"Neutrino mass spectrum and neutrinoless double β decay" (review)

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hep-ph/0003219

1. Introduction
2. Constraints from $\nu_\odot$, ν_{atm}, reactors etc.
3. Schemes with normal hierarchy
4. Schemes with partial degeneracy
5. Schemes with complete degeneracy
6. Transition regions
7. Scheme with inverse mass hierarchy
8. Four neutrino scenarios
9. Summary

$\nu=3$: items 3–7

$\nu=4$: item 8

1. Introduction

$$\nu_e = \sum_j U_{ej} \nu_j$$

$$m_{ee} = \left| \sum_j |U_{ej}|^2 e^{i\varphi_j} m_j \right|$$

$$= \left| \sum_j \underbrace{e^{i(\varphi_j - \varphi_1)} |U_{ej}|^2 \sqrt{m_1^2 + \Delta m_{j1}^2}}_{\equiv\, m_{ee}^{(j)}} \right|$$

$\varphi_1 \equiv 0$ (without loss of generality)

present experimental upper bound

$m_{ee} \lesssim 0.2$ eV Heideberg - Moscow '99

2. Constraints from $\nu_\odot$, ν_{atm}, reactors etc.

* $\nu_\odot$

$\nu_\odot$ sol.	$\Delta m_\odot^2$	$\sin^2 2\theta_\odot$
LMA	$(0.1 \sim 1.5)\times 10^{-4}$ eV^2	$0.53 \sim 1$
LOW	$(0.3 \sim 2.5)\times 10^{-7}$ eV^2	$0.8 \sim 1$
SMA	$(0.4 \sim 1.0)\times 10^{-5}$ eV^2	$(0.2 \sim 1.2)\times 10^{-2}$
quasi VO	$10^{-8} \sim 10^{-9}$ eV^2	~ 1

* ν_{atm}

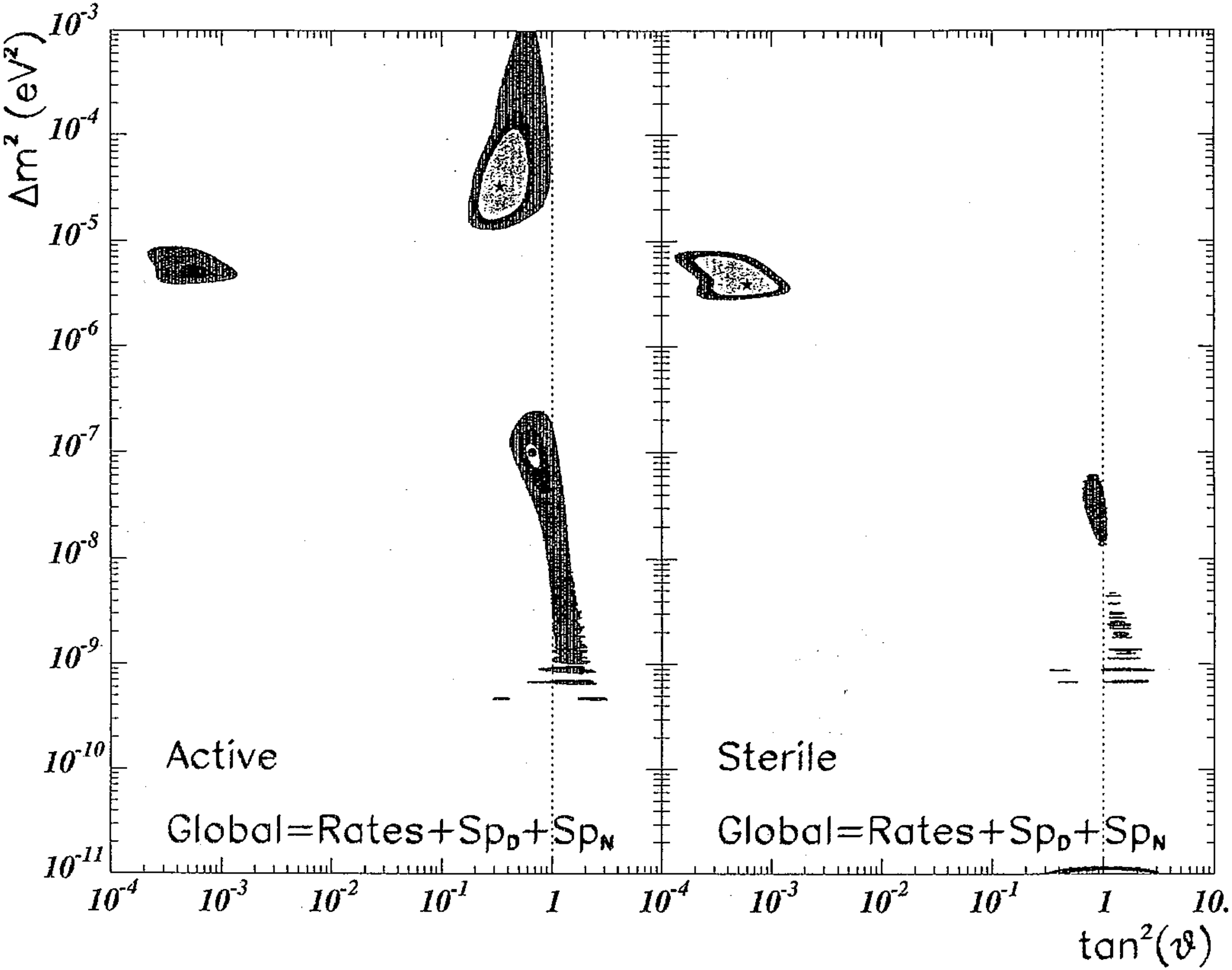
almost pure $\nu_\mu \leftrightarrow \nu_\tau$

$$\sin^2 2\theta_{atm} = 0.88 \sim 1.0$$

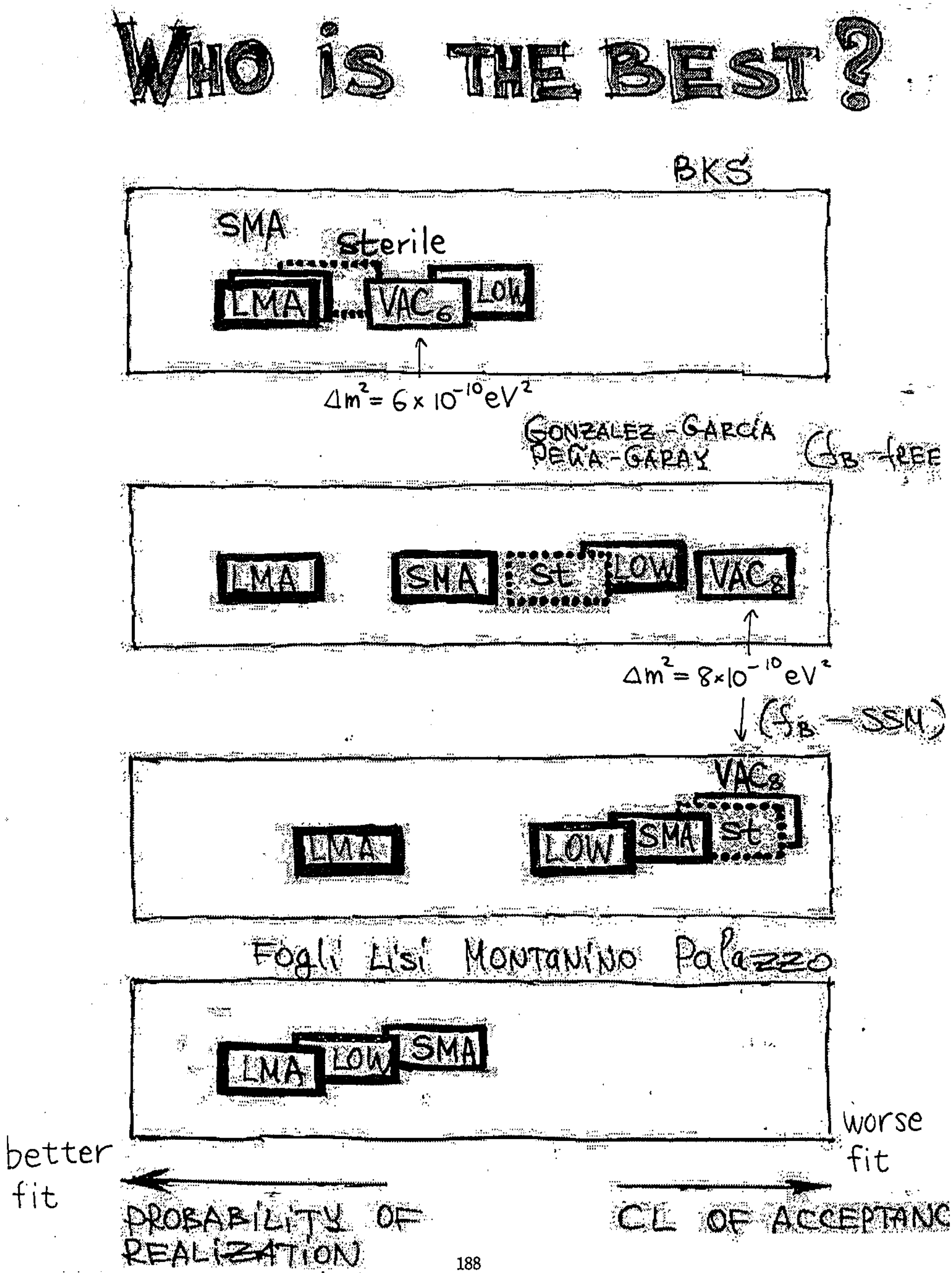
$$\Delta m_{atm}^2 = (2 \sim 6) \times 10^{-3} \text{ eV}^2$$

2.1

Gonzalez- Garcia - Peña - Garay
hep-ph/0009041



http://www.ba.infn.it/~now2000/views/slides/Smirnov/slide16.jpg R.



