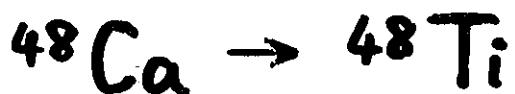
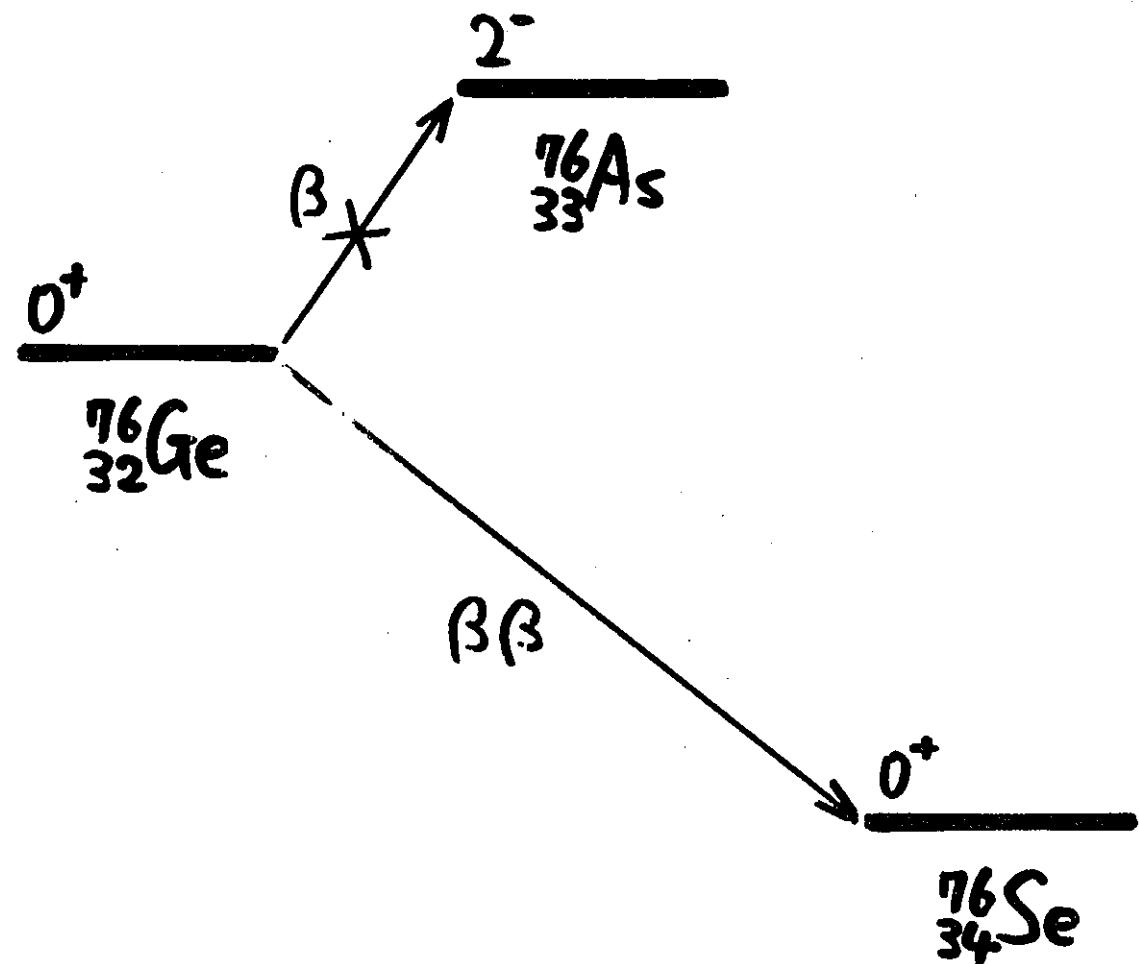


