Extra-Dimension

のニュートリノ物理

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1. Introduction

Neutrino Physics.

neutrino oscillation

solution to $\begin{pmatrix} \text{Solar} \\ \text{Atm} \end{pmatrix}$ neutrino anomaly.

$\Rightarrow$ evidence of $\left\{ \begin{array}{l} m_\nu \neq 0 \\ \text{mixing} \end{array} \right.$

New Physics beyond the standard model.!

◎ $\left. \begin{array}{l} \text{Important} \\ \text{New} \end{array} \right\}$ information to $\left\{ \begin{array}{l} \text{fermion masses} \\ \text{mixings} \end{array} \right.$

$\because$ fermion mass hierarchy problem in the SM.

However ....

$m_\nu \ll m_{lepton}, m_{quark}$ makes the problem more difficult .....

◎ Why m_ν is so small ?

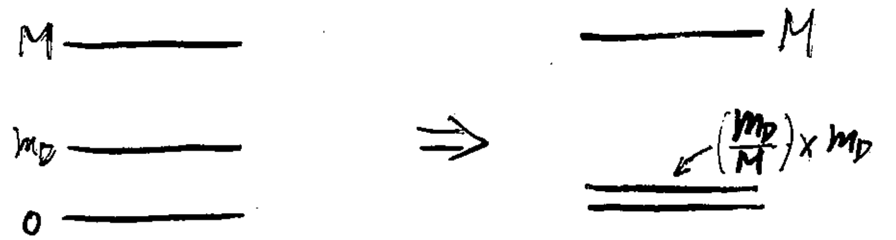
◎ What physics behind $\left\{ \begin{array}{l} m_\nu \neq 0 \\ \theta \neq 0 \end{array} \right.$?

◎ See-Saw Mechanism

ν is Majorana particle

$$\begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix} \xrightarrow[\text{See-saw}]{} \begin{pmatrix} m_D^2/M & 0 \\ 0 & M \end{pmatrix}$$

if $m_D \ll M$



Enhancement of hierarchy $m_D \ll M$

physics at scale M ? $\longleftarrow$ oscillation data

◎ Another mechanism to generate small m_ν ?

◎ Extra-Dimension ?

geometrical meaning of $\begin{cases} \text{small } m_\nu \\ \text{mixing} \end{cases}$

Dirac neutrinos

$$\left(\frac{m}{M}\right)^{\alpha} \times m$$

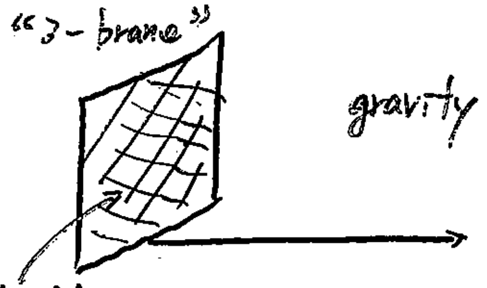
In general, $\alpha \neq \mathbb{N}$.

2. Theory with Extra-dimension(s)

∘ Old : Kaluza-Klein theory.

gravity in 4+n dim $\rightarrow$ gravity $\oplus$ gauge int. in 4-dim.

∘ New : "Brane World"



SM fields are confined on the brane

Which field lives where

◎ New approach to the gauge hierarchy problem.

$$M_W \ll M_{pl}.$$

(I) Large (flat) Extra-Dimensions. (Antoniadis et al. / Arkani-Hamed et al)

$$\frac{1}{2} M_F^{2+n} \int d^{4+n}x \, R_{4+n} \rightarrow \frac{1}{2} \underbrace{M_F^{2+n} V_n}_{= M_{pl}^2} \int d^4x \, R_4$$

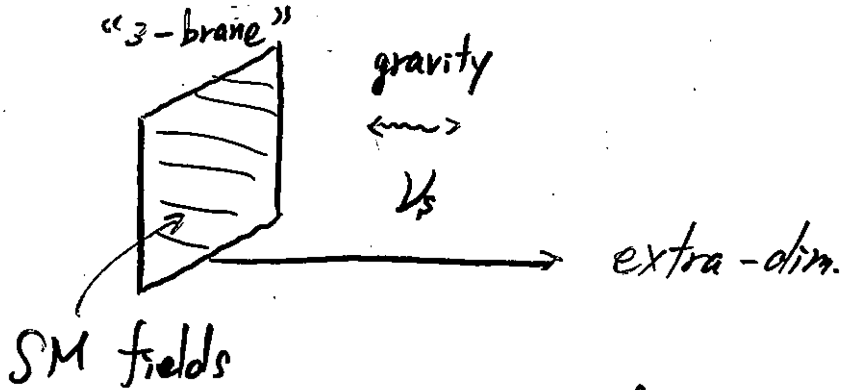
ex) $\begin{cases} M_F \sim \mathcal{O}(1\,\text{TeV}) \\ V_n \sim r^n \; : \; \underline{r \sim 1\,\text{mm}} \; (\text{for } n=2) \end{cases}$

