Phenomenology of ν oscillations at a $\bar{\nu}$ factory

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1. Introduction
   - motivation
   - $N_\nu = 3$ oscillation
   - $N_\nu = 4$ oscillation
2. ν oscillations at a ν factory
   - ν factory
   - $N_\nu = 3$ case (sign of Δm^2_{32}, value of $\theta_{13}, \theta_{23}, \delta$)
   - $N_\nu = 4$ case (sign of Δm^2_{43}, value of $\theta_{23}, \theta_{34}, \theta_{24}, \delta_1$)
3. Summary

2) $N_\nu = 3$ analysis

$N_\nu = 3 \rightarrow$ 2 independent $\Delta m_{jk}^2 \equiv m_j^2 - m_k^2$

$(\Delta m_{31}^2 = \Delta m_{32}^2 + \Delta m_{21}^2)$

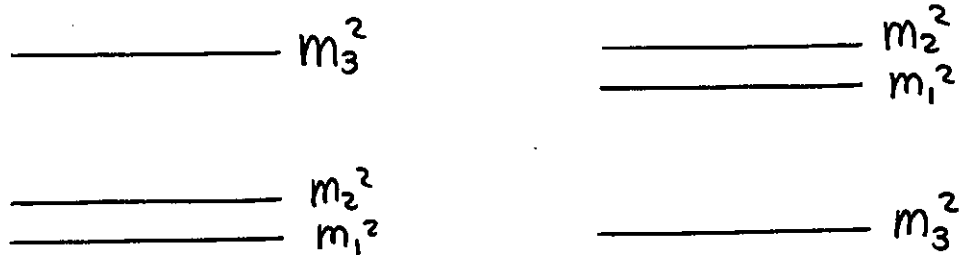
We assume mass hierarchy:

$$\underbrace{|\Delta m_{21}^2|}_{\nu_\odot} \ll \underbrace{|\Delta m_{32}^2| \simeq |\Delta m_{31}^2|}_{\nu_{atm}}$$

$O(10^{-5} eV^2)$ or $O(10^{-10} eV^2)$ $\qquad$ $O(10^{-2.5} eV^2)$

We know $\Delta m_{21}^2 > 0$ to account for $\nu_\odot$,

but don't know the sign of Δm_{32}^2:



$\Delta m_{32}^2 > 0$ $\qquad$ $\Delta m_{32}^2 < 0$

For ν_{atm} $\Delta m_{21}^2 \rightarrow 0$ is a good approximation,

so ν_{atm} depends on $(\underbrace{\Delta m_{32}^2, \theta_{23}}_{\downarrow}, \theta_{13})$ as $\theta_{13} \rightarrow 0$

$(\Delta m_{atm}^2, \theta_{atm})_{N_\nu = 2}$

For $\nu_\odot$ $|\Delta m_{32}^2| \rightarrow \infty$ is a good approximation,

so $\nu_\odot$ depends on $(\underbrace{\Delta m_{21}^2, \theta_{12}}_{\downarrow}, \theta_{13})$ as $\theta_{13} \rightarrow 0$

$(\Delta m_\odot, \theta_\odot)_{N_\nu = 2}$

If $\begin{pmatrix} |\theta_{13}| \ll 1 \\ \theta_{23} \simeq \frac{\pi}{4} \end{pmatrix}$ then MNS mixing matrix looks like

$$U_{MNS} \simeq \begin{pmatrix} C_\odot & S_\odot & \epsilon \\ -S_\odot/\sqrt{2} & C_\odot/\sqrt{2} & 1/\sqrt{2} \\ S_\odot/\sqrt{2} & -C_\odot/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \qquad |\epsilon| \ll 1$$

ν_{atm} : $\begin{pmatrix} 2\times 10^{-3}\mathrm{eV}^2 \lesssim \Delta m_{atm}^2 \lesssim 6\times 10^{-3}\mathrm{eV}^2 \\ 0.84 \lesssim \sin^2 2\theta_{atm} \leq 1.0 \end{pmatrix}$

$\nu_\odot$: $(\Delta m_\odot^2, \sin^2 2\theta_\odot) = \begin{cases} (O(10^{-10}\mathrm{eV}^2), 1.0) & \text{vacuum} \\ \text{or} & \\ (O(10^{-5}\mathrm{eV}^2), O(1)) & \text{LMA MSW} \\ \text{or} & \\ (O(10^{-5}\mathrm{eV}^2), 0.01) & \text{SMA MSW} \end{cases}$

$(\Delta m_{21}^2, \sin^2 2\theta_{12})$ and $(\Delta m_{32}^2, \sin^2 2\theta_{23})$ can be determined by $\nu_\odot$, ν_{atm} experiments to a certain precision.

$\Longrightarrow$ The next thing to do is to determine θ_{13} and furthermore δ ($\cancel{CP}$ phase).

① Determination of the sign of Δm_{32}^2

Apart from the precise value of θ_{13},
we can determine the sign of Δm_{32}^2
by looking at matter effects.

In the approximation of const. density $\left(\begin{array}{l} \text{leading order in} \\ |\Delta m_{21}^2|/|\Delta m_{32}^2| \end{array} \right)$

$$\boxed{\begin{array}{l} P(\nu_e \to \nu_\mu) \simeq s_{23}^2 \sin^2 2\theta_{13}^{M(-)} \sin^2\left(\frac{B^{(-)}L}{2}\right) \\ P(\bar{\nu}_e \to \bar{\nu}_\mu) \simeq s_{23}^2 \sin^2 2\theta_{13}^{M(+)} \sin^2\left(\frac{B^{(+)}L}{2}\right) \end{array}}$$

$$\tan 2\theta_{13}^{M(\mp)} \equiv \frac{\Delta E_{32} \sin 2\theta_{13}}{\Delta E_{32} \cos 2\theta_{13} \pm A}, \quad \Delta E_{32} \equiv \frac{\Delta m_{32}^2}{2E}$$

$$B^{(\pm)} \equiv \left[(\Delta E_{32} \cos 2\theta_{13} \pm A)^2 + (\Delta E_{32} \sin 2\theta_{13})^2\right]^{\frac{1}{2}}, \quad A \equiv \sqrt{2} G_F n_e$$

If $\Delta m_{32}^2 > 0$ then $\sin^2 2\theta_{13}^{M(-)}$ is enhanced for ${}^\exists E_\nu$
$\to P(\nu_e \to \nu_\mu)$ increases

If $\Delta m_{32}^2 < 0$ then $\sin^2 2\theta_{13}^{M(-)}$ is enhanced for ${}^\exists E_\nu$
$\to P(\bar{\nu}_e \to \bar{\nu}_\mu)$ increases

cross sections are different in inelastic scatterings.

$$\sigma_{\nu N} : \sigma_{\bar{\nu} N} = 2 : 1$$

To demonstate $\theta_{13} \neq 0$ or existence of matter effects,

$$\frac{|N_\nu - 2N_{\bar{\nu}}|}{\delta(N_\nu - 2N_{\bar{\nu}})} = \frac{|N_\nu - 2N_{\bar{\nu}}|}{\sqrt{N_\nu + 4N_{\bar{\nu}}}} \gg 1$$

is necessary. $\to$ $L \gtrsim 3{,}000$ km is preferred.

When this condition is satisfied, the sign of Δm_{32}^2 can be easily determined.

We introduce a quantity which mimics χ^2:

$$R \equiv \sum_j \frac{(N^j_\nu - 2N^j_{\bar\nu})^2}{N^j_\nu + 4N^j_{\bar\nu}}$$

To demonstrate $\mathrm{sign}(\Delta m^2_{32}) = \pm 1$

$R > 3$ is necessary.