* reactors

$N_\nu = 3 \qquad |U_{e3}|^2 \lesssim \frac{1}{40} \qquad$ CHOOZ

$N_\nu = 4 \qquad$ (3+1) - scheme $\qquad |U_{e4}|^2 \lesssim \frac{1}{100} \qquad$ Bugey

$\qquad\qquad\qquad\qquad\qquad\qquad |U_{e3}|^2 \lesssim \frac{1}{40} \qquad$ CHOOZ

$\qquad\qquad$ (2+2) - scheme $\qquad |U_{e3}|^2 + |U_{e4}|^2 \lesssim \frac{1}{100} \qquad$ Bugey

* cosmology

large scale structure of the universe

$\sum_j m_j \lesssim 2\,\mathrm{eV} \qquad$ Fukugita - Liu - Sugiyama '99

$\qquad < 4\,\mathrm{eV} \qquad$ Hu et al.

$N_\nu = 3$ schemes & $N_\nu = 4$ schemes

$*\ N_\nu = 3 \quad (\nu_\odot + \nu_{atm})$ or

$$\begin{Bmatrix} \nu_\odot : \nu_e \leftrightarrow \frac{1}{\sqrt{2}}(\nu_\mu - \nu_\tau) \\ \nu_{atm} : \nu_\mu \leftrightarrow \nu_\tau \end{Bmatrix} \qquad U_{MNS} \simeq \begin{pmatrix} C_\odot & S_\odot & \epsilon' \\ -S_\odot/\sqrt{2} & C_\odot/\sqrt{2} & 1/\sqrt{2} \\ S_\odot/\sqrt{2} & -C_\odot/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

$$|\epsilon'|^2 \lesssim \frac{1}{40}$$

$*\ N_\nu = 4 \quad (\nu_\odot + \nu_{atm} + \nu_{LSND})$

(a) (3+1) − scheme (@ 99% CL of LSND) or

$\Delta m^2_{LSND} = 0.9,\ 1.7,\ 6.0\ eV^2$

$$U_{MNS} \simeq \begin{pmatrix} \boxed{\begin{matrix} C_\odot & S_\odot & \epsilon' \\ -S_\odot/\sqrt{2} & C_\odot/\sqrt{2} & 1/\sqrt{2} \\ S_\odot/\sqrt{2} & -C_\odot/\sqrt{2} & 1/\sqrt{2} \end{matrix}}^{\;/\!/\; U_{MNS}(N_\nu=3)} & \begin{matrix} \epsilon \\ \delta \\ 0 \end{matrix} \\ \begin{matrix} \frac{\delta}{\sqrt{2}} - \frac{\epsilon}{\sqrt{2}} & -\frac{\delta}{\sqrt{2}} - \frac{\epsilon}{\sqrt{2}} & -\frac{\delta}{\sqrt{2}} \end{matrix} & 1 \end{pmatrix} \Rightarrow$$

almost the same phenomenology as $N_\nu = 3$ scheme

(b) (2+2) − scheme (@ 90% CL of LSND)

$$U_{MNS} = \begin{pmatrix} C_\odot & S_\odot & \epsilon'' & \epsilon''' \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{pmatrix} \qquad |\epsilon''|^2 + |\epsilon'''|^2 \lesssim \frac{1}{100}$$

$\nu_\odot$: hybrid of $\begin{pmatrix} \nu_e \leftrightarrow \nu_{active} \\ \nu_e \leftrightarrow \nu_s \end{pmatrix}$ $\quad C_s = 1 \Leftrightarrow$ pure $\nu_e \leftrightarrow \nu_s$

disfavored by SK

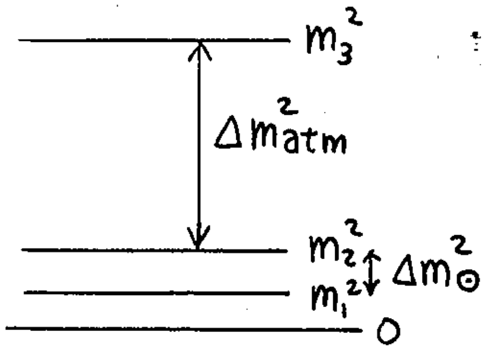
ν_{atm} : hybrid of $\begin{pmatrix} \nu_\mu \leftrightarrow \nu_\tau \\ \nu_\mu \leftrightarrow \nu_s \end{pmatrix}$ $\quad C_s = 0 \Leftrightarrow$ pure $\nu_\mu \leftrightarrow \nu_s$

$$\boxed{C_s \equiv |U_{s1}|^2 + |U_{s2}|^2} \qquad \nu_\odot + \nu_{atm} \Rightarrow \begin{Bmatrix} \text{LMA}: & 0.1 \lesssim C_s \lesssim 0.4 \\ \text{LOW}: & 0.1 \lesssim C_s \lesssim 0.2 \\ \text{SMA}: & 0.1 \lesssim C_s \lesssim 0.8 \end{Bmatrix}$$

3. Schemes with normal mass hierarchy

$$m_1^2 \ll \Delta m_{21}^2 \ll \Delta m_{31}^2$$

$$\rightarrow \quad m_3 \simeq \sqrt{\Delta m_{atm}^2}$$
$$m_2 \simeq \sqrt{\Delta m_\odot^2}$$
$$m_1 \lesssim \frac{1}{10}\sqrt{\Delta m_\odot^2}$$



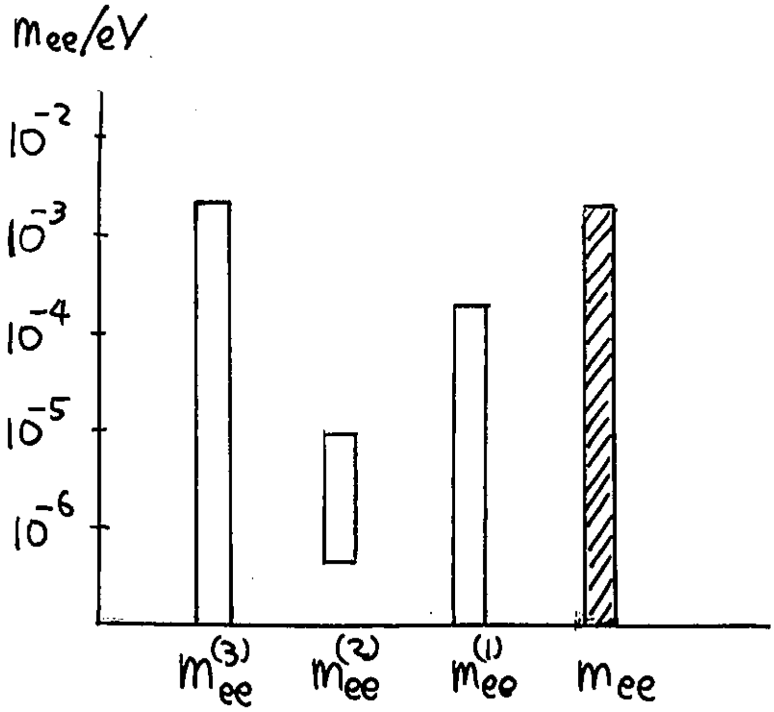
3.1 single maximal (large) mixing ($\nu_\odot$: SMA)

$$m_{ee}^{(3)} \simeq \underbrace{|U_{e3}|^2}_{\lesssim \frac{1}{40}} \underbrace{\sqrt{\Delta m_{atm}^2}}_{\sim 6\times 10^{-2}\,eV} \lesssim 2\times 10^{-3}\,eV$$

$$m_{ee}^{(2)} \simeq S_\odot^2 \sqrt{\Delta m_\odot^2} = (5\times 10^{-7} \sim 10^{-5})\,eV$$

$$m_{ee}^{(1)} = C_\odot^2\, m_1 \simeq m_1 \ll m_2 < 2\times 10^{-3}\,eV$$

$$\therefore \begin{cases} m_{ee}^{max} = (2\sim 3)\times 10^{-3}\,eV \\ m_{ee}^{min} = 0 \end{cases}$$



