

超新星中のMSW効果 — 基礎的解説 —

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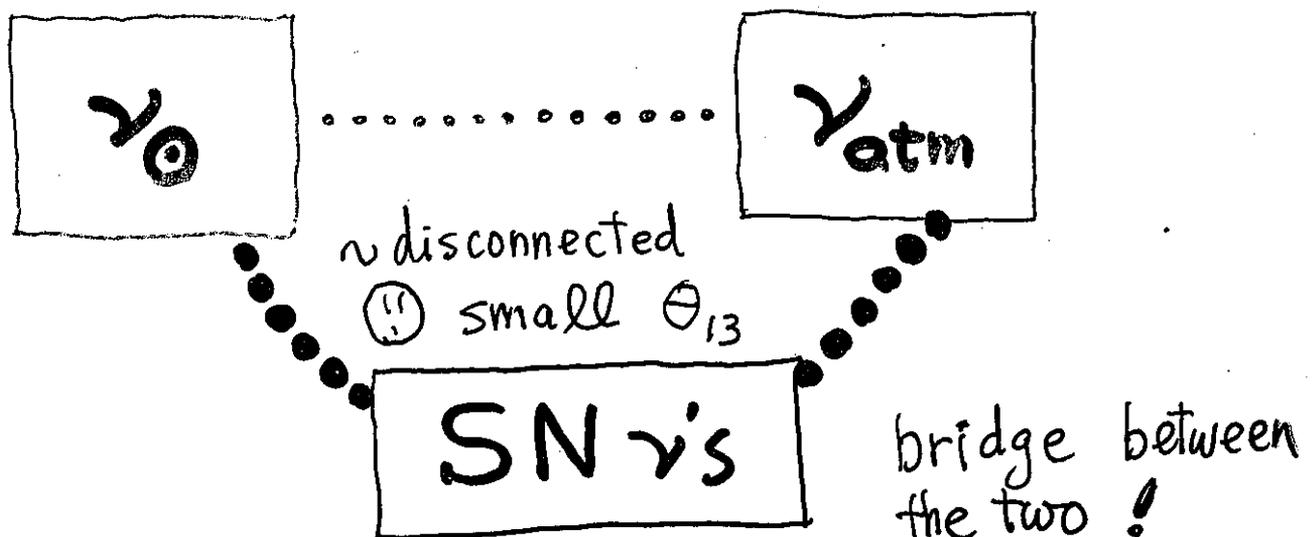
What is Special in MSW Effect in SN?

MSW Resonance Condition (2 flavor)

$$\Delta m^2 = 2\sqrt{2} G_F E \rho_m Y_e / m_p \cos 2\theta$$

$$\approx 10^6 \left(\frac{\rho_m}{10^{12} \text{ g/cm}^3} \right) \left(\frac{Y_e}{0.5} \right) \left(\frac{E}{10 \text{ MeV}} \right) \text{ eV}^2$$

⇒ All the ν species with masses \lesssim in cosmologically interesting mass range have level crossing in SN!



What Happens at SN Core?

Just outside of ν sphere

$$\left\{ \begin{array}{l} \nu_3^m \approx \nu_e \leftarrow \text{almost pure flavor state} \\ \nu_2^m \approx \nu_H \\ \nu_1^m \approx \nu_H' \end{array} \right\} \nu_\mu, \nu_\tau \text{ composition only!}$$

H_0 : unperturbed Hamiltonian

$$i \frac{d}{dx} \begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = \left\{ U \begin{bmatrix} 0 & & \\ & \frac{\Delta m_{12}^2}{2E} & \\ & & \frac{\Delta m_{13}^2}{2E} \end{bmatrix} U^\dagger + \left[\begin{array}{ccc} a(x) & & \\ & 0 & \\ & & 0 \end{array} \right] \right\} \times \begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix}$$

\Rightarrow degenerate perturbation theory

zero-order basis: \hat{H}_0 2nd-3rd 2×2 subspace diagonalization

$$\Rightarrow \begin{bmatrix} \nu_e \\ \nu_H \\ \nu_H' \end{bmatrix}$$

What Happens in SN Envelope?

“2 Level Crossings”

⇒ H and L resonances

The Key Question

Is H resonance adiabatic?

- If adiabatic: $\nu_e \rightarrow \nu_3$ conversion
irrespective of the ν_0 solution!
- If completely nonadiabatic: \approx identical with ν_0 conversion

if density profile is similar

 →
- If moderately nonadiabatic: ν conversion depends on $\left\{ \begin{array}{l} \nu_0 \text{ solution} \\ \text{SN density profile} \end{array} \right.$

given by $\rho(r) \propto r^{-n}$. In most progenitor models n is approximately 3 (see, e.g., Fig. 1 of Ref. 15) and the slope is 1.5 times steeper. Then the minimum mixing angle

(H.M. & H. Nunokawa '90)