二重ベータ崩壊の基礎

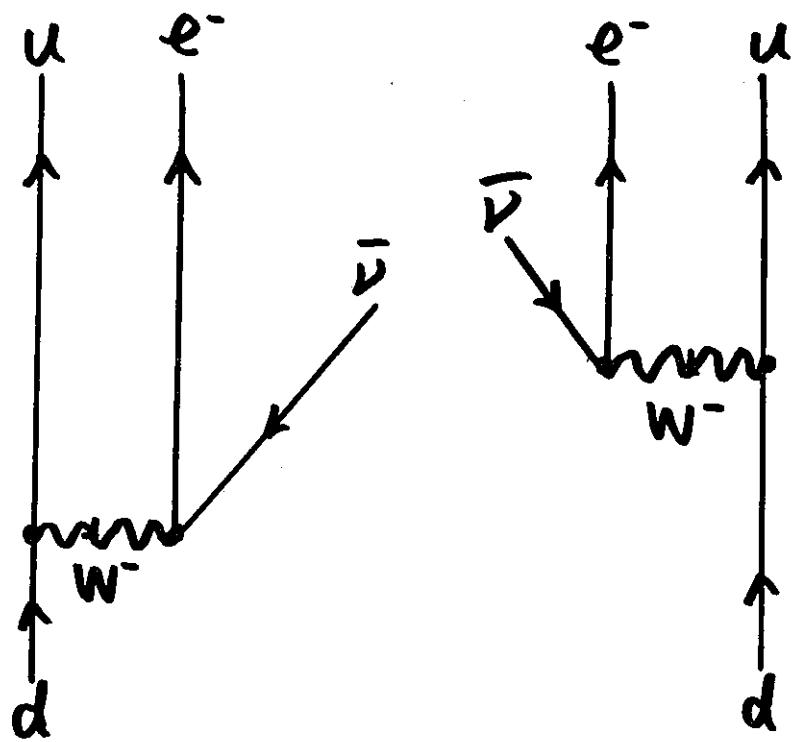
友田 敏章

- Introduction
- Majorana Neutrinos
- Theory of $\beta\beta$ Decay
 - Effective Hamiltonian
 - $OL\beta\beta$ Decay