$\Rightarrow$ for 10^{19} μ/yr · 50 kt · 1 yr

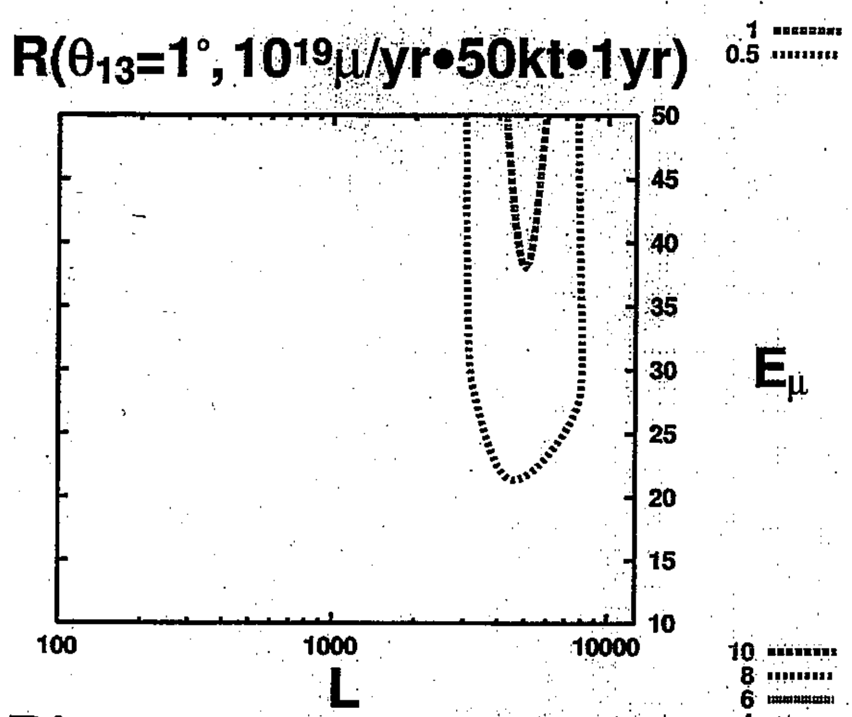
$L \gtrsim 2000$ km

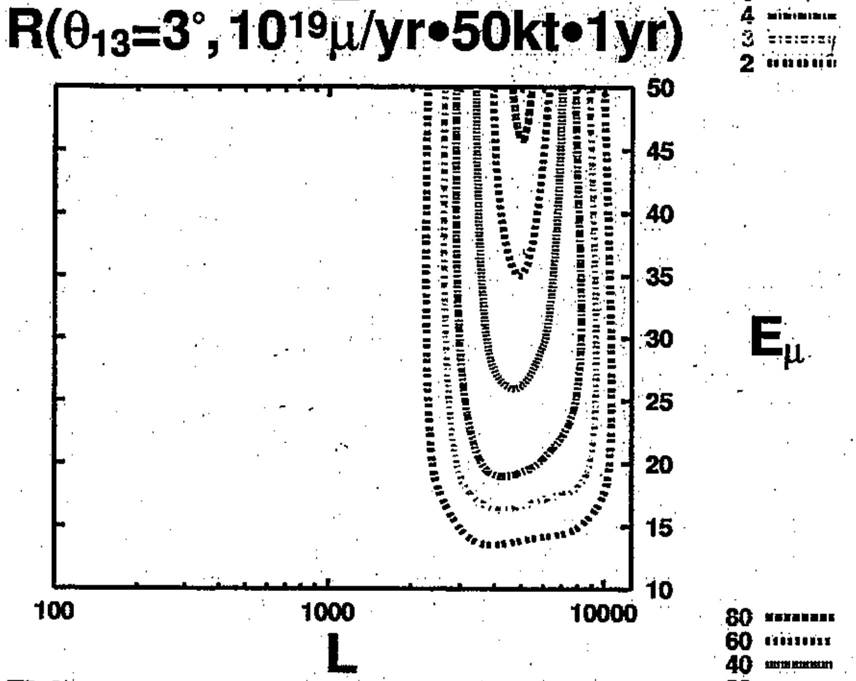
($L \simeq 5000$ km appears to be best)

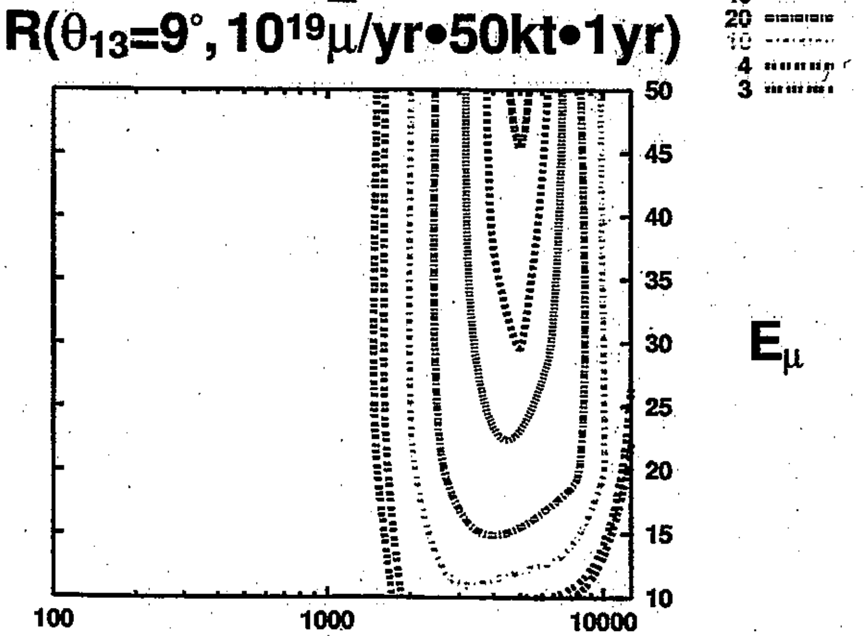
for 10^{21} μ/yr · 10 kt · 1 yr

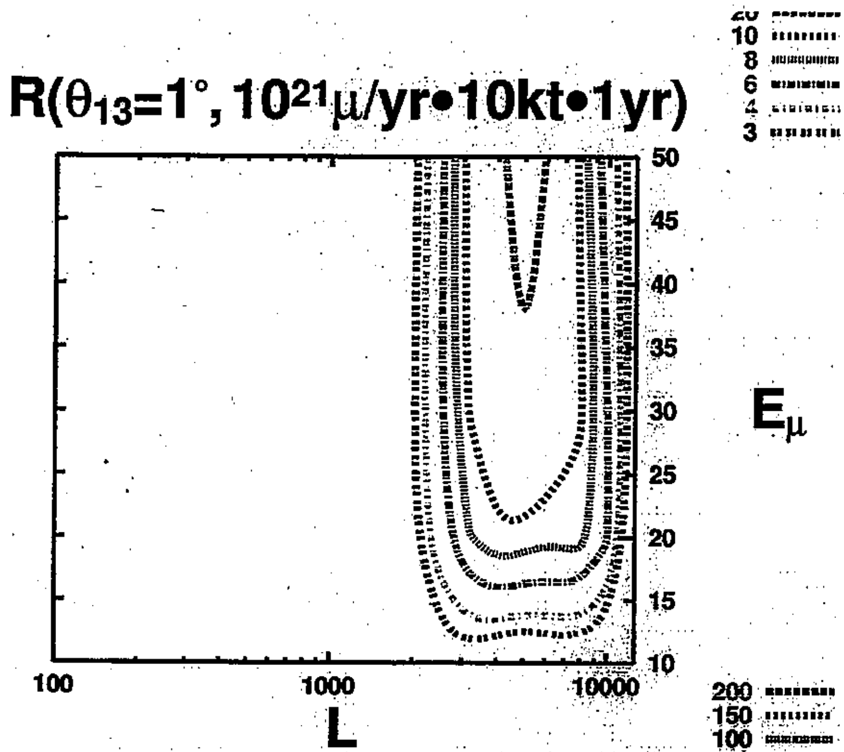
$L \gtrsim 800$ km

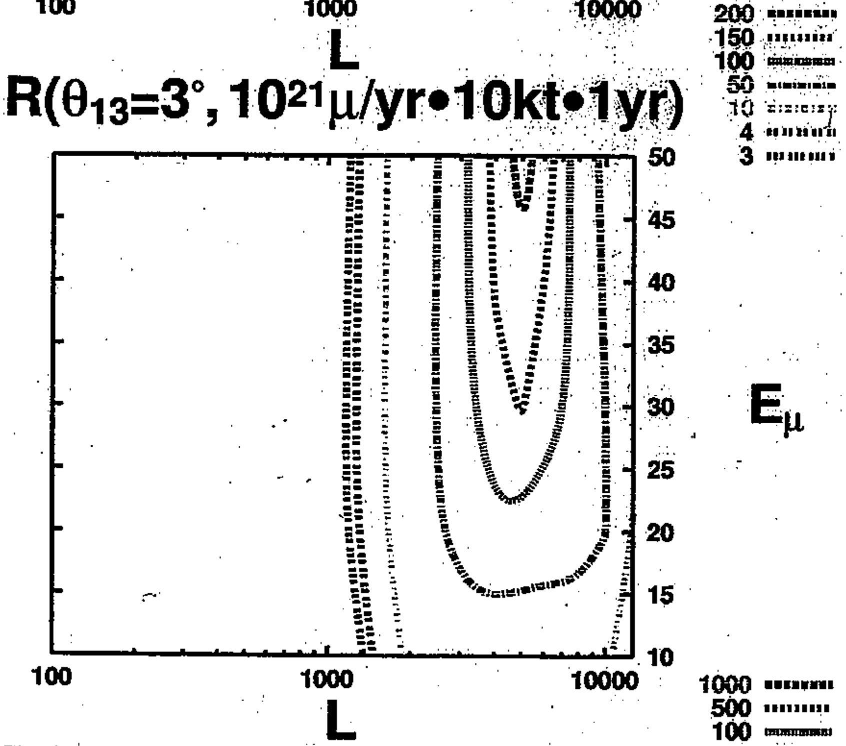
($L \simeq 5000$ km again is the best)

