New! ICRR at Kashiwa, May 12, 2000

Measuring Leptonic $\cancel{CP}$ by Low Energy ν ~~by~~ Oscillation Experiments

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with

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We know that $\begin{pmatrix} \text{leptons} \\ \text{quarks} \end{pmatrix}$ come with

3 flavors :

$$U_{CKM} = \begin{bmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} 1 & & \\ & 1 & \\ & & e^{i\delta} \end{bmatrix} \begin{bmatrix} c_{13} & & s_{13} \\ & 1 & \\ -s_{13} & & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{bmatrix}$$

Something entirely new in ν sector ?

"Anomaly cancellation in the SM"

$\rightarrow$ sterile ν

No evidence so far (LSND ?)

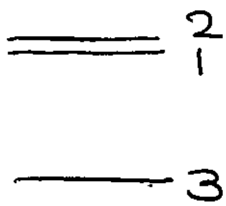
"disfavored by SuperK."

In this talk I assume :

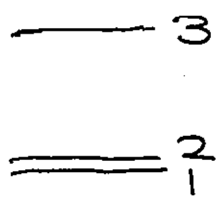
(1) 3 active flavor mixing, No steriles

(2) should accomodate

$$\begin{array}{ccc} \Delta m^2_{atm} & \gg & \Delta m^2_{solar} \\ \equiv & & \equiv \\ \Delta m^2_{13} \approx \Delta m^2_{23} & & \Delta m^2_{12} \end{array}$$



or



(3) employ standard parametrization of MNS (= Leptonic CKM) matrix

Two relevant hierarchies

$$\frac{\Delta m^2}{E} = 10^{-13}\left(\frac{\Delta m^2}{10^{-3}\,\mathrm{eV}^2}\right)\left(\frac{E}{10\,\mathrm{GeV}}\right)^{-1}\ \mathrm{eV}$$

$$a(x) = 1.04\times 10^{-13}\left(\frac{\rho}{2.7\,\mathrm{g/cm^3}}\right)\left(\frac{Y_e}{0.5}\right)\ \mathrm{eV}$$

① High energy option : $E \sim 10\,\mathrm{GeV}$

$$\frac{\Delta m^2_{13}}{E} \approx a \gg \frac{\Delta m^2_{12}}{E}$$

good for Large Angle MSW

matter enhanced θ_{13}

② Low energy option : $E \sim 100\,\mathrm{MeV}$

$$\frac{\Delta m^2_{13}}{E} \gg a \approx \frac{\Delta m^2_{12}}{E}$$

good for Small-Angle MSW

matter enhanced θ_{12}

$P(\nu_\beta \to \nu_\alpha)$ in matter : Adiabatic approximation

$$i\frac{d}{dx}\begin{bmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{bmatrix} = \left\{ U \begin{bmatrix} m_1^2/2E & & \\ & m_2^2/2E & \\ & & m_3^2/2E \end{bmatrix} U^\dagger + \begin{bmatrix} a(x) & & \\ & 0 & \\ & & 0 \end{bmatrix} \right\} \begin{bmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{bmatrix}$$

matter mass eigenstate

$$\begin{bmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{bmatrix} = V \begin{bmatrix}\nu_{m1}\\ \nu_{m2}\\ \nu_{m3}\end{bmatrix}$$

$$U \begin{bmatrix} m_1^2/2E & & \\ & m_2^2/2E & \\ & & m_3^2/2E \end{bmatrix} U^\dagger + \begin{bmatrix} a(x) & & \\ & 0 & \\ & & 0 \end{bmatrix}$$

$$= V H_d V^\dagger$$

$$H_d = \begin{bmatrix} h_1 & & \\ & h_2 & \\ & & h_3 \end{bmatrix}$$

T-violation in matter

$$P(\nu_\beta \to \nu_\alpha) - P(\nu_\alpha \to \nu_\beta)$$

$$= 4\, J_M \sum_{(i,j)} \sin\left[\int dx\,(h_j - h_i)\right]$$

$$J = c_{12}^M s_{12}^M s_{23} c_{23} c_{13}^{M\,2} s_{13}^M \sin\delta$$

neat structure remains: matter cannot create fake $\cancel{T}$ effect!