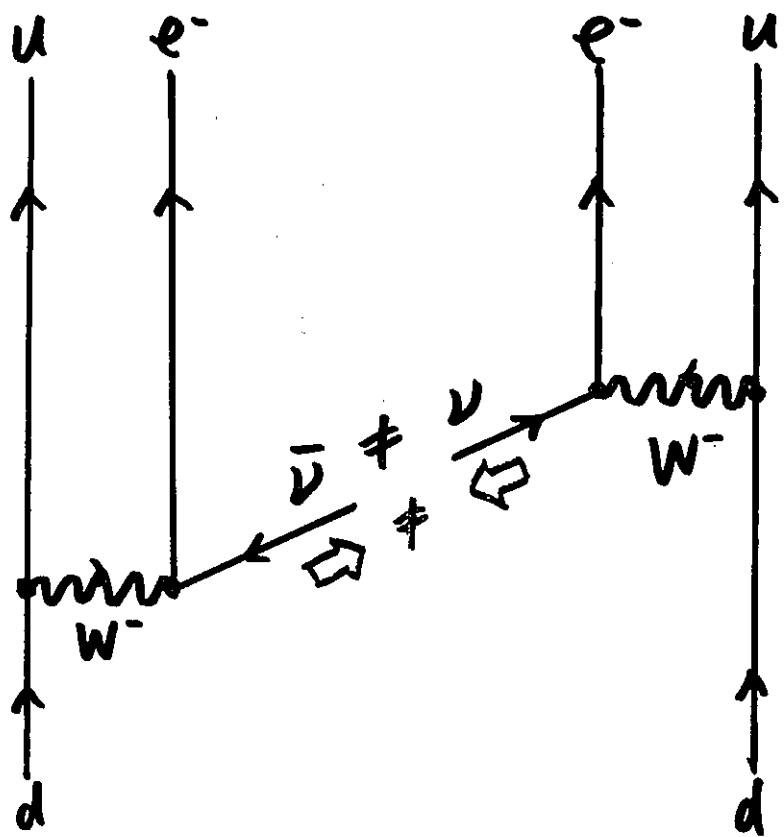
Double Beta Decay



Standard Model

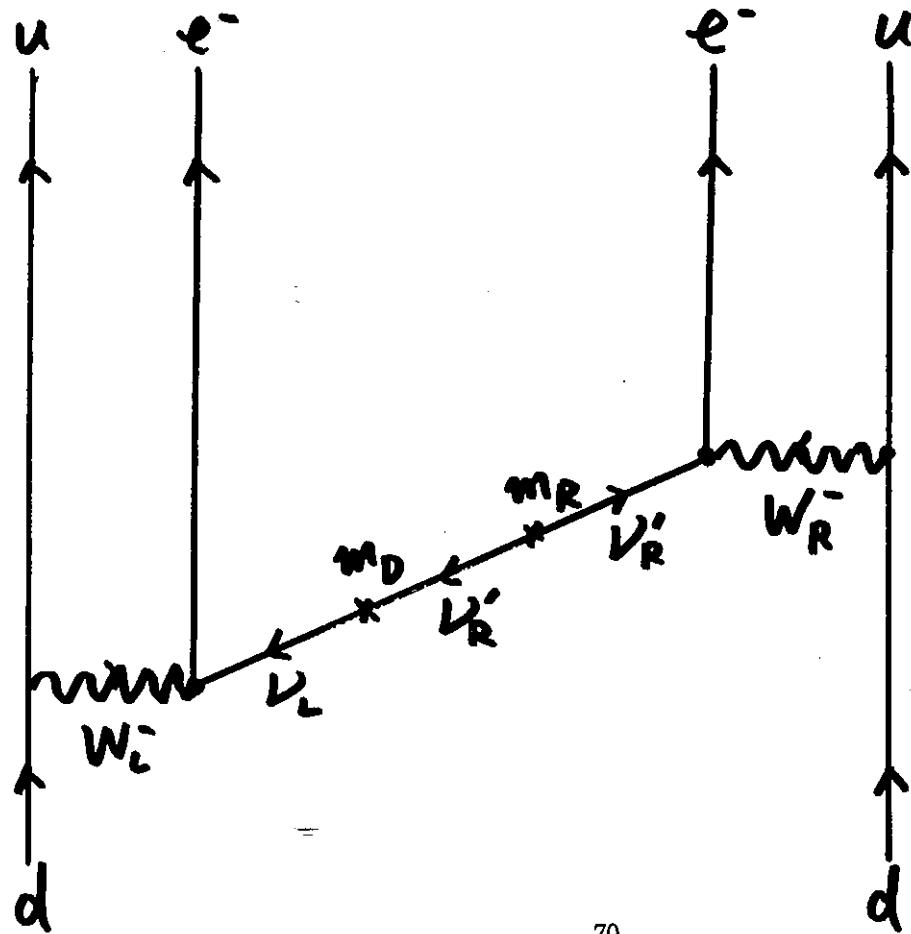
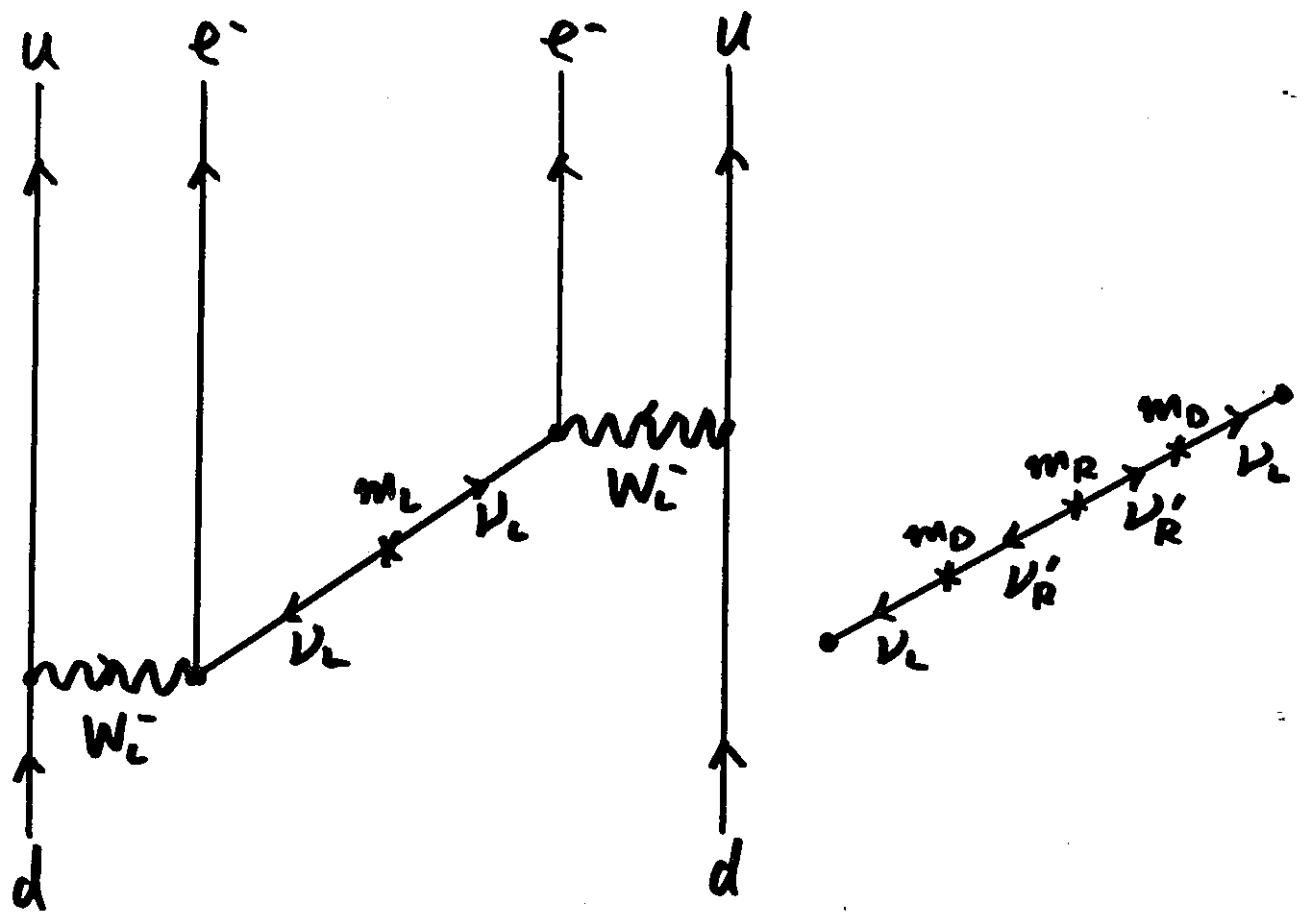