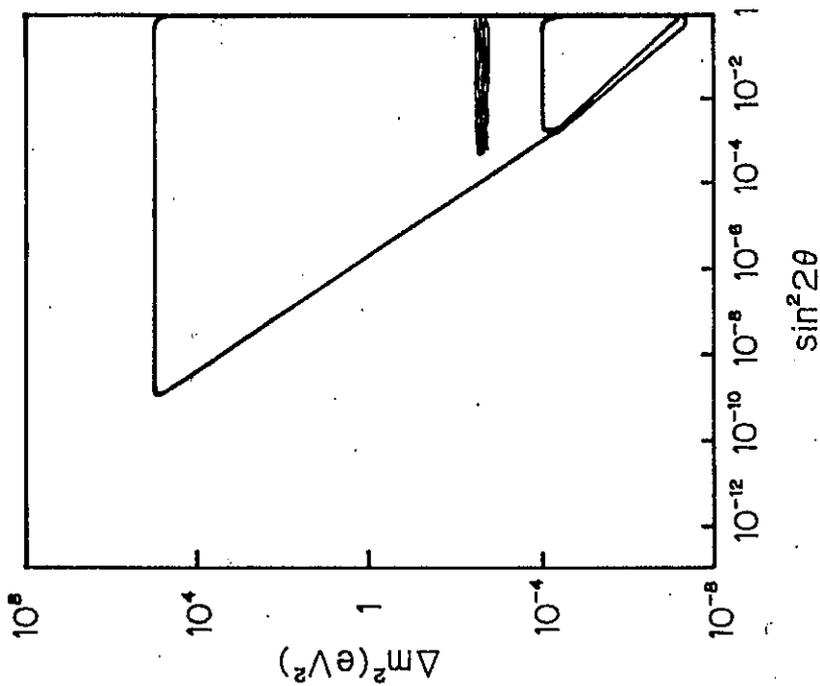


FIG. 1. The MSW triangle for supernova, the region where the resonant flavor conversion is effective, is schematically illustrated as a large triangular contour. The neutrino sphere density is tentatively taken as 10^{11} g/cm³. Also shown (small one) is the MSW triangle for the solar neutrino.

below.

In Fig. 2 we present the flow of the neutrino energy eigenstates as functions of the matter density. Notice that the ν_e is only affected by the matter. In Fig. 2 we are implicitly assuming that the mixing angles are small

complete conversion $\nu_e \leftrightarrow \nu_\tau$ ← Adiabatic if $\sin^2 2\theta \gtrsim 2.5 \times 10^{-4}$

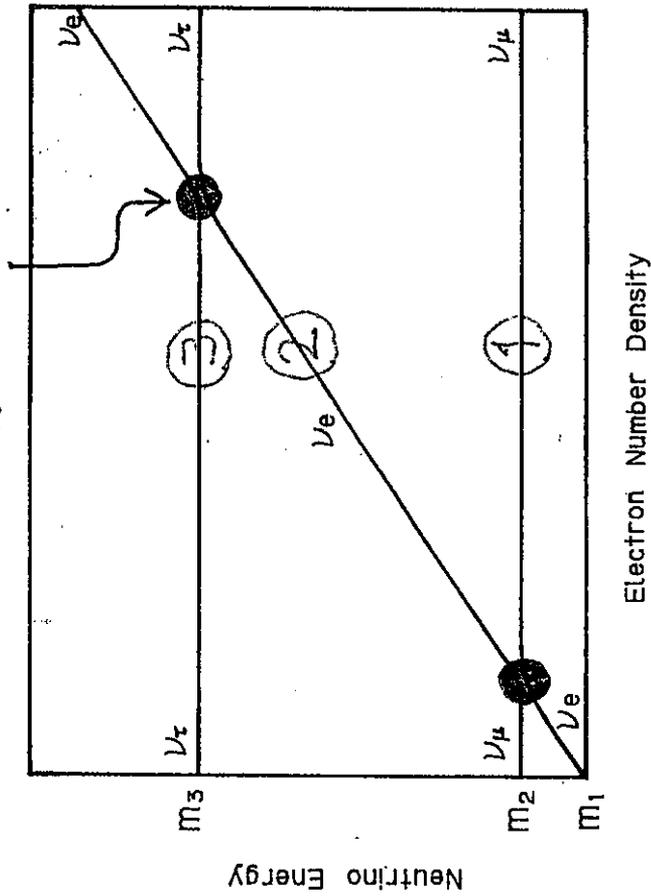


FIG. 2. Flow of neutrino energy eigenstates as functions of the electron number density under the condition of mass hierarchy (1). Vacuum mixing angles are assumed to be small so that the mass eigenstates are approximately equal to the flavor eigenstates in the vacuum. A comment on the scale of this figure: if $m_3 = 100$ eV, then $\nu_e - \nu_\tau$ crossing point is located at electron number density $10^{10} N_A$, with N_A being the Avogadro number.

Closer Look at γ Conversion in SN Envelope

- Where is the resonance ?

$$\rho_{\text{res}} \approx 1.4 \times 10^6 \text{ g/cm}^3 \left(\frac{\Delta m^2}{1 \text{ eV}^2} \right) \left(\frac{E}{10 \text{ MeV}} \right)^{-1} \left(\frac{Y_e}{0.5} \right)^{-1} \cos 2\theta$$

③

$$\rho_H \approx (2-7) \times 10^3 \text{ g/cm}^3$$

$$\rho_L = \begin{cases} 5 - 15 \text{ g/cm}^3 & \text{SMA} \\ 10 - 30 \text{ g/cm}^3 & \text{LMA} \\ \lesssim 10^{-4} \text{ g/cm}^3 & \text{Vacuum} \end{cases}$$

- ④ • No influence to { dynamics of collapse
cooling of the core

- r-process nucleosynthesis ($\rho \gtrsim 10^5 \text{ g/cm}^3$)
is NOT affected
- γ 's traverse static progenitor envelope