CP-violation in matter

$U \to U^*$ $\quad \boxed{a(x) \longrightarrow -a(x)}$

$\mathrm{Re}[UUUU] \nLeftarrow$ even under CP

$\mathrm{Im}[UUUU] \nLeftarrow$ odd "

messy structure appears

matter can create fake $\cancel{CP}$

matter effect dominates $\cancel{CP}$ in $\nu_\mu \to \nu_e$

High Energy Option : Solar Δm^2

Perturbation theory

$\tilde{\nu}$ basis

$$\begin{bmatrix} \tilde{\nu}_e \\ \tilde{\nu}_\mu \\ \tilde{\nu}_\tau \end{bmatrix} = e^{i\theta_{12}\lambda_2}\, U^\dagger \begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix}$$

$$= e^{-i\theta_{13}\lambda_5}\, \Gamma_\delta^\dagger\, e^{-i\theta_{23}\lambda_7} \begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix}$$

Hamiltonian in flavor basis

$$\tilde{H} = e^{i\theta_{12}\lambda_2} \underbrace{U^\dagger H U}_{\text{Hamiltonian in vacuum mass eigenstate basis}} e^{-i\theta_{12}\lambda_2}$$

$$= \begin{bmatrix} 0 & & \\ & 0 & \\ & & \frac{\Delta m_{13}^2}{2E} \end{bmatrix} + a(x) \begin{bmatrix} c_{13}^2 & 0 & c_{13}s_{13} \\ 0 & 0 & 0 \\ c_{13}s_{13} & 0 & s_{13}^2 \end{bmatrix}$$

$$+ \frac{\Delta m_{12}^2}{2E} \begin{bmatrix} s_{12}^2 & c_{12}s_{12} & 0 \\ c_{12}s_{12} & c_{12}^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} ac_{13}^2 & 0 & ac_{13}s_{13} \\ 0 & 0 & 0 \\ ac_{13}s_{13} & 0 & as_{13}^2 + \frac{\Delta m_{13}^2}{2E} \end{bmatrix}}_{H_0} + \underbrace{\frac{\Delta m_{12}^2}{2E} \begin{bmatrix} s_{12}^2 & c_{12}s_{12} & 0 \\ c_{12}s_{12} & c_{12}^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{H'}$$

H_0 is essentially 2×2

$\rightarrow$ matter enhanced θ_{13}

"2-flavor MSW"

$\boxed{\text{But}}$ there is no $\cancel{T}$ if unperturbed genuine $\cancel{CP}$ by H'

$\rightarrow$

$\cancel{T}/\cancel{CP}$ Effect comes with energy denominator

$\rightarrow$ suppression factor $\dfrac{\Delta m^2_{12}}{\Delta m^2_{13}} \sim 10^{-2}$

☹ High energy option

= good for Large angle MSW

$\boxed{\text{but}}$

$\cancel{T}/\cancel{CP}$ suppressed by $\sim 10^{-2}$

Low Energy Option

ν basis Hamiltonian

$$H_0 = \begin{bmatrix} 0 & & \\ & 0 & \\ & & \frac{\Delta m^2_{13}}{2E} \end{bmatrix}$$ ← degenerate eigenvalu !

$$H' = \frac{\Delta m^2_{12}}{2E} \begin{bmatrix} s^2_{12} & c_{12}s_{12} & 0 \\ c_{12}s_{12} & c^2_{12} & 0 \\ 0 & 0 & 0 \end{bmatrix} + a(x) \begin{bmatrix} c^2_{13} & 0 & c_{13}s_{13} \\ 0 & 0 & 0 \\ c_{13}s_{13} & 0 & s^2_{13} \end{bmatrix}$$

degenerate perturbation theory !

① must first diagonalize 2×2

to obtain

(energy eigenvalue to 1st order
(wave function to zeroth order !

↑

contains $\cancel{T}$ / $\cancel{CP}$ effect

"Vacuum Dominated Matter Oscillation"

= better characterization of the Low energy option (H.M. & H. Nunokawa)
hep-ph/0004114

ν conversion, though in matter, imitates vacuum ν oscillation → $\boxed{\text{Fig.}}$ $\boxed{\text{mechanism ?}}$

→ Good news because very little matter effect pollusion

→ Feasible experiments for measuring ~~CP~~ ?

