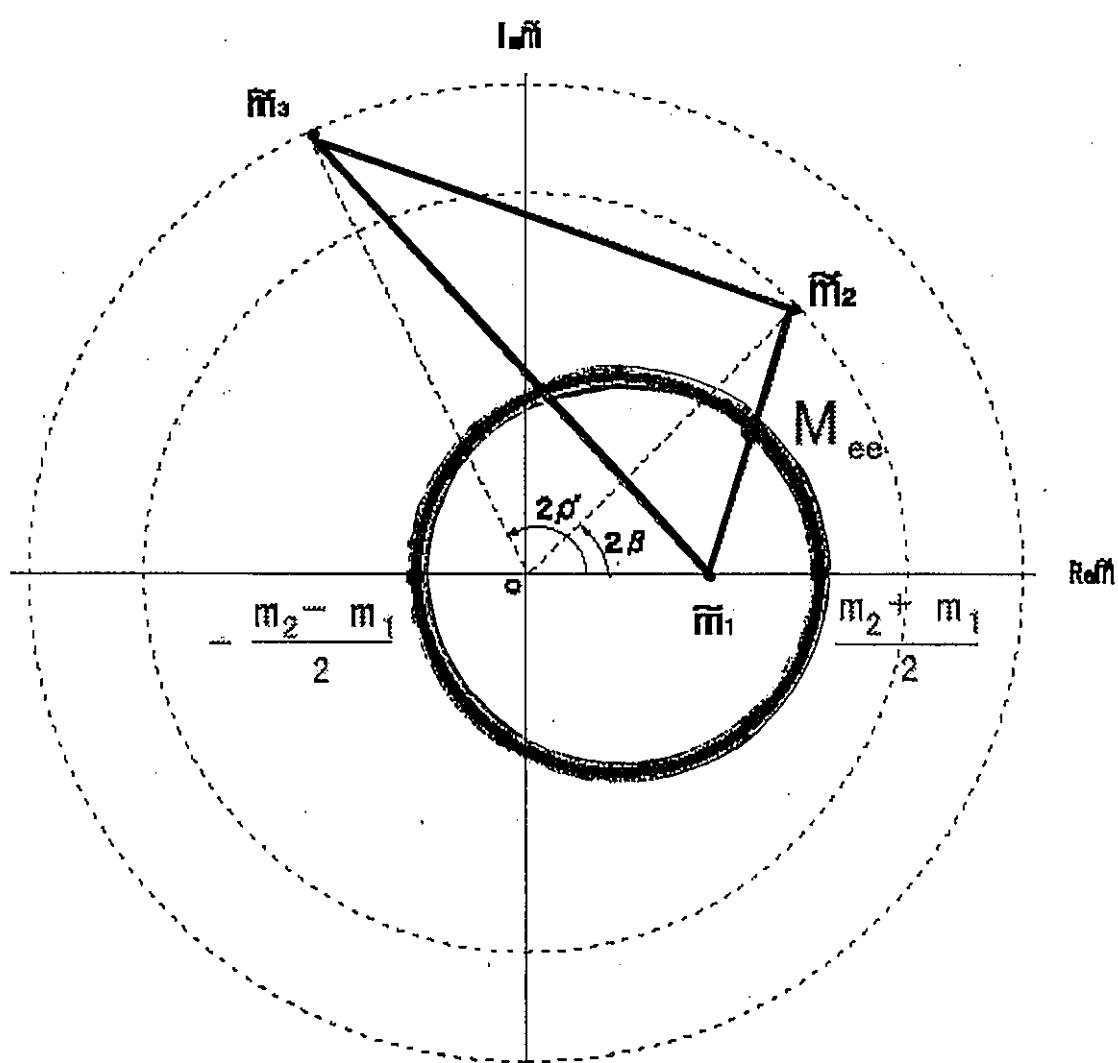
$\{|2\beta| < \alpha_{12} \text{ and } |2\rho'| < \alpha_{23} - \alpha_{12}\}$ is excluded

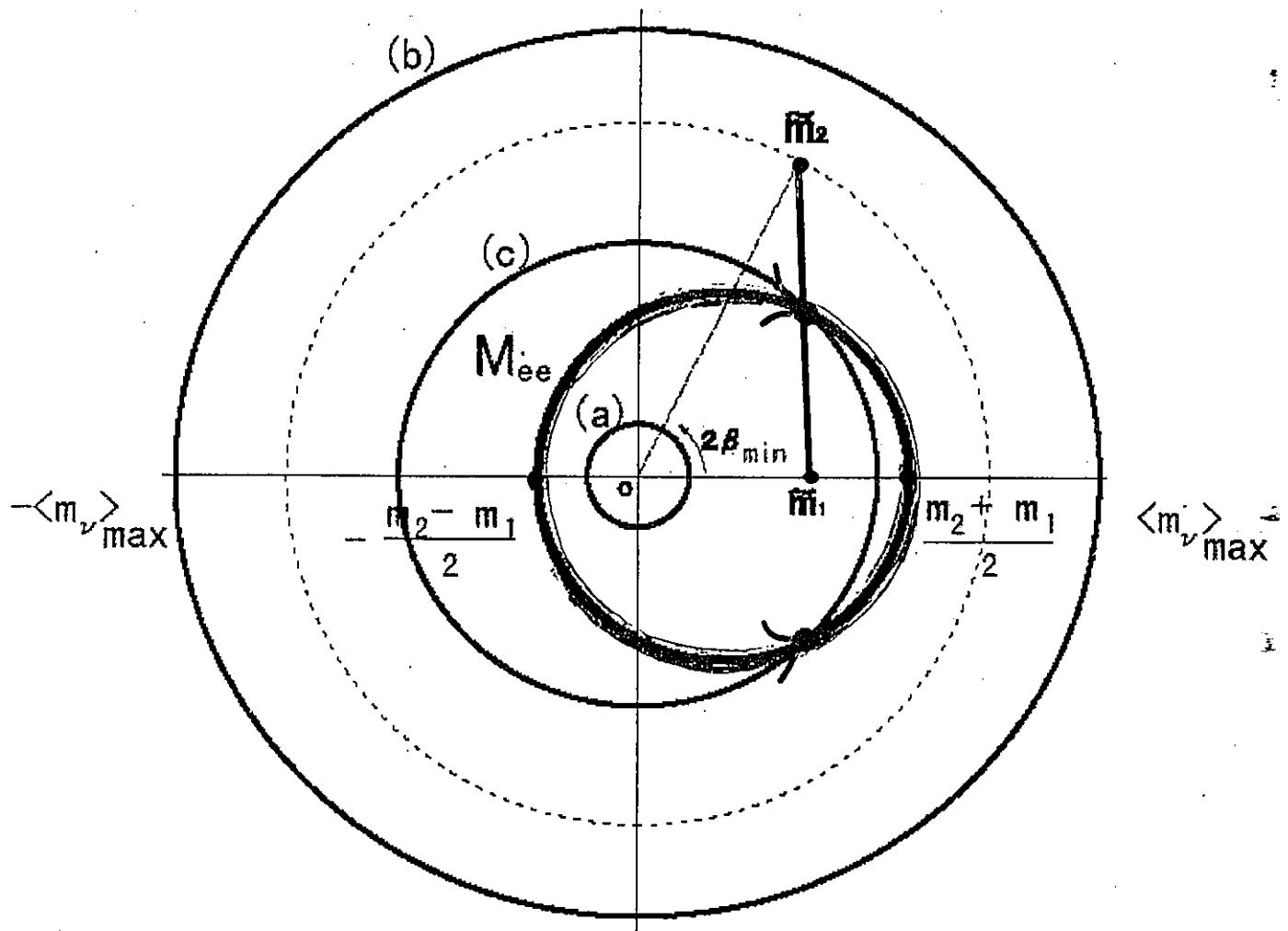
6. Constraints on CP violating phases from $(\beta\beta)_{0\nu}$ and Neutrino Oscillation]

- Allowed region for CP violating phases β and ρ' using the constraints on mixing angles from the neutrino oscillation data:

$$|U_{e1}|^2 = |U_{e2}|^2 = \frac{1}{2} \quad (\text{Solar MSWLMA})$$

$$|U_{e3}|^2 \approx 0 \quad (\text{CHOOZ})$$





For the cases

$$(a) \langle m_\nu \rangle_{\max} \leq \frac{m_2 - m_1}{2} \quad \Rightarrow \text{excluded}$$

$$(b) \frac{m_2 + m_1}{2} \leq \langle m_\nu \rangle_{\max} \quad \Rightarrow \text{no constraints on } \beta$$

$$(c) \frac{m_2 - m_1}{2} \leq \langle m_\nu \rangle_{\max} \leq \frac{m_2 + m_1}{2} \quad \Rightarrow \text{allowed bound on } \beta \text{ as:}$$

$$\cos^{-1} \left(\frac{4 \langle m_\nu \rangle_{\max}^2 - m_1^2 - m_2^2}{2m_1 m_2} \right) \leq |2\beta| \leq \pi$$

7. Averaged neutrino mass $(m_\nu)_\beta$ from β decay

- Averaged neutrino mass $(m_\nu)_\beta$ defined from β decay

$$(m_\nu)_\beta^2 \equiv |U_{e1}|^2 m_1^2 + |U_{e2}|^2 m_2^2 + |U_{e3}|^2 m_3^2$$

with unitarity relation



$$1 = |U_{e1}|^2 + |U_{e2}|^2 + |U_{e3}|^2$$

- Relations among mixing matrix elements

$$|U_{e1}|^2 = \frac{m_2^2 - (m_\nu)_\beta^2}{m_2^2 - m_1^2} + |U_{e3}|^2 \frac{m_3^2 - m_2^2}{m_2^2 - m_1^2}$$

$$|U_{e2}|^2 = \frac{(m_\nu)_\beta^2 - m_1^2}{m_2^2 - m_1^2} - |U_{e3}|^2 \frac{m_3^2 - m_1^2}{m_2^2 - m_1^2}$$



combined with $0 \leq |U_{ei}|^2 \leq 1$

For $(m_\nu)_\beta < m_2$ case:

$$0 \leq |U_{e3}|^2 \leq \frac{(m_\nu)_\beta^2 - m_1^2}{m_3^2 - m_1^2}$$

$$\frac{(m_\nu)_\beta^2 - m_1^2}{m_2^2 - m_1^2} \geq |U_{e2}|^2 \geq 0$$

$$\frac{m_2^2 - (m_\nu)_\beta^2}{m_2^2 - m_1^2} \leq |U_{e1}|^2 \leq \frac{m_3^2 - (m_\nu)_\beta^2}{m_3^2 - m_1^2}$$