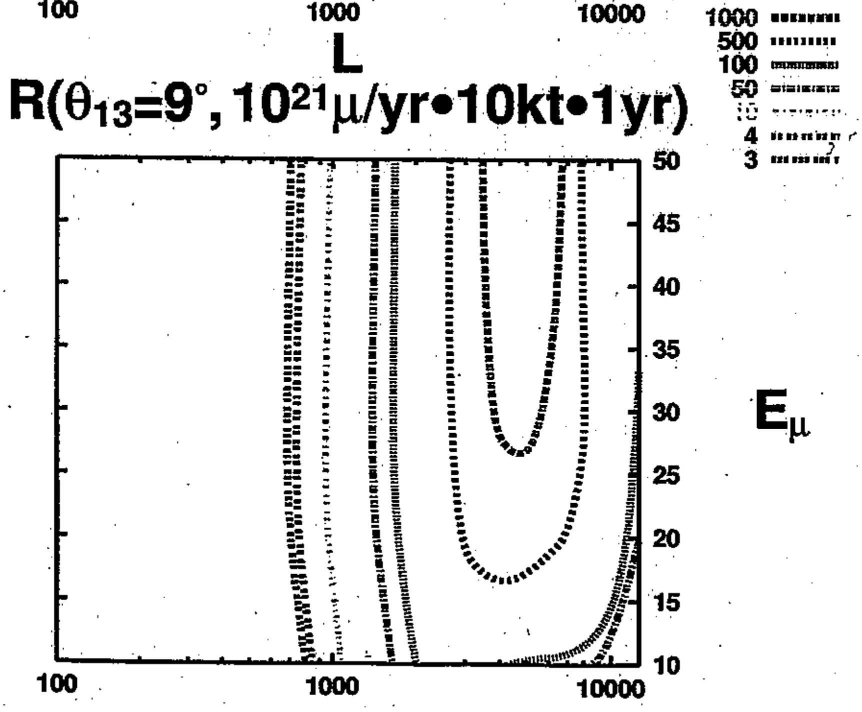

R(θ13=1°, 10^19μ/yr•50kt•1yr)
1
0.5
50
45
40
35
30
25
20
15
10
100
1000
10000
L
Eμ


R(θ13=3°, 10^19μ/yr•50kt•1yr)
10
8
6
4
3
2
50
45
40
35
30
25
20
15
10
100
1000
10000
L
Eμ


R(θ13=9°, 10^19μ/yr•50kt•1yr)
80
60
40
20
10
4
3
50
45
40
35
30
25
20
15
10
100
1000
10000
Eμ


R(θ13=1°, 10²¹μ/yr•10kt•1yr)
10
8
6
4
3
50 45 40 35 30 25 20 15 10
Eμ
100 1000 10000
L


R(θ13=3°, 10²¹μ/yr•10kt•1yr)
200
150
100
50
10
4
3
50 45 40 35 30 25 20 15 10
Eμ
100 1000 10000
L


R(θ13=9°, 10²¹μ/yr•10kt•1yr)
1000
500
100
50
4
3
50 45 40 35 30 25 20 15 10
Eμ
100 1000 10000

② relatively precise measurement of θ_{23}, θ_{13}

Again in the constant desity matter $\left(\begin{array}{c}\text{leading order in}\\ |\Delta m^2_{21}|/|\Delta m^2_{32}|\end{array}\right)$

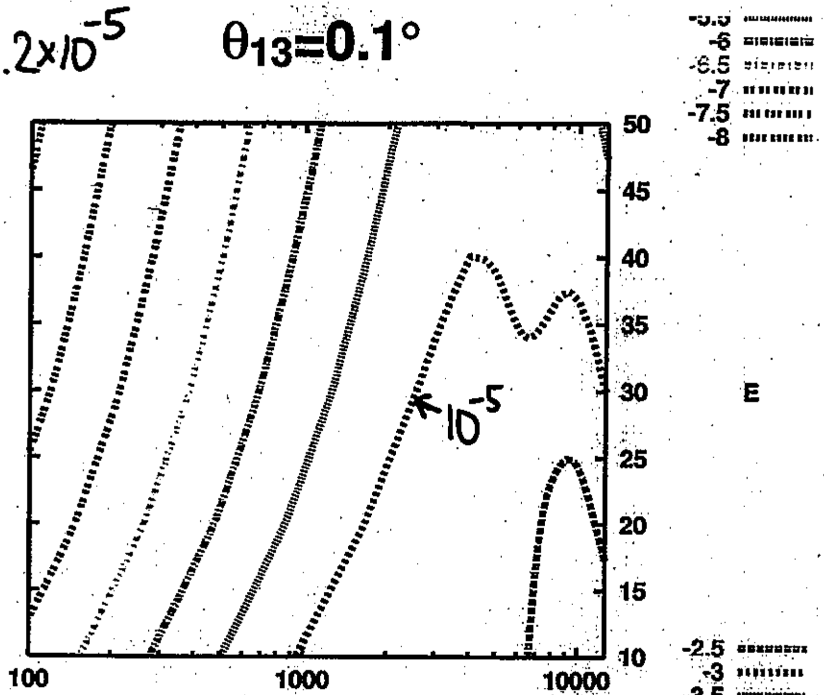
$$1 - P(\nu_\mu \to \nu_\mu) \simeq s_{23}^4 \sin^2 2\theta_{13}^{M(-)} \sin^2\left(\frac{B^{(-)}L}{2}\right)$$
$$+ \sin^2 2\theta_{23} \left[\sin^2\theta_{13}^{M(-)} \sin^2 \frac{L}{4}(\Delta E_{32} + A - B^{(-)}) + \cos^2\theta_{13}^{M(-)} \sin^2 \frac{L}{4}(\Delta E_{32} + A + B^{(-)})\right]$$

By looking at the disappearance rate for $\nu_\mu \to \nu_\mu$ and appearance rate for $\nu_e \to \nu_\mu$ we have two information on θ_{23}, θ_{13} provided that we know the density profile of the Earth exactly.

$\Longrightarrow$ In this case the distance had better be small to avoid uncertainty of the matter density. $L \lesssim 1{,}000$ km(?)

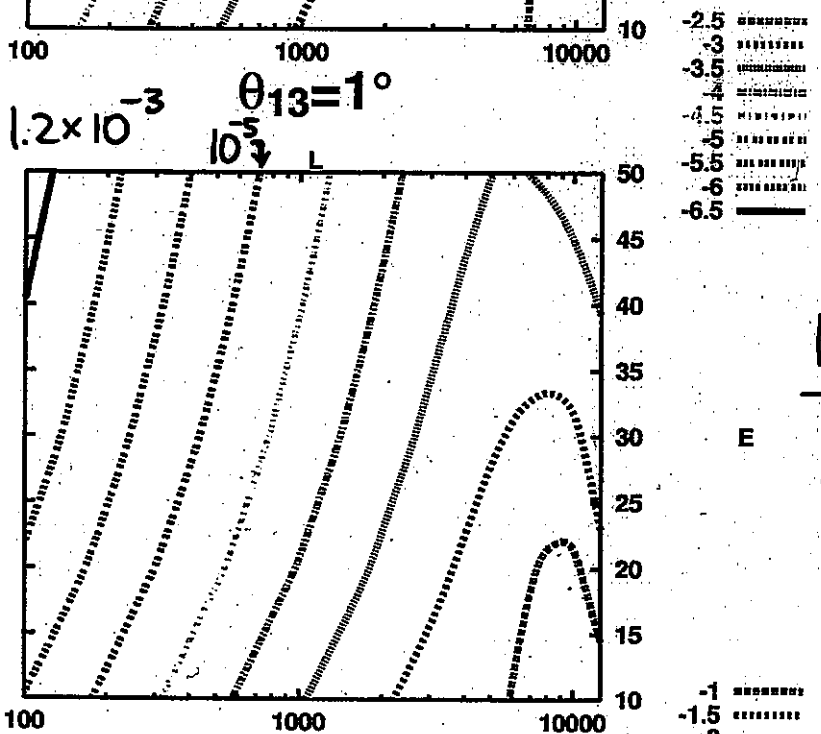
as long as N_{wrong} exceeds $N_{background}$.