⇒ YES ! if LMA $\nu_\odot$ and

if $\begin{cases} \text{100 times more intense } \nu \text{ beam than K2K} \\ \text{megaton water Cherenkov} \end{cases}$

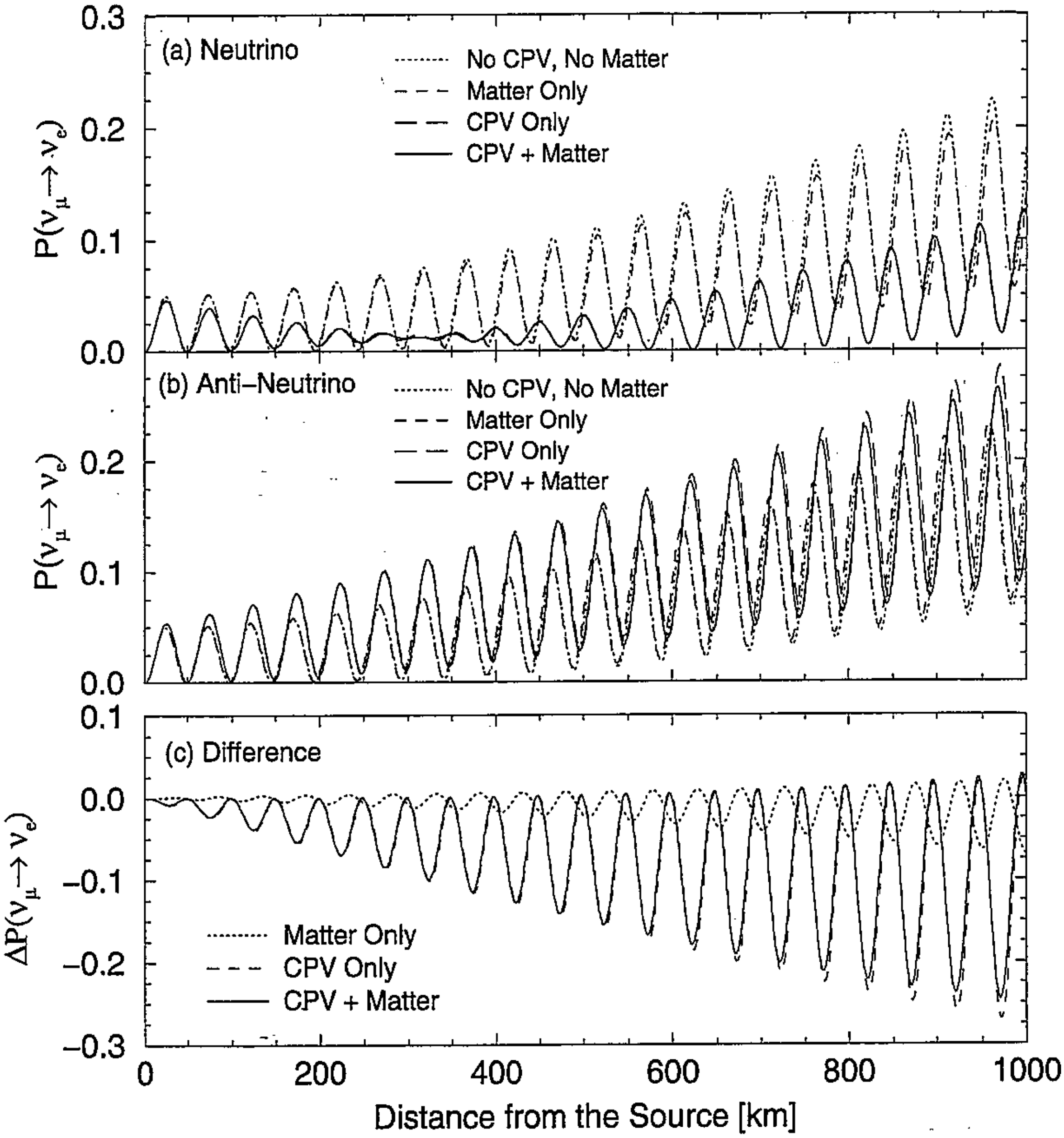


FIG. 1. Oscillation probability for (a) neutrinos, $P(\nu_\mu \to \nu_e)$, (b) anti-neutrinos, $P(\bar{\nu}_\mu \to \bar{\nu}_e)$, and (c) their difference, $\Delta P(\nu_\mu \to \nu_e) \equiv P(\nu_\mu \to \nu_e) - P(\bar{\nu}_\mu \to \bar{\nu}_e)$ with fixed neutrino energy $E_\nu =$ 60 MeV, are plotted as a function of distance from the source. The mixing parameters are fixed to be $\Delta m^2_{13} = 3 \times 10^{-3}$ eV^2, $\sin^2 2\theta_{23} = 1.0$, $\Delta m^2_{12} = 2.7 \times 10^{-5}$ eV^2, $\sin^2 2\theta_{12} = 0.79$, $\sin^2 2\theta_{13} = 0.1$ and $\delta = \pi/2$. We take the matter density as $\rho = 2.72$ g/cm^3 and the electron fraction as $Y_e = 0.5$.