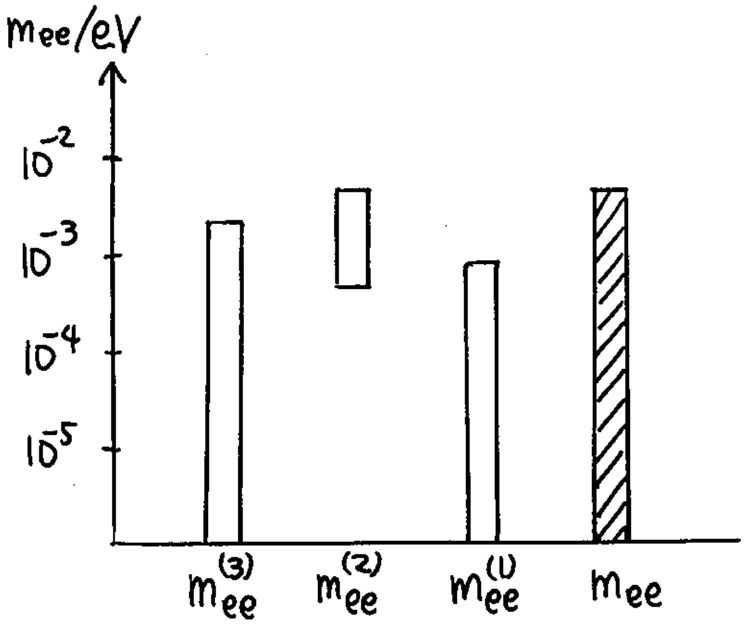
3.2 Bi-large mixing ($\nu_\odot$: LMA)

$$m_{ee}^{(3)} = |U_{e3}|^2 \sqrt{\Delta m_{atm}^2} < 2 \times 10^{-3}\,\mathrm{eV}$$

$$m_{ee}^{(2)} = \frac{1}{2}\left(1-\sqrt{1-\sin^2 2\theta_\odot}\right)\sqrt{\Delta m_\odot^2} = (0.4 \sim 4) \times 10^{-3}\,\mathrm{eV}$$

$$m_{ee}^{(1)} \simeq c_\odot^2\, m_1 < 1 \times 10^{-3}\,\mathrm{eV} \qquad \boxed{c_\odot^2 = 0.5 \sim 0.84}$$

$$\therefore \begin{cases} m_{ee}^{max} = 7 \times 10^{-3}\,\mathrm{eV} \\ m_{ee}^{min} = 0 \end{cases}$$

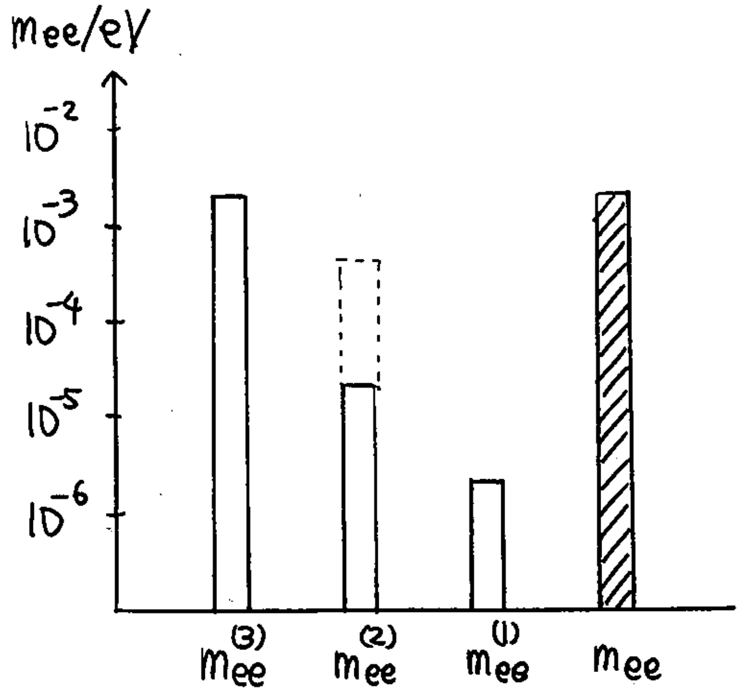


3.3 Scheme with vacuum oscillation solution ($\nu_\odot$: VO or LOW)

$$
\begin{cases}
\text{VO} & m_{ee}^{(2)} = s_\odot^2 \sqrt{\Delta m_\odot^2} < 2\times 10^{-5}\,\text{eV} \\
\text{LOW} & m_{ee}^{(2)} = s_\odot^2 \sqrt{\Delta m_\odot^2} < 3\times 10^{-4}\,\text{eV}
\end{cases}
$$

$$
m_{ee} \simeq m_{ee}^{(3)} < 2\times 10^{-3}\,\text{eV}
$$

$$
\begin{cases}
m_{ee}^{max} = 2\times 10^{-3}\,\text{eV} \\
m_{ee}^{min} = 0
\end{cases}
$$



3.4 Triple maximal mixing scheme

$$\theta_{12} = \theta_{23} = \frac{\pi}{4}, \quad \theta_{13} = \sin^{-1}\frac{1}{\sqrt{3}}, \quad \delta = \frac{\pi}{2}$$

$$U_{MNS} = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1 & -i \\ \omega i & \omega^2 i & 1 \\ \omega^2 i & \omega i & 1 \end{pmatrix} \qquad \omega = e^{\frac{2}{3}\pi i}$$

* $\nu_\odot$ $\quad P_{ee} = 1 - 4|U_{e1}|^2|U_{e2}|^2 \underbrace{\sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)}_{\to 0}$

$\Delta m_{21}^2 \ll 10^{-10}\,\mathrm{eV}^2$ $\quad - 4|U_{e3}|^2(1-|U_{e3}|^2)\underbrace{\sin^2\left(\frac{\Delta m_{32}^2 L}{4E}\right)}_{\to \frac{1}{2}}$

$$= 1 - 4\cdot\frac{1}{3}\cdot\left(1-\frac{1}{3}\right)\cdot\frac{1}{2} = \frac{5}{9}$$

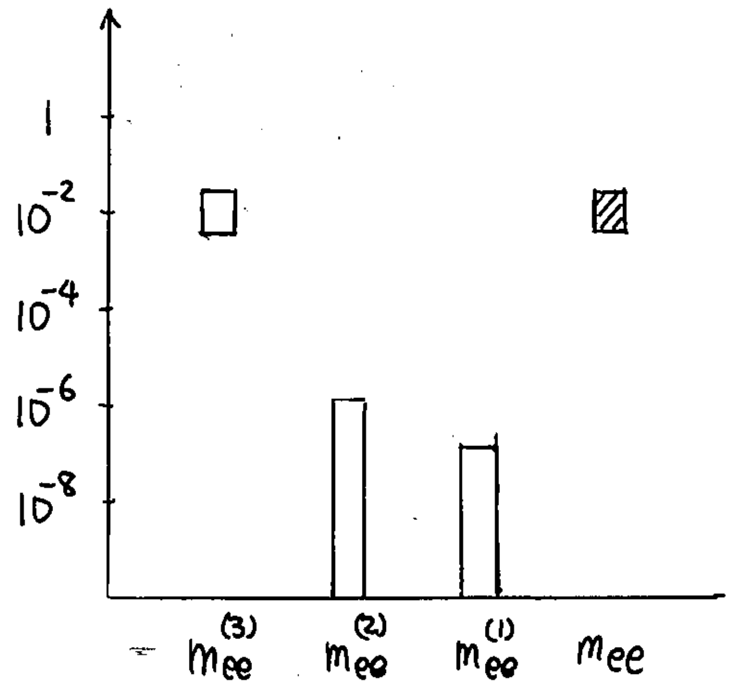
energy independent solution

fit : poor but still $\lesssim 3\sigma$ CL

* ν_{atm} $\quad \Delta m_{atm}^2 \sim 8\times 10^{-4}\,\mathrm{eV}^2$

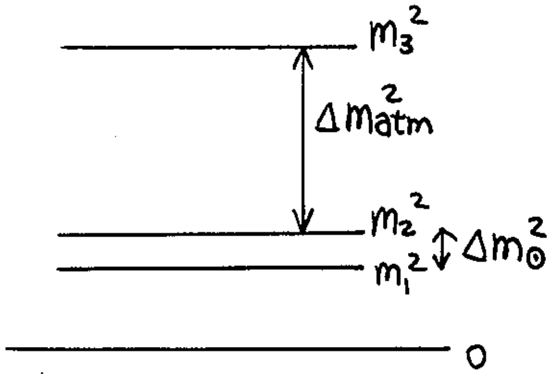
fit : poor but still $\lesssim 3\sigma$ CL

$$m_{ee} \simeq m_{ee}^{(3)} = \frac{1}{3}\sqrt{\Delta m_{atm}^2} \sim 10^{-2}\,\mathrm{eV}$$



4. Schemes with partial degeneracy

$$\Delta m_{21}^2 \ll m_1^2 \ll \Delta m_{32}^2$$



$\longrightarrow m_1 \simeq m_2$

$m_3 \simeq \sqrt{\Delta m^2_{atm}}$

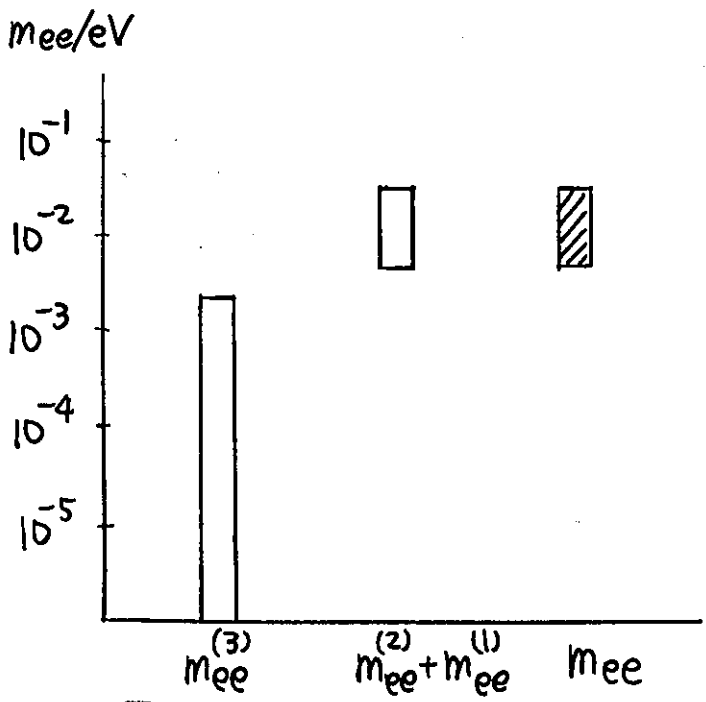
$$0.5\times 10^{-2}\,\mathrm{eV} < m_1 < 3\times 10^{-2}\,\mathrm{eV}$$

$$|m_{ee}^{(1)} + m_{ee}^{(2)}| \simeq m_1 |C_\odot^2 + e^{i\varphi_2} S_\odot| = (\cos 2\theta_\odot \sim 1)\times m_1 \, , \quad m_{ee}^{(3)} = |U_{e3}|^2\sqrt{\Delta m^2_{atm}} \lesssim 2\times 10^{-3}\,\mathrm{eV}$$