3 Flavor MSW

$$\nu_\alpha = U_{\alpha i} \nu_i \quad (\alpha = e, \mu, \tau \quad i = 1, 2, 3)$$

$$U = U_{23} U_{13} U_{12} \quad (\delta = 0)$$

$$= \begin{bmatrix} 1 & & & \\ & c_{23} & s_{23} & \\ & -s_{23} & c_{23} & \\ & & & 1 \end{bmatrix} \begin{bmatrix} c_{13} & & & \\ & 1 & & \\ & & s_{13} & \\ & & & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & & \\ -s_{12} & c_{12} & & \\ & & & 1 \end{bmatrix}$$

$$i \frac{d}{dx} \begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = \left\{ U \begin{bmatrix} m_1^2/2E & & \\ & m_2^2/2E & \\ & & m_3^2/2E \end{bmatrix} U^\dagger + \begin{bmatrix} a_e(x) & & \\ & 0 & \\ & & 0 \end{bmatrix} \right\} \begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix}$$

\leadsto basis \leftarrow crucial

$$a_e = \sqrt{2} G_F N_e(x)$$

$$\begin{bmatrix} \tilde{\nu}_e \\ \tilde{\nu}_\mu \\ \tilde{\nu}_\tau \end{bmatrix} = U_{13}^\dagger U_{23}^\dagger \begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} \left(= U_{12} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix} \right)$$

(Kuo-Pantaleone).

$$i \frac{d}{dx} \begin{bmatrix} \tilde{\nu}_e \\ \tilde{\nu}_\mu \\ \tilde{\nu}_\tau \end{bmatrix} = \left\{ \frac{\Delta m_{12}^2}{4E} \begin{bmatrix} -\cos 2\theta_{12} & \sin 2\theta_{12} & 0 \\ \sin 2\theta_{12} & \cos 2\theta_{12} & 0 \\ 0 & 0 & \frac{\Delta m_{13}^2 + \Delta m_{23}^2}{\Delta m_{12}^2} \end{bmatrix} + a_e(x) \begin{bmatrix} c_{13}^2 & 0 & c_{13} s_{13} \\ 0 & 0 & 0 \\ c_{13} s_{13} & 0 & s_{13}^2 \end{bmatrix} \right\} \begin{bmatrix} \tilde{\nu}_e \\ \tilde{\nu}_\mu \\ \tilde{\nu}_\tau \end{bmatrix}$$

* If $s_{13} \ll 1$, Effectively **2** flavor MSW!
but not between " ν_e " and " ν_{heavy} "

→ Effect of 3rd generation can be taken into account perturbatively.

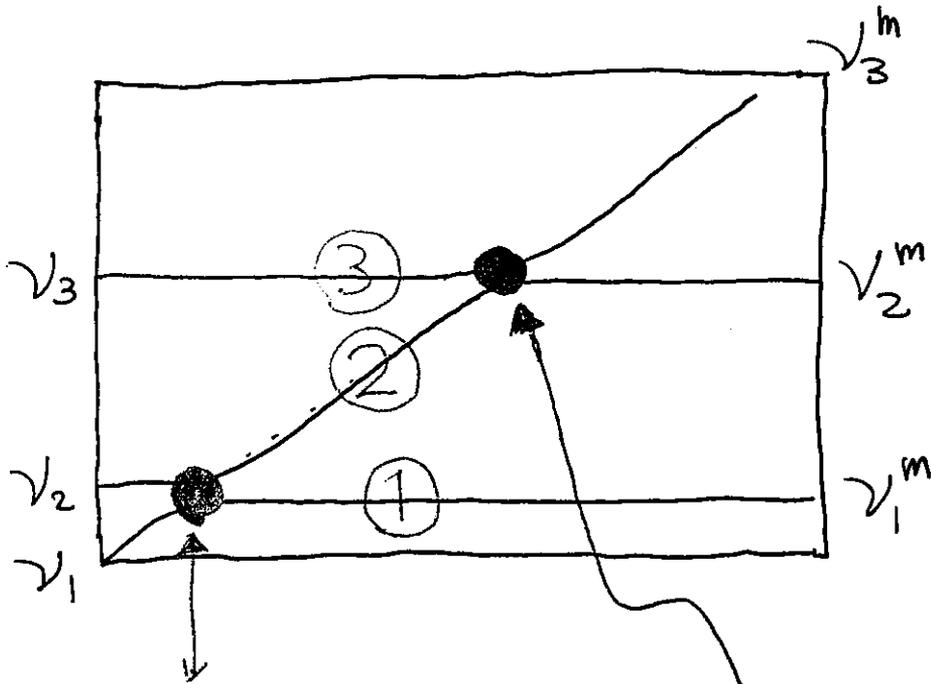
* CHOOZ bound: $\sin^2 2\theta_{13} \lesssim 0.2$
→ $s_{13}^2 \lesssim 5 \times 10^{-2}$

* $P_{3\nu} = c_{13}^4 P_{2\nu} (a_e \cos^2 \theta_{13}) + s_{13}^4$

If $s_{13} \approx 1$ → No solar ν deficit

valid also
for vacuum
oscillation

Two (Hand L) Resonances are independent with each other



For L resonance

H effect is suppressed if

$$U_{e3} \ll 1$$

↑
matter correction
small

$$U_{e3}^m = U_{e3} \left[1 + O\left(\frac{\Delta m_{12}^2}{\Delta m_{13}^2}\right) \right]$$