For $m_2 < (m_\nu)_\beta$ case:

$$\frac{(m_\nu)_\beta^2 - m_2^2}{m_3^2 - m_2^2} \leq |U_{e3}|^2 \leq \frac{(m_\nu)_\beta^2 - m_1^2}{m_3^2 - m_1^2}$$

$$\frac{m_3^2 - (m_\nu)_\beta^2}{m_3^2 - m_2^2} \geq |U_{e2}|^2 \geq 0$$

$$0 \leq |U_{e1}|^2 \leq \frac{m_3^2 - (m_\nu)_\beta^2}{m_3^2 - m_1^2}$$

- Present experimental bound for $(m_\nu)_\beta$:

$$(m_\nu)_\beta < 3 \text{ eV}$$

by Particle Data Group 2000

8. Constraints on mixing angles

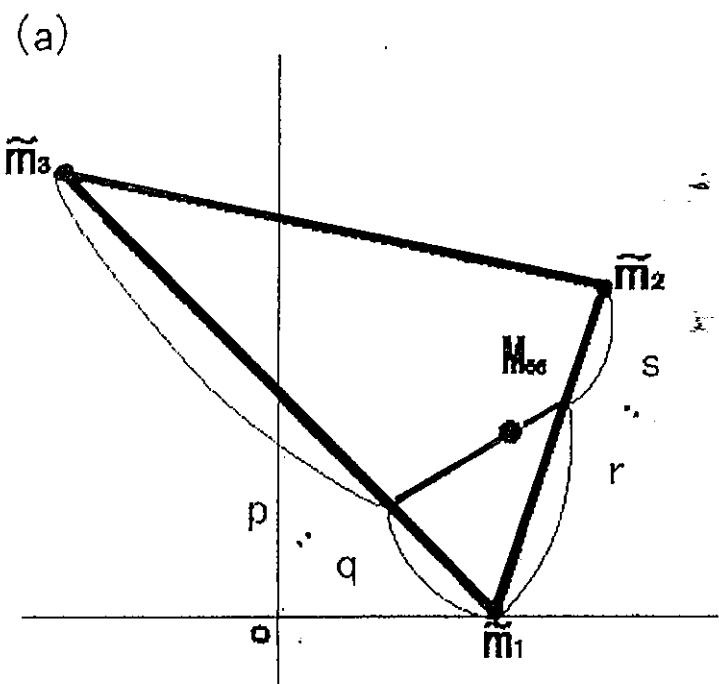
from $(\beta\beta)_{0\nu}$ + ν oscillation + β decay]

- The position of complex mass M_{ee} in $(\beta\beta)_{0\nu}$ is restricted, ^{on the line} with using the definition of the neutrino mass $(m_\nu)_\beta$ from β decay:

(a) $(m_\nu)_\beta < m_2$ case:

$$p = \frac{m_3^2 - (m_\nu)_\beta^2}{m_3^2 - m_1^2}, \quad q = \frac{(m_\nu)_\beta^2 - m_1^2}{m_3^2 - m_1^2}$$

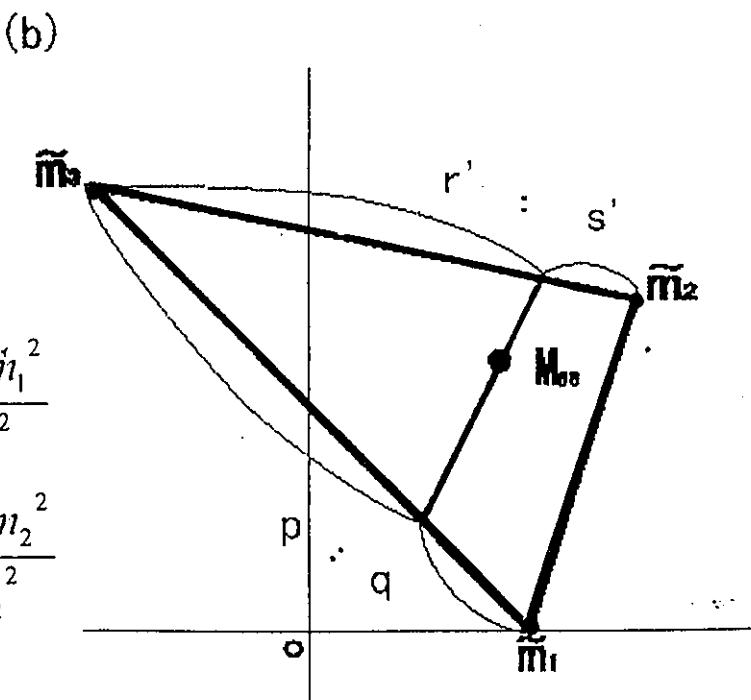
$$r = \frac{(m_\nu)_\beta^2 - m_1^2}{m_2^2 - m_1^2}, \quad s = \frac{m_2^2 - (m_\nu)_\beta^2}{m_2^2 - m_1^2}$$



(b) $m_2 < (m_\nu)_\beta$ case:

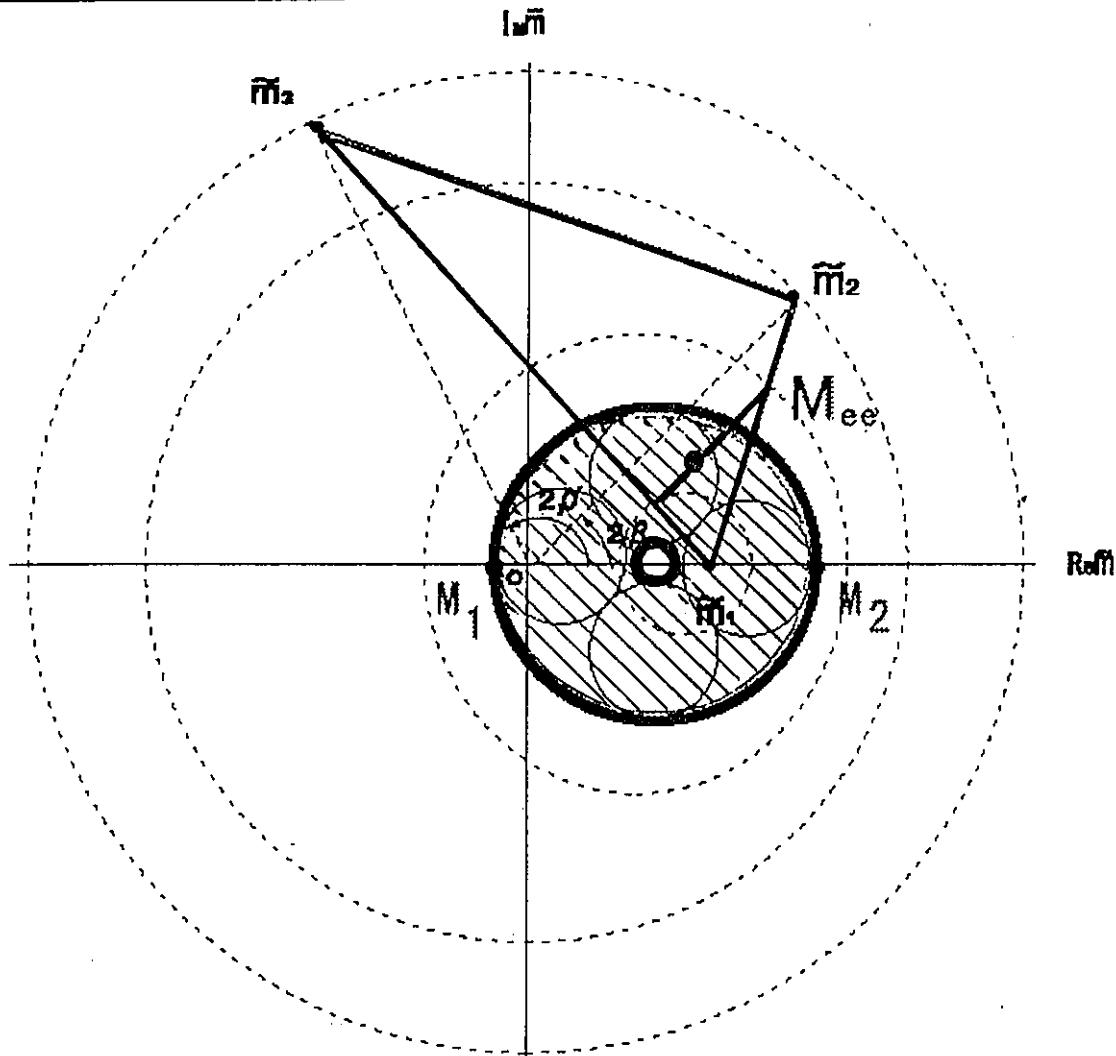
$$p = \frac{m_3^2 - (m_\nu)_\beta^2}{m_3^2 - m_1^2}, \quad q = \frac{(m_\nu)_\beta^2 - m_1^2}{m_3^2 - m_1^2}$$

$$r' = \frac{m_3^2 - (m_\nu)_\beta^2}{m_3^2 - m_2^2}, \quad s' = \frac{(m_\nu)_\beta^2 - m_2^2}{m_3^2 - m_2^2}$$



- The position of complex mass M_{ee} in $(\beta\beta)_{0\nu}$ is restricted to be inside of a circle by changing the CP violating phases β and ρ' .

(a) $(m_\nu)_\beta < m_2$ case:



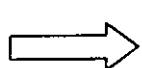
Here, $M_1 M_2$ are given by

$$M_1 = \frac{m_1 m_2 - (m_\nu)_\beta^2}{m_2 - m_1} + |U_{e3}|^2 \frac{(m_3 + m_1)(m_3 - m_2)}{m_2 - m_1}$$

$$M_2 = \frac{m_1 m_2 + (m_\nu)_\beta^2}{m_2 + m_1} - |U_{e3}|^2 \frac{(m_3 - m_1)(m_3 - m_2)}{m_2 + m_1}$$