$\sin^2 2\theta_{13} = 1.2\times 10^{-5}$ $\theta_{13}=0.1°$



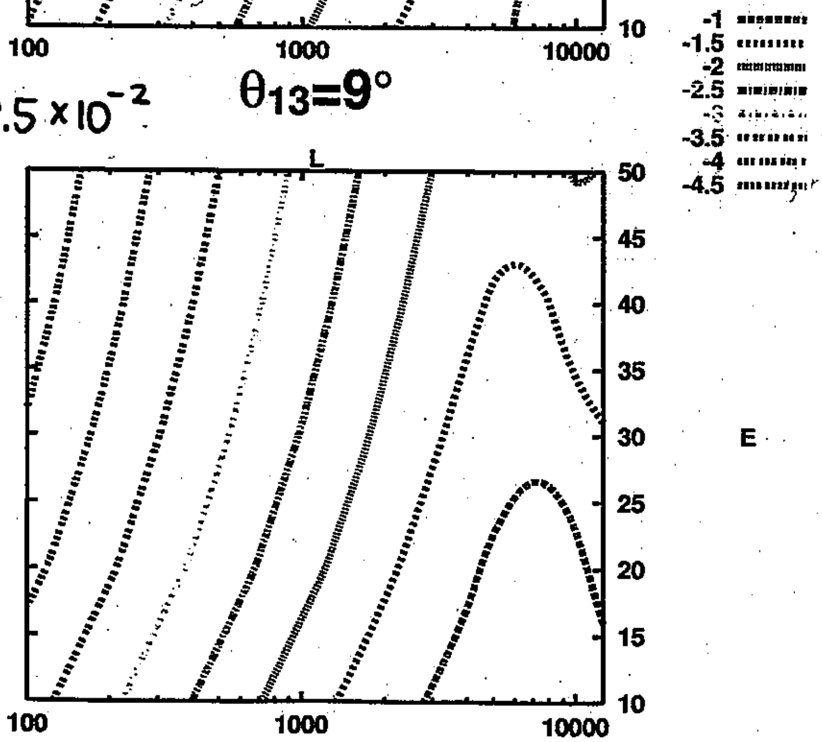
$\theta_{13}=1°$

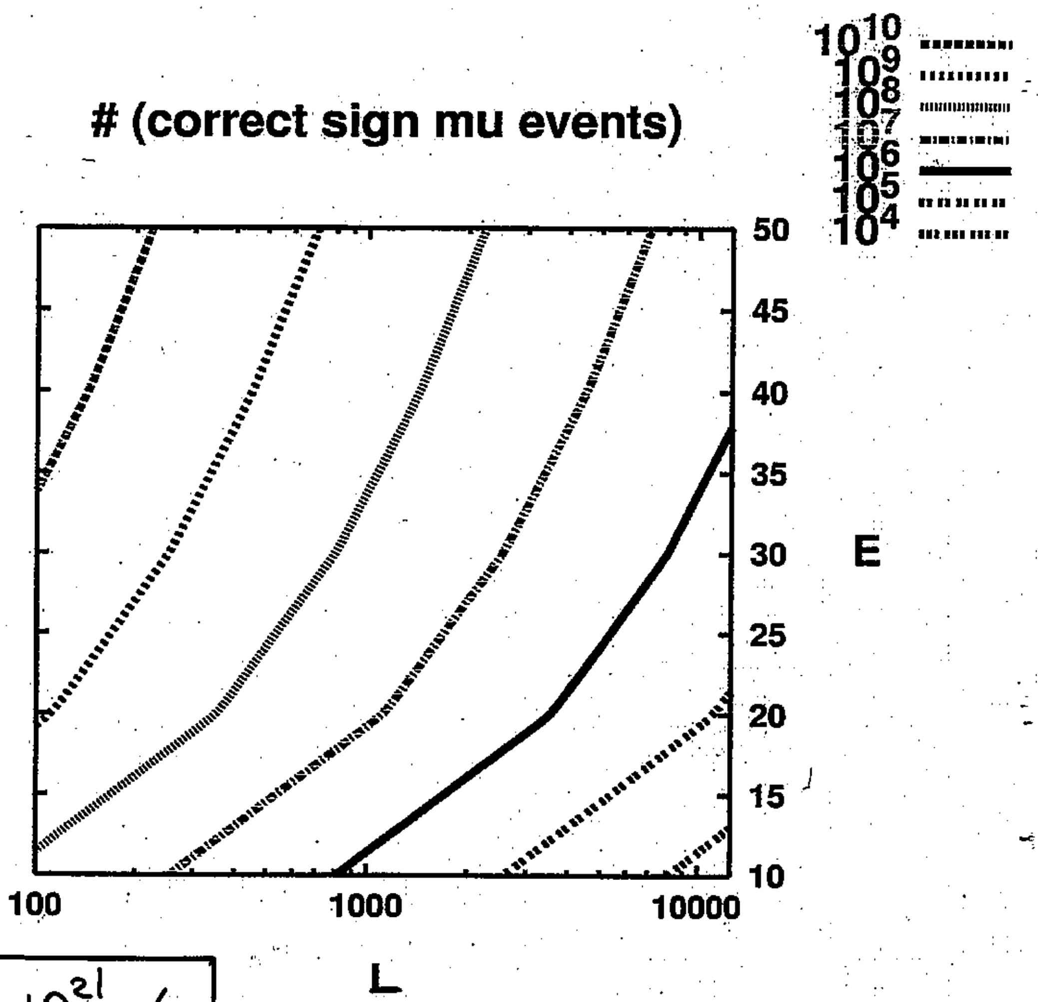
$\sin^2 2\theta_{13} = 1.2\times 10^{-3}$



$\frac{N_{wrong}}{N_{correct}}$

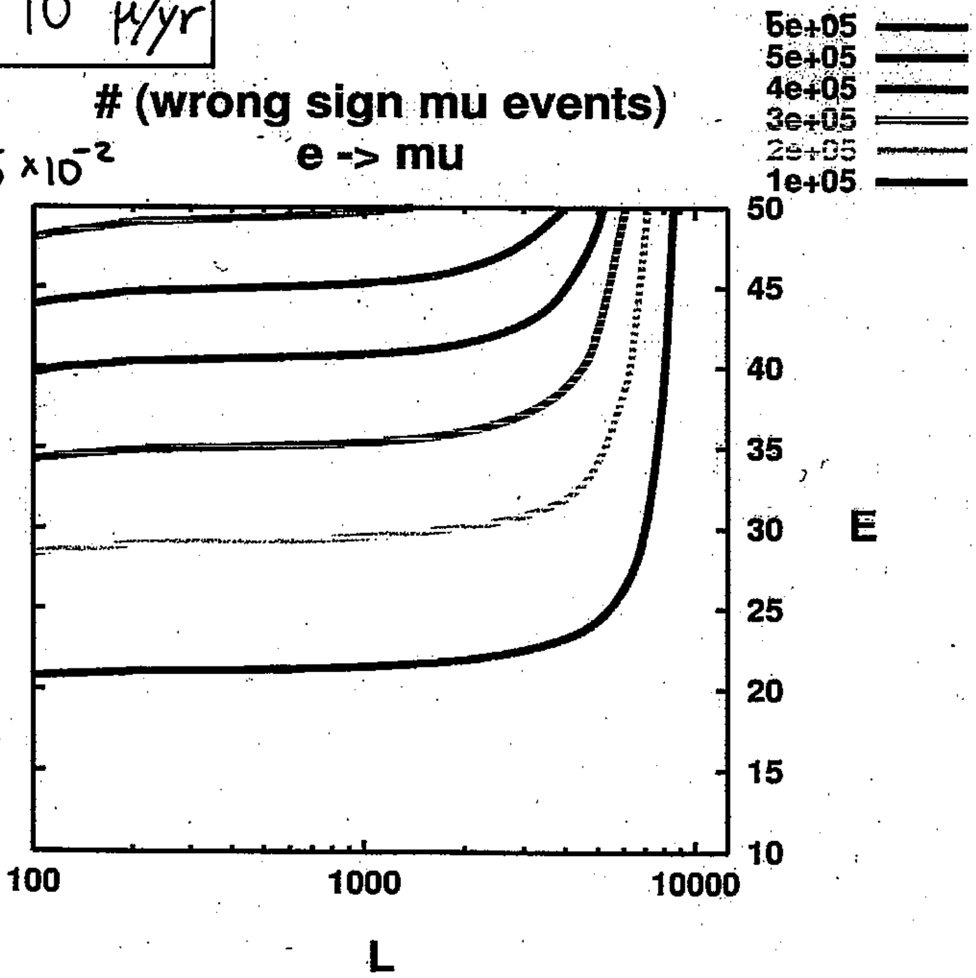
$\sin^2 2\theta_{13} = 9.5\times 10^{-2}$ $\theta_{13}=9°$





