# 大気ニュートリノの4世代解析とsterile neutrinoシナリオの現状

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## 要旨

sterile neutrino シナリオは
まだ全然死んでいない！

sterile neutrino シナリオ

標準的シナリオ　：SNOデータの統計がたまるまで死なない。

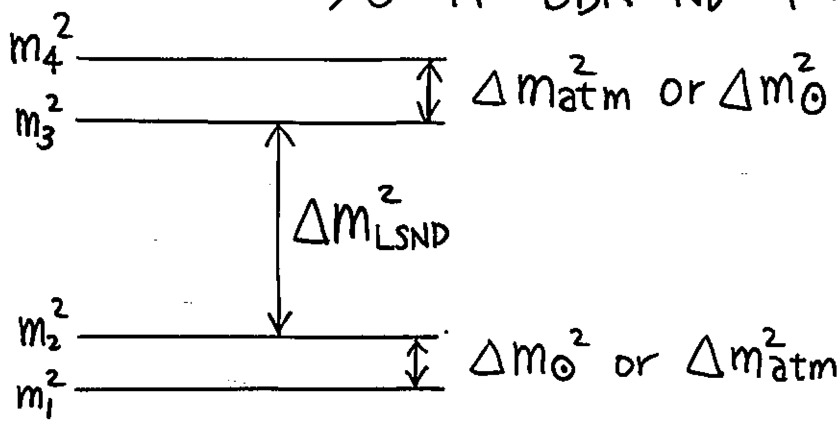
もっと一般的なシナリオ（exoticな）：LSNDが完全に死ぬまで死なない。

3) $N_\nu = 4$ case

$$U_{MNS} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{pmatrix}$$

$U_{e3}\ U_{e4} \to 0$ ($\because$ Bugey)

$U_{s3}\ U_{s4} \to 0$ if BBN $N_\nu < 4$ is assumed




6 mixing angles

(if BBN $N_\nu<4$)

$\theta_{12}$, $\theta_{13}$, $\theta_{14}$, $\theta_{23}$, $\theta_{24}$, $\theta_{34}$

$\theta_{12} \to \theta_\odot$

$\theta_{13}, \theta_{14} \to$ almost $0$ ($\because$ Bugey)

$\theta_{23} \to 0$ (if BBN $N_\nu<4$); $\theta_{23} \to$ mixing of $\begin{pmatrix} \sin^2(\Delta m_{atm}^2 L/4E) \\ \& \\ \sin^2(\Delta m_{LSND}^2 L/4E) \end{pmatrix}$ in $\nu_{atm}$