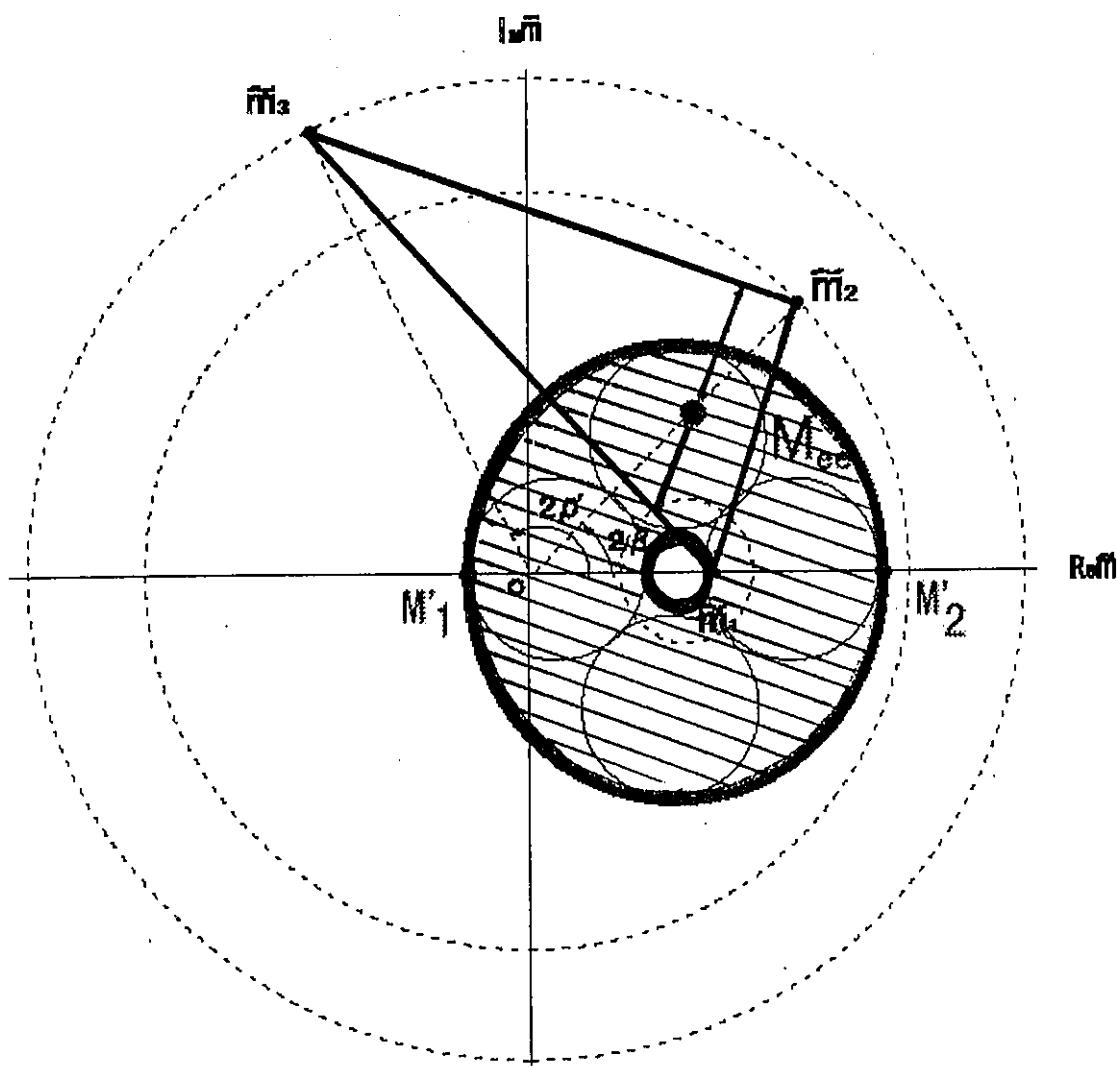
Note that

$$0 \leq |U_{e3}|^2 \leq \frac{(m_\nu)_\beta^2 - m_1^2}{m_3^2 - m_1^2}$$



$$\frac{m_1 m_2 - (m_\nu)_\beta^2}{m_2 - m_1} \leq M_1, \quad M_2 \leq \frac{m_1 m_2 + (m_\nu)_\beta^2}{m_2 + m_1}$$

(b) $m_2 < (m_v)_\beta$ case:



Here, M'_1, M'_2 are given by

$$M'_1 = \frac{(m_v)_\beta^2 - m_3 m_2}{m_3 - m_2} + |U_{e1}|^2 \frac{(m_3 + m_1)(m_2 - m_1)}{m_3 - m_2}$$

$$M'_2 = \frac{m_3 m_2 + (m_v)_\beta^2}{m_3 + m_2} - |U_{e1}|^2 \frac{(m_3 - m_1)(m_2 - m_1)}{m_3 + m_2}$$

Note that $0 \leq |U_{e1}|^2 \leq \frac{m_3^2 - (m_v)_\beta^2}{m_3^2 - m_1^2}$

→ $\frac{(m_v)_\beta^2 - m_1 m_2}{m_3 - m_2} \leq M'_1, \quad M'_2 \leq \frac{m_3 m_2 + (m_v)_\beta^2}{m_3 + m_2}$

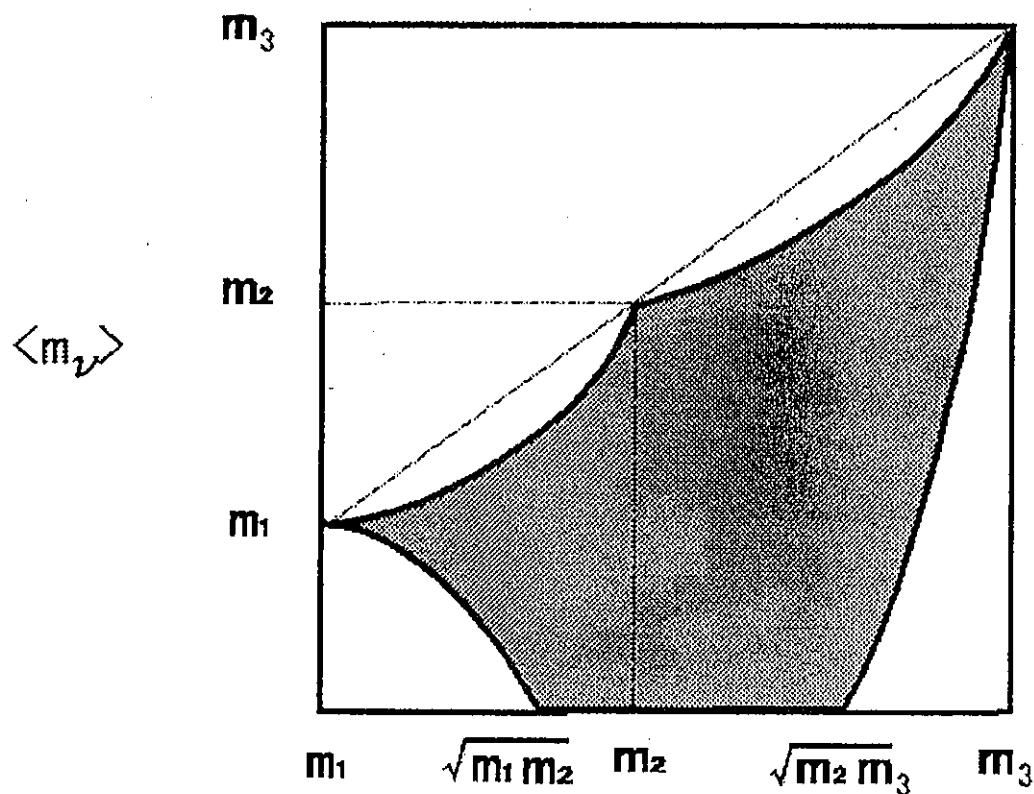
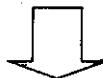
- The allowed region in the $\langle m_\nu \rangle - (m_\nu)_\beta$ plane obtained irrespectively of the CP violating phases and mixing angles.

(a) $(m_\nu)_\beta < m_2$ case:

$$\max \left\{ \frac{m_2 m_1 - (m_\nu)_\beta^2}{m_2 - m_1}, 0 \right\} \leq \langle m_\nu \rangle \leq \frac{m_2 m_1 + (m_\nu)_\beta^2}{m_2 + m_1}$$

(b) $m_2 < (m_\nu)_\beta$ case:

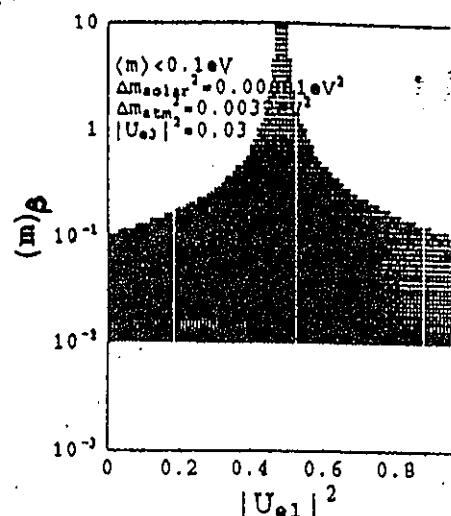
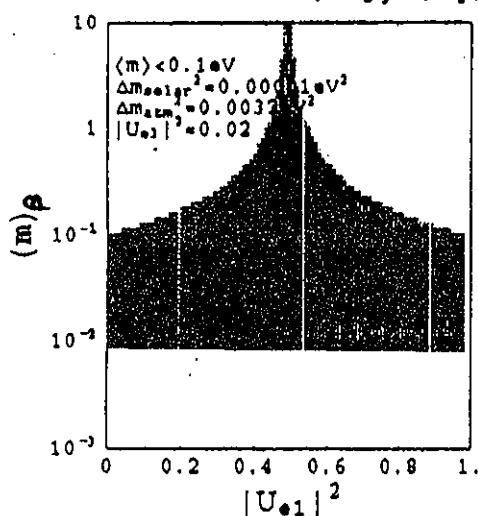
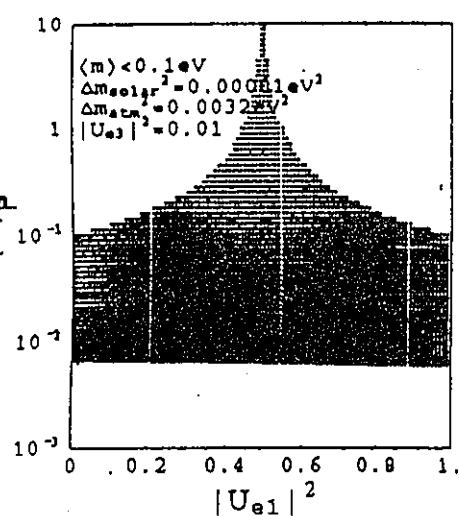
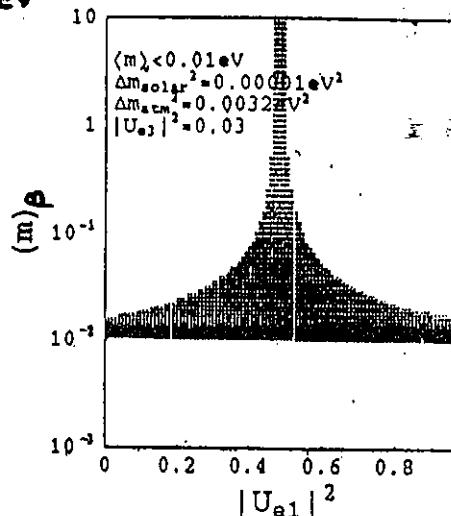
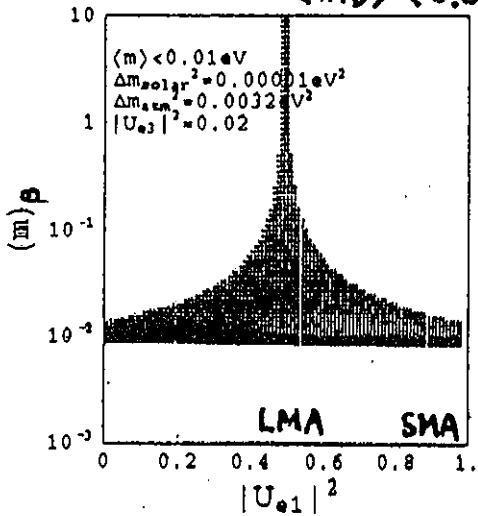
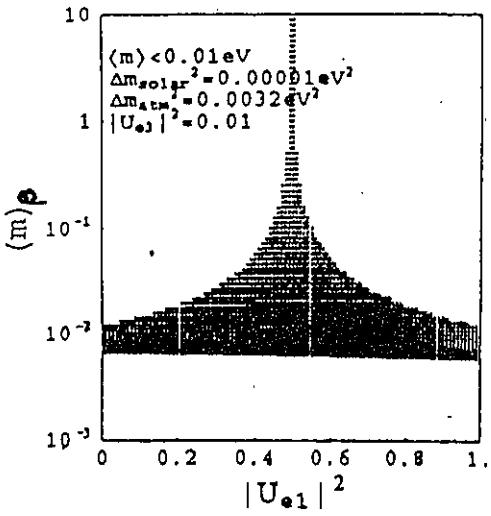
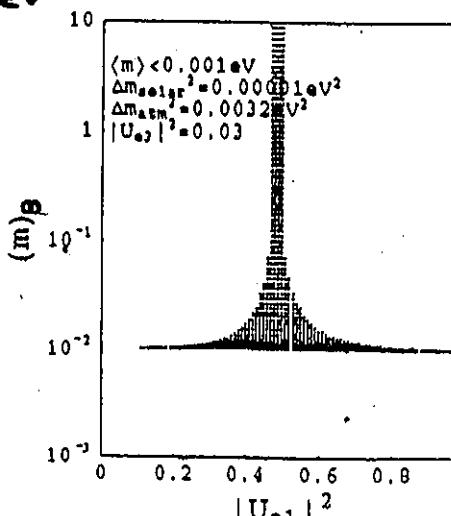
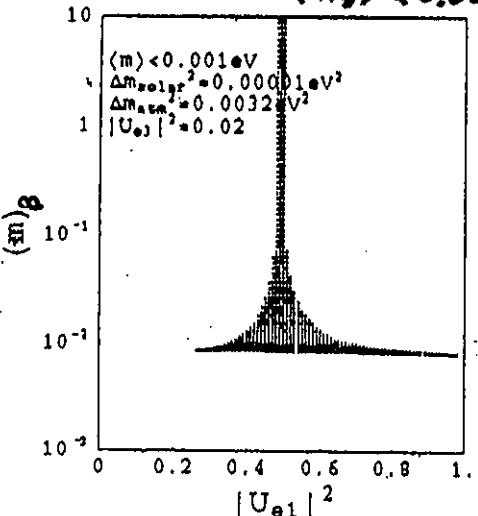
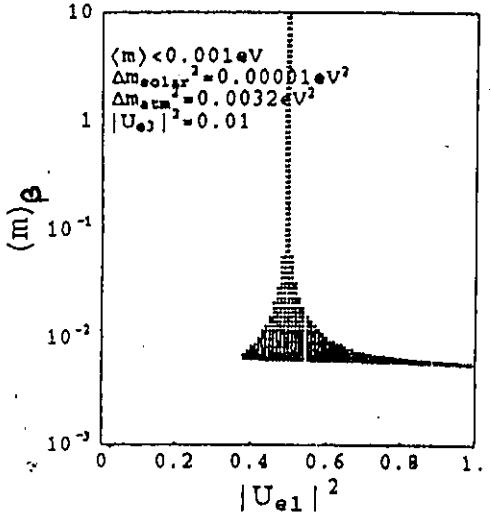
$$\max \left\{ \frac{(m_\nu)_\beta^2 - m_1 m_2}{m_3 - m_2}, 0 \right\} \leq \langle m_\nu \rangle \leq \frac{m_3 m_2 + (m_\nu)_\beta^2}{m_3 + m_2}$$



$(m_\nu)_\beta$

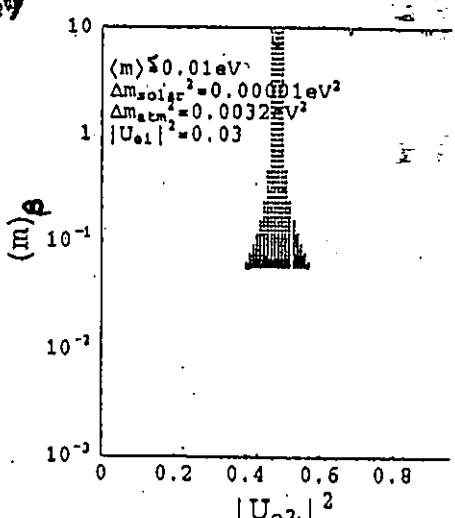
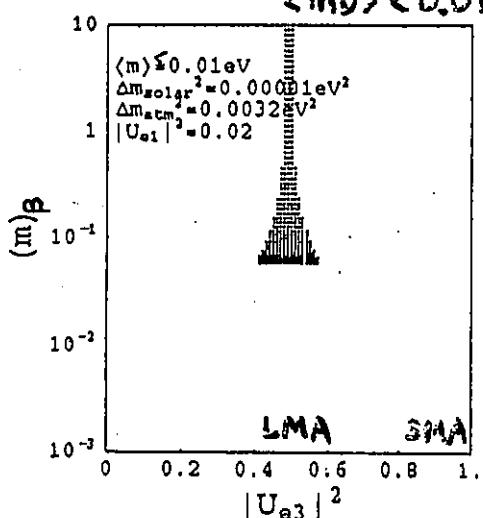
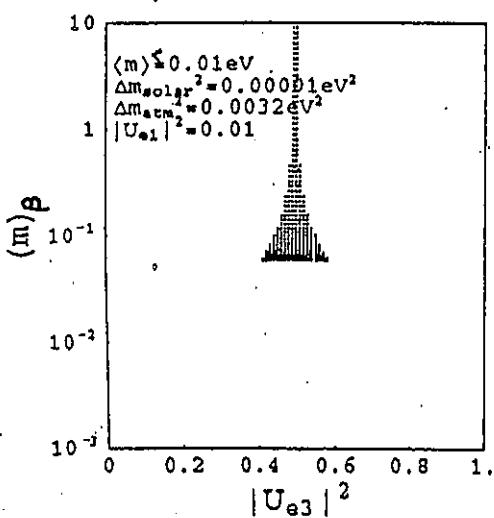
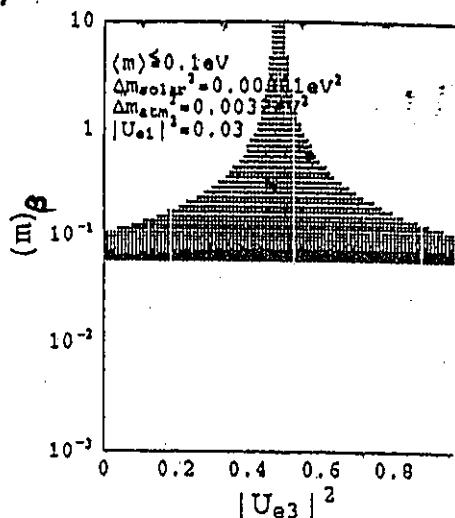
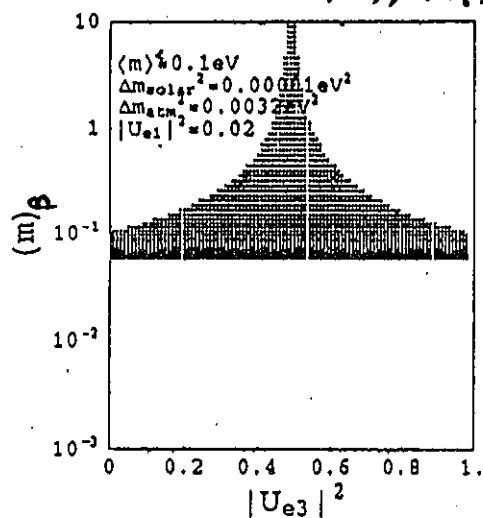
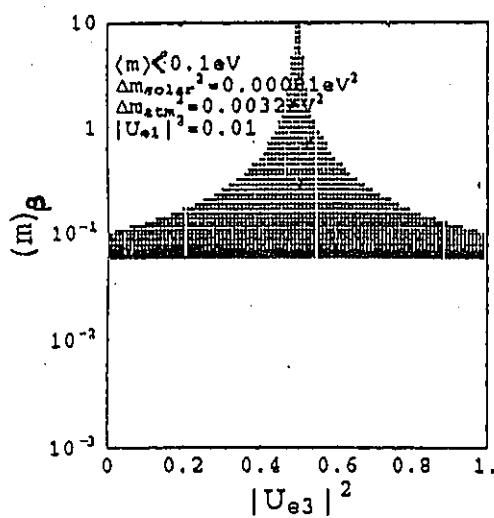
m_3
 m_2
 m_1

MSW

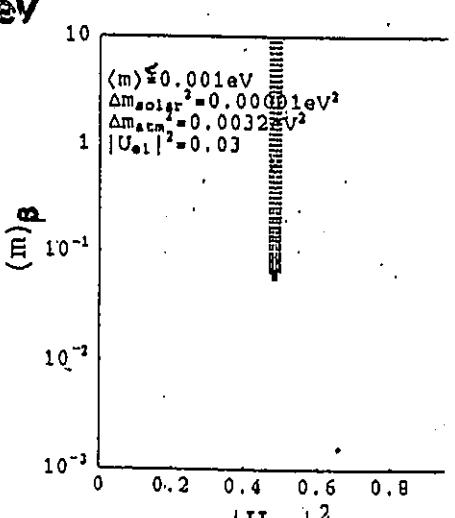
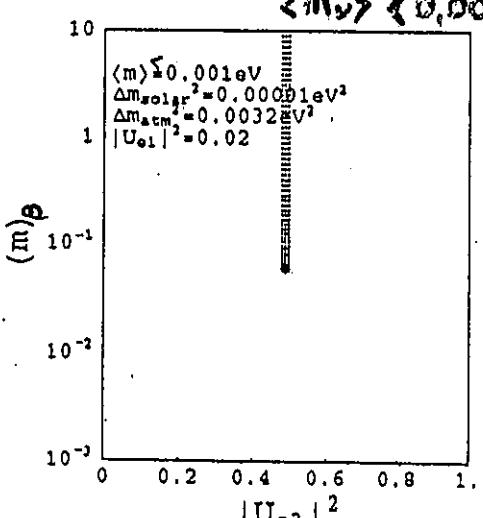
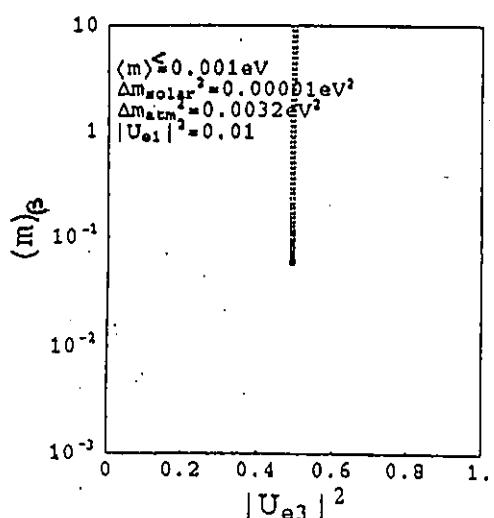
 $\langle m_\nu \rangle < 0.1 \text{ eV}$  $\langle m_\nu \rangle < 0.01 \text{ eV}$  $\langle m_\nu \rangle < 0.001 \text{ eV}$ 