$10kt \cdot 1yr \cdot 10^{21} \mu/yr$

$\sin^2 2\theta_{13} = 9.5 \times 10^{-2}$



③ measurement of δ ($\cancel{CP}$)

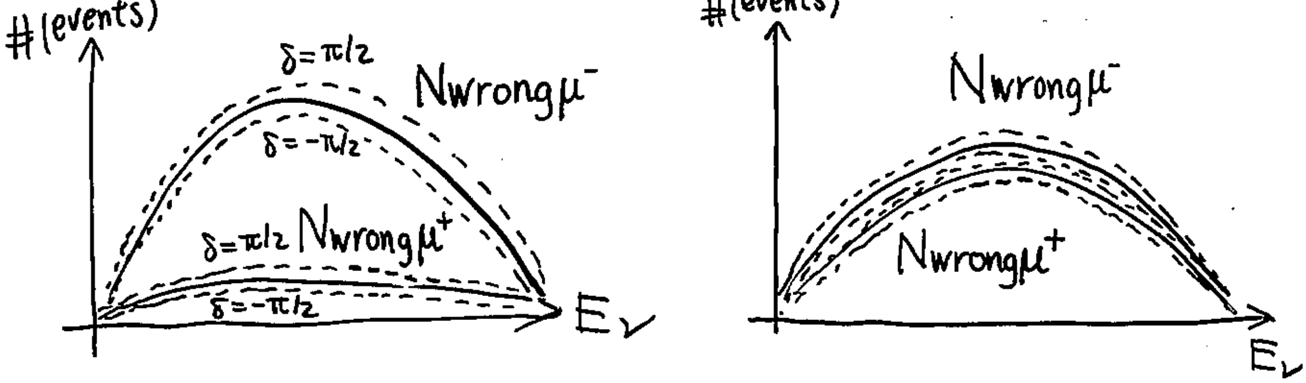
In the hierarchical limit, to first order in $\left|\frac{\Delta m^2_{21}}{\Delta m^2_{32}}\right|$ and/or $\left|\frac{\Delta E_{21}}{A}\right|$, the appearance probability is

$$P(\nu_e \to \nu_\mu) \simeq s^2_{23} \sin^2 2\theta^{M(-)}_{13} \sin^2\left(\frac{B^{(-)}L}{2}\right)$$
$$- \frac{1}{2} \frac{\Delta E_{21} \Delta E_{32}}{\lambda^{(-)}_+ \lambda^{(-)}_-} \sin\delta \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta^{M(-)}_{13}$$
$$\times \sin\left(\frac{\lambda^{(-)}_+ L}{2}\right) \sin\left(\frac{\lambda^{(-)}_- L}{2}\right) \sin\left(\frac{B^{(-)}L}{2}\right)$$

$$P(\bar\nu_e \to \bar\nu_\mu) \simeq s^2_{23} \sin^2 2\theta^{M(+)}_{13} \sin^2\left(\frac{B^{(+)}L}{2}\right) \qquad \lambda^{(\pm)}_\pm = \frac{1}{2}\left(A - \Delta E_{32} \pm B^{(\pm)}\right)$$
$$+ \frac{1}{2} \frac{\Delta E_{21} \Delta E_{32}}{\lambda^{(+)}_+ \lambda^{(+)}_-} \sin\delta \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta^{M(+)}_{13}$$
$$\times \sin\left(\frac{\lambda^{(+)}_+ L}{2}\right) \sin\left(\frac{\lambda^{(+)}_- L}{2}\right) \sin\left(\frac{B^{(+)}L}{2}\right)$$

Because of the matter effect, one of these is enhanced while the other suppressed.
For the enhanced channel, # of events are large, so we don't want to subtract the matter effect which leads to smaller #.

e.g. the case of $\Delta m^2_{32} > 0$ the case of $\Delta m^2_{32} < 0$



In 1st order in $\Delta E_{21}/\Delta E_{32}$

$$P_{even} \simeq s_{23}^2 \sin^2 2\theta_{13}^{M(-)} \sin^2\left(\frac{B^{(-)}L}{2}\right)$$

$$P_{odd} \simeq -\frac{J_M^{(-)}}{2} \frac{\Delta E_{21}\Delta E_{32}}{\lambda_+^{(-)}\lambda_-^{(-)}} \sin\left(\frac{\lambda_+^{(-)}L}{2}\right)\sin\left(\frac{\lambda_-^{(-)}L}{2}\right)\sin\left(\frac{B^{(-)}L}{2}\right)$$

$$B^{(-)} \equiv \left[(\Delta E_{32}\cos 2\theta_{13} - A)^2 + (\Delta E_{32}\sin 2\theta_{13})^2\right]^{\frac{1}{2}}$$

$$\lambda_\pm^{(-)} = \frac{1}{2}\ (A - \Delta E_{32} \pm B^{(-)})\ ,\quad \Delta E_{32} \equiv \frac{\Delta m_{32}^2}{2E_\nu}$$

$$\tan 2\theta_{13}^{M(-)} = \frac{\Delta E_{32}\sin 2\theta_{13}}{\Delta E_{32}\cos 2\theta_{13} - A}$$

$$\sin 2\theta_{13}^{M(-)} = \Delta E_{32}\sin 2\theta_{13} / B^{(-)}$$

Off resonance

$$P_{even} \propto \sin^2 2\theta_{13}$$

$$P_{odd} \propto \sin 2\theta_{13}$$

$$R_\delta = \sum_j \frac{(N_{odd}^j)^2}{N_{even}^j}$$: almost independent of $\sin 2\theta_{13}$

To have $R_\delta > 3$

it seems that we need

$10^{21}\ \mu/yr \cdot 10\,kt \cdot 1\,yr$

3 flavor oscillation in constant matter

A2.5

$$U\, \mathrm{diag}(E_1, E_2, E_3)\, U^{-1} + \mathrm{diag}(A, 0, 0)$$

$$= V\, \mathrm{diag}\,(\xi_1, \xi_2, \xi_3)\, V^{-1} \qquad V: \text{unitary}$$

$$i \frac{d}{dx} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = V\, \mathrm{diag}\,(\xi_1, \xi_2, \xi_3)\, V^{-1} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

$$\nu_\alpha(x) = \sum_j V_{\alpha j}\, e^{-i \xi_j x}\, V_{\beta j}^{*}\, \nu_\beta(0)$$

$$\Delta \xi_{jk} \equiv \xi_j - \xi_k$$

$$A(\nu_\beta \to \nu_\alpha ; L) = \sum_j V_{\alpha j}\, e^{-i \xi_j L}\, V_{\beta j}^{*}$$

