U_{e3} 測定が理論に与えるインパクト

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目　次

PRL vol 84 (2000) 3535
Akhmedov, Branco, Rebelo
U_{e3}の一般的予言 ???

	U_{e3}
LMA MSW	$\lambda \sim \lambda^2$
SMA MSW	$10^{-2} \sim 10^{-3}$
VO	very small

1 はじめに

Quark Masses and Mixings

$$\frac{m_c}{m_t} \sim \lambda^4 \quad \frac{m_u}{m_t} \sim \lambda^8 \qquad \lambda \simeq 0.2$$

$$\frac{m_s}{m_b} \sim \lambda^2 \quad \frac{m_d}{m_b} \sim \lambda^4$$

$$|V_{ud}| = \lambda \quad |V_{cb}| \simeq A\lambda^2$$

$$|V_{ub}| \simeq |A\lambda^3(\rho + i\eta)| \qquad \lambda, A, \rho, \eta$$

Small CKM elements are related to Quark Mass Hierarchy

Lepton Masses and Mixings

$$\frac{m_\mu}{m_\tau} \sim \lambda^2 \qquad \frac{m_e}{m_\tau} \sim \lambda^4$$

$U_{\alpha i}$ MNS Mixing ? $\quad m_{\nu_i}$? $i = 1 \sim 3$?

What is the unified picture?

ニュートリノ セクター

知っている情報（ニュートリノ振動）

$\Delta m^2_{atm} = (2 \sim 5) \times 10^{-3} \mathrm{eV}^2$

$\sin^2 2\theta_{atm} > 0.88$

	$\Delta m^2_{\odot}$ (eV^2)	$\sin^2 2\theta_{\odot}$
SMA	$(4 \sim 10) \times 10^{-6}$	$(0.1 \sim 1.0) \times 10^{-2}$
LMA	$(2 \sim 20) \times 10^{-5}$	$0.65 \sim 0.97$
VO	$(0.5 \sim 5) \times 10^{-10}$	$0.6 \sim 1.0$

3 family $\Delta m^2_{atm} = |\Delta m^2_{32}| \gg \Delta m^2_{\odot} = |\Delta m^2_{21}|$

- $m_3 \gg m_2 \gtrsim m_1$ quark-like
- $m_3 \simeq m_2 \simeq m_1$ degenerate
  Non-Abelian Flavor Symmetry が必要
  $\beta\beta_{0\nu}$ $\langle m_{ee} \rangle \lesssim 0.36 \mathrm{eV}$ 90% C.L.
- $m_3 \ll m_2 \simeq m_1$ Inverse Hierarchy
  Zee-Model $\beta\beta_{0\nu}$

$$\sin^2\theta_{atm} = 4U_{\mu 3}^2(1-U_{\mu 3}^2)^2 \simeq 1$$

$$|U_{\mu 3}| \simeq \frac{1}{\sqrt{2}}$$

$$U_{MNS} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

↑ Large Mixing Sector

U_{e1}, U_{e2} depend on Solar Neutrino Solutions

SMA $\quad |U_{e1}| \simeq 1 \quad |U_{e2}| \simeq \lambda^2$

$\left.\begin{array}{l}\text{LMA}\\ \text{VO}\end{array}\right\}$ $|U_{e1}| \simeq |U_{e2}| \simeq \frac{1}{\sqrt{2}}$

$|U_{e3}|$ is constrained by CHOOZ

$$|U_{e3}| \lesssim 0.16 \quad (\Delta m_{32}^2 \simeq 3\times 10^{-3}\,\mathrm{eV}^2)$$

$J_{CP} \propto |U_{e3}|$ $\cancel{CP}$, $\cancel{T}$ 実験

$P(\nu_\mu \to \nu_e) = 4|U_{e3}|^2|U_{\mu 3}|^2 \sin^2 \frac{\Delta m_{31}^2 L}{4E}$ LBL 実験