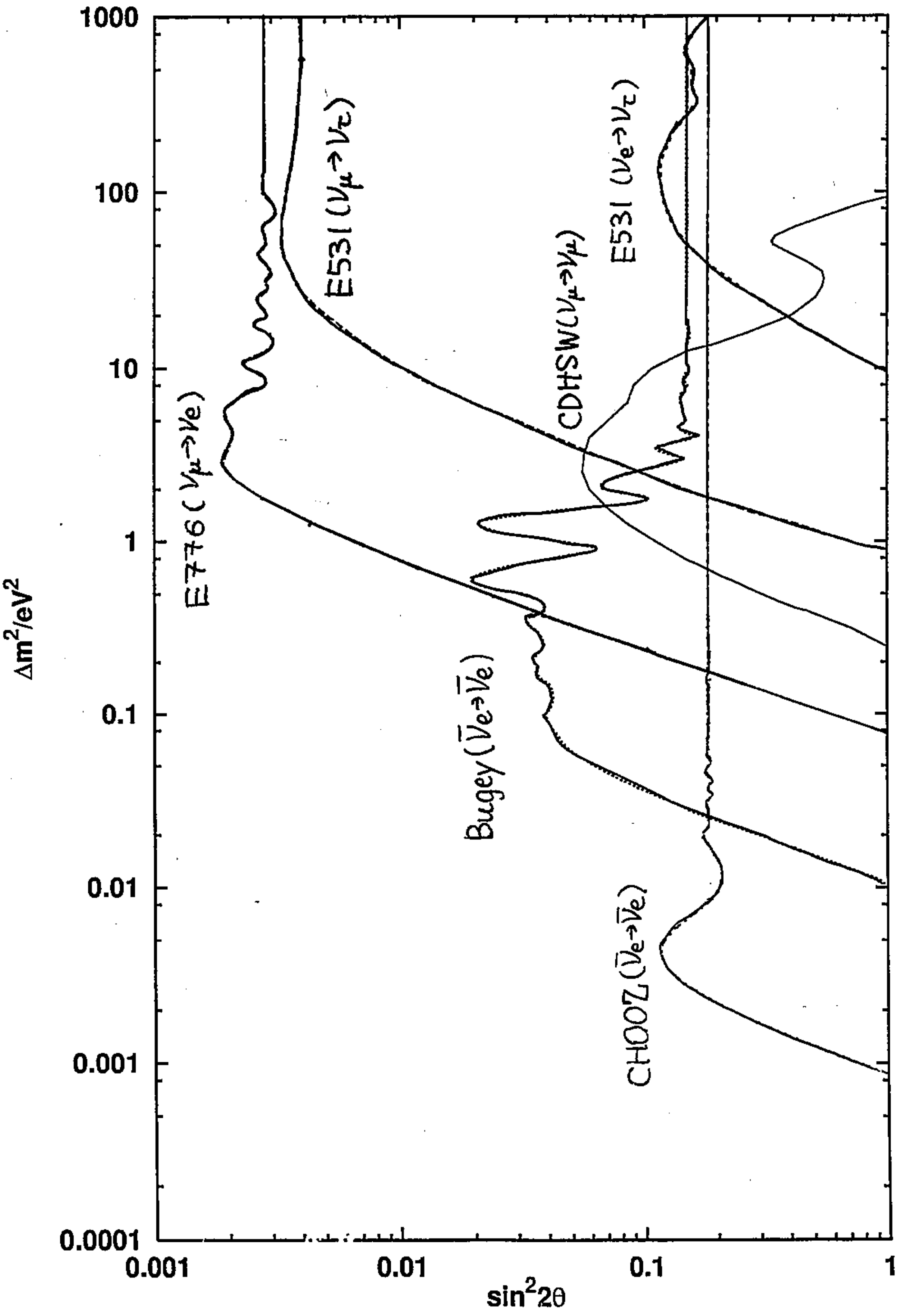
$\theta_{24} \to \theta_{atm}$

$\theta_{34} \to 0$ (if BBN $N_\nu<4$); $\theta_{34} \to$ mixing of $\begin{pmatrix} \nu_\mu \leftrightarrow \nu_\tau \\ \& \\ \nu_\mu \leftrightarrow \nu_s \end{pmatrix}$ in $\nu_{atm}$

3 CP phases $\delta_1$, $\delta_2$, $\delta_3$

$\delta_1 \to$ can be measurable if $\theta_{23}, \theta_{34}$ are large

$\delta_2, \delta_3 \to$ decouple as $\theta_{13}, \theta_{14} \to 0$


1000
100
10
1
0.1
0.01
0.001
0.0001
0.001
0.01
0.1
1
$\Delta m^2/eV^2$
$\sin^2 2\theta$
E776 ($\nu_\mu \to \nu_e$)
E531 ($\nu_\mu \to \nu_\tau$)
CDHSW ($\nu_\mu \to \nu_\mu$)
E531 ($\nu_e \to \nu_\tau$)
Bugey ($\bar{\nu}_e \to \bar{\nu}_e$)
CHOOZ ($\bar{\nu}_e \to \bar{\nu}_e$)

○ 宇宙論からの制限

Hubble パラメーター　$H=\left(\frac{4\pi^3}{45}\frac{g_*(T)}{M_{pl}^2}T^4\right)^{1/2}$

弱い相互作用による衝突距離　$l_{coll}=\frac{1}{\Gamma_\nu}=\frac{1}{C(T)G_F^2T^5}$　$\left(C(T)\sim\frac{1}{2}\right)$