m_3
 m_2

MSW

 m_1  $\langle m_\nu \rangle < 0.01 \text{ eV}$

LMA SMA

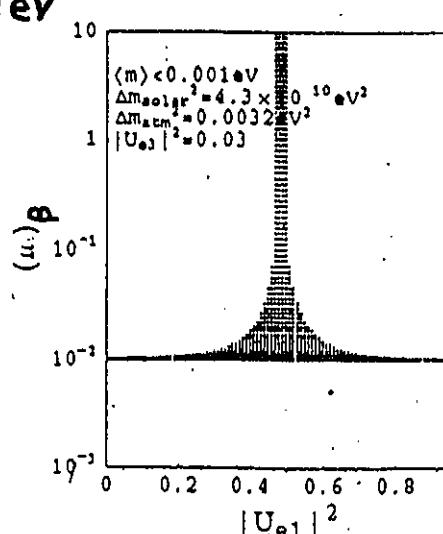
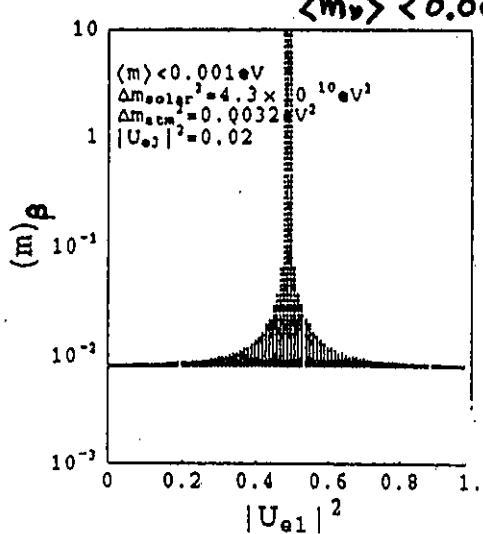
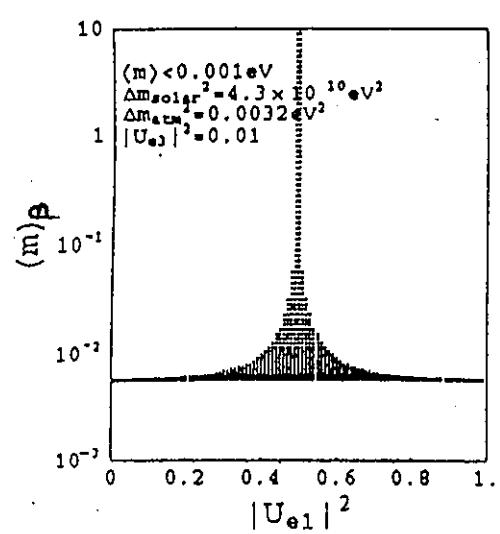
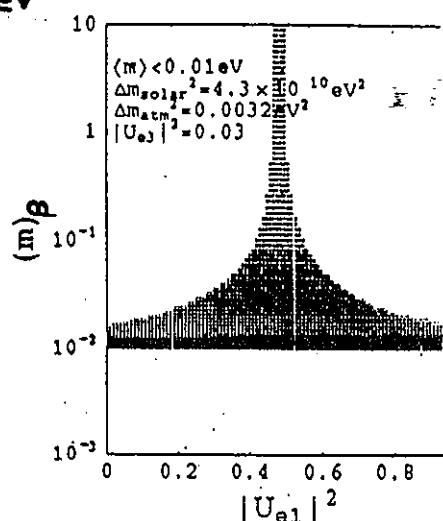
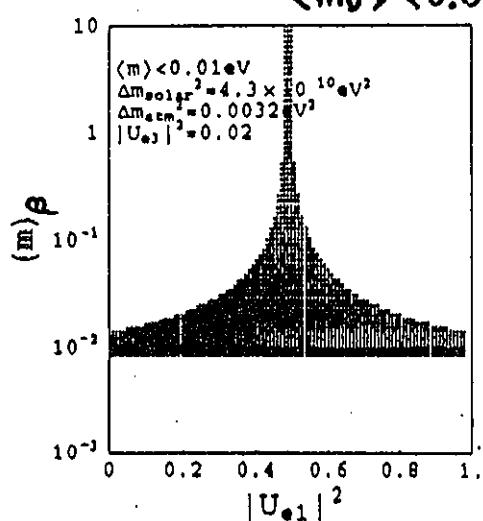
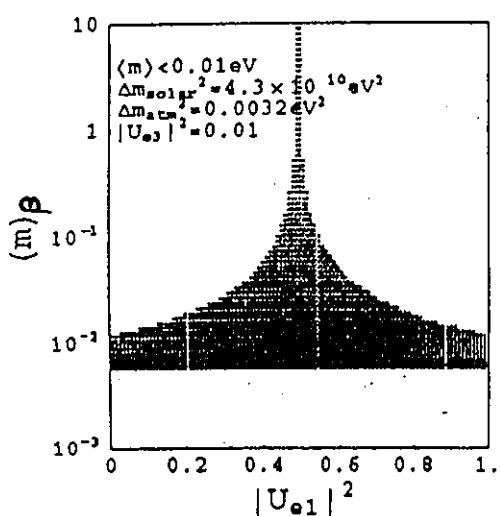
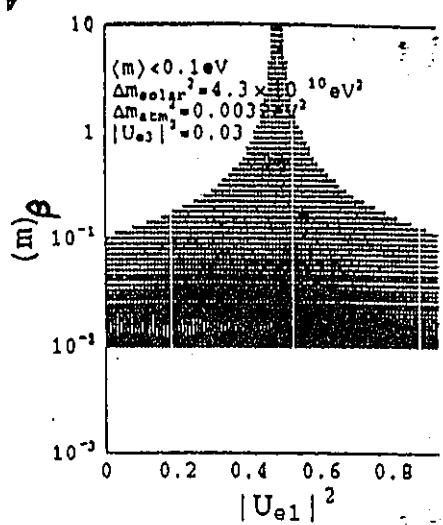
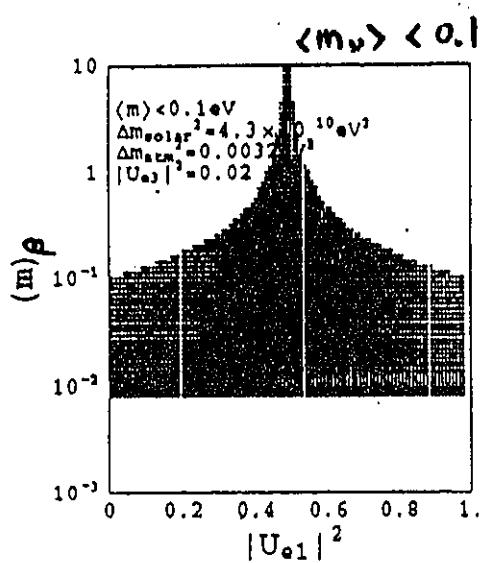
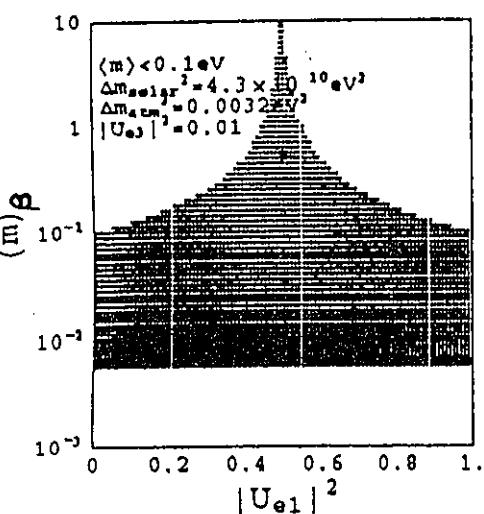
 $\langle m_\nu \rangle < 0.001 \text{ eV}$

SMA solution for the solar neutrino problem
is disfavored for small $\langle m_\nu \rangle$ ($\langle m_\nu \rangle < 0.01 \text{ eV}$)