$2\nu\beta\beta$



$2\nu\beta\beta$
impossible

Majorana Neutrinos



Charged Current Weak Interaction

$$\mathcal{L}_{cc}(x) = \frac{g}{2\sqrt{2}} [\bar{l} \gamma^\mu (1 - \gamma_5) \nu_L W_{L\mu}^- + \bar{l} \gamma^\mu (1 + \gamma_5) \nu'_R W_{R\mu}^-] + h.c.$$

$$l = \begin{pmatrix} e \\ \mu \\ \tau \\ \vdots \end{pmatrix}, \quad \nu_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \\ \vdots \end{pmatrix}, \quad \nu'_R = \begin{pmatrix} \nu'_{eR} \\ \nu'_{\mu R} \\ \nu'_{\tau R} \\ \vdots \end{pmatrix}$$

General Neutrino Mass Term

$$\mathcal{L}_m(x) = -\frac{1}{2} (\overline{(\nu_L)^c} \nu'_R) M^0 \begin{pmatrix} \nu_L \\ (\nu'_R)^c \end{pmatrix} + h.c.$$

$$M^0 = \begin{pmatrix} M_L^0 & {M_D^0}^T \\ M_D^0 & M_R^0 \end{pmatrix}, \quad M_L^0 = {M_L^0}^T, \quad M_R^0 = {M_R^0}^T$$

Diagonalization of the Mass Matrix

$$O_\nu M^0 {O_\nu}^T = MS$$

$$M_{jk} = \delta_{jk} m_j, \quad m_j \geq 0$$

$$S_{jk} = \delta_{jk} S_j, \quad S_j = \pm 1$$

$$\mathcal{L}_m = -\frac{1}{2} \overline{NMN}$$

Mass Eigenstate Majorana Neutrinos

$$N = N^c = \Lambda O_\nu \begin{pmatrix} \nu_L \\ (\nu'_R)^c \end{pmatrix} + \Lambda^* O_\nu \begin{pmatrix} (\nu_L)^c \\ \nu'_R \end{pmatrix}, \quad \Lambda = \sqrt{S}$$

CP Transformation Property

$$(\mathcal{CP})N(\mathbf{x}, \bar{t})(\mathcal{CP})^{-1} = iS\gamma^0 N(-\mathbf{x}, t).$$

Propagators of Majorana Neutrinos

$$\langle T[N_j(x)\bar{N}_k(y)] \rangle = iS_F(x-y; m_j)\delta_{jk}$$

$$\langle T[N_j(x)N_k^T(y)] \rangle = iS_F(x-y; m_j)C^T \cancel{\epsilon_j^2} \delta_{jk}$$

Current Neutrinos

$$\nu_L = UN_L \quad \nu'_R = VN_R$$

$$N_L = P_L N, \quad P_L = \frac{1}{2}(1 - \gamma_5)$$

$$N_R = P_R N, \quad P_R = \frac{1}{2}(1 + \gamma_5)$$

$$\begin{pmatrix} U \\ V^* \end{pmatrix} = O_\nu^T \Lambda^*$$

Propagators of Current Neutrinos

$$\langle T[\nu_{eL}(x)\nu_{eL}^T(y)] \rangle = \sum_{j=1}^{2F} U_{ej}^2 P_L i S_F(x-y; m_j) C^T P_L$$

$$\langle T[\nu_{eL}(x)\nu'_{eR}^T(y)] \rangle = \sum_{j=1}^{2F} U_{ej} V_{ej} P_L i S_F(x-y; m_j) C^T P_R$$

Neutrino Mass Matrices

1. Dirac Neutrinos

$$M^0 = \begin{pmatrix} 0 & M_D^{0\ T} \\ M_D^0 & 0 \end{pmatrix}$$

Diagonalization

$$O_\nu = \frac{1}{\sqrt{2}} \begin{pmatrix} A & B \\ A & -B \end{pmatrix}, \quad BM_D^0 A^T = M_I$$

$$M = \begin{pmatrix} M_I & 0 \\ 0 & M_{II} \end{pmatrix}, \quad S = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad N = \begin{pmatrix} N_I \\ N_{II} \end{pmatrix}$$

Dirac Neutrinos

$$\psi = \frac{1}{\sqrt{2}}(N_I - iN_{II})$$

$$\begin{pmatrix} U_I & U_{II} \\ V_I^* & V_{II}^* \end{pmatrix} = O_\nu^T \Lambda^* = \frac{1}{\sqrt{2}} \begin{pmatrix} A^T & -iA^T \\ B^T & iB^T \end{pmatrix}$$