For ν

$$P(\nu_\beta \to \nu_\alpha ; L) = \delta_{\alpha\beta} - 4 \sum_{j<k} V_{\alpha j} V_{\beta j}^{*} V_{\alpha k}^{*} V_{\beta k} \sin^2\left(\frac{\Delta \xi_{jk} L}{2}\right)$$
$$-2i \sum_{j<k} V_{\alpha j} V_{\beta j}^{*} V_{\alpha k}^{*} V_{\beta k} \sin(\Delta \xi_{jk} L)$$

$$P(\nu_\beta \to \nu_\alpha ; L) - P(\nu_\alpha \to \nu_\beta ; L)$$
$$= 4\, \underline{\mathrm{Im}\left(V_{\alpha 1} V_{\alpha 2}^{*} V_{\beta 1}^{*} V_{\beta 2}\right)} \left[\sin(\Delta \xi_{12} L) - \sin(\Delta \xi_{13} L) + \sin(\Delta \xi_{23} L)\right]$$

modified Jarlskog factor

For $\bar{\nu}$

$$U^{*} \mathrm{diag}\,(E_1, E_2, E_3)\, U^{*-1} + \mathrm{diag}\,(-A, 0, 0)$$

$$= W\, \mathrm{diag}\,(\zeta_1, \zeta_2, \zeta_3)\, W^{-1} \qquad W: \text{unitary}$$

$$P(\bar{\nu}_\beta \to \bar{\nu}_\alpha ; L) = \delta_{\alpha\beta} - 4 \sum_{j<k} W_{\alpha j} W_{\beta j}^{*} W_{\alpha k}^{*} W_{\beta k} \sin^2\left(\frac{\Delta \zeta_{jk} L}{2}\right)$$
$$-2i \sum_{j<k} W_{\alpha j} W_{\beta j}^{*} W_{\alpha k}^{*} W_{\beta k} \sin(\Delta \zeta_{jk} L)$$

$$(W_{\alpha j}, \Delta \zeta_{jk}) \longleftrightarrow (V_{\alpha j}, \Delta \xi_{jk})$$

completely
unrelated

$$\boxed{\frac{1}{2}\left[P(\nu_\alpha\to\nu_\beta;\delta)+P(\nu_\alpha\to\nu_\beta;-\delta)\right]}$$

$$=\delta_{\alpha\beta}-4\sum_{j<k}\mathrm{Re}\left(V_{\alpha j}V_{\beta j}^{*}V_{\alpha k}^{*}V_{\beta k}\right)\sin^2\left(\frac{\Delta\xi_{jk}L}{2}\right) \quad :\text{T-even}$$

$$\boxed{\frac{1}{2}\left[P(\nu_\alpha\to\nu_\beta;\delta)-P(\nu_\alpha\to\nu_\beta;-\delta)\right]}$$

$$=2\sum_{j<k}\mathrm{Im}\left(V_{\alpha j}V_{\beta j}^{*}V_{\alpha k}^{*}V_{\beta k}\right)\sin\left(\Delta\xi_{jk}L\right) \quad :\text{T-odd}$$

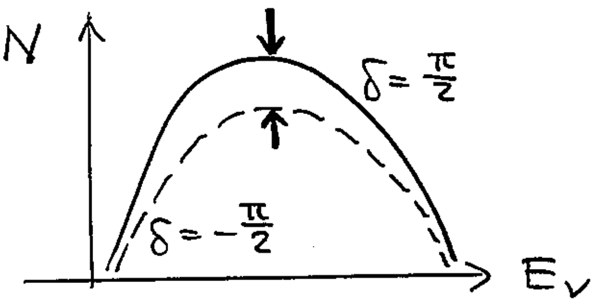
$$N^j_{even}=\frac{E_\mu^2}{\pi m_\mu^2 L^2}\int_{y_j}^{y_{j+1}}dy\,12y^2(1-y)\sigma(E_\nu)\frac{1}{2}\left[P(\delta)+P(-\delta)\right]$$

$$N^j_{odd}=\frac{E_\mu^2}{\pi m_\mu^2 L^2}\int_{y_j}^{y_{j+1}}dy\,12y^2(1-y)\underbrace{\sigma(E_\nu)}_{=\,0.67\times10^{-38}\,\mathrm{cm}^2\frac{E_\nu}{\mathrm{GeV}}}\frac{1}{2}\left[P(\delta)-P(-\delta)\right]$$

$$y\equiv E_\nu/E_\mu$$

$\delta\to\frac{\pi}{2}$

$$R_\delta\equiv\sum_j\frac{\left(N^j_{odd}\right)^2}{N^j_{even}} \quad \leftarrow \quad \sum_j\frac{\left[N(\delta=\frac{\pi}{2})-N(\delta=0)\right]^2}{(\delta N)^2}$$



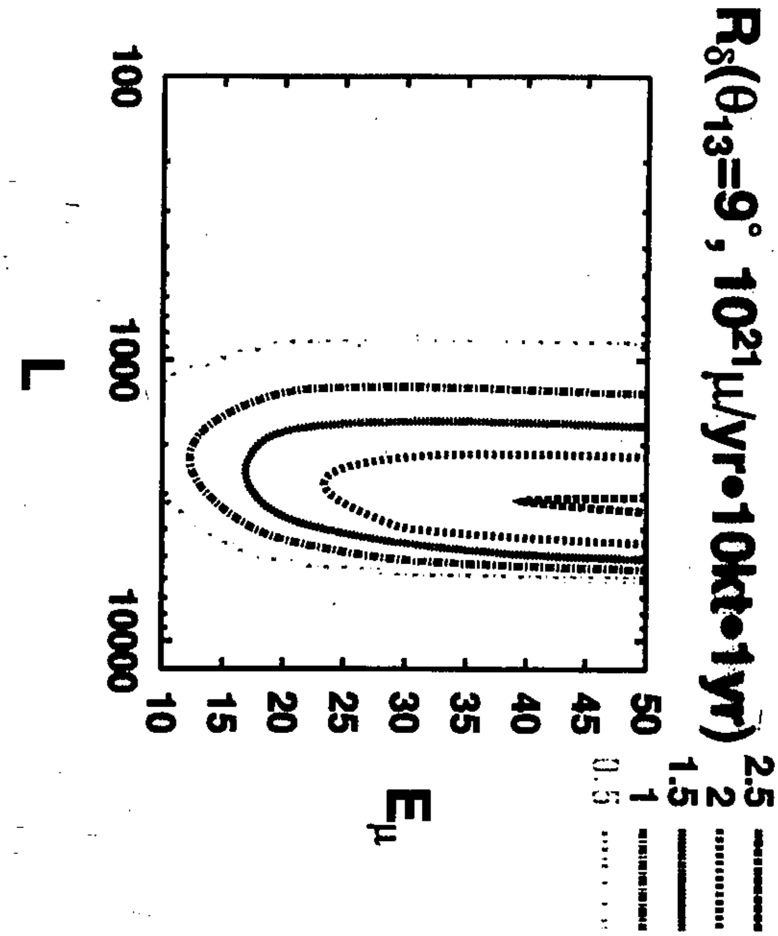
Is the difference bigger than stastical errors?

confirmation of $\delta\neq0$

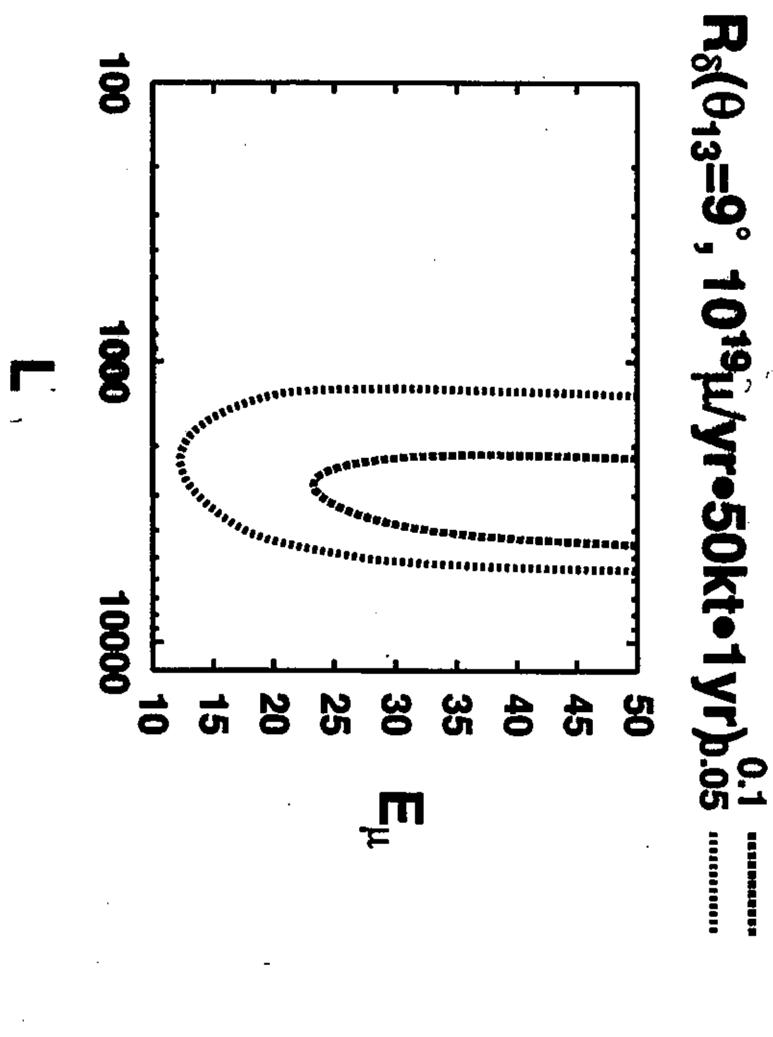
↑

It is necessary $\boxed{R_\delta>3}$

even when systematic errors or existence of background is ignored.