For H resonance

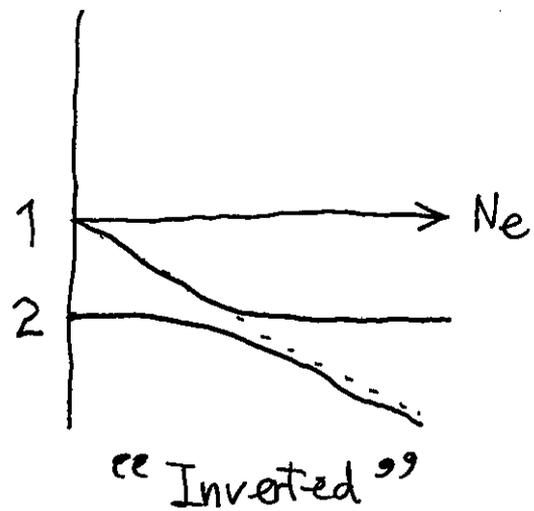
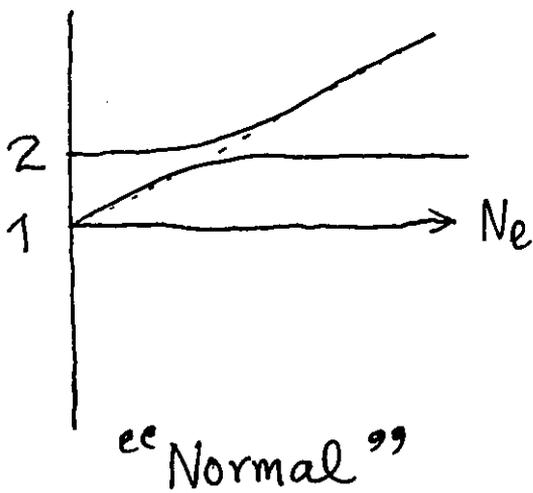
L effect is suppressed ☹️

$U_{e2}^m =$ matter suppressed

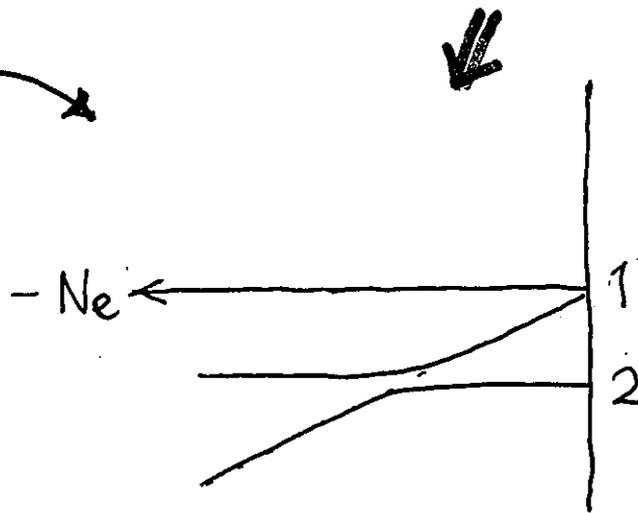
$$U_{e2}^m \approx \frac{\Delta m_{12}^2}{\Delta m_{13}^2} U_{e2}$$

What Happens in the Case of Inverted Mass Hierarchy?

⇒ Resonance in antineutrino channel



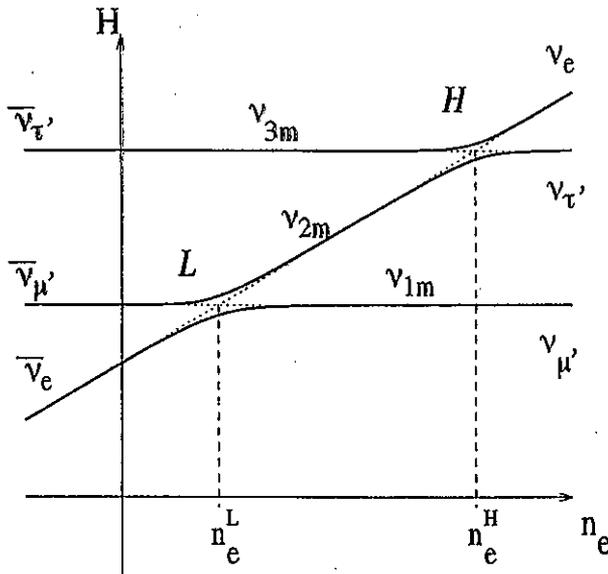
Looks similar!



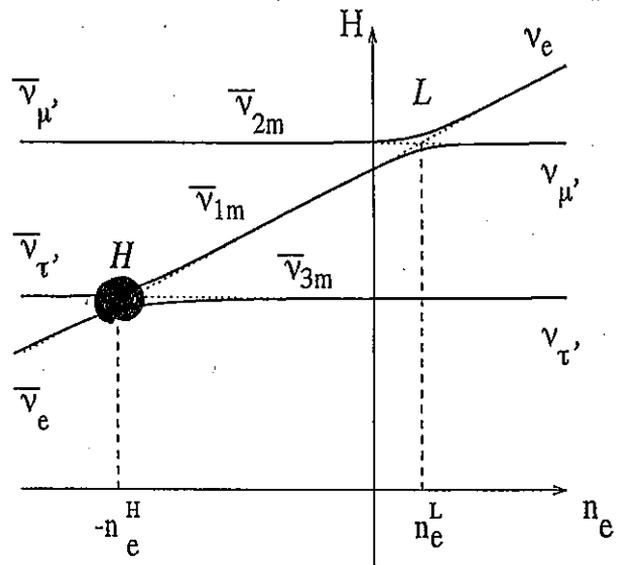
“negative density diagram”

“Normal”

“Inverted”