2 大角度ニュートリノフレーバー混合とモデルの分類

しかけによる分類

hep-ph/0003058 Barr, Dorsner

ニュートリノフレーバー混合行列（MNS行列）

$$U_{MNS} \equiv L_E^{\dagger} L_\nu$$

$$\left.\begin{array}{l} L_E^{\dagger} M_E R_E = M_E^{diag} \\ L_\nu^T M_\nu L_\nu = M_\nu^{diag} \end{array}\right\}$$ in some prefered basis given by underlying theory of that model

I Large mixing from M_ν (L_ν)

(1) Non see saw

(2) See saw

A Large mixing from M_R (Majorana Mass Matrix)

B Large mixing from N (Dirac Mass Matrix)

C Large mixing from $-N^T M_R^{-1} N$

II Large mixing from M_E (L_E)

(1) CKM small by cancellation

(2) lopsided M_E

5'

M_U, M_D, M_E $m_\nu = 0$ in SM

$\underbrace{18 \quad 18 \quad 18}$

パラメータの数 54

観測量 m_{qi}, V_{CKM}, m_{ei} 13

Quark Sector の例

$M_U : 18$ $M_D : 18$] 36

6つの masses と 4 CKM 要素] 10 観測量

<u>26</u> コは観測量ではない。

基底のつけかえの自由度

$$\underset{\psi_L}{\underline{3}} \quad \underset{\psi_{RU}}{\underline{1}}, \quad \underset{\psi_{RD}}{\underline{1}}$$

U(3) 回転の自由度 $9 + 9 + 9 - 1 = 26$

この自由度は SM では意味のあるものではない!!

しかし、Beyond SM では意味をもつ。

Mass Matrix の形を決めるということは、

Beyond SM の領域に入っている。

Model は preferred basis をもつ。

I (1) Large mixing from M_ν

= Non See saw = $\underline{M_E : \text{Diagonal}}$

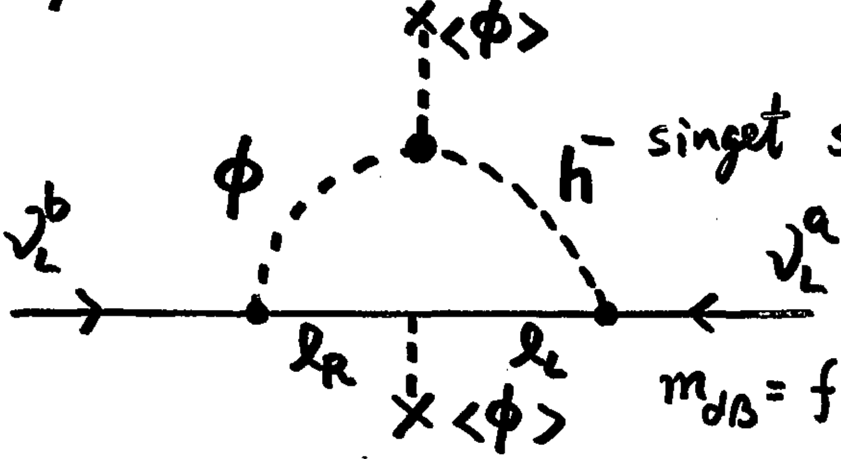
Left handed majorana masses are given

by $\nu_L^T C \langle \Delta^0 \rangle \nu_L$ SU(2) triplet Higgs

$2 \times 2 = 1,\ 3$

Additional Symmetry : $L_e - L_\mu - L_\tau$, ...

by Zee-like models SU(2) singlet scalar



$$m_{\alpha\beta} = f^{\alpha\beta}(m_{L\alpha}^2 - m_{L\beta}^2) \times \frac{v_2}{v_1}\mu F(M_1^2, M_2^2)$$

by R-parity and lepton-number-violation in SUSY $\boxed{\epsilon_i L_i H}$

+ loop diagrams

Typical form for M_ν

$$M_\nu = \begin{pmatrix} m_{11} & cM & sM \\ cM & m_{22} & m_{23} \\ sM & m_{23} & m_{33} \end{pmatrix}$$

$M \gg m_{ij}$, $s \equiv \sin\theta$ $c \equiv \cos\theta$ $\theta \sim \frac{\pi}{4}$