GENIUS

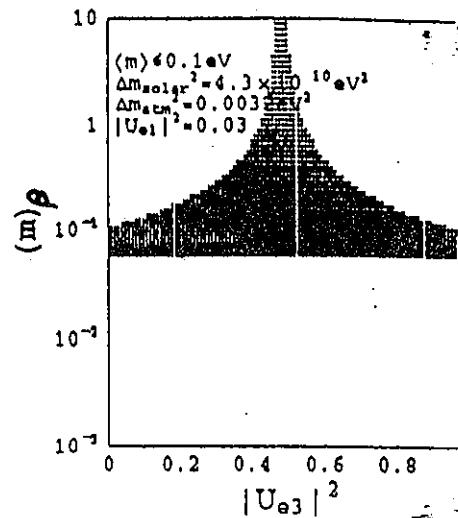
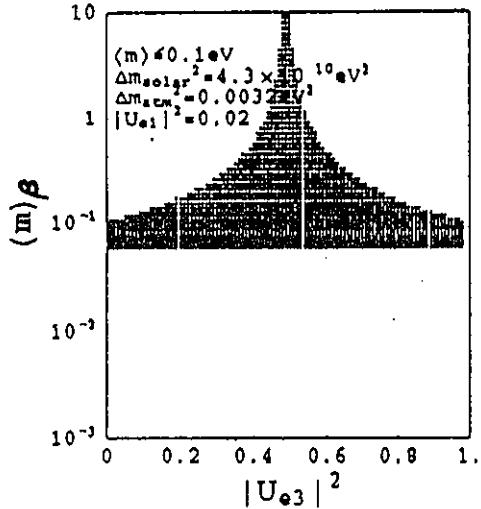
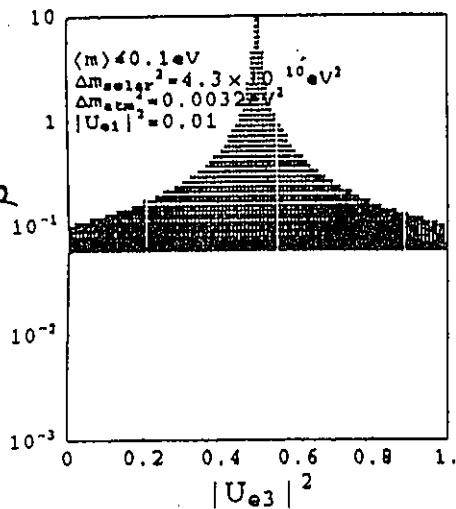
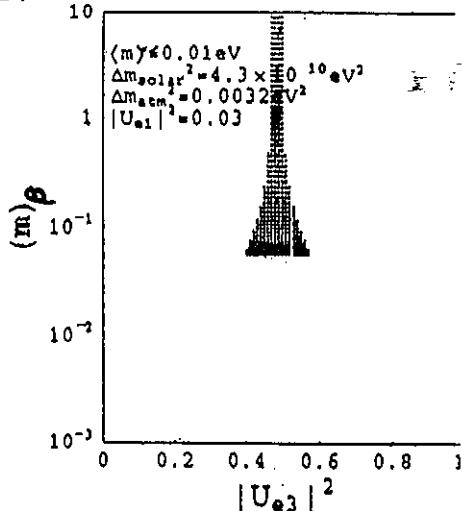
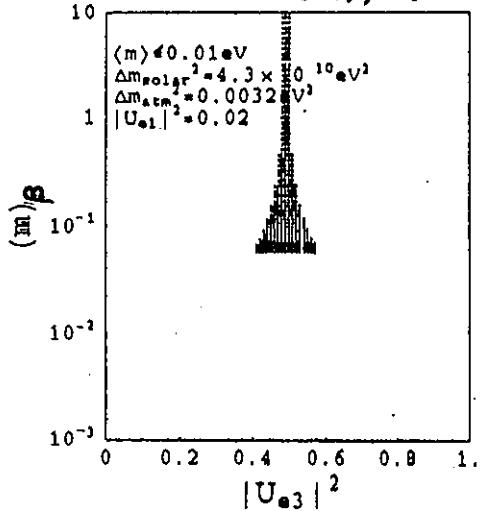
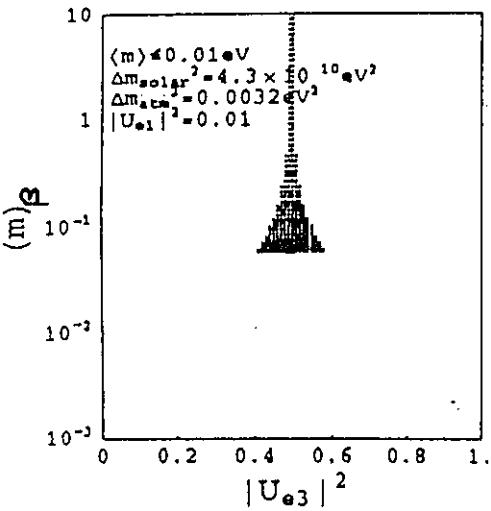
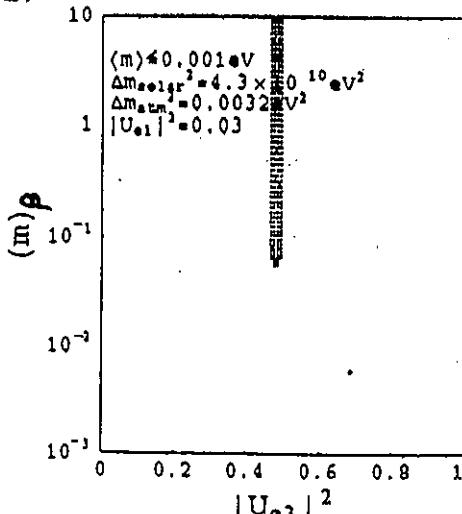
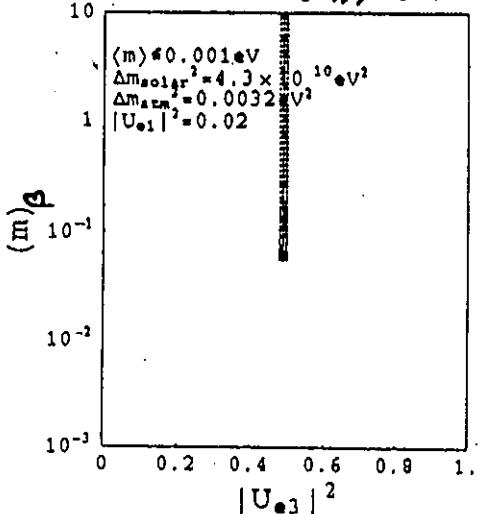
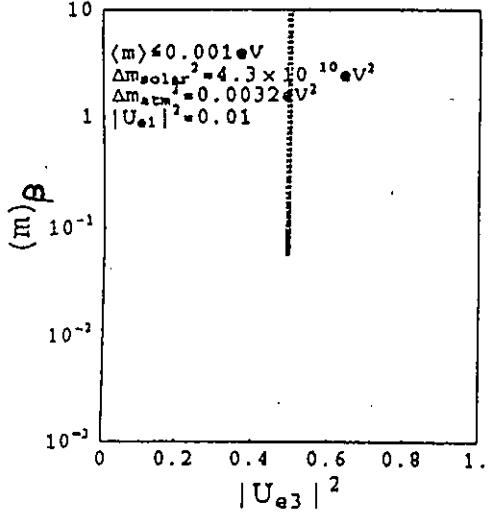
m_3
 m_2
 m_1

Just so



m_3
 m_2

Just so

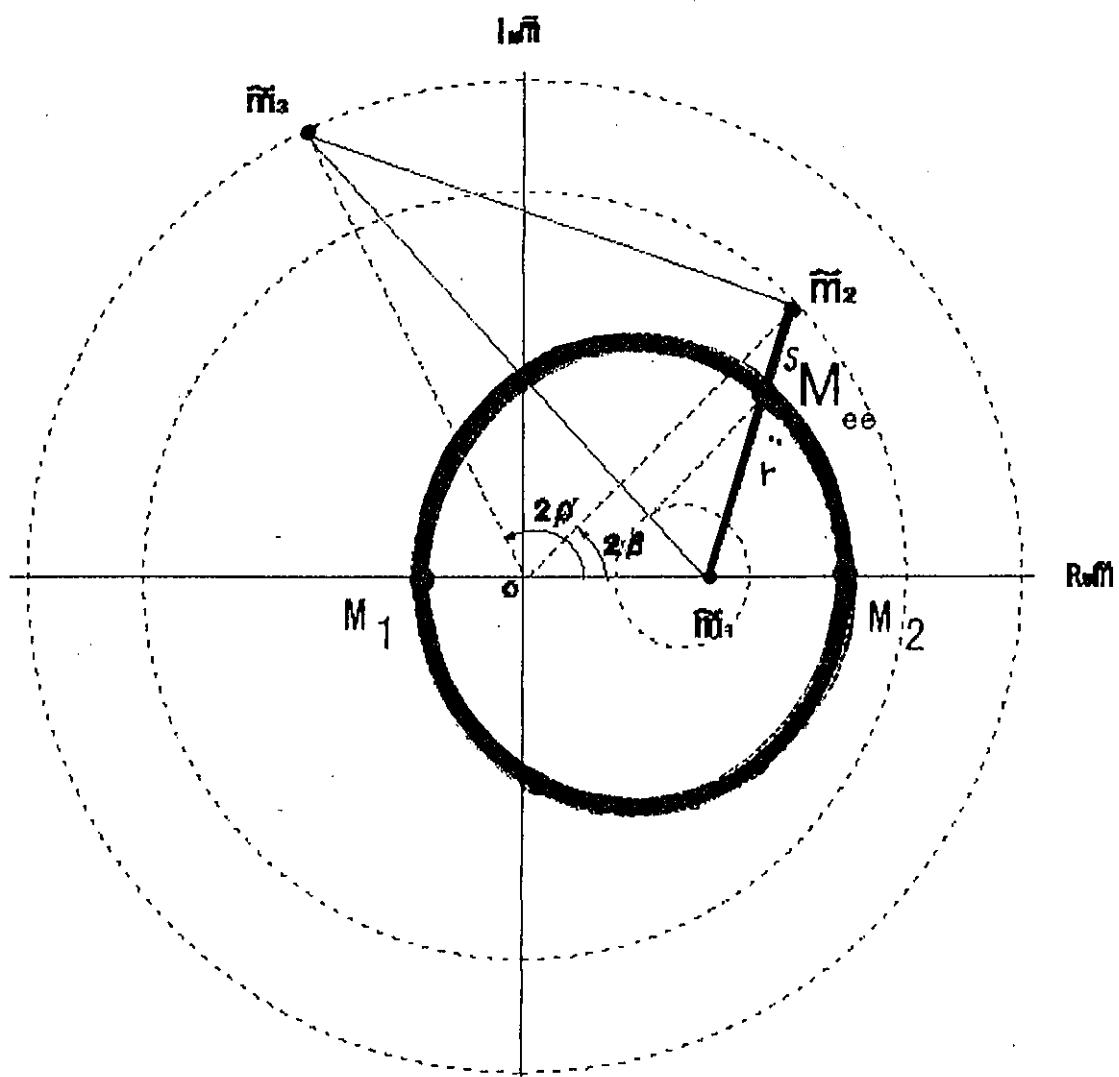
 m_1 $\langle m_\nu \rangle < 0.1 \text{ eV}$  $\langle m_\nu \rangle < 0.01 \text{ eV}$  $\langle m_\nu \rangle < 0.001 \text{ eV}$ 

9. Constraints on CP violating phases from $(\beta\beta)_{0\nu} + \beta$ decay]

- The constraints on the CP violating phases for typical mixing angles.