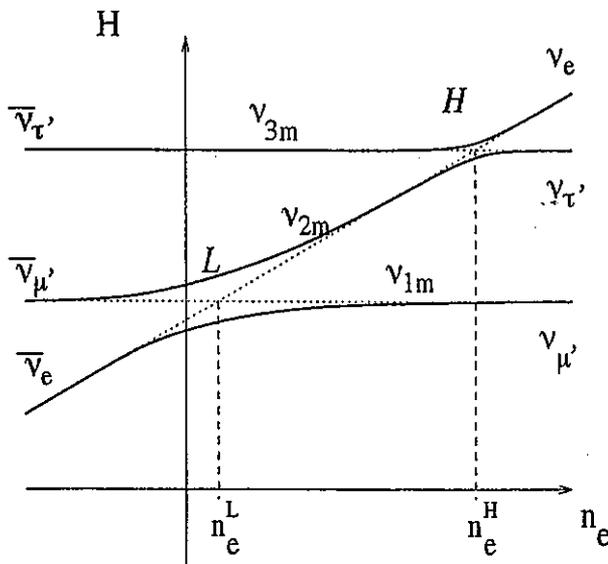


(a)

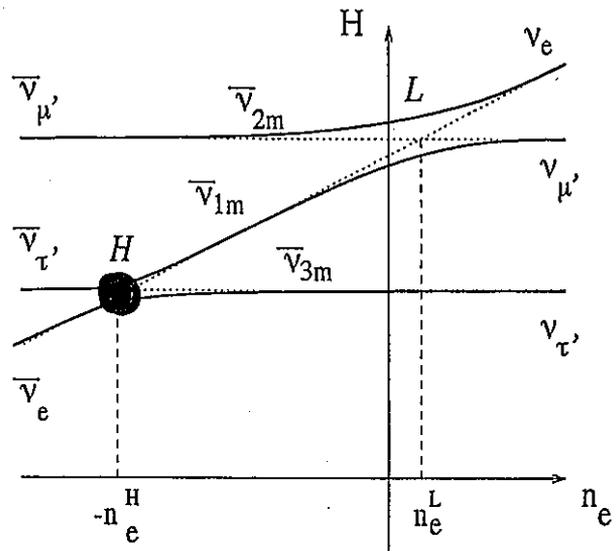


(b)

“Resonance in antineutrino channel”



(c)



(d)

Figure 5: The level crossing diagrams for (a) the normal mass hierarchy and small θ_\odot , (b) the inverted mass hierarchy and small θ_\odot , (c) the normal mass hierarchy and large θ_\odot , (d) the inverted mass hierarchy and large θ_\odot . Solid lines show the eigenvalues of the effective Hamiltonian as functions of the electron number density. The dashed lines correspond to energies of flavor levels ν_e , $\nu_{\mu'}$, and $\nu_{\tau'}$. The part of the plot with $n_e < 0$ corresponds to the antineutrino channel.

If $s_{13}^2 \gtrsim 2.5 \times 10^{-4} \rightarrow$ complete conversion
of $\bar{\nu}_e \leftrightarrow \bar{\nu}_{\tau'}$

1st Conclusion

IF H resonance is adiabatic

then { $\nu_e \leftrightarrow \nu_H$ conversion
for Normal mass hierarchy
 $\bar{\nu}_e \leftrightarrow \bar{\nu}_H$ conversion
for Inverted mass hierarchy

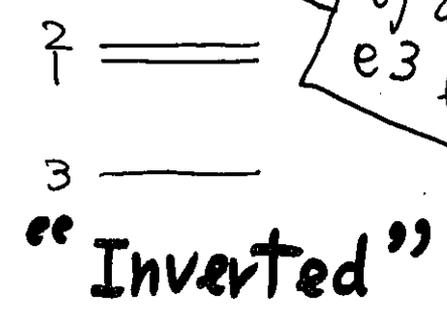
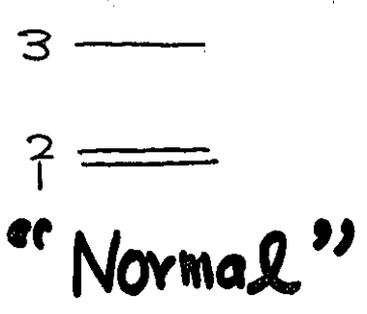
irrespective of the ν_θ solution



Higher { " ν_e " (NMH) Temperature!
" $\bar{\nu}_e$ " (IMH)