Rδ(θ13=9°, 10^21μ/yr•10kt•1yr)
2.5
2
1.5
1
0.5
100
1000
10000
L
10
15
20
25
30
35
40
45
50
Eμ



Rδ(θ13=9°, 10^19μ/yr•50kt•1yr)
0.1
0.05
100
1000
10000
L
10
15
20
25
30
35
40
45
50
Eμ

comparison of $\cancel{CP}$ in conventional beam

LOI of JHF

	N_{cc}	R_δ
WIDE	3162	0.22
LE2π	444	0.11

To have $R_\delta > 3$
it takes 15 yrs.

But comparable to
ν factory w/ 10^{19} μ/yr. 50kt. 1yr.

general case of $N_\nu = 4$ (w/o BBN constraint)

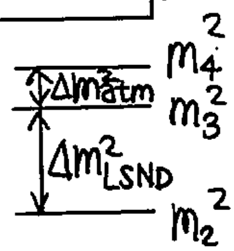
In this case, $\underbrace{C_s \equiv |U_{s1}|^2 + |U_{s2}|^2}_{\text{depending on}} \gtrsim 0.4$

$\begin{pmatrix} \text{LMA} \\ \text{vacuum} \end{pmatrix}$ solutions may exist.

But here we assume SMA solution for simplicity: $\theta_{12} = \theta_\odot|_{SMA} \ll 1$

$$U_{MNS} \sim \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -c_{atm} s_{23} e^{i\delta_1} & c_{23} c_{atm} & s_{atm} \\ 0 & -c_{23} s_{34} + c_{34} s_{23} s_{atm} e^{i\delta_1} & -c_{23} c_{34} s_{atm} - s_{23} s_{34} e^{-i\delta_1} & c_{atm} c_{34} \\ 0 & c_{23} c_{34} + s_{23} s_{34} s_{atm} e^{i\delta_1} & -c_{23} s_{34} s_{atm} - s_{23} c_{34} e^{-i\delta_1} & c_{atm} s_{34} \end{pmatrix}$$

$\hookrightarrow$ full mixing between $\begin{pmatrix} \nu_\mu \\ \nu_\tau \\ \nu_s \end{pmatrix}$ & $\begin{pmatrix} \nu_2 \\ \nu_3 \\ \nu_4 \end{pmatrix}$



$\downarrow$

One can play the same game as in $N_\nu = 3$ before CHOOZ data appeared.

① determination of the sign of Δm^2_{43}

② determination of θ_{atm}, θ_{23}, θ_{34}

③ determination of δ_1

① determination of the sign of Δm^2_{43}

$$\mathcal{M} \equiv U \,\mathrm{diag}(-\Delta E_{21}, 0, \Delta E_{32}, \Delta E_{32}+\Delta E_{43})\, U^{-1} + \mathrm{diag}(V_{CC}, 0, 0, -V_{NC})$$

Consider the case

$$|\Delta E_{43}| \simeq |V_{NC}| \quad , \quad \begin{array}{l} |\Delta E_{21} L/E| \ll 1 \\ |\Delta E_{32} L/E| \gg 1 \end{array}$$

ΔE_{21}: $\Delta E_\odot$, ΔE_{32}: ΔE_{LSND}

Then (lower 3x3 matrix)

$$\mathcal{M}_3 = U^M \mathrm{diag}(-\Delta E_{32}, F_0 + \sqrt{F^2+G^2}, F_0 - \sqrt{F^2+G^2})\, U^{M-1}$$

$$F_0 \equiv \frac{1}{2}[s_3^2(\tilde{c}_2^2\lambda_- + \tilde{s}_2^2\lambda_+) + \tilde{c}_2^2\lambda_+ + \tilde{s}_2^2\lambda_-]$$

$$F \equiv \frac{1}{2}[s_3^2(\tilde{c}_2^2\lambda_- + \tilde{s}_2^2\lambda_+) - \tilde{c}_2^2\lambda_+ - \tilde{s}_2^2\lambda_-]$$

$$G \equiv \tilde{c}_2\tilde{s}_2 s_3(\lambda_- - \lambda_+)$$

$$\tilde{s}_2 \equiv \sin(\theta_2 - \theta_2^M) \quad , \quad \tilde{c}_2 \equiv \cos(\theta_2 - \theta_2^M)$$

$$\tan 2\theta_2^M = \frac{\Delta E_{43}\sin 2\theta_2}{\Delta E_{43}\cos 2\theta_2 - V_{NC}}$$

$$\lambda_\pm \equiv \frac{1}{2}\left\{\Delta E_{43} - V_{NC} \pm \sqrt{(\Delta E_{43}\cos 2\theta_2 - V_{NC})^2 + (\Delta E_{43}\sin 2\theta_2)^2}\right\}$$

$$U = (1, 1, e^{i\delta'})\, e^{i\theta_1\lambda_2}\, e^{i\theta_2\lambda_5}\, (e^{i\delta'}, 1, 1)\, e^{i\theta_3\lambda_2}$$

$$\boxed{\begin{aligned} P(\nu_\mu \to \nu_\tau) = & -4 U^M_{\mu 3} U^{M*}_{\tau 3} U^{M*}_{\mu 4} U^M_{\tau 4} \sin^2\left(\sqrt{F^2+G^2}\,L\right) \\ & + 4|U^M_{\mu 2}|^2 |U^M_{\tau 2}|^2 \sin^2\left(\frac{\Delta E_{32} L}{2}\right) \end{aligned}}$$

$$U^M \equiv \begin{pmatrix} U_{\mu 2} & c_\omega U_{\mu 3} + s_\omega U_{\mu 4} e^{i\delta'} & c_\omega U_{\mu 4} - s_\omega U_{\mu 3} e^{-i\delta'} \\ U_{\tau 2} & c_\omega U_{\tau 3} + s_\omega U_{\tau 4} e^{i\delta'} & c_\omega U_{\tau 4} - s_\omega U_{\tau 3} e^{-i\delta'} \\ U_{s 2} & c_\omega U_{s 3} + s_\omega U_{s 4} e^{i\delta'} & c_\omega U_{s 4} - s_\omega U_{s 3} e^{-i\delta'} \end{pmatrix}$$

$$\tan 2\omega = \frac{G}{F}$$

$$U \equiv \begin{pmatrix} c_1 c_2 c_3 e^{i\delta'} - s_1 s_3 & s_1 c_3 + c_1 c_2 s_3 e^{i\delta'} & c_1 s_2 \\ -s_1 c_2 c_3 e^{i\delta'} - c_1 s_3 & c_1 c_3 - s_1 c_2 s_3 e^{i\delta'} & -s_1 s_2 \\ -s_2 c_3 & -s_2 s_3 & c_2 e^{-i\delta'} \end{pmatrix}$$

$$P(\bar{\nu}_\mu \to \bar{\nu}_\tau) = P(\nu_\mu \to \nu_\tau)\Big|_{V_{NC} \to -V_{NC},\ \delta' \to -\delta'}$$

difference of N_{ν_τ} and $2N_{\bar{\nu}_\tau}$

$\longrightarrow$ mainly matter effects

$\longrightarrow$ sign of Δm^2_{43} should be obtained

2 determination of $\theta_{atm}, \theta_{23}, \theta_{34}$

$$\begin{cases} P(\nu_\mu \to \nu_\tau) \\ P(\bar{\nu}_\mu \to \bar{\nu}_\tau) \\ 1 - P(\nu_\mu \to \nu_\mu) = 4|U^M_{\mu 3}|^2 |U^M_{\mu 4}|^2 \sin^2(\sqrt{F^2+G^2}L) \\ \qquad + 4|U^M_{\mu 2}|^2 (1 - |U^M_{\mu 2}|^2) \sin^2\left(\frac{\Delta E_{32} L}{2}\right) \end{cases}$$

$\longrightarrow$ in principle $\theta_{atm}, \theta_{23}, \theta_{34}$ can be determined with certain precision

To avoid uncertainty shorter L is desired.