① $\nu_\odot$: SMA

$$m_{ee}^{(1)} + m_{ee}^{(2)} \simeq m_1 \lesssim 3\times 10^{-2}\,\mathrm{eV}$$

$$\begin{cases} m_{ee}^{max} \simeq 3\times 10^{-2}\,\mathrm{eV} \\ m_{ee}^{min} \simeq 10^{-3}\,\mathrm{eV} \end{cases}$$



② $\nu_\odot$: LMA , LOW , VO

LMA $\quad m_1 = (1 \sim 3) \times 10^{-2}\,\mathrm{eV}$

$$m_{ee}^{(1)} + m_{ee}^{(2)} \simeq m_1 \left(C_\odot^2 + S_\odot^2 e^{i\varphi_2} \right) + e^{i\varphi_2} \frac{\Delta m_\odot^2}{2m_1} S_\odot^2$$

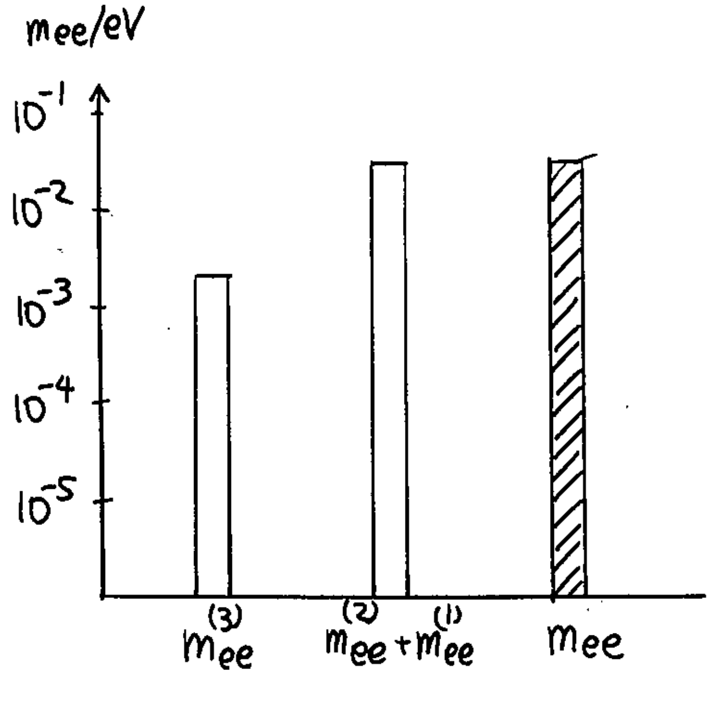
if $1 - \sin^2 2\theta_\odot > 0.1$ then $m_{ee}^{min} = 10^{-3}\,\mathrm{eV}$

LOW $\quad m_1 = (0.1 \sim 3) \times 10^{-2}\,\mathrm{eV}$

$$\begin{cases} m_{ee}^{max} \simeq 3 \times 10^{-2}\,\mathrm{eV} \\ m_{ee}^{min} \simeq 0 \end{cases}$$

VO $\quad m_1 = (10^{-4} \sim 3 \times 10^{-2})\,\mathrm{eV}$

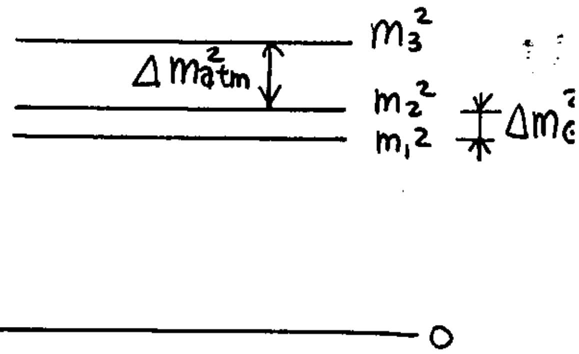
$$\begin{cases} m_{ee}^{max} \simeq 3 \times 10^{-2}\,\mathrm{eV} \\ m_{ee}^{min} \simeq 0 \end{cases}$$



5. Schemes with complete mass degeneracy

$$\Delta m_{21}^2 \ll \Delta m_{32}^2 \ll m_1^2$$

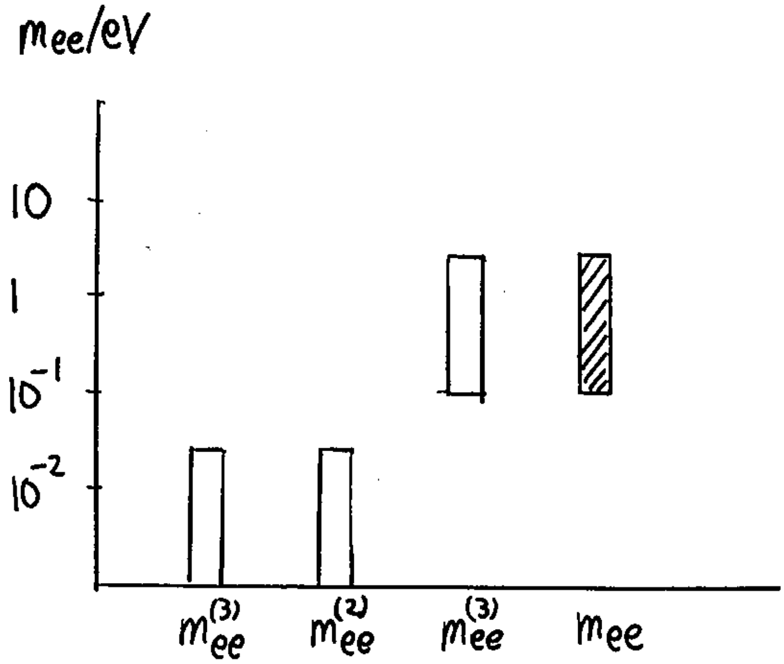
$$\longrightarrow \quad m_1 > 0.1\ \mathrm{eV}$$



① $\nu_\odot$: SMA

$$m_{ee}^{(1)} \simeq C_\odot^2 m_1 \gg \underset{\underset{S_\odot^2 m_1}{\wr\wr}}{m_{ee}^{(2)}}\ , \quad \underset{\underset{|U_{e3}|^2 m_3}{\wr\wr}}{m_{ee}^{(3)}}$$

$$m_{ee} \simeq m_{ee}^{(1)} \simeq m_1 > 0.1\ \mathrm{eV}$$: close to the present bound



② $\nu_\odot$: LMA, LOW, VO

$$m_{ee}^{(1)} + m_{ee}^{(2)} \simeq m_1 (c_\odot^2 + s_\odot^2 e^{i\varphi_2})$$
$$= (\cos 2\theta_\odot \sim 1) \times m_1$$

$$m_{ee}^{(3)} = |U_{e3}|^2 e^{i\varphi_3} m_1$$

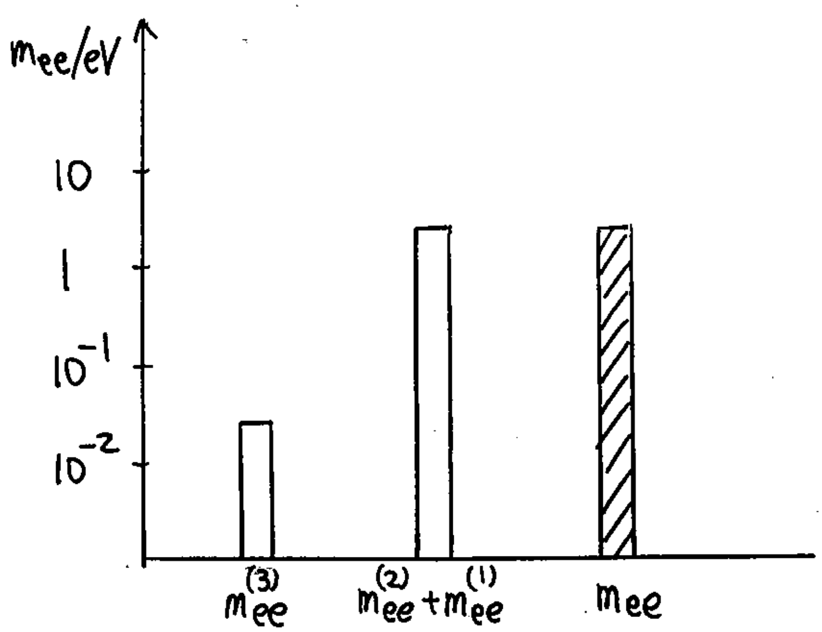
if $||1-\sin^2 2\theta_\odot| > 10^{-3}$

then
$$m_{ee} \simeq |m_{ee}^{(1)} + m_{ee}^{(2)}| = (0.2 \sim 1.0) \times m_1$$
$$\gtrsim 2 \times 10^{-2}\,\text{eV}$$