should be
easy to
detect

SN ν 's May Judge $\Delta m_{13}^2 \gtrless 0$



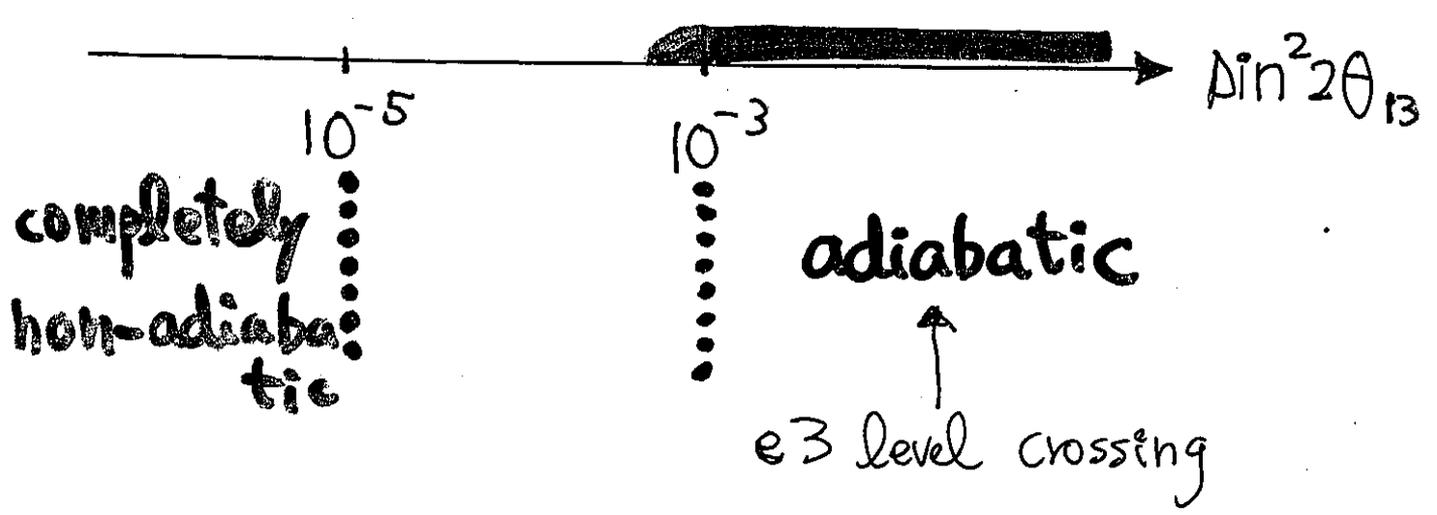
if adiabatic
e3 level crossing

makes great difference
in supernova!

Suppose $m_\nu = \sqrt{\Delta m_{atm}^2}$,
largest for clarity

- Smirnov
- Spergel
- Bahcall '94
- Jegerlehner
- Neubig
- Raffelt '96
-

Physics of SN ν depends on θ_{13}



(H.M. & H. Nunokawa; to appear)

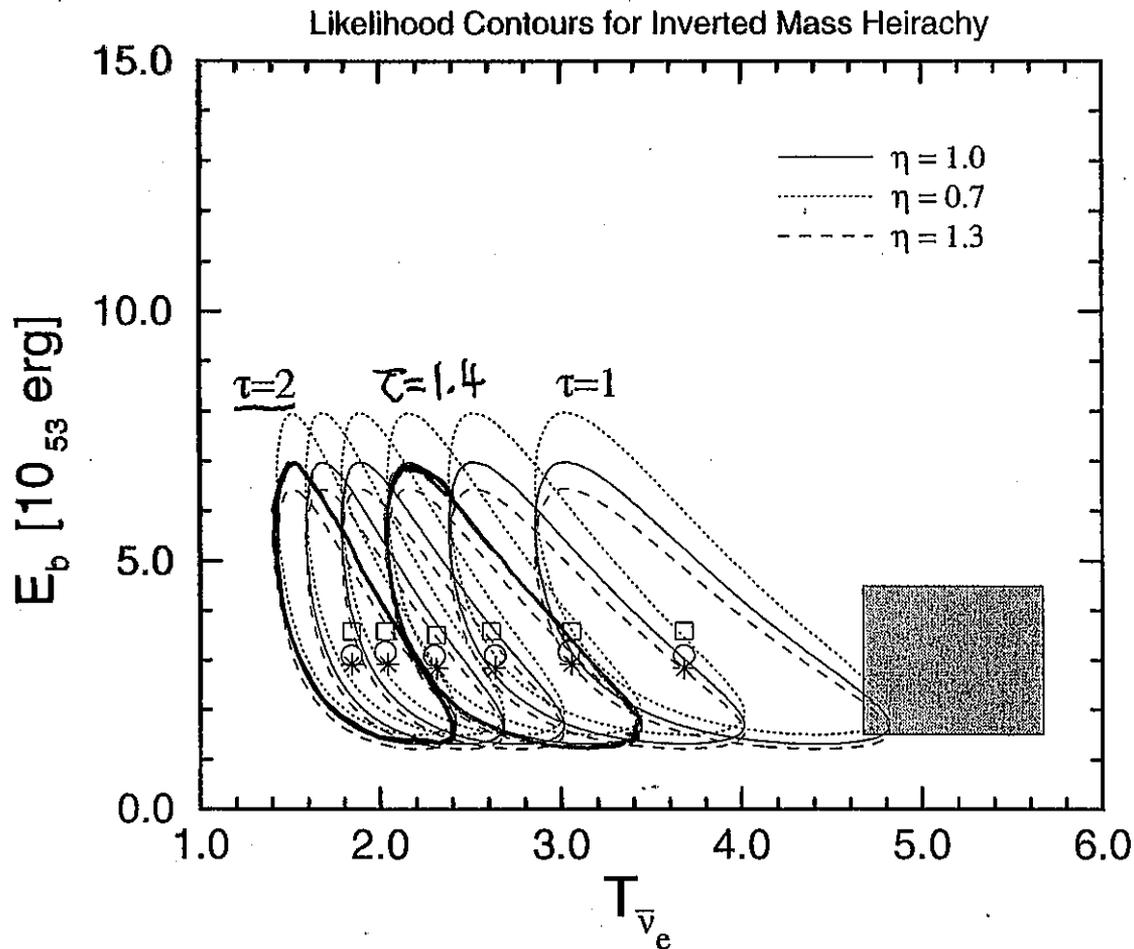


Figure 1: Contours of constant likelihood which correspond to 95.4 % confidence regions. Best-fit points for $T_{\bar{\nu}_e}$ and E_b are also shown by the open circles. From left to right, $\tau \equiv T_{\bar{\nu}_x}/T_{\bar{\nu}_e} = T_{\nu_x}/T_{\bar{\nu}_e} = 2, 1.8, 1.6, 1.4, 1.2$ and 1.0 where $x = \mu, \tau$. This plot corresponds to Fig. 5 in Raffelt's paper in PRD 54, 1194 (1996) but inverted mass hierarchy and complete conversion of $\bar{\nu}_e \leftrightarrow \bar{\nu}_\tau$ at the first resonance is assumed (so this result does not depend on which solution we assume for the solar neutrino problem as discussed in the Dighe-Smirnov paper [hep-ph/9907423]). The dotted lines (open squares) and dashed lines (stars) correspond to the cases where energy equipartition between electron type neutrinos and other neutrinos is violated, as $L_{\nu_x} = L_{\bar{\nu}_x} = \eta L_{\nu_e} = \eta L_{\bar{\nu}_e}$ ($x = \mu, \tau$) with $\eta = 0.7$ and 1.3 , respectively.

$$\tau \equiv \frac{T_{\bar{\nu}_x}}{T_{\bar{\nu}_e}}$$

$$\eta \equiv \frac{L_{\bar{\nu}_x}}{L_{\bar{\nu}_e}}$$

How to Observe MSW Effect in SN?