$\Downarrow$ rotation in the 2-3 plane by θ

$$M'_\nu = \begin{pmatrix} m_{11} & M & 0 \\ M & m'_{22} & m'_{23} \\ 0 & m'_{23} & m'_{33} \end{pmatrix}$$ 1-2 plane Large Mixing

(1-3) mixing depends on m_{ij} Δm^2_{21}

$$\theta_{13}^{MNS} \simeq m_{ij} \simeq \frac{\Delta m^2_{21}}{\Delta m^2_{32}}$$

$\simeq 10^{-2}$ for LMA MSW

$\simeq 10^{-7}$ for VO

(Bi-maximal $\theta_{23} \sim \theta_{12} \sim \frac{\pi}{4}$

$m_3 \ll m_2 \simeq m_1$)

I (2) A = See-Saw =

Large Mixing from M_R

Dirac Neutrino matrix $N \simeq U, D, E$

Specific Structure in M_R

$$N = \begin{pmatrix} x^2 y & 0 & 0 \\ 0 & x & x \\ 0 & O(x^2) & 1 \end{pmatrix} m_D, \quad M_R = \begin{pmatrix} 0 & 0 & A \\ 0 & 1 & 0 \\ A & 0 & 0 \end{pmatrix} m_R$$

$$\underset{-N^T M_R^{-1} N}{M_\nu} = -\begin{pmatrix} 0 & O(x^4 y/A) & x^2 y A \\ O(x^4 y/A) & x^2 & x^2 \\ x^2 y/A & x^2 & x^2 \end{pmatrix} \frac{m_D^2}{m_R}$$

$x, y \ll 1$

θ_{23} は x, y, A に無関係に $\frac{\pi}{4}$。

θ_{12} は x, y, A の値によって $\frac{\pi}{4}$ になる。

Dirac-Majorana Conspiracy problem should be avoided!

θ_{13} depends on x, y, A

Jezabek, Sumino, PLB 440 (1998) 327

I (2) B = See-saw =

Large Mixing from N

less natural in U, D, E, N unified picture

Oda, Takasugi, Tanaka, Yoshimura, PR D59 (1999) 055001

Shafi, Tavartkiladze (hep-ph/9905202)

Why does only N have large off-diagonal elements?

linear combination of certain matrices ⇒ N

$U_{e3} = \lambda, \lambda^4$ (Oda, Takasugi ---)

I (2) C = See-saw =

See-saw enhancement

$$N = \begin{pmatrix} \varepsilon' & \varepsilon' & \varepsilon'' \\ \varepsilon' & \varepsilon & \varepsilon \\ \varepsilon'' & \varepsilon & 1 \end{pmatrix} m \qquad M_R^{-1} = \begin{pmatrix} r_1 & 0 & 0 \\ 0 & r_2 & 0 \\ 0 & 0 & r_3 \end{pmatrix} M^{-1}$$

$$M_\nu \simeq \begin{pmatrix} (\varepsilon'/\varepsilon)^2 & \varepsilon'/\varepsilon & \varepsilon'/\varepsilon \\ \varepsilon'/\varepsilon & 1 & 1 \\ \varepsilon'/\varepsilon & 1 & 1 \end{pmatrix} r_2 \varepsilon^2 \frac{m^2}{M}$$

但し, $r_2 \varepsilon^2 \gg r_3, r_1 \varepsilon'^2$

$M_{R3} \gg M_{R2}, M_{R1}$

$U_{e3} \simeq U_{e2}$

II (1) Large mixing from M_E

= CKM small by cancellation =

" flavor - democratic " $S_{3L} \times S_{3R}$

$$M_{FD} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}_{LR} m$$

rank 1

$(0, 0, 3)\, m$

$\Downarrow$

$$U_{FD} = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ -1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ 0 & -2/\sqrt{6} & 1/\sqrt{3} \end{pmatrix}$$

$$M_Q = c_Q M_{FD} + m_Q^{break}$$

$$V_{CKM}^{Q} = (U_{FD} U_U^{break})^\dagger (U_{FD} U_D^{break}) = U_U^{break\dagger} U_D^{break}$$