(II) Small Warped Extra-Dimension (Randall-Sundrum)

$$ds^2 = \underbrace{e^{-kr_c|\phi|}}_{\text{warp factor}} (dt^2 - d\vec{x}^2) - r_c^2 \, d\phi^2$$

$$\langle H \rangle \sim M_p \times e^{-kr_c\pi}$$

For neutrino physics



fermion
gauge
Higgs.

* In general.

All singlet fields under G_{SM} can live in the bulk.

(ex) Sterile neutrinos.

- geometrical meanings of { small m_ν, mixing.
- v.s. oscillation data

※ SM with ν_R

In general, we can write down Majorana mass $\underline{M \, \overline{\nu_R^c} \, \nu_R}$

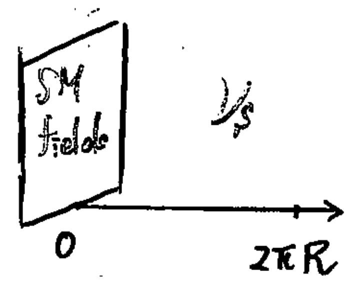
$\because$ ν_R is singlet under G_{SM}.

If $m_D \ll M$ $\Rightarrow$ sea-saw

3. Neutrino Physics
in theory with Large Extra-Dimensions.

(Dvali – Smirnov
NP B563 (1999) 63).

(3-1) Set Up.



- All SM fields are localized on the brane.
- singlet neutrino lives in the bulk
- extra-dimensions are compactified on S^n

(3-2) Neutrino mixing with the bulk modes. 0/21

(i) bulk mode (5-dim).

$$S_5^{free} = \int d^4x \int_0^{2\pi R} dy \; \overline{\Psi} \; i \Gamma^A \partial_A \Psi$$

$$\Gamma^A = (\underset{\mu = 0,1,2,3.}{\gamma^\mu}, \; i\gamma_5)$$

∘ K-K Mode decomposition

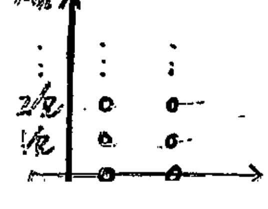
$$\Psi(x^\mu, y) = \sum_n \left[\psi_L^n(x)\, \xi_n(y) + \psi_R^n(x)\, \eta_n(y) \right]$$

$$\bullet\ \mathcal{L}_{(4)} = \sum_n \Big[\overline{\psi_L^n}\, i\partial\!\!\!/\, \psi_L^n + \overline{\psi_R^n}\, i\partial\!\!\!/\, \psi_R^n - \big(\underset{\text{K-K mass}}{m_n}\, \overline{\psi_L^n}\, \psi_R^n + h.c. \big) \Big]$$

$\Rightarrow$

- $\left(\frac{d^2}{dy^2} + m_n^2 \right) \begin{pmatrix} \xi_n \\ \eta_n \end{pmatrix} = 0$
- boundary condition
  $\begin{matrix} \xi \\ \eta \end{matrix}(y) = \begin{matrix} \xi \\ \eta \end{matrix}(y + 2\pi R)$ compactification on S^1
- orthonormality condition
  $\int dy\; \xi_n \xi_m = \int dy\; \eta_n \eta_m = \delta_{nm}$

$$\Rightarrow \begin{cases} m_n = \dfrac{n}{R} \quad : n = 0, \pm 1, \pm 2, \cdots \\ \xi_n(y) = \eta_n(y) = \dfrac{1}{\sqrt{2\pi R}}\, e^{i \frac{n}{R} y} \end{cases}$$



(ii) neutrino mass matrix.

$$\mathcal{L}_{mass} = \int dy \, \frac{g_Y}{\sqrt{M_F}} \left(\overline{\nu_{eL}} \, H \, \delta(y) \right) \Psi$$

live on brane.

$$\Psi = \sum_n \left(\psi_L^n(x) \, \xi_n(y) + \psi_R^n(x) \, \eta_n(y) \right)$$

$$= \frac{g_Y}{\sqrt{2\pi R \cdot M_F}} \cdot H \cdot \overline{\nu_{eL}} \left(\sum_{n=-\infty}^{\infty} \psi_R^n \right)$$

effective Yukawa coupling

$$g_Y^{eff} \sim \frac{1}{\sqrt{2\pi R \cdot M_F}} = \frac{M_F}{M_{pl}} \ll 1.$$

$\because$ Gauss' law

$$M_F^{2+1} \cdot 2\pi R = M_p^2$$

volum suppression !