probabilities in vacuum but with θ_{12} replaced by θ_{12}^M, and Δ_{ij} by integrals over the energy eigenvalues, $h_{1,2}$ and $h_3 \simeq \Delta_{13}$. For example, the appearance probability $P(\nu_\mu \to \nu_e)$ reads

$$
\begin{aligned}
P(\nu_\mu \to \nu_e) = {} & 4s_{23}^2 c_{13}^2 s_{13}^2 \sin^2(\frac{1}{2}\Delta_{13}L) \\
& + c_{13}^2 \sin 2\theta_{12}^M \left[(c_{23}^2 - s_{23}^2 s_{13}^2)\sin 2\theta_{12}^M + 2c_{23}s_{23}s_{13}\cos\delta\cos 2\theta_{12}^M\right] \\
& \times \sin^2\left[\frac{1}{2}\sqrt{(\cos 2\theta_{12} - \frac{a}{\Delta_{12}}c_{13}^2)^2 + \sin^2 2\theta_{12}}\Delta_{12}L\right] \\
& - 2J_M(\theta_{12}^M, \delta)\sin\left[\sqrt{(\cos 2\theta_{12} - \frac{a}{\Delta_{12}}c_{13}^2)^2 + \sin^2 2\theta_{12}}\Delta_{12}L\right]
\end{aligned} \tag{16}
$$

where J_M is the matter enhanced Jarlskog factor, $J_M(\theta_{12}^M, \delta) = \cos\theta_{12}^M \sin\theta_{12}^M c_{23}s_{23}c_{13}^2 s_{13}\sin\delta$, and we have averaged the rapidly oscillating piece in the CP violating term. The antineutrino transition probability $P(\bar{\nu}_\mu \to \bar{\nu}_e)$ is given by the same expressions as above but replacing θ_{12}^M by $\bar{\theta}_{12}^M$ and δ by $-\delta$.

Notice that at relatively short baseline, $L < 1{,}000$ km or so, the approximation $\sin x \simeq x$ is valid. Then, the expressions of the oscillation probabilities approximately reduce to those in the vacuum because

$$
\sin 2\theta_{12}^M (or, \bar{\theta}_{12}^M)\sqrt{(\cos 2\theta_{12} \mp \frac{a}{\Delta_{12}}c_{13}^2)^2 + \sin^2 2\theta_{12}}\Delta_{12} = \sin 2\theta_{12}\Delta_{12}. \tag{17}
$$

Only mild a-dependence would remain due to the $\cos\delta$ term in (16). Hence, as long as $\Delta_{12}L$