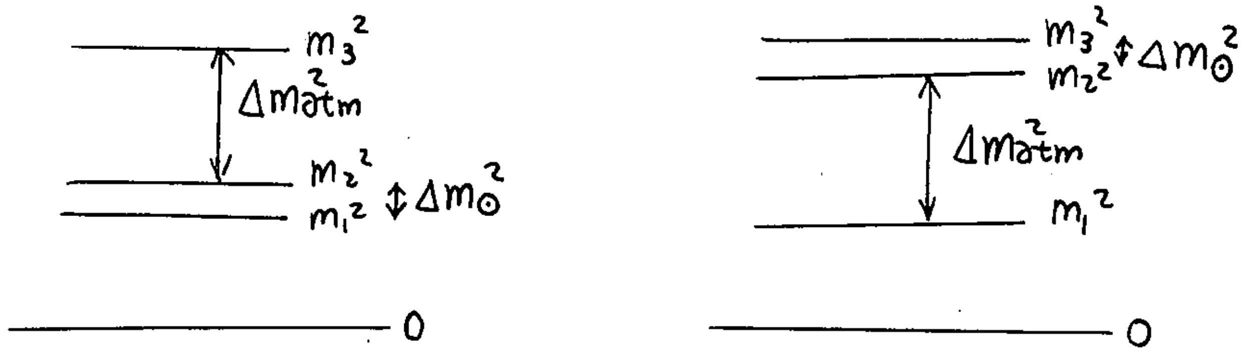
if $U_{e1}^2 = U_{e2}^2$ and $U_{e3}^2 = 0$

then
$$m_{ee} = 0$$

$$\begin{cases} m_{ee}^{max} = m_{ee} \text{ (experimental upper bound)} \\ m_{ee}^{min} = 0 \end{cases}$$



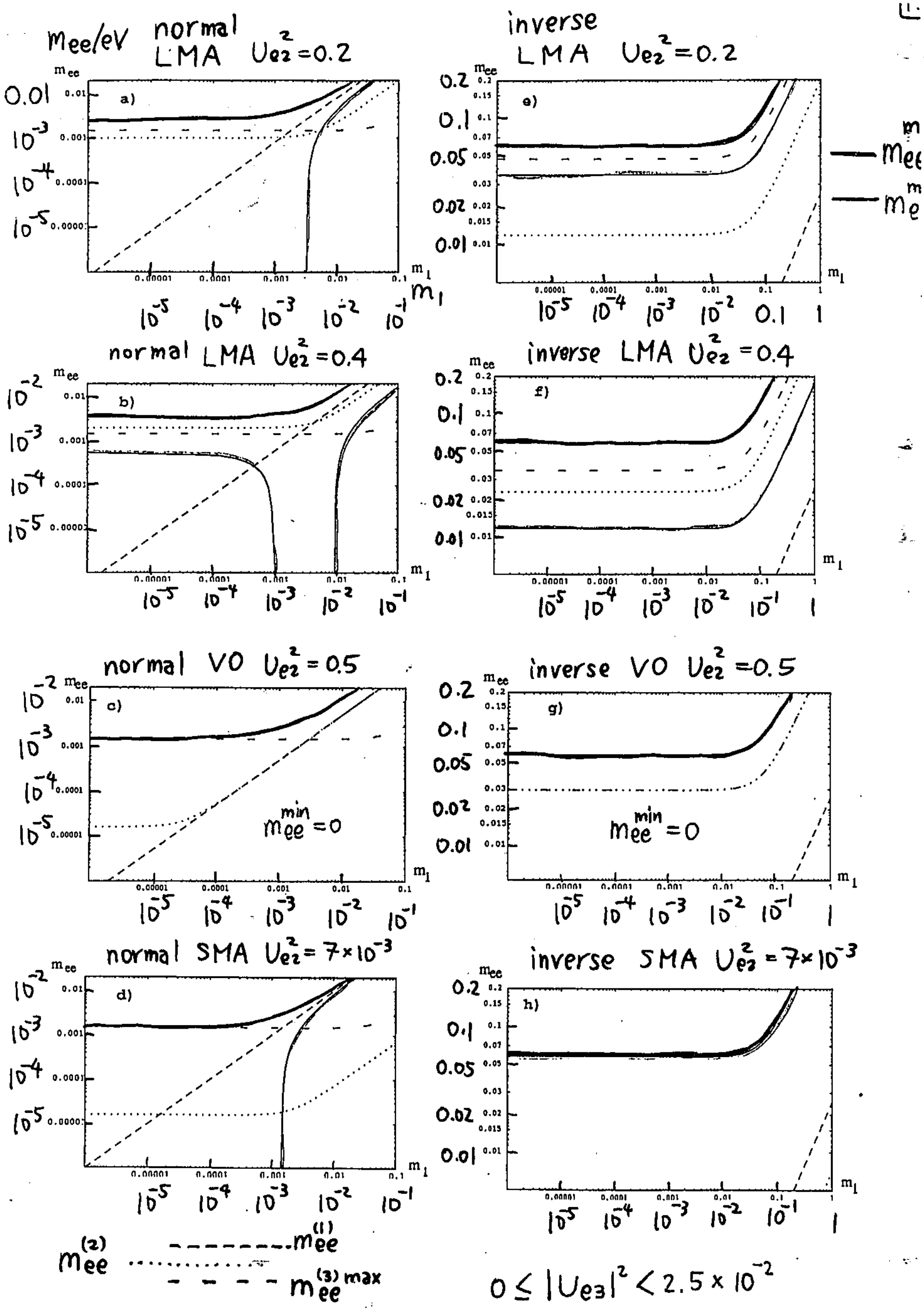
6. Transition regions



$$0 \le m_1^2 < \infty$$

$$m_{ee}^{(3)} = |U_{e3}|^2 \sqrt{\Delta m_{atm}^2} \sqrt{1 + \underbrace{\frac{m_1^2}{\Delta m_{atm}^2}}}$$

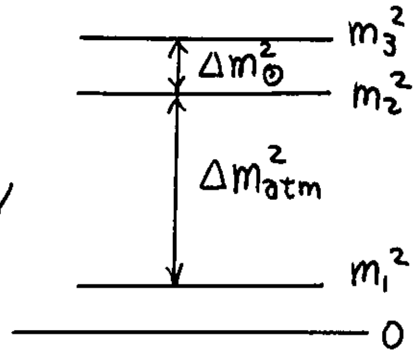
this term can be large as $m_1^2 \to$ large



Mee/eV normal LMA $U_{e2}^2=0.2$
inverse LMA $U_{e2}^2=0.2$
a)
e)
m_1
normal LMA $U_{e2}^2=0.4$
inverse LMA $U_{e2}^2=0.4$
b)
f)
normal VO $U_{e2}^2=0.5$
inverse VO $U_{e2}^2=0.5$
c)
g)
$m_{ee}^{min}=0$
normal SMA $U_{e2}^2=7\times10^{-3}$
inverse SMA $U_{e2}^2=7\times10^{-3}$
d)
h)
$m_{ee}^{(1)}$
$m_{ee}^{(2)}$
$m_{ee}^{(3)max}$
$0 \le |U_{e3}|^2 < 2.5\times10^{-2}$

7. Scheme with inverse mass hierarchy

$$\Delta m^2_{atm} \simeq m_3^2 \simeq m_2^2 \gg m_1^2$$



$$m_1^2 \ll \Delta m^2_{atm} \rightarrow m_1 < 2\times 10^{-2}\,\text{eV}$$

$$\text{for } \frac{m_1^2}{m_2^2} < 0.1$$

$$\text{CHOOZ} : U_{e1}^2 < 2.5\times 10^{-2}$$

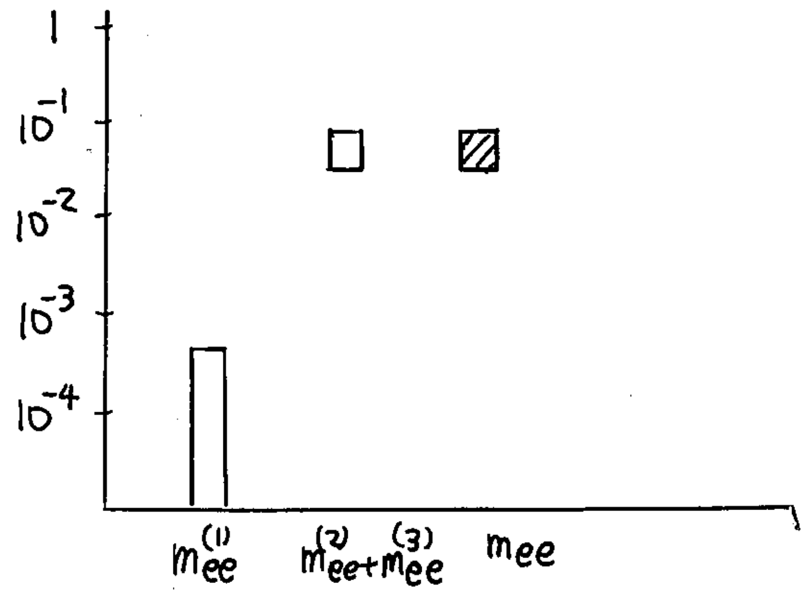
$$m_{ee}^{(1)} < 5\times 10^{-4}\,\text{eV}$$

$$m_{ee}^{(2)} + m_{ee}^{(3)} \simeq \left(s_{\odot}^2 + c_{\odot}^2 e^{i(\varphi_2-\varphi_3)}\right)\sqrt{\Delta m^2_{atm}}$$