- Mass Matrix.

$$\begin{cases} \nu_R^0 \equiv \psi_R^0 \\ \nu_R^n \equiv \frac{1}{\sqrt{2}} \, \psi_R^n + \psi_R^{-n} \\ \nu_L^n \equiv \frac{1}{\sqrt{2}} \, \psi_L^n - \psi_L^{-n} \end{cases}$$

$$(\nu_{eL}, \nu_L^1, \nu_L^2 \cdots) \begin{bmatrix} m_D & \sqrt{2}\, m_D & \sqrt{2}\, m_D & \cdots \\ 0 & 1/R & 0 & \cdots \\ \vdots & 0 & 2/R & \\ \vdots & \vdots & & \ddots \end{bmatrix} \begin{pmatrix} \nu_R^0 \\ \nu_R^1 \\ \nu_R^2 \\ \vdots \end{pmatrix}$$

$$m_D \sim \frac{M_F}{M_{pl}} \times v$$

★ $\nu_e \to \nu_{KK}$ oscillation !!

$$\nu_{eL} \longrightarrow \times \longrightarrow \nu_R^{(n)} \longrightarrow \times \longrightarrow \nu_L^{(n)}$$

(3-3) Oscillation Phenomena

(i) vacuum oscillation.

- diagonalize (Assume $m_D \ll 1/R$)

- $\begin{cases} m_0 \sim m_D \\ m_n \sim n/R \end{cases}$

- mixing angle $(\nu_L - \nu_L^n)$

$$\tan\theta_n \sim \xi/n \quad ; \quad \xi \equiv \sqrt{2}\, m_D \cdot R \ll 1$$

$$\therefore \quad \nu_e \sim \frac{1}{N}\left(\nu_0 + \xi \sum_{n=1}^{\infty} \frac{1}{n} \nu_n \right)$$

$$N^2 = 1 + \xi^2 \sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{\pi^2}{6}\xi^2.$$

$$\therefore \quad \nu_e(t) = \frac{1}{N}\left(\nu_0 + \xi \sum_{n=1}^{\infty} \frac{1}{n} e^{i\phi_n} \nu_n \right)$$

$$\phi_n = \frac{\Delta m_n^2 t}{2E} \sim \frac{m_n^2 t}{2E} = \frac{n^2}{2R^2 E} t.$$

◎ $P_{\nu_e \to \nu_e} = \left| \langle \nu_e(0) | \nu_e(t) \rangle \right|^2$

$$= \frac{1}{N^4} \left| 1 + \xi^2 \sum_{n=1}^{\infty} \frac{e^{i\phi_n}}{n^2} \right|^2$$

$$\overline{P}_{\nu_e \to \nu_e} = \frac{1}{\left(1 + \frac{\pi^2}{6}\xi^2\right)^2} \left(1 + \frac{\pi^4}{90}\xi^4 \right)$$

$$\sim 1 - \frac{\pi^2}{3}\xi^2$$

※ Note two ν oscillation case

$$\overline{P} \sim 1 - 2\,\xi^2.$$

(ii) Oscillation in Matter.

MSW effect.