$\Gamma_\nu/H$　$>1$ : $\nu$ は熱平衡
　　　　$<1$ : $\nu$ は熱平衡に達しない

sterile $\nu$ が理論に存在して $\nu_\alpha(\alpha=e,\mu,\tau)\to\nu_s$ の $\nu$ 振動が起こり, $\frac{\Gamma_{\nu_s}}{H}\equiv\frac{\Gamma_\nu}{H}P(\nu_\alpha\to\nu_s)\geq 1$ が満たされる場合, 元素合成に対する Big Bang 宇宙論の成功は危うくなる.

$$\Rightarrow\quad \frac{\Gamma_{\nu_s}}{H}\equiv\frac{\Gamma_\nu}{H}P(\nu_\alpha\to\nu_s)<1$$ が必要

今の場合の質量行列

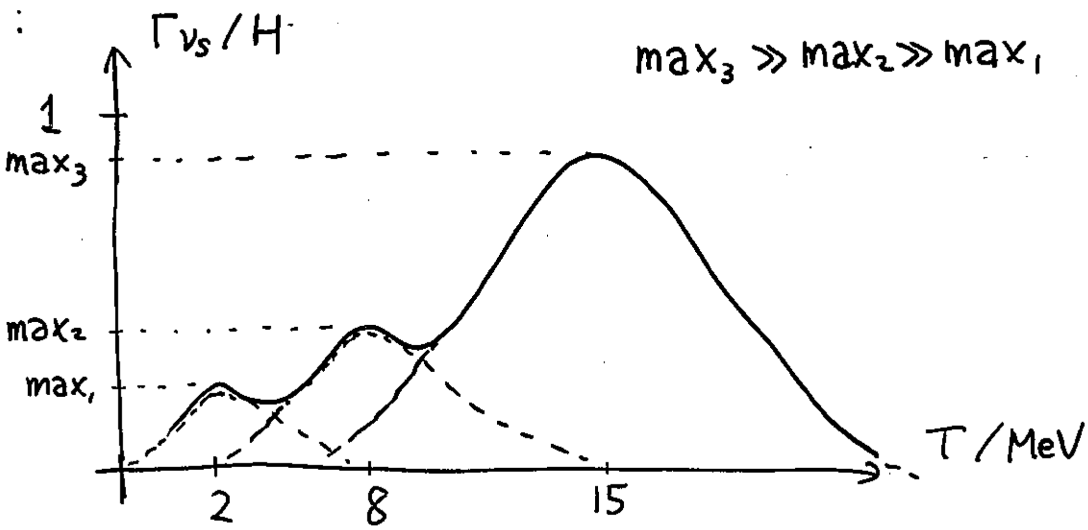
$$U\,\mathrm{diag}(E_j)\,U^{-1}+\mathrm{diag}(V,cV,cV,0)$$

$$\Delta E_{ji}=\Delta m_{ji}^2/6.3T$$

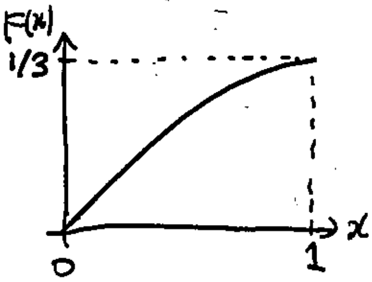
$$V=-\frac{\sqrt{2}}{\zeta(3)}\left(\frac{7}{90}\right)^2\pi^6\frac{G_F}{m_W^2}T^5(2+\cos^2\theta_W)$$

$$C=\frac{\cos^2\theta_W}{2+\cos^2\theta_W}$$

結果：



$$\max x_3 \sim 870 \left(\frac{\Delta m_{31}^2}{eV^2}\right)^{1/2} F\left(|U_{s3}|^2 + |U_{s4}|^2\right) < 1$$




$$F(x) \sim \frac{\sqrt{3}}{4} x \quad (\text{as } x \to 0)$$

$$\Rightarrow \quad |U_{s3}|^2 + |U_{s4}|^2 \lesssim 1.3 \times 10^{-2}$$

with $0.27\,eV^2 \lesssim \Delta m_{31}^2 = \Delta m_{LSND}^2 \lesssim 2.3\,eV^2$

$$\Rightarrow \quad \boxed{\begin{array}{l} |\theta_{23}| \lesssim 6^\circ \\ |\theta_{34}| \lesssim 11^\circ \end{array}}$$

結論： N. Okada - O.Y. Int. J. Mod. Phys.