$$M_E = c_E M_{FD} + m_E^{break} \qquad S_{3L} \times S_{3R}$$

$$M_\nu = c_\nu I + m_\nu^{break} \qquad S_{3L} \quad \text{Majorana Neutrino}$$

$$U_{MNS} = (U_{FD} U_E^{break})^\dagger U_\nu^{break} = U_E^{break\dagger} U_{FD}^T U_\nu^{break}$$

$$= U_E^{break\dagger} \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{6} & 1/\sqrt{6} & -2/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{pmatrix} U_\nu^{break}$$

$$\sin^2 2\theta_{32} \approx \frac{8}{9}$$

II (2) Large mixing from lopsided M_E

SU(5) – like model

$M_E = M_D^T$ Down Mass $10 . 5^* \langle \phi^P \rangle$

Lepton Mass $5^* . 10 \langle \phi^P \rangle$

$$M_D = C \begin{pmatrix} 0 & \lambda^3 & \lambda^4 \\ x & \lambda^2 & \lambda^2 \\ y & \boxed{z} & 1 \end{pmatrix}$$

m_d, m_s, m_b

V_{us}, V_{cb}, V_{ub}

$\lambda = 0.22$

$V_{us} \simeq \sqrt{\frac{m_d}{m_s}} \Rightarrow x \simeq \lambda^3$

$$(M_D M_D^\dagger)_{LL} \simeq C^2 \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^4 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix}$$

x, y, z independent

$$(M_E M_E^\dagger)_{LL} = (M_D^\dagger M_D)_{RR} = C^2 \begin{pmatrix} x^2+y^2 & yz+\lambda^2 x & y+\lambda^2 x \\ yz+\lambda^2 x & z^2 & z \\ y+\lambda^2 x & z & 1 \end{pmatrix}$$

$x \simeq \lambda^3$ $y \lesssim 1$ $z \lesssim 1$

If $z \simeq 1$, we get $\theta_{23}^E \simeq \frac{\pi}{4}$

"lopsided" M_E

Neutrino Mass Pattern is decoupled from $\theta_{23}^{MNS} = \frac{\pi}{4}$.

Why is $z \simeq 1$ choosen? E twisted ?

$E(6) \supset SO(10) \supset SU(5)$ Bando – Kugo

$27 = 16 + 10 + 1$ SO(10)

$16 = 10 + \underset{\sim}{5^*} + 1$ $10 = 5 + \underset{\sim}{5^*}$ SU(5)

3 U_{e3}の予言のフレームワーク

$$U_{MNS} \equiv L_E^{\dagger} L_\nu$$

$$L_E = L_E^0 L_E^1 , \quad L_\nu = L_\nu^0 L_\nu^1$$

0: symmetric limit　　1: breaking

$$U_{MNS} = L_E^{1\dagger} \left(L_E^{0\dagger} L_\nu^0 \right) L_\nu^1$$

$$L_E^{0\dagger} L_\nu^0 \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & c & -s \\ 0 & s & c \end{pmatrix}$$ とおく　$c \sim s \sim \frac{1}{\sqrt{2}}$

$\sin^2 2\theta_{atm} \simeq 1$

$$L_E^1 = \begin{pmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{pmatrix} \quad L_\nu^1 = \begin{pmatrix} N_{11} & N_{12} & N_{13} \\ N_{21} & N_{22} & N_{23} \\ N_{31} & N_{32} & N_{33} \end{pmatrix}$$

$L_{22} \simeq L_{33} \simeq 1$　　$L_{23} \simeq L_{32} \ll 1$

$N_{22} \simeq N_{33} \simeq 1$　　$N_{23} \simeq N_{32} \ll 1$

L_{12} and N_{12} could be large $O(1)$!
(L_{21})　　(N_{21})