① $\nu_{\odot}$: SMA

$$m_{ee} \simeq m_{ee}^{(2)} + m_{ee}^{(3)} \simeq \sqrt{\Delta m^2_{atm}}$$

$$= (4 \sim 8)\times 10^{-2}\,\text{eV}$$



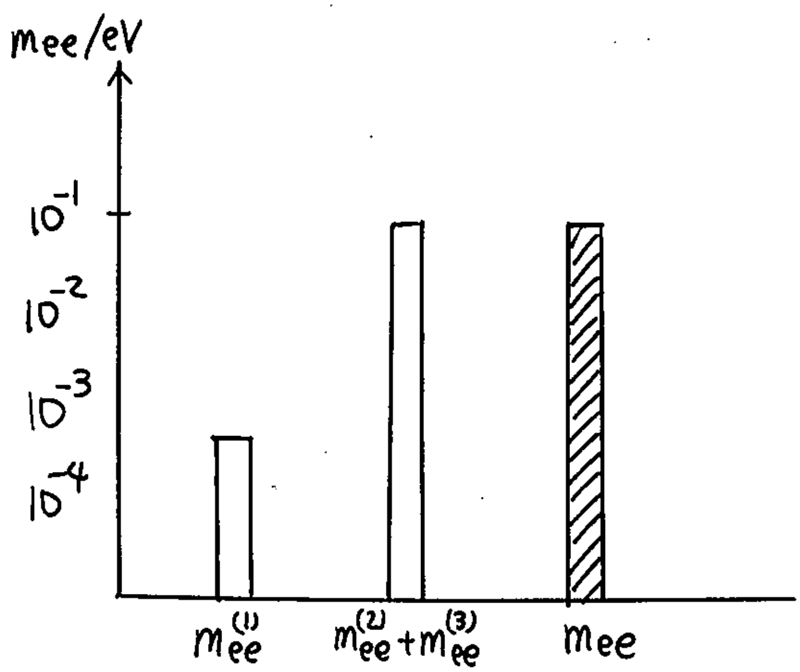
② $\nu_\odot$: LMA, LOW, VO

$$m_{ee}^{(2)} + m_{ee}^{(3)} = (\cos 2\theta_\odot \sim 1) \times \sqrt{\Delta m^2_{atm}}$$

for $\sin^2 2\theta_\odot < 0.98$

$$m_{ee} > 4 \times 10^{-3}\,\mathrm{eV} \gg m_{ee}^{(1)}$$

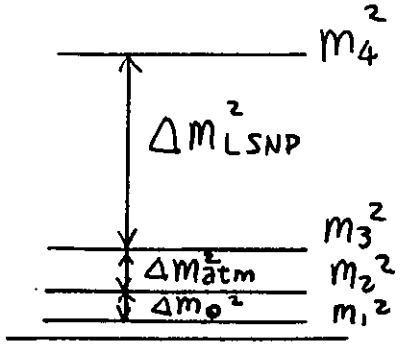
①, ② $\Omega_\nu = \frac{2m_1}{91.5\,\mathrm{eV}}\, h^{-2} \sim 0.01$: small



8. Four neutrino scenarios

8.1 (3+1) − scheme $\quad \Delta m^2_{LSND} = 0.9,\ 1.7,\ 6.0\ eV^2$

① nomal hierarchy



$$m_4 = \sqrt{\Delta m^2_{LSND}} = (0.9 \sim 2.4)\,eV$$

$$m_3 = \sqrt{\Delta m^2_{atm}}$$

$$m_2 = \sqrt{\Delta m_\odot^2}$$

$$m_1 \ll m_2$$

ordinary 3×3 MNS matrix for $N_\nu = 3$

$$U_{MNS} \simeq \begin{pmatrix} \boxed{\begin{matrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{matrix}} & \begin{matrix} \epsilon \\ \delta \\ -\frac{\delta}{\sqrt{2}} \end{matrix} \\ \begin{matrix} -\frac{\epsilon}{\sqrt{2}}+\frac{\delta}{\sqrt{2}} & -\frac{\epsilon}{\sqrt{2}}-\frac{\delta}{\sqrt{2}} & 0 \end{matrix} & 1 \end{pmatrix}$$

$$m_{ee}^{(1)} = U_{e1}^2 m_1 = C_\odot^2 m_1 \ll m_{ee}^{(2)}$$

$$m_{ee}^{(2)} = U_{e2}^2 m_2 = S_\odot^2 \sqrt{\Delta m_\odot^2} = \begin{cases} (1\sim 9)\times 10^{-6}\,eV & \text{SMA} \\ (0.4\sim 3)\times 10^{-3}\,eV & \text{LMA} \\ (0.3\sim 1)\times 10^{-4}\,eV & \text{LOW} \\ 10^{-4}\,eV & \text{VO} \end{cases}$$

$$m_{ee}^{(3)} = |U_{e3}|^2 \sqrt{\Delta m^2_{atm}} \lesssim 2\times 10^{-3}\,eV$$

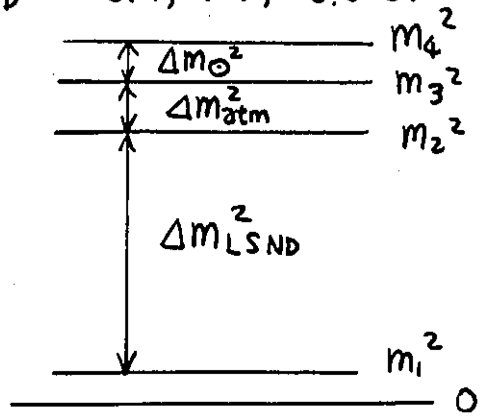
$$m_{ee}^{(4)} = |U_{e4}|^2 m_4 = \epsilon^2 \sqrt{\Delta m^2_{LSND}}$$

Δm^2_{LSND}	$0.9\,eV^2$	$1.7\,eV^2$	$6.0\,eV^2$
ϵ^2_{max}	1.4×10^{-2}	2.6×10^{-2}	3.6×10^{-2}
ϵ^2_{min}	1.4×10^{-2}	1.4×10^{-2}	1.9×10^{-2}
$\epsilon^2_{max}\sqrt{\Delta m^2_{LSND}}/eV$	1.3×10^{-2}	3.4×10^{-2}	8.8×10^{-2}
$\epsilon^2_{min}\sqrt{\Delta m^2_{LSND}}/eV$	1.3×10^{-2}	1.8×10^{-2}	4.7×10^{-2}

$$\therefore\ m_{ee} \simeq m_{ee}^{(4)} = (1\sim 9)\times 10^{-2}\,eV$$ for all $\nu_\odot$ solutions

② inverse hierarchy

$\Delta m^2_{LSND} = 0.9,\ 1.7,\ 6.0\ \mathrm{eV}^2$



$$m_2 \simeq m_3 \simeq m_4 \simeq \sqrt{\Delta m^2_{LSND}} \gg m_1$$

$\longrightarrow$ reduced to the $N_\nu=3$ scheme with complete degeneracy

i) $\nu_\odot$: SMA

$$m_{ee} \simeq C_\odot^2 m_2 \simeq m_2 \simeq \sqrt{\Delta m^2_{LSND}}$$

$$= 0.9,\ 1.3,\ 2.4\ \mathrm{eV}$$

: <u>already excluded</u>

ii) $\nu_\odot$: LMA, LOW, VO

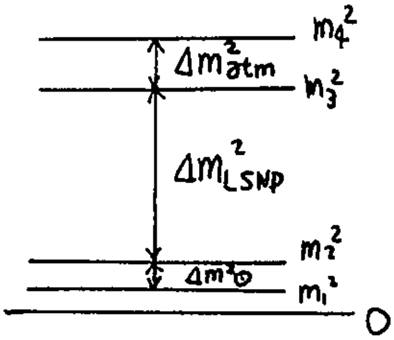
$$m_{ee} \simeq (\cos 2\theta_\odot \sim 1) \times \sqrt{\Delta m^2_{LSND}}$$