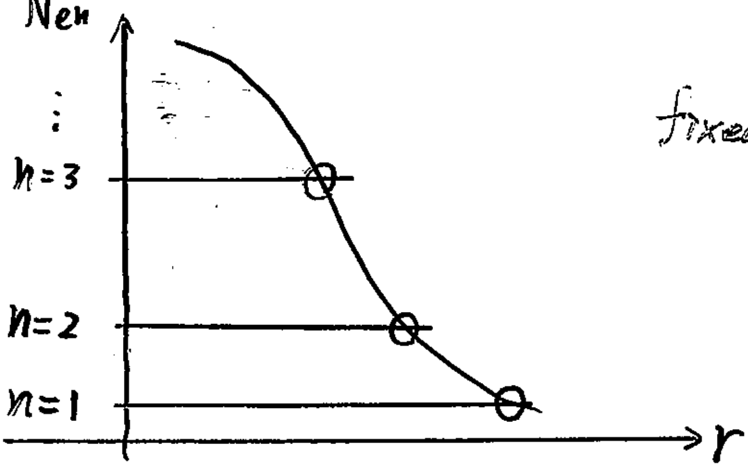
resonance condition

$$N_{en} \equiv \frac{\rho}{m_N}\left(Y_e - \frac{1}{2}Y_n\right)$$

$$\frac{\Delta m_n}{2E} \sim V = G_F N_{en}$$

$$\circledS \quad \Delta m_n^2 = m_n^2 - m_D^2 \sim \left(\frac{n}{R}\right)^2$$

$$\Rightarrow N_{en}^{res.} = \frac{n^2}{2E G_F R^2} \propto n^2$$



fixed E.

There are many resonance points in the medium!

Survival probability $\nu_e \to \nu_e$ after crossing of k resonances is product in each resonance

$$P = P_1 \cdot P_2 \cdot \cdots \cdot P_k$$

∘ Using Landau - Zenner formula.

$$P_n = \begin{cases} 1 & : E < E_{nR} \\ e^{-\frac{\pi}{2} K_n} & : E > E_{nR} \end{cases}$$

$$\begin{cases} E_{nR} \sim \dfrac{m_n^2}{2 G_F \cdot N_{en}^{int.}} \propto n^{-2} \\ K_n = \dfrac{m_n^2}{2E} \dfrac{\sin^2 2\theta_n}{\underbrace{\cos 2\theta_n}_{\simeq 1}} \cdot \dfrac{N_{en}}{dN_{en}/dr} \end{cases}$$

$n^2 \propto m_n^2$

$$\equiv K = \frac{4\xi^2}{E \cdot R^2} \cdot \frac{N_{en}}{dN_{en}/dr} \leftarrow \text{independent of } n$$

$$\therefore P \sim e^{-\frac{\pi}{2} K f(E)}$$

$$f(E) = \begin{cases} 0 & E < E_{1R} \\ n & E_{n,R} < E < E_{n+1,R} \end{cases}$$

$$E_{n,R} = n^2 E_{1R}$$

(3-4) Solution to Solar ν problem

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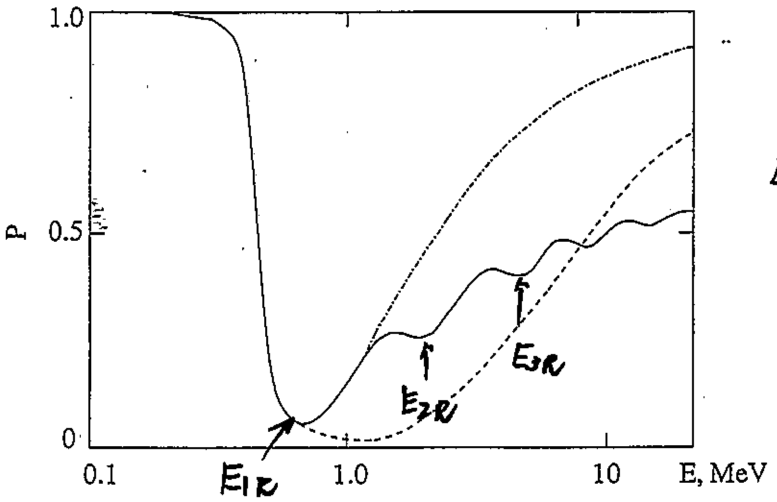
neutrino oscillation

$\nu_e \to \nu_{KK}$ with small angle MSW.

— Fit —

$$\begin{cases} m_1 = \frac{1}{R} \sim (2\text{-}3) \times 10^{-3}\ eV \\ \xi^2 \sim 0.25 \times 10^{-3} \end{cases}$$

$$\Rightarrow M_F \sim 5\ TeV$$



$E_{nR} = n^2 E_{1R}$

Figure 1: The survival probability as the function of neutrino energy for the electron neutrino conversion to the bulk states in the Sun (solid line), $4\xi^2 = 10^{-3}$. Dot-dashed line shows the survival probability of the two neutrino conversion for the equivalent mixing $\sin^2 2\theta = 10^{-3}$. Dashed line corresponds to the survival probability of the two neutrino conversion for $\sin^2 2\theta = 4 \cdot 10^{-3}$ which gives good fit of the total rates in all experiments.

(3-5) Summary.

◎ Implication of theory with Large Extra-Dimensions for neutrino physics are considered.