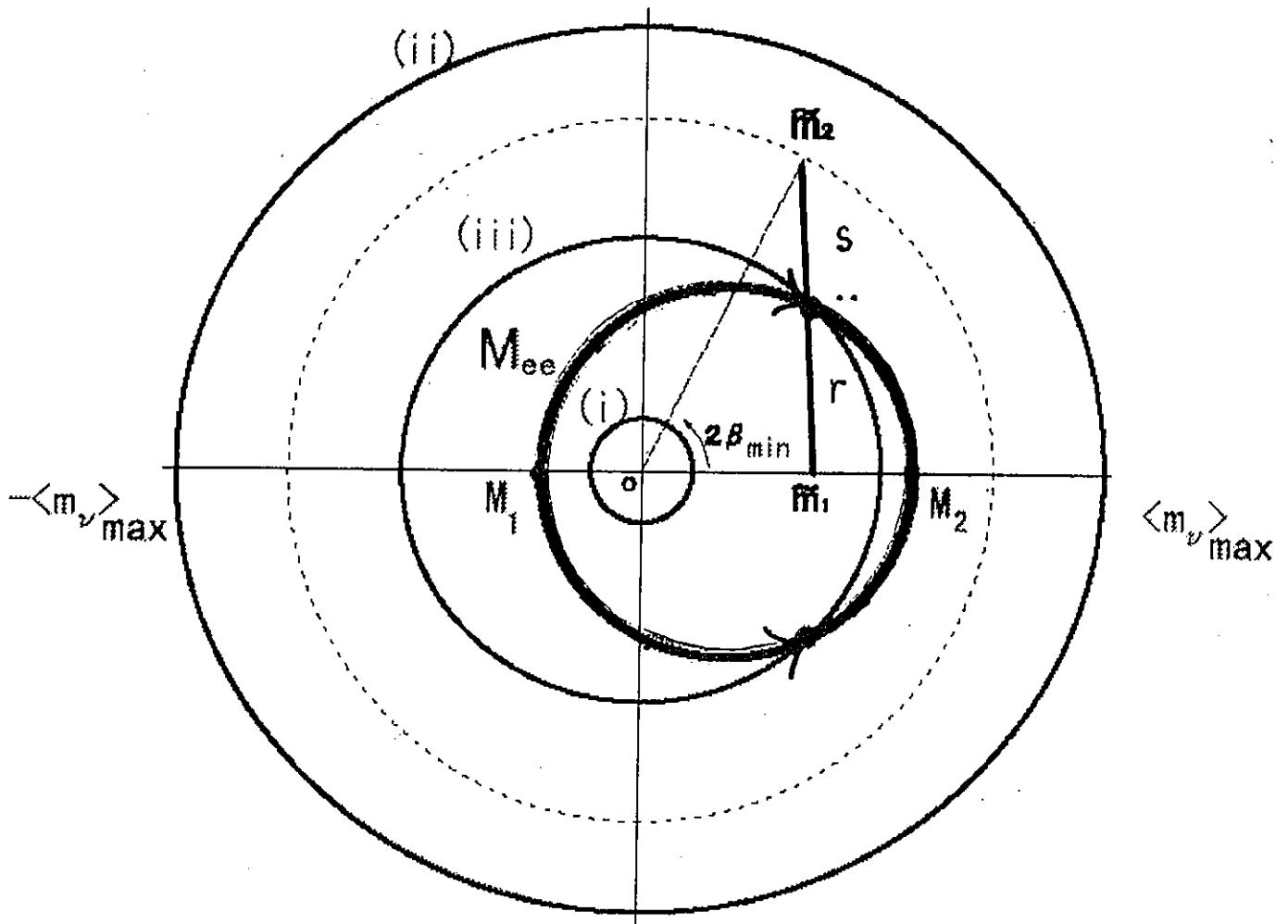
For $|U_{e3}| = 0$ case in (a):

The position of complex mass M_{ee} moves along a circle by changing the CP violating phases β , independently of ρ' .



with
$$M_1 = \frac{m_1 m_2 - (m_\nu)_\beta^2}{m_2 - m_1}$$

$$M_2 = \frac{m_1 m_2 + (m_\nu)_\beta^2}{m_2 + m_1}$$



For the cases

$$(i) \quad -\langle m_\nu \rangle_{\max} \leq -\frac{m_1 m_2 - (m_\nu)_\beta^2}{m_2 - m_1} \quad \Rightarrow \quad \text{excluded}$$

$$(ii) \quad \frac{m_1 m_2 + (m_\nu)_\beta^2}{m_2 + m_1} \leq \langle m_\nu \rangle_{\max} \quad \Rightarrow \quad \text{no constraints on } \beta$$

$$(iii) \quad \frac{m_1 m_2 - (m_\nu)_\beta^2}{m_2 - m_1} \leq \langle m_\nu \rangle_{\max} \leq \frac{m_1 m_2 + (m_\nu)_\beta^2}{m_2 + m_1}$$

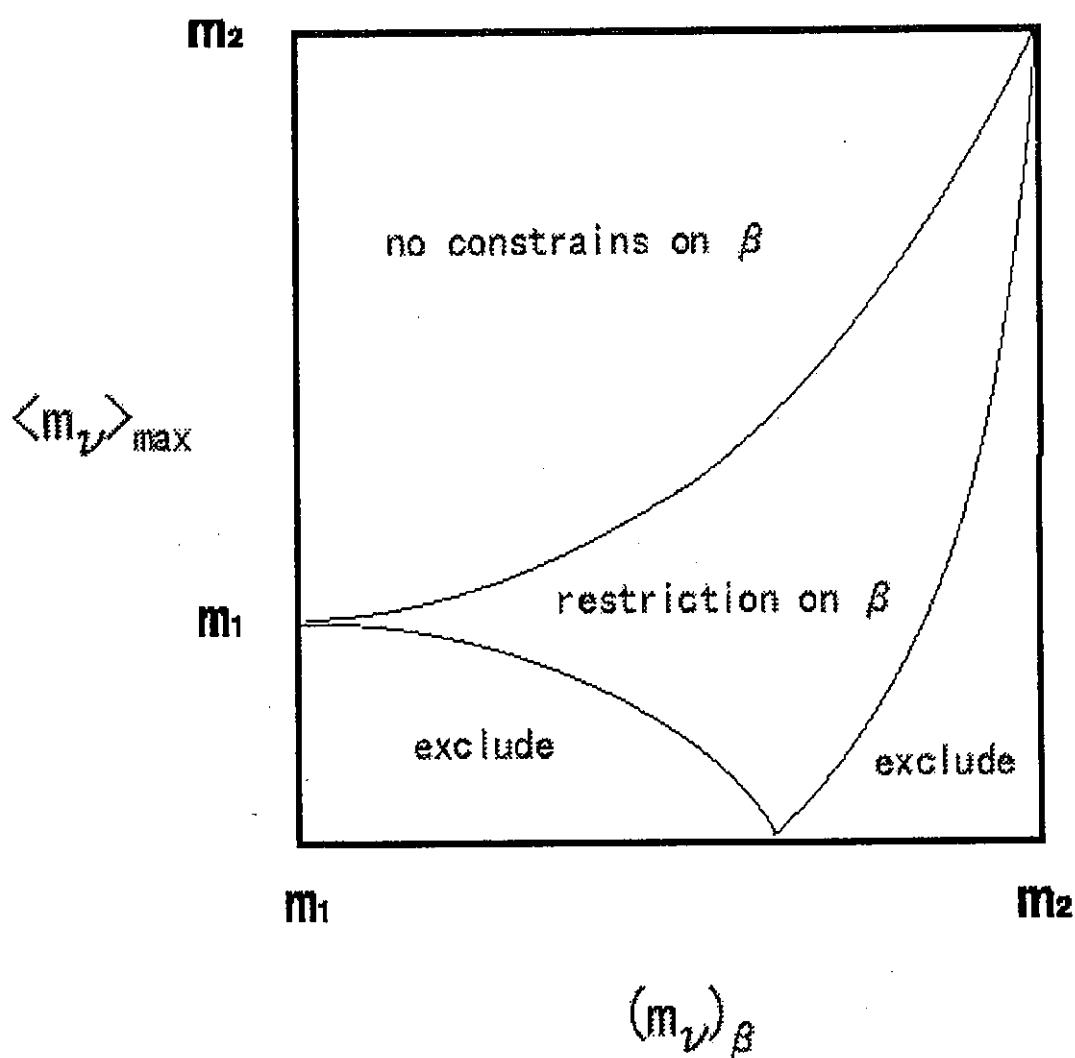
allowed bound on β as:



$$\cos^{-1} \left(\frac{\langle m \rangle_{\max}^2 - (m_1 r)^2 - (m_2 s)^2}{2 m_1 m_2 r s} \right) \leq |2\beta| \leq \pi$$

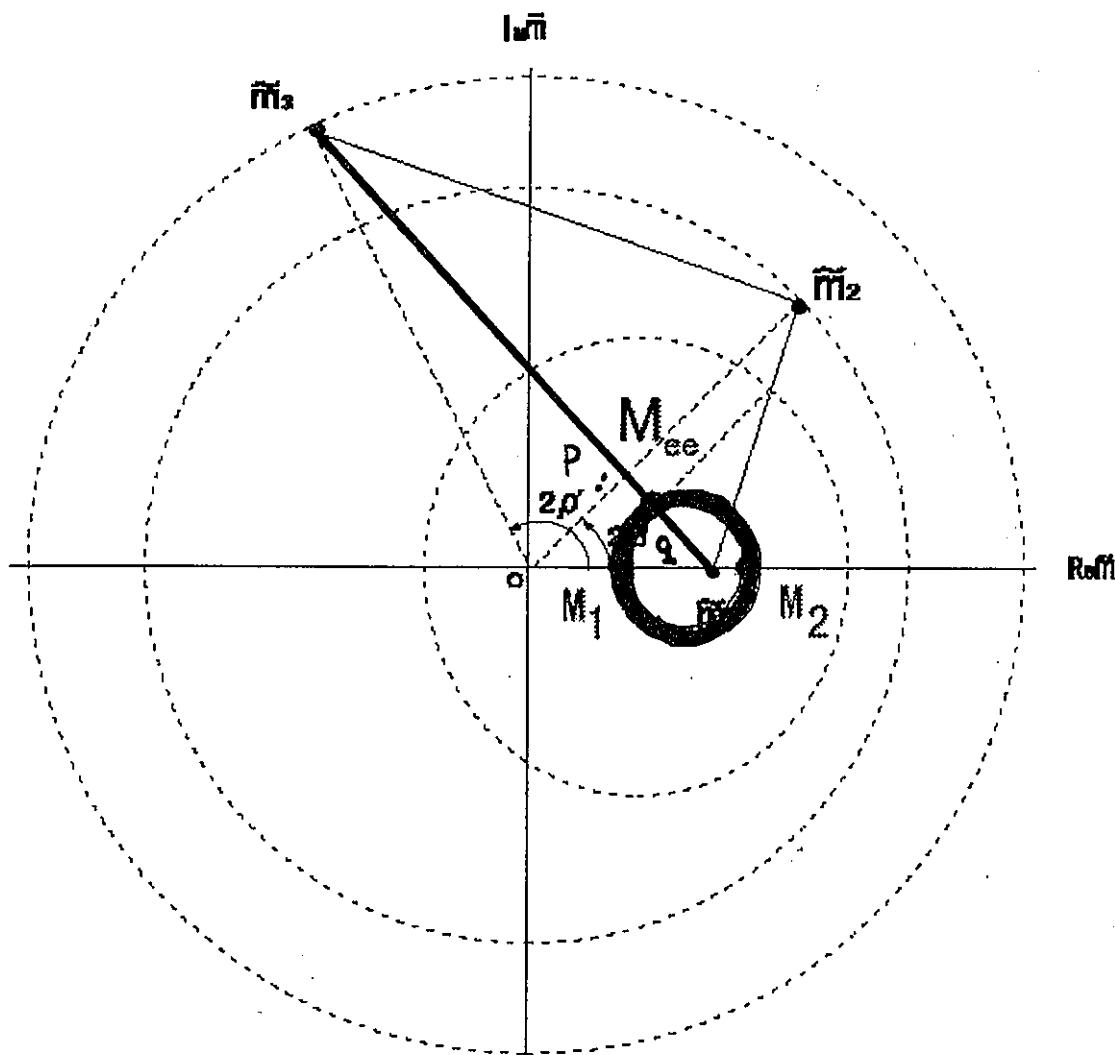
Restriction on CP violating phase β in the $\langle m_\nu \rangle_{\max}$ and $(m_\nu)_\beta$ plane