Normal mass Hierarchy

① Enhancement of high energy

$\nu_e e$ elastic scattering

forward peak

(H.M. & H. Nunokawa 990)

② Steep rise of oxygen induced

events: $\nu_e {}^{16}\text{O} \rightarrow e^- F$

backward peak

(Quian-Fuller 994)

$$P(\phi) \approx 0.5 - 0.4 \cos \phi \quad \text{at } T_\nu = 5 \text{ MeV}$$

tures of ν_e and ν_H results in a 22–33 % difference in the number of scattering events within the ν_e temperatures between 3 and 4 MeV. Thanks to better statistics with 200–300 events it should be possible to distinguish the cases with and without neutrino mixing. It is good news that the global feature of the burst such as the event number is sensitive to the presence of the flavor mixing. The signature in the ^{37}Cl detector is even more prominent. The expected number of ^{37}Ar atoms is a factor of ~ 3 larger with the neutrino mixing. These sensitivities are, however, due to the assumed factor 2 difference in the ν_e and ν_H temperatures.

Minokata - Nunokawa

TABLE I. Expected numbers of events in the proposed super-Kamiokande detector from the neutrino burst from a future supernova at distance of 10 kpc. The table include the case with $T_{\nu_e} = T_{\bar{\nu}_e} = 3$ and 4 MeV, each with and without the effect of neutrino mixing. The event numbers are calculated by two different detection efficiencies (see text) with effective threshold at electron energy of about 7.5 and 5 MeV, and the latter results are presented in parentheses.

T_{ν_e} (MeV)	Neutrino mixing	Absorption	ν_e and $\bar{\nu}_e$ scattering events		Total scattering events $(a)+(b)+(c)+(d)$	^{37}Cl	
		$\bar{\nu}_e p \rightarrow n e^+$	$\nu_e^{(a)}$	$\bar{\nu}_e^{(b)}$			$\nu_\mu + \nu_\tau^{(c)}$
3	No	4024 (4496)	85 (133)	17 (33)	49 (60)	39 (49)	190 (275)
	Yes		160 (192)	17 (33)	37 (51)	39 (49)	253 (325)
4	No	6167 (6463)	118 (160)	27 (44)	57 (66)	46 (55)	248 (325)
	Yes		183 (209)	27 (44)	46 (58)	46 (55)	302 (366)

$\sim 20\% \uparrow$

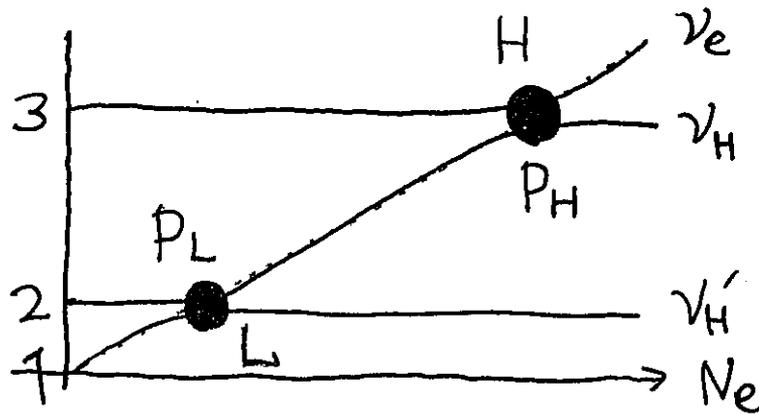
pending on the temperature of the neutrino. With such large numbers of events it should be possible to do more detailed analyses of the neutrino events. They include the energy distributions and the time profile of the events which are the topics of the following two subsections.

To reveal the sensitivity to the detection threshold we have repeated the same calculation with $q_{th} = 1.7$ MeV. This value corresponds to 50% detection efficiency at electron energy ~ 5 MeV, whereas our "official" choice in (4) corresponds to ~ 7.5 MeV. The latter one will be used throughout our subsequent analysis unless otherwise stated. The results of the computation with lower thresh-

Normal Mass Hierarchy

Smirnov - Dighe
hep-ph/9907423

Conversion Probabilities and ν Fluxes at the Earth



P_H, P_L : Jump probabilities

well approximated by Landau-Zener formula

mass eigenstate 1
 ν flux at the earth

$$F_1 = P_H P_L F_e^0 + (1 - P_H) P_L F_x^0 + (1 - P_L) F_x^0$$

$$= P_H P_L F_e^0 + (1 - P_H P_L) F_x^0$$

ν_e flux at SN ν sphere

ν_H flux at SN ν sphere

$$F_2 = P_H(1-P_L) \underset{\uparrow v_e}{F_e^0} + (1-P_H)(1-P_L) \underset{\uparrow v_H}{F_x^0} + P_L \underset{\uparrow v_H}{F_x^0}$$

$$= P_H(1-P_L) F_e^0 + \{1 - P_H(1-P_L)\} F_x^0$$

$$F_3 = (1-P_H) \underset{\uparrow v_e}{F_e^0} + P_H \underset{\uparrow v_H}{F_x^0}$$