- Phenomenology is determined by

  (i) bulk fermion = sterile neutrino

  (ii) large number of ν_s is involved in physical process.

  (iii) for $m_D \ll 1/R$, active neutrino is the combinations of mass eigenstates with $\begin{cases} \text{increasing masses} & n/R \\ \text{decreasing mixing angles.} & \propto \frac{1}{n} \end{cases}$

- The resonance conversion to the bulk states can solve the solar ν problem. with $R \sim 0.06 - 0.1$ mm.

"Probing Large Extra Dimensions with Neutrinos."

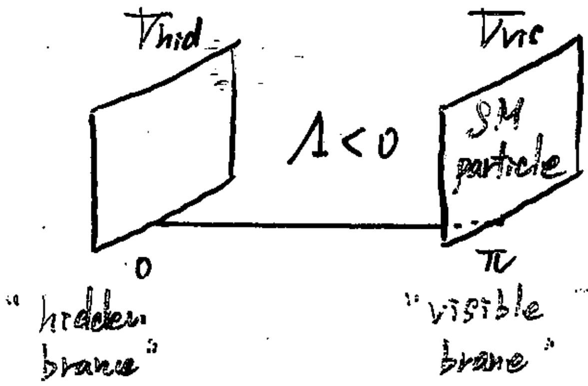
4. Neutrino Physics in theory with Small Warped Extra-Dimension.

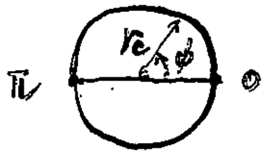
(Grossman - Neubert
PL B474 (2000) 361

(4-1) Randall - Sundrum Model (Randall-Sundrum '99)

— Set up —

5-dim theory, compactified on S^1/Z_2





Z_2-parity $\phi \to -\phi$.

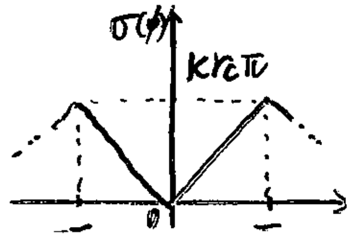
$$\begin{cases} S_{bulk} = -\frac{1}{2} M_F^3 \int d^5x \sqrt{g}\,(R + 2\Lambda) \\ S_{hid} = \int d^4x \sqrt{-g_{hid}}\,(\mathcal{L}_{hid} - V_{hid}) \\ S_{vis} = \int d^4x \sqrt{-g_{vis}}\,(\mathcal{L}_{vis} - V_{vis}) \end{cases}$$

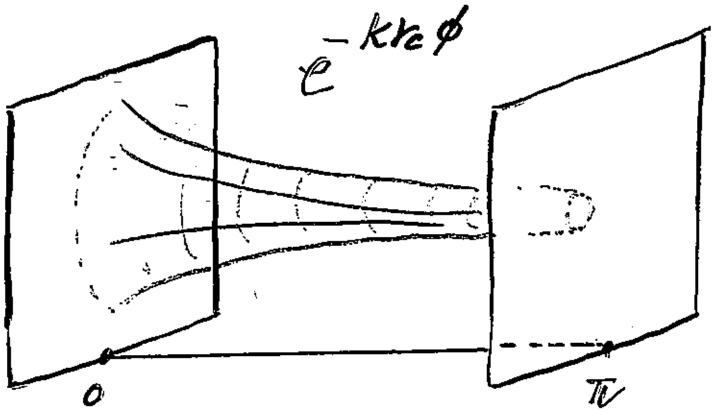
— Metric ansatz —

$$ds^2 = e^{-2\sigma(\phi)} \eta_{\mu\nu} dx^\mu dx^\nu - r_c^2 d\phi^2$$

◎ Solution of 5-dim Einstein eqs.

$$\begin{cases} \sigma(\phi) = k r_c |\phi| \\ k^2 = \frac{-\Lambda}{24 M_F^3} \Leftrightarrow \Lambda < 0 \\ V_{hid} = -V_{vis} = 24 k M_F^3 \end{cases}$$





slice of AdS_5

◎ 4-dim M_{pl}

$$M_{pl}^2 = \frac{M_F^3}{k}\left(1 - e^{-2kr_c\pi}\right)$$

$$\sim M_F^3/k \qquad (\text{for } kr_c\pi \gg 1)$$

◎ Solution to hierarchy problem

$$S_{Higgs} = \int d^4x \sqrt{-g_{vis}}\left[g_{vis}^{\mu\nu} (D_\mu H_0)^\dagger (D_\nu H_0) - \lambda\left(H_0^\dagger H_0 - v_0^2\right)^2 \right]$$

$$\sqrt{-g_{vis}}\, g_{vis}^{\mu\nu} = \underbrace{e^{-2kr_c\pi}}_{\text{warp factor}}\eta^{\mu\nu}$$