Bi-maximal case

$$U_{e2} = L_{11}^* N_{12} + N_{22}(cL_{21}^* + sL_{31}^*) + N_{32}(cL_{31}^* - sL_{21}^*)$$
$$\simeq L_{11}^* N_{12} + cL_{21}^* N_{22}$$

$$U_{e3} = L_{11}^* N_{13} + N_{23}(cL_{21}^* + sL_{31}^*) + \underbrace{N_{33}}_{\approx 1}(cL_{31}^* - sL_{21}^*)$$
$$\simeq L_{11}^* N_{13} - sL_{21}^*$$

SMA MSW solution $L_{11}^* \simeq 1$ $N_{22} \simeq 1$

$U_{e2} \simeq N_{12} + cL_{21}^* = \lambda^2$ solar neutrino

$U_{e3} \simeq N_{13} - sL_{21}^* < 0.16$ CHOOZ

Since $L_{21} = \lambda \sim \lambda^2$ in SU(5), we get $U_{e3} = \lambda \sim \lambda^2$

$(N_{12} \lesssim \lambda^2)$ $\quad (N_{12} \gtrsim N_{13})$ $\quad$ in SU(5) $M_D \simeq M_E^T$

LMA MSW, VO solutions

$U_{e2} \simeq L_{11}^* N_{12} + cL_{21}^* N_{22} \simeq \frac{1}{\sqrt{2}}$

$U_{e3} \simeq L_{11}^* N_{13} - sL_{21}^*$

If $L_{21} \simeq \lambda^2$, we get $U_{e3} \gtrsim \lambda^2$

depends on N_{13} !

However, if $L_{21} \ll \lambda^2$ we get $U_{e3} \ll \lambda^2$!!

Model dependent !!

4 U_{e3}によるモデルの識別はどこまで可能か

I (1) Non see saw

Zee-like model : Bi-maximal

$L_{ij}\,(i \neq j) = 0$ Base $\Rightarrow$ $U_{e3} \simeq N_{13} \simeq 10^{-3},\ 10^{-8}$

$U_{e2} = L_{11}^{*} N_{12} + c L_{21}^{*} N_{22}$

$U_{e3} = L_{11}^{*} N_{13} - s L_{21}^{*}$

$\dfrac{\Delta m_{\odot}^2}{\Delta m_{atm}^2}$ LMA VO

Bilinear R-parity Violation : SMA MSW

Takayama, Yamaguchi, hep-ph/9910320

2 parameter in U_{MNS}

$U_{e2} \simeq N_{12} = s_1 c$ $\quad U_{e3} \simeq N_{13} = s_1 s$ $\quad c \simeq s \simeq \frac{1}{\sqrt{2}}$

$\simeq U_{e2} \simeq \lambda^2$

$L_e - L_\mu - L_\tau$ flavor symmetry

$$m_\nu = m_0 \begin{pmatrix} \varepsilon' & 1 & 1 \\ 1 & \varepsilon & \varepsilon \\ 1 & \varepsilon & \varepsilon \end{pmatrix}$$

$L_{ij}(i \neq j) = 0$ Basis

hep-ph/9807235

Barbieri, Hall … et al.

Bi-maximal

$U_{e3} = 10^{-2}$ for LMAMSW

$= 10^{-7}$ for VO

I (2) See - saw

$$M_R = \begin{pmatrix} M & 0 & 0 \\ 0 & 0 & M' \\ 0 & M' & 0 \end{pmatrix}$$

hep-ph/0003139

Miura, Takasugi, Yoshimura

Z_3 symmetry

$$\left.\begin{aligned} U_{e2} &= N_{12} = -\sqrt{\tfrac{2}{3}}\, c' \\ U_{e3} &= N_{13} = i\sqrt{\tfrac{2}{3}}\, s' \end{aligned}\right\} \quad U_{e3} \text{ is unknown.}$$