原子炉 (Bugey)

$$U \simeq \begin{pmatrix} C_\odot & S_\odot & \epsilon & \epsilon \\ \epsilon & \epsilon & C_{atm} & S_{atm} \\ \epsilon & \epsilon & -S_{atm} & C_{atm} \\ -S_\odot & C_\odot & \epsilon & \epsilon \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & -1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

hep-ph/9606411

Big Bang 元素合成

すなわち　4種類の $\nu$ の混合は　ほとんど次の2チャンネル

$$\nu_e \leftrightarrow \nu_s \qquad \nu_\mu \leftrightarrow \nu_\tau$$

太陽 $\nu$　　　大気

$\Downarrow$

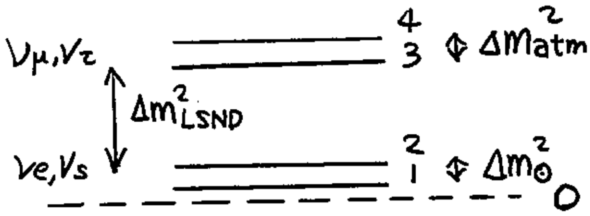
small-angle MSW解のみが許される

$N_\nu = 3+1\ (\nu_s)$ の場合 $\nu_\odot, \nu_{atm}, \nu_{LSND}$ を全部エネルギー依存解で説明し，$\nu-\bar{\nu}$ 非対称を仮定せずに元素合成からの制約を課するのが標準的と言える：

N. Okada − O.Y.
Int. J. Mod. Phys. A12 ('97) 3669

$$U_{MNS} \simeq \begin{pmatrix} C_\odot & S_\odot & \epsilon & \epsilon \\ \epsilon & \epsilon & 1/\sqrt{2} & 1/\sqrt{2} \\ \epsilon & \epsilon & -1/\sqrt{2} & 1/\sqrt{2} \\ -S_\odot & C_\odot & \epsilon & \epsilon \end{pmatrix}$$

$$(\Delta m^2_{21}, \sin^2 2\theta_{12}) = (\Delta m^2_\odot, \sin^2 2\theta_\odot)_{\text{small MSW}}$$
$$\Delta m^2_{43} = \Delta m^2_{atm}$$
$$\Delta m^2_{31} = \Delta m^2_{LSND}$$




Majorana型と仮定するとHDM成分はない。
(Dirac型の場合には全体に下駄をはかせてHDMを3成分に出来る.)

この時 $\nu_\odot$ : $\nu_e \leftrightarrow \nu_s$ (small angle MSW解のみ)

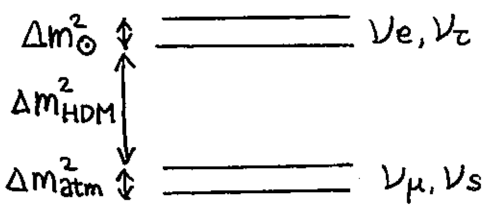
$\nu_{atm}$ : $\nu_\mu \leftrightarrow \nu_\tau$ (100%)

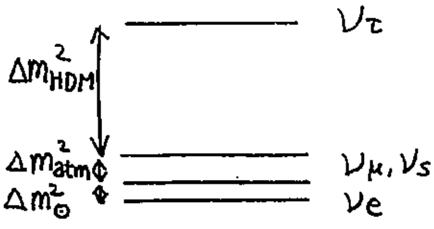
$\nu_{LSND}$ : $\nu_e \leftrightarrow \nu_\mu$ ( off-diagonal の小さい成分で説明できる)

今のように

NB ・$\Delta m^2_{31} = \Delta m^2_{LSND}$ とした場合には，$\nu-\bar{\nu}$ 非対称性を考慮してもしなくても元素合成の制約は同じ結論を与える。 R. Foot, hep-ph/9809315