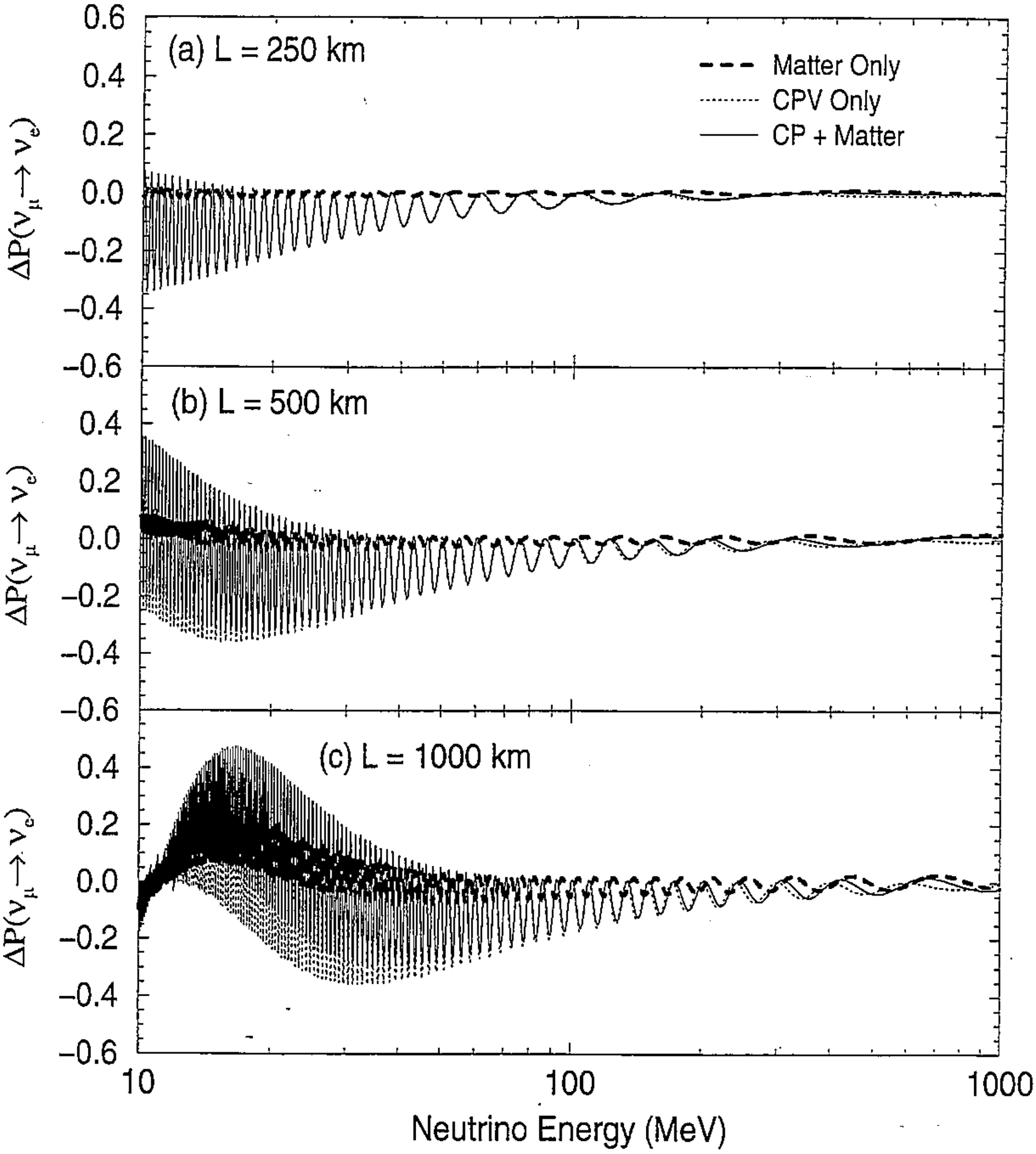


FIG. 2. The difference of the probability $\Delta P(\nu_\mu \to \nu_e) \equiv P(\nu_\mu \to \nu_e) - P(\bar{\nu}_\mu \to \bar{\nu}_e)$ is plotted as a function of neutrino energy. The mixing parameters as well as the electron number density are fixed to be the same as in Fig. 1.

1 Mton Water Cherenkov

⊕

$$\nu \text{ flux} = \underbrace{\text{K2K}}_{\sim 3\times 10^{6}\ \text{POT}^{-1}\ \text{at SK.}} \times 10^2$$

$$\left.\begin{array}{l}\sigma(\nu_e e \to \nu_e e) \simeq 10^{-42}\ \text{cm}^2 \\ \sigma(\nu_e {}^{16}O \to F e^-) \simeq 10^{-39}\ \text{cm}^2\end{array}\right) \text{ at } E = 100 \text{ MeV}$$

But $\#({}^{16}O) = \frac{1}{10}\#(e^-)$ in water

∴ No. of events due to ^{16}O reaction is 100 times larger than $\nu_e e$ elastic scatt.

$\#$ of $^{16}O \doteq 3.34 \times 10^{31}$ / kton

$$N = 10000 \left(\frac{L}{250\ \text{km}}\right)^{-2} \left(\frac{V}{\text{Mton}}\right)\left(\frac{\text{POT}}{10^{22}}\right)$$

assuming 100% $\nu_\mu \to \nu_e$ conversion

Summary

$\boxed{1.}$ I tried to give an overview description of the structure of ~~CP~~ within the framework of

$\begin{pmatrix} \text{atmospheric } \nu \\ \text{solar } \nu \\ \text{CHOOZ} \end{pmatrix}$ constrained 3 flavor ν mixing

New!

$\boxed{2.}$ Phenomenon of

"Vacuum dominated matter oscillation" allows us to design a feasible experiment (if $\nu_\odot$ = LMA MSW) with Low energy (~ 100 MeV) ν beam.