2. The See-Saw Mechanism (1-Generation)

$$M^0 = \begin{pmatrix} 0 & m_D \\ m_D & m_R \end{pmatrix}$$

Diagonalization

$$O_\nu = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}, \quad \frac{1}{2} \tan 2\theta = \frac{m_D}{m_R}$$

$$M \approx \begin{pmatrix} \frac{m_D^2}{m_R} & 0 \\ 0 & m_R + \frac{m_D^2}{m_R} \end{pmatrix}, \quad S = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} U_{e1} & U_{e2} \\ V_{e1}^* & V_{e2}^* \end{pmatrix} = O_\nu^T \Lambda^* = \begin{pmatrix} -i \cos \theta & \sin \theta \\ i \sin \theta & \cos \theta \end{pmatrix}$$

$$\begin{cases} U_{e1}^2 m_1 \approx -\frac{m_D^2}{m_R} \\ U_{e1} V_{e1} \approx -\frac{m_D}{m_R} \end{cases}$$

Theoretical Description of $\beta\beta$ Decay

Effective Hamiltonian

Charged Current Interaction for Leptons (β Decay)

$$\mathcal{L}_{cc}(x) = \frac{g}{2\sqrt{2}} [j_L^\mu W_{L\mu}^- + j_R^\mu W_{R\mu}^-] + h.c.$$

$$j_L^\mu = \bar{e}\gamma^\mu(1 - \gamma_5)\nu_{eL}, \quad j_R^\mu = \bar{e}\gamma^\mu(1 + \gamma_5)\nu'_{eR}$$

$$\nu_{eL} = \sum_{i=1}^{2F} U_{ei} N_{iL}, \quad \nu'_{eR} = \sum_{i=1}^{2F} V_{ei} N_{iR}$$

Mixing of W_L and W_R

$$\begin{pmatrix} W_L^- \\ W_R^- \end{pmatrix} = \begin{pmatrix} \cos \zeta & \sin \zeta \\ -\sin \zeta & \cos \zeta \end{pmatrix} \begin{pmatrix} W_1^- \\ W_2^- \end{pmatrix}$$

Effective Weak Interaction Hamiltonian

$$H_W = (G \cos \theta_C / \sqrt{2}) (-j_{L\mu} J_L^{\mu\dagger} + \kappa j_{L\mu} J_R^{\mu\dagger} + \eta j_{R\mu} J_L^{\mu\dagger} + \lambda j_{R\mu} J_R^{\mu\dagger}) + h.c.$$

$$\frac{G}{\sqrt{2}} = \frac{g^2}{8M_1^2} [\cos^2 \zeta + (M_1/M_2)^2 \sin^2 \zeta]$$

$$\lambda = \frac{(M_1/M_2)^2 + \tan^2 \zeta}{1 + (M_1/M_2)^2 \tan^2 \zeta}$$

$$\eta = \kappa = -\frac{[1 - (M_1/M_2)^2] \tan \zeta}{1 + (M_1/M_2)^2 \tan^2 \zeta}$$

Nucleon Currents

$$J_L^{\mu\dagger}(x) = \bar{\psi}(x)\tau^+(g_V\gamma^\mu - ig_W\sigma^{\mu\nu}q_\nu - g_A\gamma^\mu\gamma_5 + g_P\gamma_5q^\mu)\psi(x)$$

$$J_R^{\mu\dagger}(x) = \bar{\psi}(x)\tau^+(g_V\gamma^\mu - ig_W\sigma^{\mu\nu}q_\nu + g_A\gamma^\mu\gamma_5 - g_P\gamma_5q^\mu)\psi(x)$$

$$\psi = \begin{pmatrix} p \\ n \end{pmatrix}$$

$$g_V(0) = 1$$

$$g_A(0) = 1.254$$

$$g_W(0) = \frac{\kappa_\beta}{2M}, \quad \kappa_\beta = 3.70$$

$$g_V(q^2)/g_V(0) = \left(\frac{\Lambda^2}{\Lambda^2 - q^2}\right)^2 \text{ etc,} \quad \Lambda \sim 1 \text{ GeV}$$

$$g_P(q^2) = \frac{2Mg_A(0)}{m_\pi^2 - q^2}$$

Nonrelativistic Impulse Approximation

$$\begin{aligned}
J_L^{\mu\dagger}(\mathbf{x}) &= \sum_{n=1}^A \tau_n^+ \delta(\mathbf{x} - \mathbf{r}_n) \sum_{k=0,1,\dots} [g_V V^{(k)\mu} + g_W W^{(k)\mu} \\
&\quad - g_A A^{(k)\mu} - g_P P^{(k)\mu}]_n \\
J_R^{\mu\dagger}(\mathbf{x}) &= \sum_{n=1}^A \tau_n^+ \delta(\mathbf{x} - \mathbf{r}_n) \sum_{k=0,1,\dots} [g_V V^{(k)\mu} + g_W W^{(k)\mu} \\
&\quad + g_A A^{(k)\mu} + g_P P^{(k)\mu}]_n \\
(k: \text{order in } 1/M)
\end{aligned}$$