$\Rightarrow$ rescale

$$H_0 \to e^{kr_c\pi} H$$

$$\therefore \int d^4x\ \eta^{\mu\nu}(D_\mu H)^\dagger (D_\nu H) - \lambda\left(H^\dagger H - v^2\right)^2$$

$$v = v_0\, e^{-kr_c\pi}$$

suppression by warp factor

$$v \sim O(M_W)\ !!$$

$$\text{for}\ \begin{cases} v_0 \sim M_{pl} \\ kr_c \sim 12 \end{cases}$$

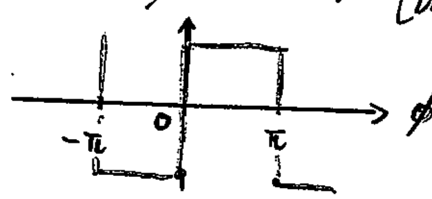
◎ $M_F \sim k \sim M_{..}$ ...

(4-2) Bulk fermion in R-S background. 5/21

$$\mathcal{L} = \sqrt{g}\Big[\bar{\Psi}\, i\Gamma^a \underbrace{e_a^{\ M}}\big(\partial_M + \underbrace{\tfrac{1}{8}\,\omega_M^{\ bc}[\Gamma_b,\Gamma_c]}_{\text{spin connection}}\big)\Psi - \underbrace{m\,\theta(\phi)}_{\text{bulk mass}}\,\bar{\Psi}\Psi\Big]$$

- $\Gamma^a = (\gamma^\mu,\ i\gamma_5)$
- $\omega_\mu^{\ \nu 5} = e^{-kr_c|\phi|}\, kr_c\, \theta(\phi)\, \delta_\mu^\nu$
- bulk mass term.

  $m\,\theta(\phi)$ ＊ kink profile of $\begin{cases} \omega \\ \text{bulk mass} \end{cases}$



◎ $\mathbb{Z}_2$ - parity.

$\mathcal{L}$ is invariant w.r.t $\phi \to -\phi$ ∵ $S^1/\mathbb{Z}_2$ orbifolding.

⇕

$$\gamma_5 \Psi(x^\mu, \phi) = \pm\Psi(x^\mu, -\phi)$$

if we take even parity

$$\Rightarrow \begin{cases} \Psi_L : \text{odd} & \Psi_L(x,\phi) = -\Psi_L(x,-\phi) \\ \Psi_R : \text{even} & \Psi_R(x,\phi) = \Psi_R(x,-\phi) \end{cases}$$

Note consistent bulk mass term.

$$\mathcal{L}_{mass} = -\underbrace{m\,\theta(\phi)}_{\text{odd}}\ \underbrace{\bar{\Psi}_L}_{\text{odd}}\ \underbrace{\Psi_R}_{\text{even}} + h.c.$$

◎ Bulk fermion solution

- K-K decomposition

$$\begin{cases} \Psi(x^\mu, \phi) = \sum_n \left[\psi_L^n(x)\, \xi_n(\phi) + \psi_R^n(x)\, \eta_n(\phi) \right] \\ S_{(4)} = \sum_n \int d^4x \left[\bar{\psi}^n i\partial\!\!\!/ \psi^n - \underbrace{m_n}_{\text{K-K mode mass.}} \bar{\psi}^n \psi^n \right] \end{cases}$$

$$\begin{cases} \xi_n(\phi) \equiv \dfrac{e^{2kr_c\phi}}{\sqrt{r_c}}\, f_L^n(\phi) \\ \eta_n(\phi) \equiv \dfrac{e^{2kr_c\phi}}{\sqrt{r_c}}\, f_R^n(\phi) \end{cases}$$

with

- mode equation

$$\left(\pm \frac{1}{r_c} \partial_\phi - m \right) f^n_{\substack{L \\ R}}(\phi) = - m_n e^{kr_c\phi} f^n_{\substack{R \\ L}}(\phi)$$

- orthonormality condition.

$$\int_0^\pi d\phi\; e^{kr_c\phi} f^n_{\substack{L \\ R}} f^m_{\substack{L \\ R}} = \delta^{nm}$$