Large Mixing from N

U_{e3} is also unknown

See - saw enhancement

$$N = \begin{pmatrix} \varepsilon' & \varepsilon' & \varepsilon'' \\ \varepsilon' & \varepsilon & \varepsilon \\ \varepsilon'' & \varepsilon & 1 \end{pmatrix} m \qquad M_R^{-1} = \begin{pmatrix} r_1 & 0 & 0 \\ 0 & r_2 & 0 \\ 0 & 0 & r_3 \end{pmatrix} M^{-1}$$

$$M_\nu \simeq \begin{pmatrix} x^2 & x & x \\ x & 1 & 1 \\ x & 1 & 1 \end{pmatrix} r_2 \varepsilon^2 \frac{m^2}{M} \qquad x = \frac{\varepsilon'}{\varepsilon}$$

assume $\quad r_2 \varepsilon^2 \gg r_3,\ r_1 \varepsilon'^2 \cdots (M_{R3} \gg M_{R2} M_{R1}$

$$U_{e3} \simeq U_{e2} = \lambda^2 \qquad \text{SMA MSW}$$

II (1) (2) Large Mixing from M_E

$$U_{e2} \simeq L_{11}^* N_{12} + c L_{21}^* N_{22}$$

$$U_{e3} \simeq L_{11}^* N_{13} - s L_{21}^*$$

Many models predict $L_{21} \simeq \lambda^{1\sim2}$ as in Quarks.

Then $U_{e3} = \lambda^{1\sim2}$ is predicted

= Exceptions =

Anti - GUT $\quad SMG^3 \times U(1)_f \times U(1)_{B-L}$

Nielsen - Takanishi hep-ph/0004137

$$U_{e3} = \lambda^3 \sim \lambda^4 \qquad \underline{L_{21} \ll \lambda^2}$$

U(1) Flavor Symmetry $\times Z_m (\times Z_2)$

hep-ph/9808355 Grosman, Nir, Shadmi

PL B456 (1999) 200 Tanimoto

In order to get VO solution, $L_{21}^* \ll \lambda^2$!

$$\left.\begin{aligned} U_{e3} &= \lambda^4 \sim \lambda^6 \quad \text{for VO} \\ &= \lambda \sim \lambda^2 \quad \text{for SMA MSW} \end{aligned}\right\} \frac{\Delta m_\odot^2}{\Delta m_{atm}^2}$$

LMA MSW is impossible !

$$M_D = C \begin{pmatrix} \overset{\nearrow \lambda^5}{0} & \lambda^3 & \lambda^4 \\ x & \lambda^2 & \lambda^2 \\ y & z & 1 \end{pmatrix}_{LR} \qquad \begin{array}{ccccc} m_d & : & m_s & : & m_b \\ \lambda^4 & & \lambda^2 & & 1 \end{array}$$

$$V_{us} \simeq \sqrt{\frac{m_d}{m_s}} \simeq 0.2 \quad \Rightarrow \quad x \simeq \lambda^3$$

$$M_E = M_D^T = C \begin{pmatrix} \overset{\nearrow \lambda^5}{0} & \lambda^3 & y \\ \lambda^3 & \lambda^2 & z \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix}$$

$$L_{21} \simeq \sqrt{\frac{m_e}{m_\mu}} \simeq \lambda \sim \lambda^2$$

See Okamura – Hagiwara

5 まとめ

- Quark-Lepton Unified Models predict

  $U_{e3} = \lambda \sim \lambda^2$ close to CHOOZ bound

  "lopsided" M_E in $SU(5)$

  as well as "Flavor Democracy"

- Unconventional Models predict

  Zee model, $L_e - L_\mu - L_\tau$ model (VO)

  $U(1) \times Z_m$ (VO) Anti-GUT

  $U_{e3} \ll \lambda^2$ very small

- Impossible to distinguish whether

  $\theta_{23}^{MNS} = \frac{\pi}{4}$ is derived from M_E or M_ν

  by U_{e3} $\left(\begin{array}{l} U_{e2} \simeq L_{11}^* N_{12} + C L_{21}^* N_{22} \\ U_{e3} \simeq L_{11}^* N_{13} - S L_{21}^* \end{array} \right)$

If U_{e3} will be not observed in $\lambda \sim \lambda^2$ with MSW solution, we need new ideas to explain $\theta_{atm} \sim \frac{\pi}{4}$!