NO contribution from v_H'

They have common form !

$$F_i = a_i F_e^0 + (1-a_i) F_x^0$$

$$a_1 = P_H P_L, \quad a_2 = P_H(1-P_L), \quad a_3 = 1 - P_H$$

$\Rightarrow v_e$ at the Earth :

$$F_e = \sum_i |U_{ei}|^2 F_i$$

$$= F_e^0 \left(\sum_i |U_{ei}|^2 a_i \right) + F_x^0 \left(\sum_i |U_{ei}|^2 (1-a_i) \right)$$

||
 $1 - \sum_i |U_{ei}|^2 a_i$

$$\odot F_e = p F_e^0 + (1-p) F_x^0$$

$$p \equiv \sum_i |U_{ei}|^2 a_i \quad (p = v_e \text{ survival probability})$$

$$= |U_{e1}|^2 P_H P_L + |U_{e2}|^2 P_H (1-P_L) + |U_{e3}|^2 (1-P_H)$$

To compute ν_μ and ν_τ flux, there is a clever way!

$$\text{original total flux} = F_e^0 + 2F_x^0$$

$$\text{flux at the earth} = F_e + F_\mu + F_\tau$$



$$F_\mu + F_\tau = F_e^0 + 2F_x^0 - \underbrace{F_e}_{p F_e^0 + (1-p) F_x^0}$$

$$= (1-p) F_e^0 + (1+p) F_x^0$$

$\left. \begin{matrix} F_e \\ F_\mu + F_\tau \end{matrix} \right\}$ can be written only by p !

- 2 independent level crossing
- ν_μ and ν_τ is indistinguishable in **SN**

Repeating similar treatment for $\bar{\nu}_\nu$, ($\bar{p} = |U_{e1}|^2$)

$$\begin{bmatrix} F_e \\ F_{\bar{e}} \\ 4F_x \end{bmatrix} = \begin{bmatrix} p & 0 & 1-p \\ 0 & \bar{p} & 1-\bar{p} \\ 1-p & 1-\bar{p} & 2+p+\bar{p} \end{bmatrix} \begin{bmatrix} F_e^0 \\ F_{\bar{e}}^0 \\ F_x^0 \end{bmatrix}$$

(generally, $\bar{p} = |U_{e1}|^2 (1 - \bar{p}_L) + |U_{e2}|^2 \bar{p}_L$)

Inverted Mass Hierarchy

" ν_e is like $\bar{\nu}$ in normal hierarchy case"

$$P = |U_{e1}|^2 P_L + |U_{e2}|^2 (1 - P_L)$$

$$\bar{P} = |U_{e1}|^2 \bar{P}_H (1 - \bar{P}_L) + |U_{e2}|^2 \bar{P}_H \bar{P}_L$$

$$+ |U_{e3}|^2 (1 - \bar{P}_H)$$

$a_1 \leftrightarrow a_2$
exchange in
 ν case in
NMH

Earth Matter Effect on ν_e

$$F_e^D = \sum_i P_i^E F_i$$

← ~~survival~~ probability of i -th mass eigenstate reach at D as ν_e

$$= F_e^0 \sum_i a_i P_i^E + F_x^0 (1 - \sum_i a_i P_i^E)$$

$$= P^D F_e^0 + (1 - P^D) F_x^0$$

$$P^D \equiv \sum_i a_i P_i^E$$

Difference between F_e^D and F_e : earth matter effect

$$F_e^D - F_e = (P^D - P)(F_e^0 - F_x^0)$$

$$P^D - P = P_H (1 - 2P_L) (P_{20}^E - |U_{e2}|^2)$$

$$+ \frac{(1 - P_H - P_H P_L) (P_{30}^E - |U_{e3}|^2)}{\sqrt{10^{-3}}}$$

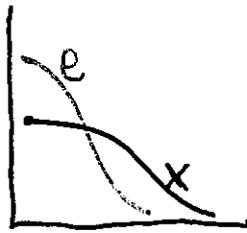


$$F_e^D - F_e = P_H (1 - 2P_L) P_{20}^E (F_e^0 - F_x^0)$$

Difference between 2 detectors:

$$F_e^{D1} - F_e^{D2} \approx P_H (1 - 2P_L) (P_{2E}^{(1)} - P_{2E}^{(2)}) (F_e^0 - F_x^0)$$

① $F_e^0 - F_x^0 \gtrless 0$ at $\begin{pmatrix} \text{low } E \\ \text{high } E \end{pmatrix}$!



② If $P_H = 0$ (adiabatic H resonance)
only very small effect

we have ignored

③ $P_{2E}^{(1)} - P_{2E}^{(2)} \approx \sin 2\theta_{e2}^m \sin (2\theta_{e2}^m - 2\theta_{e2}) \begin{matrix} > 0 \\ \text{for SMA} \\ \text{LMA} \end{matrix}$
 $\times \left[\sin^2 \left(\frac{\pi d_1}{l_m} \right) - \sin^2 \left(\frac{\pi d_2}{l_m} \right) \right]$

④

$d_1 > d_2 \Rightarrow P_{2E}^{(1)} > P_{2E}^{(2)}$

⇒ $\begin{cases} \text{high } E: & \text{matter suppression of } \nu_e \text{ flux} \\ \text{low } E: & \text{matter enhancement of } \nu_e \text{ flux} \end{cases}$