Retained terms:

$$\begin{aligned}
V^{(0)0} &= 1 \\
\mathbf{A}^{(0)} &= \boldsymbol{\sigma} \\
A_n^{(1)0} &= C_n \\
\mathbf{V}_n^{(1)} + (g_W/g_V) \mathbf{W}_n^{(0)} &= \mathbf{D}_n
\end{aligned}$$

where

$$\begin{aligned}
C_n &= (\mathbf{p}_n + \mathbf{p}'_n) \cdot \boldsymbol{\sigma}_n / 2M \\
\mathbf{D}_n &= [\mathbf{p}_n + \mathbf{p}'_n - i\mu_\beta \boldsymbol{\sigma}_n \times (\mathbf{p}_n - \mathbf{p}'_n)] / 2M \\
(\mu_\beta = \kappa_\beta + 1 = 4.70)
\end{aligned}$$

$0\nu\beta\beta$ Decay

Effective Hamiltonian

$$H_W = (G \cos \theta_C / \sqrt{2}) \sum_{i=1}^{2F} [j_{Li\mu} \tilde{J}_{Li}^{\mu\dagger} + j_{Ri\mu} \tilde{J}_{Ri}^{\mu\dagger}] + h.c.$$

$$j_{Li}^\mu = \bar{e} \gamma^\mu (1 - \gamma_5) N_{iL}$$

$$j_{Ri}^\mu = \bar{e} \gamma^\mu (1 + \gamma_5) N_{iR}$$

$$\tilde{J}_{Li}^{\mu\dagger} = U_{ei} (J_L^{\mu\dagger} + \kappa J_R^{\mu\dagger})$$

$$\tilde{J}_{Ri}^{\mu\dagger} = V_{ei} (\lambda J_R^{\mu\dagger} + \eta J_L^{\mu\dagger})$$

Differential $0\nu\beta\beta$ Decay Rate

$$dW_{0\nu} = 2\pi \sum_{\text{spin}} |R_{0\nu}|^2 \delta(\epsilon_1 + \epsilon_2 + E_F - E_I) \frac{d\mathbf{p}_1}{(2\pi)^3} \frac{d\mathbf{p}_2}{(2\pi)^3}$$

$$R_{0\nu} = \sqrt{\frac{1}{2}} \left(\frac{G \cos \theta_C}{\sqrt{2}} \right)^2 \sum_{i=1}^{2F} \sum_N \sum_{\alpha, \beta = L, R} \sum_s \int d\mathbf{x} d\mathbf{y} \int \frac{d\mathbf{k}}{(2\pi)^3}$$

$$\times \langle F | \tilde{J}_{\beta i}^{\nu\dagger}(\mathbf{y}) | N \rangle \langle N | \tilde{J}_{\alpha i}^{\mu\dagger}(\mathbf{x}) | I \rangle [1 - P(e_1, e_2)]$$

$$\times \frac{\bar{e}_{p_2 s'_2}(\mathbf{y}) \gamma_\nu 2P_\beta N_{iks}(\mathbf{y}) \bar{e}_{p_1 s'_1}(\mathbf{x}) \gamma_\mu 2P_\alpha N_{iks}^c(\mathbf{x})}{\epsilon_1 + \omega + E_N - E_I}$$

Summation Σ_s of the Numerator over Neutrino Spin

$$4\bar{e}_{p_2 s'_2}(\mathbf{y}) \gamma_\nu P_\beta \frac{1}{2\omega} (\omega \gamma^0 - \mathbf{k} \cdot \boldsymbol{\gamma} + m_i) e^{i\mathbf{k} \cdot (\mathbf{y} - \mathbf{x})} P_\alpha \gamma_\mu e_{p_1 s'_1}^c(\mathbf{x})$$

$$P_L (\omega \gamma^0 - \mathbf{k} \cdot \boldsymbol{\gamma} + m_i) P_L = m_i P_L$$

$$P_L (\omega \gamma^0 - \mathbf{k} \cdot \boldsymbol{\gamma} + m_i) P_R = (\omega \gamma^0 - \mathbf{k} \cdot \boldsymbol{\gamma}) P_R$$