Depending on how small $(1-\sin^2 2\theta_\odot)$ is, it is still acceptable

8.2 Scenario with two heavy degenerate neutrinos

$m_3 \simeq m_4 \simeq \sqrt{\Delta m^2_{LSND}}$

$m_2 \simeq \sqrt{\Delta m^2_\odot} \gg m_1$



$$m_1 \ll m_2 \simeq \sqrt{\Delta m^2_\odot} = \begin{cases} (0.3 \sim 1.2) \times 10^{-2}\,\mathrm{eV} & \mathrm{LMA} \\ (2 \sim 5) \times 10^{-4}\,\mathrm{eV} & \mathrm{LOW} \\ (2 \sim 3) \times 10^{-3}\,\mathrm{eV} & \mathrm{SMA} \\ (3 \sim 10) \times 10^{-5}\,\mathrm{eV} & \mathrm{VO} \end{cases}$$

$$m_3 \simeq m_4 \simeq \sqrt{\Delta m^2_{LSND}} \doteq (0.4 \sim 1.4)\ \mathrm{eV}$$

extreme cases

a) $|U_{e4}|^2 \gg |U_{e3}|^2$

LSND $\quad \sin^2 2\theta_{e\mu} = 4|U_{e4}|^2|U_{\mu4}|^2$

$$|U_{e4}|^2 = \frac{\sin^2 2\theta_{e\mu}}{4|U_{e4}|^2}$$

$$|U_{\mu4}|^2 = S_{24}^2 = (0.3 \sim 0.7) \quad \left(\because \nu_{atm}\ \ 30^\circ \lesssim \theta_{24} \lesssim 55^\circ\right)$$

$$m_{ee}^{(3)} + m_{ee}^{(4)} \simeq |U_{e4}|^2 \sqrt{\Delta m^2_{LSND}}$$

$$= \frac{\sin^2 2\theta_{e\mu}}{4|U_{\mu4}|^2}\sqrt{\Delta m^2_{LSND}} \doteq (0.7 \sim 1.7) \times 10^{-3}\,\mathrm{eV}$$

if $\begin{Bmatrix} \Delta m^2_{LSND} = 1\,\mathrm{eV}^2 \\ \sin^2 2\theta_{e\mu} = 2 \times 10^{-3} \end{Bmatrix}$

b) $U_{e3} \simeq U_{e4}$, $U_{\mu3} \simeq U_{\mu4}$

LSND $\quad \sin^2 2\theta_{e\mu} = 4\,|U_{e3}^* U_{\mu3} + U_{e4}^* U_{\mu4}|$

$$= 16\,|U_{e4}|^2|U_{\mu4}|^2 \quad (\text{if } U_{\alpha j} = \text{real})$$

$$m_{ee}^{(3)} + m_{ee}^{(4)} \simeq 2\,|U_{e4}|^2\sqrt{\Delta m^2_{LSND}} = \frac{\sin^2 2\theta_{e\mu}}{8|U_{\mu4}|^2}\sqrt{\Delta m^2_{LSND}}$$

if $\varphi_3 = \varphi_4$

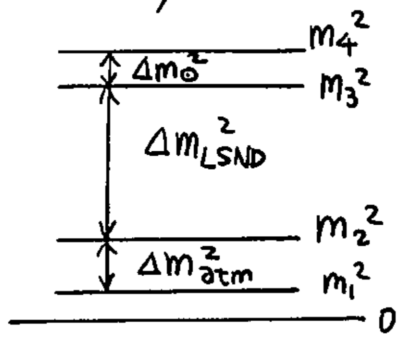
$$= (0.4 \sim 0.8) \times 10^{-3}\,\mathrm{eV}$$

From Bugey

$$|U_{e3}|^2 + |U_{e4}|^2 \lesssim \frac{1}{100}$$

$$m_{ee}^{(3)} + m_{ee}^{(4)} \simeq \left(|U_{e3}|^2 + |U_{e4}|^2\right)\sqrt{\Delta m^2_{LSND}} \lesssim 10^{-2}\,\mathrm{eV}$$

8.3 Scenario with inverse mass hierarchy



① $\nu_\odot$: SMA

$$m_{ee} \simeq m_{ee}^{(3)} \simeq \sqrt{\Delta m^2_{LSND}}$$

$$= (0.4 \sim 1.4)\ \mathrm{eV}$$

: <u>already excluded</u>

② $\begin{cases} \text{LMA} : & 0 \le |U_{s3}|^2 + |U_{s4}|^2 \lesssim 0.4 \\ \text{Low} : & 0 \le |U_{s3}|^2 + |U_{s4}|^2 \lesssim 0.2 \end{cases}$ (from ν_{atm})

in either case

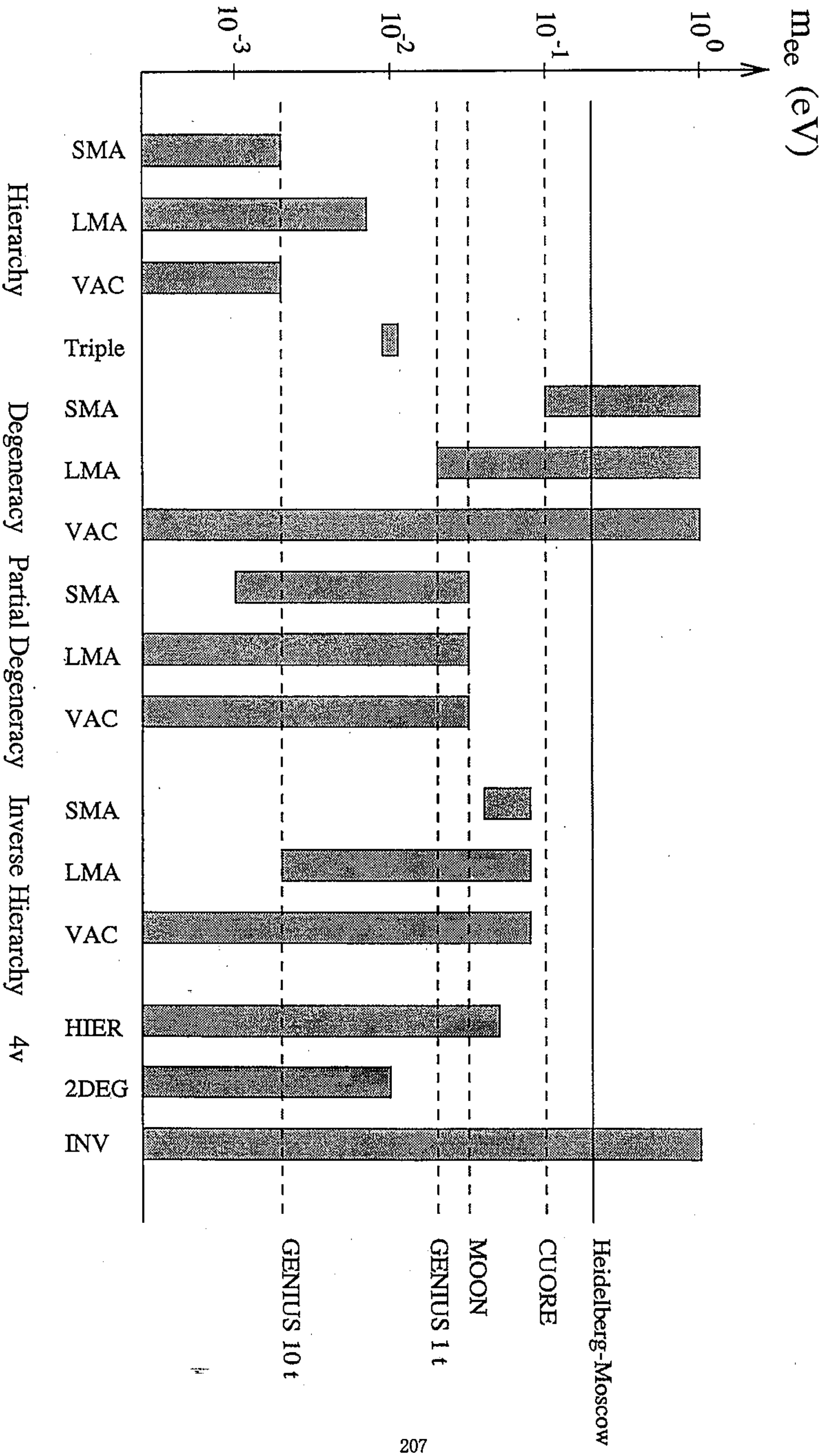
$$m_{ee}^{(3)} + m_{ee}^{(4)} = \left(S_\odot^2 + C_\odot^2 e^{i(\varphi_3 - \varphi_4)}\right)\sqrt{\Delta m^2_{LSND}}$$

$$= (\cos 2\theta_\odot \sim 1) \times \sqrt{\Delta m^2_{LSND}}$$

9. Summary

* To estimate $\langle m_{ee} \rangle$, it is very important to determine the type of the $\nu_\odot$ solution and deviation $\boxed{1 - \sin^2 2\theta_\odot}$.