・$\nu-\bar{\nu}$ 非対称性を考慮して状況が変わるのは $\Delta m^2_{31} = \Delta m^2_{HDM}$ とした時で，次のようなシナリオが許される：



Hot + CDM が観測をよく再現するという話は

J.R. Primack et. al. PPL 74 ('95) 2160

の解析に基いたものであるが，これは主として

近距離の星を対象としている。

その後 J.R. Primack, M.A.K. Gross

(astro-ph/9810204) は，遠くの星

まで含めると，$m_\nu \sim$ 数 eV の Hot + CDM

のシナリオは必ずしも良い fit を与えないことが

指摘されている（むしろ $m_\nu = 0 \sim 0.5$ eV 程度の

方が良い）。

最近 福来-Liu-杉山 (hep-ph/9908450) は

$$m_\nu \lesssim \frac{1.5\,\mathrm{eV}}{N_\nu} \quad (= 0.5\,\mathrm{eV} \text{ if } N_\nu = 3)$$

という制限を与えている。

$\longrightarrow$ CDM + $\Lambda$ モデルの方が

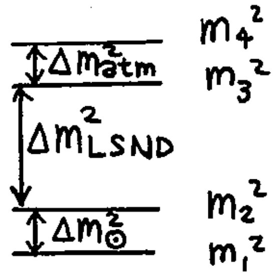
今は主流となっている。

# $N_\nu = 4$ analysis of $\nu_{atm}$ w/o BBN constraints

O. Y.

$$U\,\mathrm{diag}(-\Delta E_{21}, 0, \Delta E_{32}, \Delta E_{32}+\Delta E_{43})\,U^{-1} + \mathrm{diag}(A_{cc}, 0, 0, -A_{NC})$$

$$A_{cc} \equiv \sqrt{2}\,G_F N_e, \quad A_{NC} \equiv \frac{1}{\sqrt{2}} G_F N_n$$




$$U = R_{34}\left(\tfrac{\pi}{2}-\theta_{34}\right) R_{24}(\theta_{24}) R_{23}\left(\tfrac{\pi}{2}\right) e^{i\delta_1\lambda_3} R_{23}(\theta_{23}) e^{-i\delta_1\lambda_3}$$

$$\times \boxed{\cancel{e^{i\delta_3\lambda_{15}} R_{14}(\theta_{14}) e^{-i\delta_3\lambda_{15}}}}\ \boxed{\cancel{e^{i\delta_2\lambda_8} R_{13}(\theta_{13}) e^{-i\delta_2\lambda_8}}}\ \boxed{\cancel{R_{12}(\theta_{12})}}$$

reactor (Bugey) $\Rightarrow \theta_{13} = \theta_{14} = 0$

$\Delta E_{21} = \Delta E_\odot \to 0 \Rightarrow \theta_{12}$ decouples

Thus

$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -C_{24}S_{23}e^{i\delta_1} & C_{23}C_{24} & S_{24} \\ 0 & -C_{23}S_{34}+C_{34}S_{23}S_{24}e^{i\delta_1} & -C_{23}C_{34}S_{24}-S_{23}S_{34}e^{-i\delta_1} & C_{24}C_{34} \\ 0 & C_{23}C_{34}+S_{23}S_{24}S_{34}e^{i\delta_1} & -C_{23}S_{34}S_{24}+S_{23}C_{34}e^{-i\delta_1} & C_{24}S_{34} \end{pmatrix}$$

<u>w/ BBN $N_\nu<4$</u>
$$\begin{cases} |\theta_{23}|, |\theta_{34}| \ll 1 \\ \theta_{24} = \theta_{atm} \simeq \frac{\pi}{4} \end{cases}$$

<u>w/o BBN $N_\nu<4$</u> $\quad \theta_{24}, \theta_{23}, \theta_{34}, \delta_1$ : can be of $O(1)$

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## $N_\nu = 4$ analysis of $\nu_\odot$ — Giunti-Gonzalez-Garcia-Peña-Gray

$$C_s \equiv |U_{s1}|^2 + |U_{s2}|^2 = |C_{23}C_{34} + S_{23}S_{24}S_{34}e^{i\delta_1}|^2$$