- boundary condition

$$Z_2\text{-parity} \Rightarrow f_L^n(\phi) = -f_L^n(-\phi)$$

$$\Rightarrow \underbrace{f_L^n(0) = f_L^n(\pi) = 0}$$

(i) Zero Mode ($m_n = 0$)

$$\begin{cases} \cdot \left(\frac{1}{r_c}\partial_\phi - m\right) f_L^0 = 0 \Rightarrow f_L^0(\phi) \propto e^{m r_c \phi} \\ \cdot \left(\frac{1}{r_c}\partial_\phi + m\right) f_R^0 = 0 \Rightarrow f_R^0(\phi) \propto e^{-m r_c \phi} \end{cases}$$

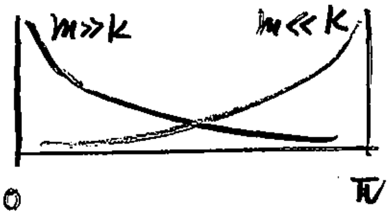
* Z_2 - projection

$$\begin{cases} f_L^0(\phi) = 0 \\ f_R^0(\phi) \propto e^{-m r_c \phi} \end{cases}$$

← project out by boundary cond.

- ∘ chiral fermion ⇐ orbifolding S^1/Z_2
- ∘ configulation of chiral fermion.

$$\eta_0(\phi) \propto e^{(2k-m) r_c \phi}$$



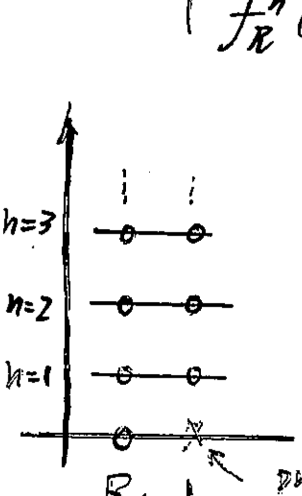
localization of chiral fermion.

※ domain wall fermion.

(ii) massive mode

$$\nu \equiv m/k$$

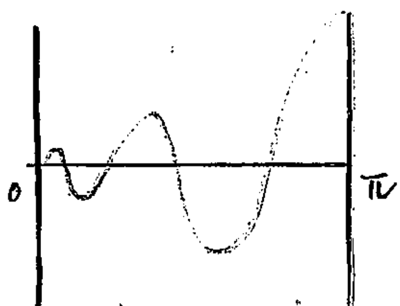
$$\begin{cases} f_L^n(\phi) = e^{\frac{1}{2}k r_c \phi}\left(a_L^n J_{\frac{1}{2}-\nu}\left(\frac{m_n}{k}e^{k r_c \phi}\right) + a_R^n J_{-\frac{1}{2}+\nu}\left(\frac{m_n}{k}e^{k r_c \phi}\right)\right) \\ f_R^n(\phi) = e^{\frac{1}{2}k r_c \phi}\left(a_R^n J_{\frac{1}{2}+\nu}(\ \text{"}\ ) - a_L^n J_{-\frac{1}{2}-\nu}(\ \text{"}\ )\right) \end{cases}$$



KK Mode eigenvalue

$$f_L^n(0) = f_L^n(\pi) = 0$$

$$m_n \sim n \underbrace{k e^{-k r_c \pi}}_{\mathcal{O}(M_W)}$$



K-K modes are localized around ϕ = π

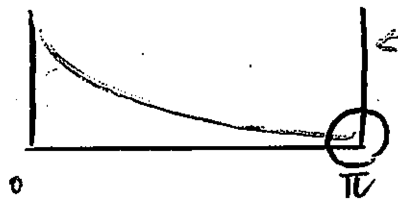
(4-3) Yukawa interaction.

geometrical meaning of small m_ν

$\Updownarrow$

wave function overlapping.

IF. bulk ν localizes around $\phi = 0$.



$\leftarrow$ visible brane ν_L, H.

$\leftarrow$ overlapping is small

$$\mathcal{L}_Y^{(4)} = \int d\phi \sqrt{-g_{vis}} \frac{g_Y}{\sqrt{M_F}} \overline{\nu_L} \cdot H \cdot \underbrace{\Psi_R(x^\mu, \phi=\pi)}_{\Uparrow \; \nu_R^n(x) \; \eta_n(\phi=\pi)}$$

$$\begin{cases} \sqrt{-g_{vis}} = e^{-4kr_c\pi} \\ \nu_L \to e^{\frac{3}{2}kr_c\pi} \nu_L \\ H \to e^{kr_c\pi} H \end{cases} \Leftarrow \text{canonical normalization}$$