(a)



For $|U_{e3}|^2 = \frac{(m_\nu)_\beta^2 - m_1^2}{m_3^2 - m_1^2}$ case in (a):

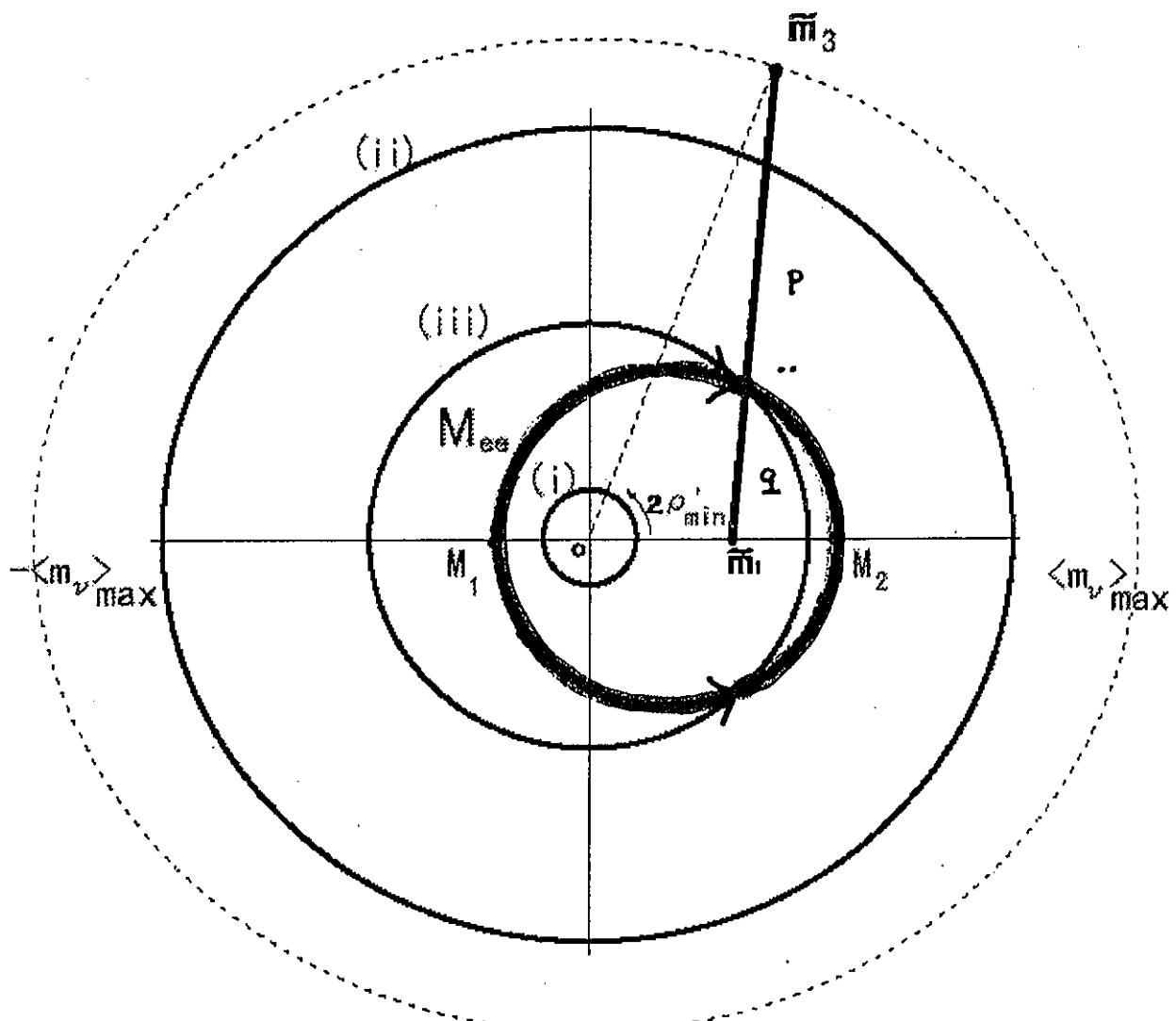
The position of complex mass M_{ee} moves along a circle by changing the CP violating phases ρ' , independently of β ρ' .



with

$$M_1 = \frac{m_1 m_3 - (m_\nu)_\beta^2}{m_3 - m_1}$$

$$M_2 = \frac{m_1 m_3 + (m_\nu)_\beta^2}{m_3 + m_1}$$



For the cases

$$(i) \quad \langle m_\nu \rangle_{\max} \leq -\frac{m_1 m_3 - (m_\nu)_\beta^2}{m_3 - m_1} \quad \Rightarrow \quad \text{excluded}$$

$$(ii) \quad \frac{m_1 m_3 + (m_\nu)_\beta^2}{m_3 + m_1} \leq \langle m_\nu \rangle_{\max} \quad \Rightarrow \quad \text{no constraints on } \beta'$$

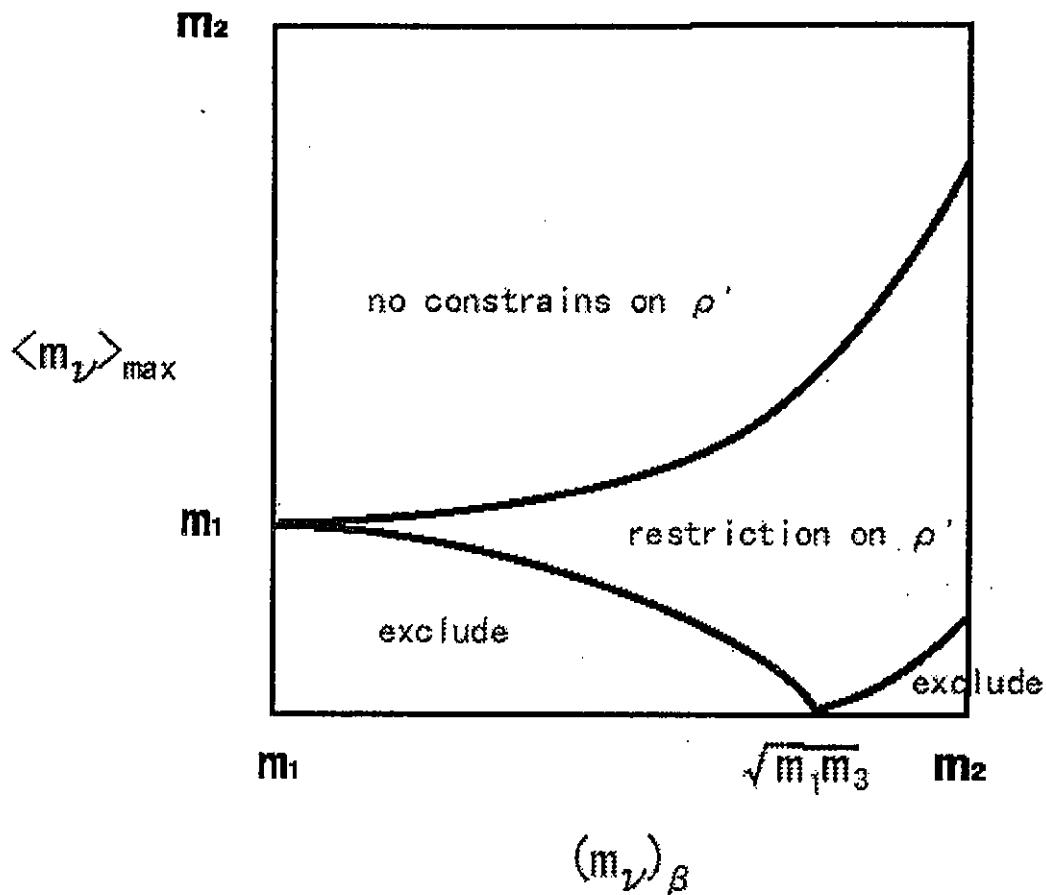
$$(iii) \quad \frac{(m_\nu)_\beta^2 - m_1 m_3}{m_3 - m_1} \leq \langle m_\nu \rangle_{\max} \leq \frac{m_1 m_3 + (m_\nu)_\beta^2}{m_3 + m_1} \quad \Rightarrow$$

allowed bound on ρ' as:

$$\cos^{-1} \left(\frac{\langle m \rangle_{\max}^2 - (m_1 p)^2 - (m_3 q)^2}{2 m_1 m_3 p q} \right) \leq |2\rho'| \leq \pi$$

Restriction on CP violating phase ρ' in the $\langle m_\nu \rangle_{\max}$ and $(m_\nu)_\beta$ plane

(a)



9. Summary

- We have proposed a graphical method ("complex-mass triangle") for obtaining the constraints on Majorana CP violating phases, and mixing angles from the neutrinoless double beta decay $(\beta \beta)_{0\nu}$.
- We have obtained, in terms of neutrino masses, allowed regions for
 - mixing angles S_1^2 vs. S_3^2 (independent of CP phases).
 - CP phases β vs. ρ' (useful only for $\langle m_\nu \rangle$ less than m_1).
- From $(\beta \beta)_{0\nu} + \nu$ oscillation, we have obtained allowed regions of Majorana CP phases, for MSW-LMA solution for solar neutrino problem.
- From $(\beta \beta)_{0\nu} + \beta$ decay, we have obtained allowed regions for their observable neutrino-masses,
 - $\langle m_\nu \rangle$ vs. $(m_\nu)_\beta$ (independently of CP phases).
- From $(\beta \beta)_{0\nu} + \nu$ oscillation + β decay, we have obtained allowed regions of mixing angles for two scenarios for mass hierarchy,

$$m_1 \ll m_2 \sim m_3 \quad \text{and} \quad m_1 \sim m_2 \ll m_3 .$$

It turns out that, for $m_1 \ll m_2 \sim m_3$, the small mixing angle (SMA) solutions for solar neutrino problem is disfavored for small $(\beta \beta)_{0\nu}$ neutrino mass, $\langle m_\nu \rangle < 0.01 \text{eV}$.
- From $(\beta \beta)_{0\nu} + \nu$ oscillation + β decay, we have obtained allowed regions for Majorana CP phases for fixed mixing angles.