$C_s = 0 \quad \nu_\odot : \nu_e \leftrightarrow \nu_{active}$ only

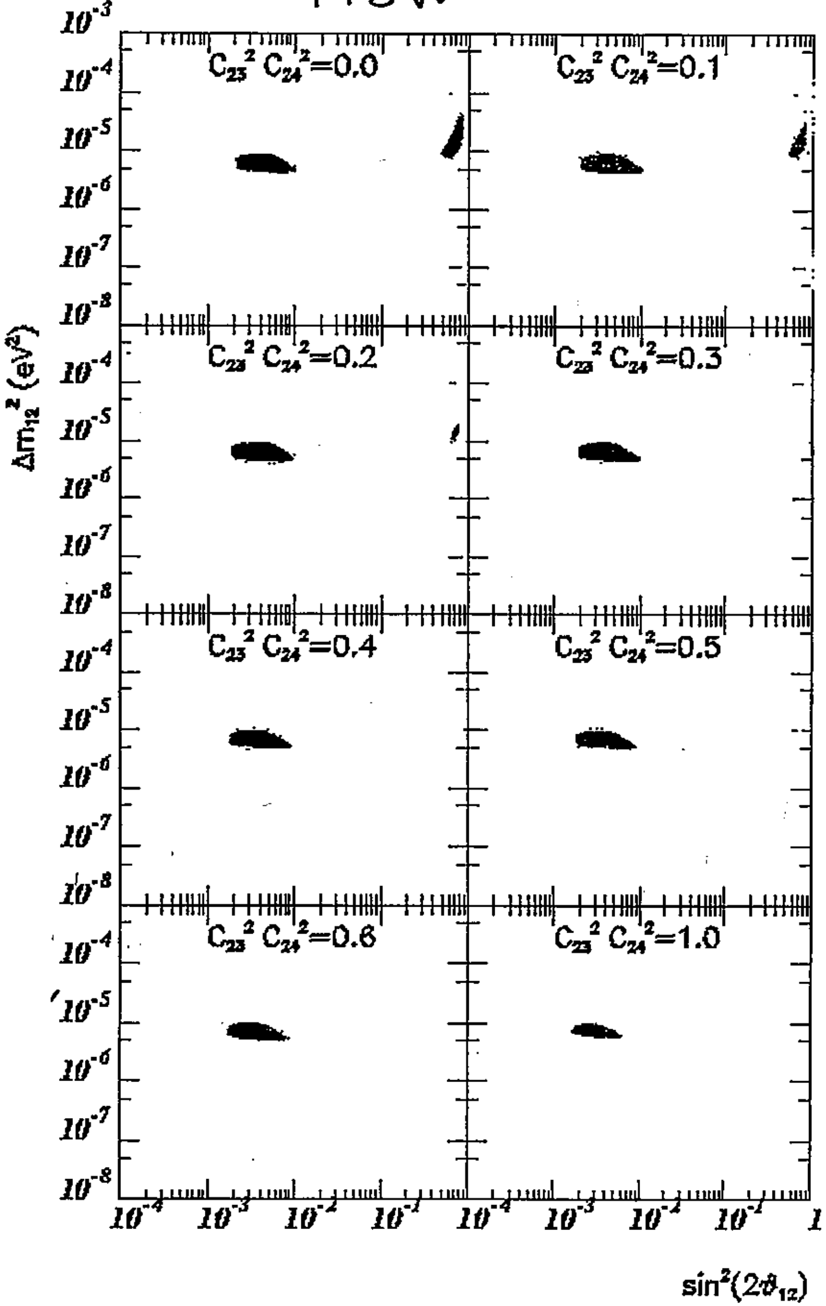
$C_s = 1 \quad \nu_\odot : \nu_e \leftrightarrow \nu_s$ only

$$\nu_\odot \begin{cases} 0 \le C_s \le 0.2 & : \text{SMA, VO, LMA} \\ 0.2 \le C_s \le 0.4 & : \text{SMA, VO} \\ 0.4 \le C_s \le 1 & : \text{SMA} \end{cases}$$

c)