$0^+ \rightarrow 0^+ 0\nu\beta\beta$ Decay

Differential Decay Rate

$$\frac{d^2W_{0\nu}}{d\epsilon_1 d \cos \theta_{12}} = (a^{(0)} + a^{(1)} \cos \theta_{12}) w_{0\nu}$$

$$w_{0\nu} = \frac{(g_A G \cos \theta_C)^4 m_e^2}{16\pi^5} p_1 p_2 \epsilon_1 \epsilon_2, \quad (\epsilon_1 + \epsilon_2 + E_F = E_I)$$

$$a^{(i)} = \sum_{j,k=1}^6 f_{jk}^{(i)} \text{Re}[X_j X_k^*] \quad (i = 0, 1)$$

where

$$X_1 = (\langle m_\nu \rangle / m_e) (\chi_F - 1) M_{\text{GT}}^{(0\nu)}$$

$$X_3 = (\langle \lambda \rangle \tilde{\chi}_- + \langle \eta \rangle \tilde{\chi}_+) M_{\text{GT}}^{(0\nu)}$$

$$X_4 = (\langle \lambda \rangle \chi'_- + \langle \eta \rangle \chi'_+) M_{\text{GT}}^{(0\nu)}$$

$$X_5 = \langle \eta \rangle \chi'_P M_{\text{GT}}^{(0\nu)}$$

$$X_6 = \langle \eta \rangle \chi'_R M_{\text{GT}}^{(0\nu)}$$

and

$$\langle m_\nu \rangle = \sum_i' m_i U_{ei}^2$$

$$\langle \lambda \rangle = \lambda \sum_i' U_{ei} V_{ei}$$

$$\langle \eta \rangle = \eta \sum_i' U_{ei} V_{ei}$$

\sum' over light neutrinos
 $(m_i \ll 100 \text{ MeV})$

Approximation

$$\epsilon_j + \omega + E_N - E_I \approx \omega + A_j$$

where

$$A_j = \epsilon_j + \langle E_N \rangle - E_I$$

This is good because

$$\omega \sim 100 \text{ MeV} \gg E_N - E_I \sim \mathbf{k}^2/M \sim 10 \text{ MeV}$$

Summation Σ_N over Nuclear States

$$\sum_N |N\rangle \frac{1}{\epsilon_j + \omega + E_N - E_I} \langle N| \approx \frac{1}{\omega + A_j}$$

Transition Amplitude in Closure Approximation

$$\begin{aligned}
 R_{0\nu} = & 4 \sqrt{\frac{1}{2}} \left(\frac{G \cos \theta_C}{\sqrt{2}} \right)^2 \sum_i \sum_{\alpha, \beta} \int d\mathbf{x} d\mathbf{y} \int \frac{d\mathbf{k}}{(2\pi)^3} \\
 & \times \langle F | \tilde{J}_{\beta i}^{\nu\dagger}(\mathbf{y}) \tilde{J}_{\alpha i}^{\mu\dagger}(\mathbf{x}) | I \rangle e^{i\mathbf{k}\cdot(\mathbf{y}-\mathbf{x})} \bar{e}_{p_2 s'_2}(\mathbf{y}) \gamma_\nu \\
 & \times P_\beta \frac{1}{2\omega} \left[\frac{\omega \gamma^0 - \mathbf{k} \cdot \boldsymbol{\gamma} + m_i}{\omega + A_1} - \frac{\omega \gamma^0 + \mathbf{k} \cdot \boldsymbol{\gamma} - m_i}{\omega + A_2} \right] P_\alpha \\
 & \times \gamma_\mu e_{p_1 s'_1}^c(\mathbf{x})
 \end{aligned}$$

Nuclear Matrix Elements

$$M_{\text{GT}}^{(0\nu)} = \langle H(r_{12}) \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \rangle$$

$$\left. \begin{array}{l} \tilde{\chi}_{\text{GT}} \\ \chi'_{\text{GT}} \end{array} \right\} = \left\langle \begin{Bmatrix} \tilde{H}(r_{12}) \\ -r_{12}H'(r_{12}) \end{Bmatrix} \right\rangle \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 / M_{\text{GT}}^{(0\nu)}$$

$$\left. \begin{array}{l} \chi_F \\ \tilde{\chi}_F \\ \chi'_F \end{array} \right\} = (g_V/g_A)^2 \left\langle \begin{Bmatrix} H(r_{12}) \\ \tilde{H}(r_{12}) \\ -r_{12}H'(r_{12}) \end{Bmatrix} \right\rangle / M_{\text{GT}}^{(0\nu)}$$

$$\chi'_T = \langle -r_{12}H'(r_{12})S_{12} \rangle / M_{\text{GT}}^{(0\nu)}$$

$$\chi'_P = (g_V/g_A) \langle -\frac{1}{2}r_{+12}H'(r_{12})i(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2) \cdot (\hat{\mathbf{r}}_{12} \times \hat{\mathbf{r}}_{+12}) \rangle / M_{\text{GT}}^{(0\nu)}$$

$$M_R'^{(0\nu)} = \langle -\frac{1}{2}m_e^{-1}H'(r_{12})\hat{\mathbf{r}}_{12} \cdot (\boldsymbol{\sigma}_1 \times \mathbf{D}_2 - \boldsymbol{\sigma}_2 \times \mathbf{D}_1) \rangle$$