③ determination of δ_1

in vacuum

$$P(\nu_\mu \to \nu_\tau) - P(\bar{\nu}_\mu \to \bar{\nu}_\tau)$$

$$\sim \frac{1}{8} \boxed{C_{atm} \sin 2\theta_{atm} \sin 2\theta_{23} \sin 2\theta_{34} \sin\delta} \sin^2\left(\frac{\Delta m_{43}^2 L}{4E}\right)$$

↳ could be $O(1)$

But for $L \sim 4000$ km $\left(\text{such that } \frac{\Delta m_{atm}^2 L}{4E} \sim 1, E = 10\,\text{GeV}\right)$

matter effect matters

$\longrightarrow \begin{cases} E \text{ should be as close as } E_{th} \sim 3.5\,\text{GeV} \\ L \text{ should be shorter than } 1000\text{ km} \end{cases}$

④ future problems

* how to measure small angles θ_{13}, θ_{14}

$\longrightarrow$ enhancement due to matter effect?

In $N_\nu = 4$ case analytical treatment is difficult.

* If $\nu_\odot$ is $\begin{pmatrix} \text{LMA} \\ \text{or} \\ \text{vacuum} \end{pmatrix}$ solutions, then we have full 4×4 mixings.

$\longrightarrow$ again analysis is complicated.

3. Conclusions

$N_\nu = 3$

* the sign of Δm_{32}^2 will be determined by looking at which of $\nu, \bar{\nu}$ is enhanced in long-base line experiments.

* θ_{13}, θ_{23} may be determined with some precision in not-so-long-base line experiments.

* $\not{CP}$ effects are not easy to observe for most the set of the oscillation parameters.

  In any case, to determine δ one has to know $\sigma_{\nu N}, \sigma_{\bar{\nu} N}$, θ_{23}, θ_{13}, the density profile of the Earth very accurately so that these errors have to be smaller than the systematic ones $(\sqrt{N(\delta=0)})$.

  (measurement of $\not{CP}$ is only possible for LMA MSW $\nu_\odot$ solution. For SMA MSW, vacuum sols., it's hopeless.)

$N_\nu = 4$

* If BBN constraint $N_\nu < 4$ holds, we have even more boring pattern:

$$\nu_\odot : \nu_e \leftrightarrow \nu_s \quad \text{(SMA MSW)}$$
$$\nu_{atm} : \nu_\mu \leftrightarrow \nu_\tau$$

* If we lift the BBN constraint, there may be a lot of interesting phenomena.

future problems

* more detailed discussions on general $N_\nu = 4$ case

* investigation of more symmetric schemes, such as $N_\nu = 6$ (3 active + 3 sterile) and their implications

http://www-jhf.kek.jp/JHF_WWW/LOI/jhfnu_loi.ps

Letter of Intent: A Long Baseline Neutrino Oscillation Experiment using the JHF 50 GeV Proton-Synchrotron and the Super-Kamiokande Detector

February 3, 2000

—V1.0—

JHF Neutrino Working Group

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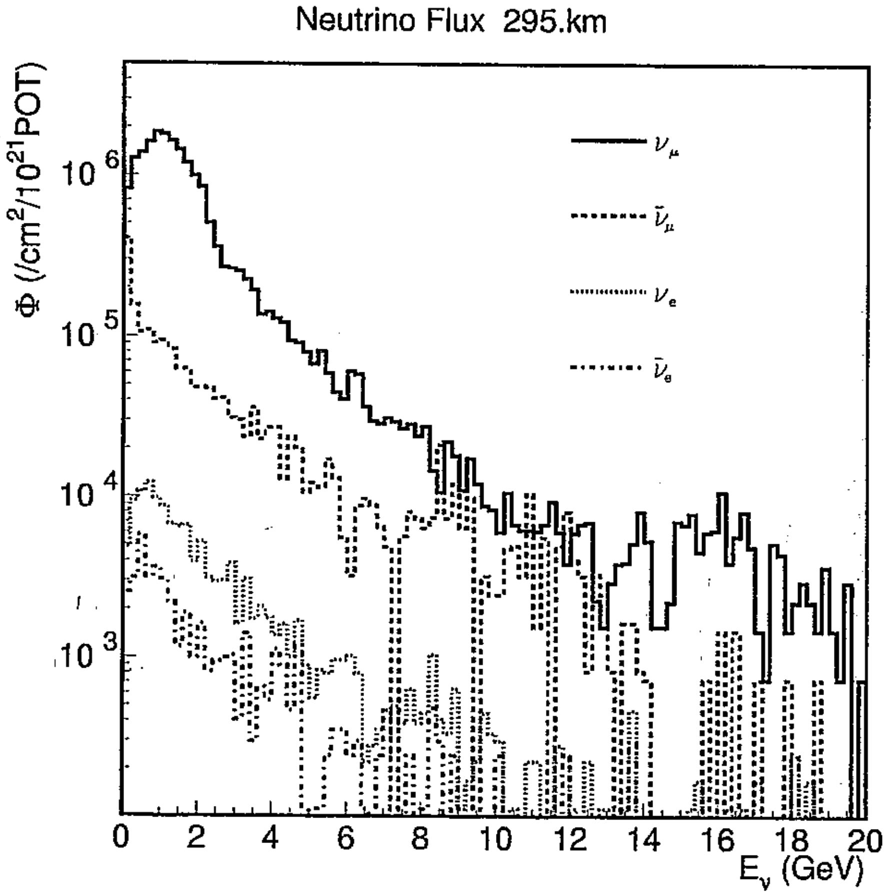
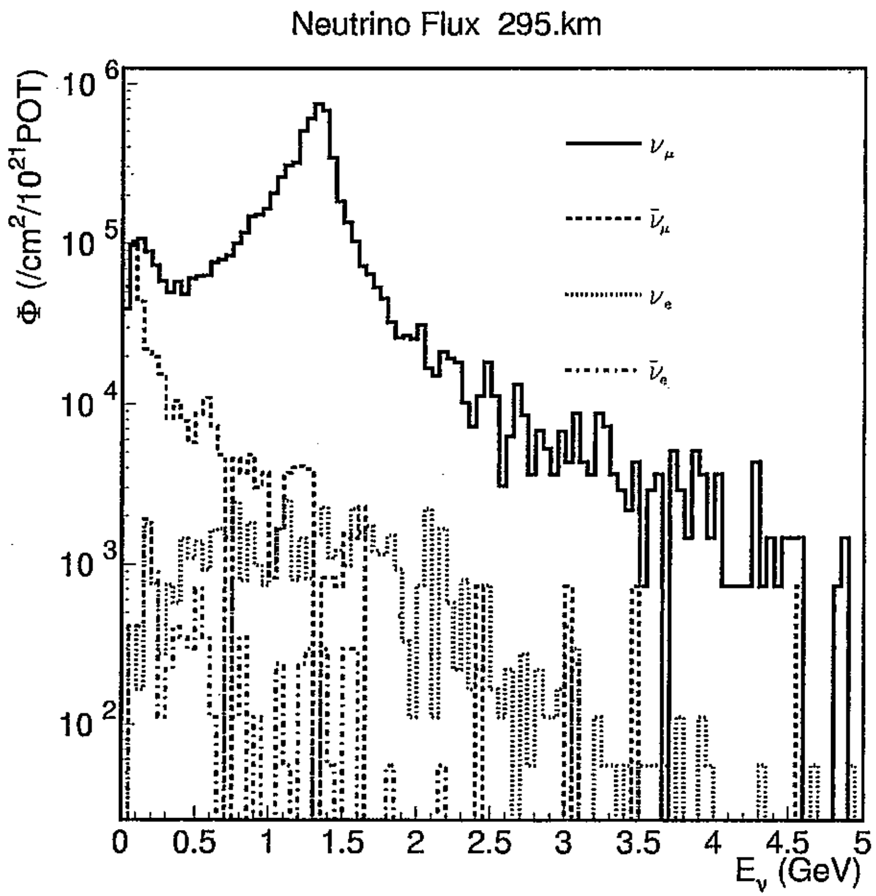
WIDE


Neutrino Flux 295.km
Φ (/cm²/10²¹POT)
10 6
10 5
10 4
10 3
0
2
4
6
8
10
12
14
16
18
20
E_ν (GeV)
ν_μ
ν̄_μ
ν_e
ν̄_e

LE 3π


Neutrino Flux 295.km
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10 6
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2
2.5
3
3.5
4
4.5
5
E_ν (GeV)
ν_μ
ν̄_μ
ν_e
ν̄_e