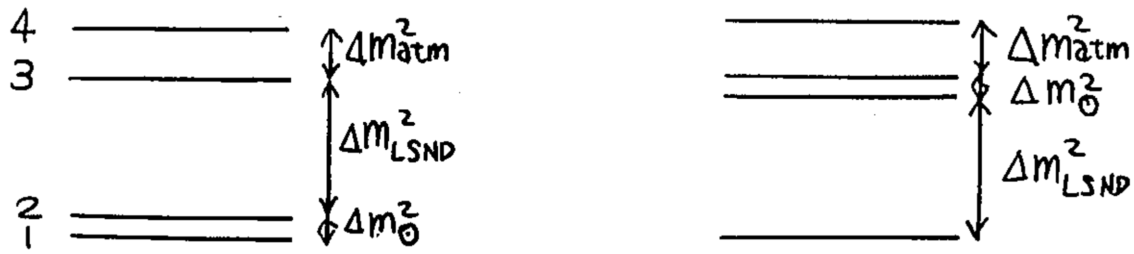
20



m_{ee} (eV)
10^{-3}
10^{-2}
10^{-1}
10^{0}
Hierarchy
SMA
LMA
VAC
Triple
Degeneracy
SMA
LMA
VAC
Partial Degeneracy
SMA
LMA
VAC
Inverse Hierarchy
SMA
LMA
VAC
4ν
HIER
2DEG
INV
GENIUS 10 t
GENIUS 1 t
MOON
CUORE
Heidelberg-Moscow

2. Mass patterns

Two distinct patterns :

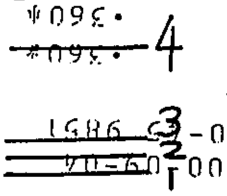


2+2 schemes 3+1 schemes

Prop1 3+1 schemes do NOT satisfy the constraints of Bugey ($\bar{\nu}_e \to \bar{\nu}_e$), CDHSW ($\nu_\mu \to \nu_\mu$), LSND ($\bar{\nu}_\mu \to \bar{\nu}_e$) (90%C

N. Okada - O.Y. Int. J. Mod. Phys. A12 ('97) 3669,
Bilenky - Giunti - Grimus Eur. Phys. J. C1 ('98) 247

∴ For 3+1 schemes in the limit $\Delta m^2_{\odot}, \Delta m^2_{atm} \to 0$



$\boxed{\text{Bugey}}$: $1 - P(\bar{\nu}_e \to \bar{\nu}_e) = \underline{4|U_{e4}|^2(1-|U_{e4}|^2)\,\Delta_{43}}$
negaive result $\leq \sin^2 2\theta_{Bugey}$

$$\Rightarrow |U_{e4}|^2 \leq \frac{1}{2}\left(1 - \sqrt{1 - \sin^2 2\theta_{Bugey}}\right) \quad \text{(a)}$$

$$\Delta_{43} \equiv \sin^2\left(\frac{\Delta m^2_{43} L}{4E}\right)$$

$\boxed{\text{CDHSW}}$: $1 - P(\nu_\mu \to \nu_\mu) = \underline{4|U_{\mu4}|^2(1-|U_{\mu4}|^2)\,\Delta_{43}}$
negative result $\leq \sin^2 2\theta_{CDHSW}$

$$\Rightarrow |U_{\mu4}|^2 \leq \frac{1}{2}\left(1 - \sqrt{1 - \sin^2 2\theta_{CDHSW}}\right) \quad \text{(b)}$$

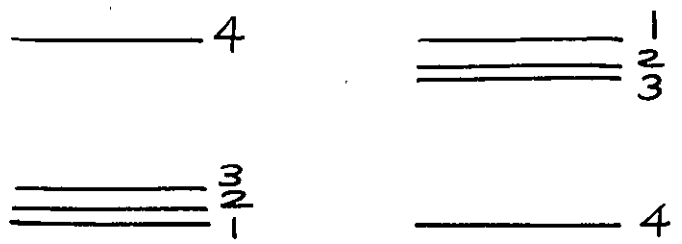
$\boxed{\text{LSND}}$: $P(\bar{\nu}_\mu \to \bar{\nu}_e) = \underline{4|U_{e4}|^2|U_{\mu4}|^2\,\Delta_{43}}$
affirmative result $= \sin^2 2\theta_{LSND}$ (c)

(a), (b), (c) $\Longrightarrow$ $\boxed{\sin^2 2\theta_{LSND} \leq \left(1-\sqrt{1-\sin^2 2\theta_{Bugey}}\right)\left(1-\sqrt{1-\sin^2 2\theta_{CDHSW}}\right)}$

This condition is NOT satisfied for any Δm^2_{LSND} @ 90% CL //

<u>Prop2</u> 3+1 schemes do satisfy the constraints of Bugey, CDHSW, LSND (99% CL) for $\Delta m^2_{LSND} \simeq 0.3,\ 0.9,\ 1.7,\ 6\ \mathrm{eV}^2$

Barger - Kayser - Learned - Weiler - Whisnant
hep-ph/0008019

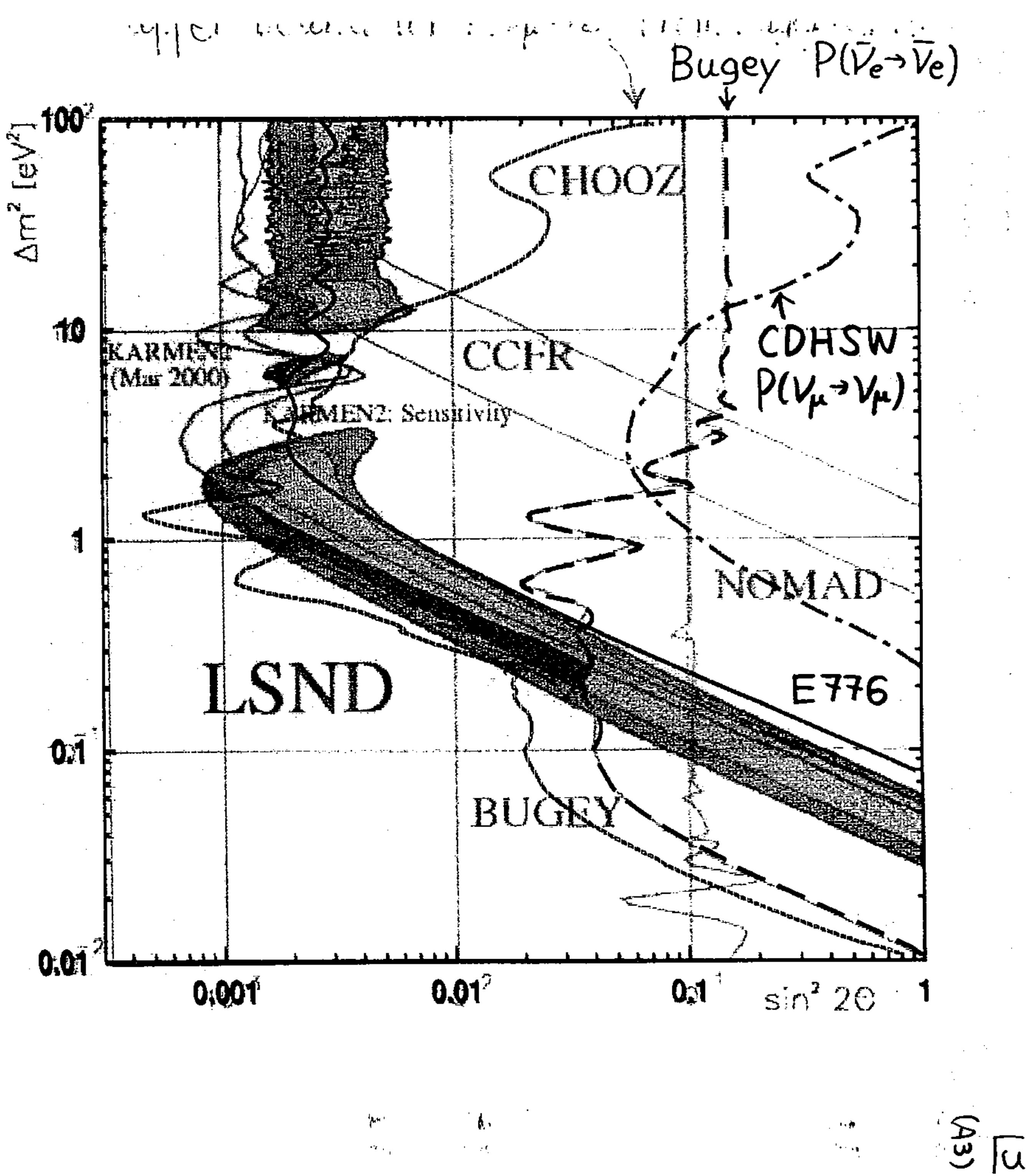


$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\\ \nu_s\end{pmatrix} = U \begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\\ \nu_4\end{pmatrix}, \quad U = \left(\begin{array}{ccc|c} & U^{3\times 3}_{MNS} & & \epsilon\\ & & & \delta\\ & & & 0\\ \hline \frac{\delta}{2}-\frac{\epsilon}{\sqrt{2}} & -\frac{\delta}{2}-\frac{\epsilon}{\sqrt{2}} & -\frac{\delta}{\sqrt{2}} & 1\end{array}\right)$$

These 3+1 schemes are indistinguishable from ordinary (3+0) scheme in most of experiments except in high energy neutrinos for $L \sim 10^4$ km :

$$P(\nu_\mu \to \nu_\tau) \begin{cases} = 4\,|U_{\mu 4}|^2 |U_{\tau 4}|^2 \sin^2\left(\dfrac{\Delta m^2_{43} L}{4E}\right) & \text{in vacuum} \\ = \underline{4\,|U^M_{\mu 4}|^2 |U^M_{\tau 4}|^2} \sin^2\left(\dfrac{BL}{2}\right) & \text{in matter} \end{cases}$$

enhancement due to matter effects


Bugey $P(\bar{\nu}_e \to \bar{\nu}_e)$
CDHSW
$P(\nu_\mu \to \nu_\mu)$
CHOOZ
CCFR
KARMEN2 (Mar 2000)
KARMEN2: Sensitivity
NOMAD
E776
LSND
BUGEY
Δm^2 [eV^2]
$\sin^2 2\theta$
100
10
1
0.1
0.01
0.001
0.01
0.1
1
(A3)