$$\chi'_R = (g_V/g_A)M_R'^{(0\nu)} / M_{\text{GT}}^{(0\nu)}$$

and

$$\tilde{\chi}_{\pm} = \tilde{\chi}_F \pm \tilde{\chi}_{\text{GT}}$$

$$\chi'_{\pm} = -\chi'_F \pm (\frac{1}{3}\chi'_{\text{GT}} - 2\chi'_T).$$

where

$$\mathbf{r}_{nm} = \mathbf{r}_n - \mathbf{r}_m, \quad \mathbf{r}_{+nm} = \mathbf{r}_n + \mathbf{r}_m$$

$$S_{nm} = (\boldsymbol{\sigma}_n \cdot \hat{\mathbf{r}}_{nm})(\boldsymbol{\sigma}_m \cdot \hat{\mathbf{r}}_{nm}) - \frac{1}{3}\boldsymbol{\sigma}_n \cdot \boldsymbol{\sigma}_m$$

$$\langle O_{12} \rangle = \langle O_F^+ | \frac{1}{2} \sum_{nm} \tau_n^+ \tau_m^+ O_{nm} | O_I^+ \rangle$$

Neutrino Propagation Function

$$H(r) = \frac{1}{2} [H(r, A_1) + H(r, A_2)]$$

$$\approx H(r, \bar{A})$$

$$= 4\pi \int \frac{dk}{(2\pi)^3} \frac{e^{ik \cdot r}}{\omega(\omega + \bar{A})}$$

$$\bar{A} = \frac{1}{2} (A_1 + A_2)$$

$$= \frac{1}{2} (Q_{\beta\beta} + 2m_e) + \langle E_N \rangle - E_I$$

$$\tilde{H}(r) = \frac{4\pi}{\varepsilon_2 - \varepsilon_1} \int \frac{dk}{(2\pi)^3} e^{ik \cdot r} \left[\frac{1}{\omega + A_1} - \frac{1}{\omega + A_2} \right]$$

$$= \frac{1}{\varepsilon_2 - \varepsilon_1} [A_2 H(r, A_2) - A_1 H(r, A_1)]$$

$$\approx H(r, \bar{A}) + \bar{A} \frac{\partial}{\partial \bar{A}} H(r, \bar{A})$$

$$= 2H(r, \bar{A}) + r \frac{\partial}{\partial r} H(r, \bar{A})$$

$0^+ \rightarrow 0^+$ OV $\beta\beta$ Decay by Heavy Neutrinos

$$m_i \gg \Lambda \sim 1 \text{ GeV}$$

$$H_h(r) \approx \frac{4\pi}{m_i^2} \int \frac{d\mathbf{k}}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{r}} \left(\frac{\Lambda^2}{\Lambda^2 + k^2} \right)^4 \\ = \frac{\Lambda^3}{16m_i^2} e^{-\Lambda r} [1 + \Lambda r + \frac{1}{3}(\Lambda r)^2]$$

$$\langle m_\nu \rangle \rightarrow \langle m_\nu \rangle + m_e^2 \langle \langle m_{\nu^-} \rangle \rangle (\chi_{FH} - \chi_{GTh}) / (\chi_F - 1)$$

$$\langle \langle m_{\nu^-} \rangle \rangle = \sum_i'' m_i^{-1} V_{ei}^2 \quad (m_i \gg 1 \text{ GeV})$$

$$\begin{aligned} & \chi_{FH} \\ & \chi_{GTh} \end{aligned} \Big\} = \left\langle H_r(r_{12}) \left\{ \frac{(g_V/g_A)^2}{\sigma_1 \cdot \sigma_2} \right\} \right\rangle / M_{GT}^{(OV)}$$

Summary

- $0\nu\beta\beta$ Decay Amplitude
= \sum (Particle Physics Parameter)
x (Nuclear Matrix Element)

Assumption : $m_i \ll 100 \text{ MeV}$ or $m_i \gg 1 \text{ GeV}$

- Particle Physics Parameters

$$\left. \begin{aligned} \langle m_\nu \rangle &= \sum_i' U_{ei}^2 m_i \\ \langle \lambda \rangle &= \lambda \sum_i' U_{ei} V_{ei} \\ \langle \eta \rangle &= \eta \sum_i' U_{ei} V_{ei} \\ \langle \langle m_\nu' \rangle \rangle &= \sum_i'' U_{ei}^2 m_i^{-1} \end{aligned} \right\} \quad \begin{matrix} \textcircled{1} \\ \textcircled{2} \end{matrix}$$

- Relative Sensitivity

$$\langle m_\nu \rangle \sim 1 \text{ eV}$$

$$\langle \lambda \rangle \sim 10^{-6}$$

$$\langle \eta \rangle \sim 10^{-8}$$

$$\langle \langle m_\nu' \rangle \rangle \sim 10^{-7} \text{ GeV}^{-1} \quad (\sim \frac{\langle m_\nu \rangle}{w^2})$$