$$\eta_0(\pi) = \frac{1}{\sqrt{r_c}} e^{2kr_c\pi} \cdot N \cdot e^{-mr_c\pi}$$

$$; \; N = \sqrt{\frac{k-2m}{e^{(k-2m)r_c\pi}-1}}$$

$$m_\nu = \underbrace{g_Y\sqrt{\frac{k}{M_F}}}_{O(1)} \cdot \langle H \rangle \left(\frac{2\nu - 1}{e^{(2\nu-1)\pi r_c k} - 1} \right)^{1/2} ; \; \nu = \frac{m}{k}$$

$$\sim v \times \left(\frac{v}{M_F} \right)^{\nu - \frac{1}{2}}$$

for $\nu > \frac{1}{2}$

using $\langle H \rangle \sim M_F e^{-kr_c\pi}$

$v/M_F \sim 10^{-16} \Rightarrow 10^{-5}\,\mathrm{eV} < m_\nu < 10\,\mathrm{eV}$

for $1.1 < m/k < 1.5$

◎ factor difference of m $\Rightarrow$ hierarchy.

(4-4) Phenomenological model. 9/21

Solution to $\left\{\begin{array}{l}\text{Solar}\\ \text{Atm}\end{array}\right\}$ neutrino anomaly.

◎ introduce two bulk sterile neutrinos with bulk masses m_1, m_2 $\quad (m_1 > m_2)$

$$-\mathcal{L}_{mass} = (\,\overline{\nu_L^e}\ \ \overline{\nu_L^\mu}\ \ \overline{\nu_L^\tau}\,)\ M \begin{pmatrix} \nu_{1R}^{0} \\ \nu_{2R}^{0} \end{pmatrix}$$

$\nu_L^e, \nu_L^\mu, \nu_L^\tau$: live on brane

ν_{1R}^0, ν_{2R}^0: live in the bulk.

$$M = v \begin{pmatrix} \varepsilon^{\nu_1-\frac{1}{2}}\, x_e & \varepsilon^{\nu_2-\frac{1}{2}}\, y_e \\ \varepsilon^{\nu_1-\frac{1}{2}}\, x_\mu & \varepsilon^{\nu_2-\frac{1}{2}}\, y_\mu \\ \varepsilon^{\nu_1-\frac{1}{2}}\, x_\tau & \varepsilon^{\nu_2-\frac{1}{2}}\, y_\tau \end{pmatrix}$$

$$\left\{\begin{array}{l} \varepsilon \equiv v/M_F \sim 10^{-16} \\ \nu_i = m_i/k \end{array}\right.$$

$$\Rightarrow \left\{\begin{array}{l} m_{\nu_1}^2 = 0 \\ m_{\nu_2}^2 \sim v^2 \left(\frac{v}{M_F}\right)^{2\nu_1-1} \\ m_{\nu_3}^2 \sim v^2 \left(\frac{v}{M_F}\right)^{2\nu_2-1} \end{array}\right.$$

$$\left\{\begin{array}{llll} \cdot\ \Delta m_\odot^2 & < \begin{array}{l}\text{MSW} \\ \text{vacuum}\end{array} & \begin{array}{l} 10^{-6}-10^{-5}\ \text{eV}^2 \\ 10^{-10}\ \text{eV}^2 \end{array} & \begin{array}{c} \nu_1 \\ 1.34-1.37 \\ 1.5 \end{array} \\ \cdot\ \Delta m_\oplus^2 & & 5\times 10^{-4} - 6\times 10^{-3}\ \text{eV}^2 & \begin{array}{c} \nu_2 \\ 1.27-1.29 \end{array} \end{array}\right.$$

◎ We can understand the hierarchy $\Delta m_\odot^2 \ll \Delta m_\oplus^2$ in terms of a small difference of the bulk fermion masses.

(4-5) Summary

1. bulk fermion solution in R-S model

   - existence of zero-mode ← chiral fermion
   - new mechanism to generate

     small m_ν
     hierarchy without fine-tuning.

     ⟺ wave function overlapping.

2. Phenomenological Model

   $3 \times \nu_L \oplus 2 \times \nu_R$
   on brane   in bulk.

5. Summary

Implication of theories with Extra-Dimensions for Neutrino Physics.

- bulk sterile neutrino
- geometrical meaning for small m_ν
  - Dirac ν
  - See-Saw like relation

$$v\left(\frac{v}{M_F}\right)^{\alpha} \qquad \alpha \neq 1 \text{ in general}$$

(∘ volume suppression
(∘ wave function overlapping