"$N_\nu=4$ analysis of $\nu_\odot$ Giunti-Gonzalez-Garcia-Peña-Gray '00

MSW



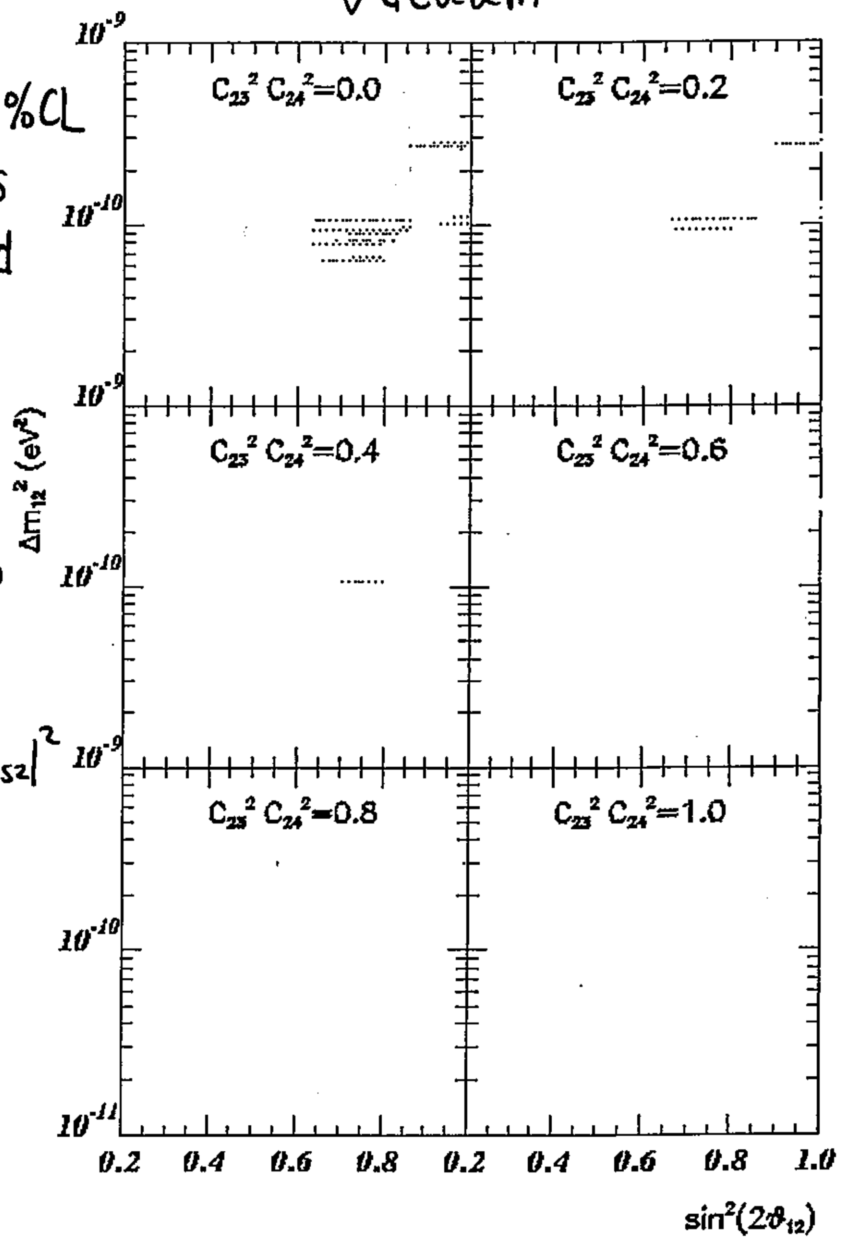

only 90%CL region is displayed

"$C_{23}^2 C_{24}^2$"

$= C_s$

$\equiv |U_{s1}|^2 + |U_{s2}|^2$

vacuum



NB LSND ('95) $\bar{\nu}_\mu \to \bar{\nu}_e$ $\quad 0.2\,\mathrm{eV}^2 \lesssim \Delta m^2_{LSND} \lesssim 2.5\,\mathrm{eV}^2$

CDHSW ('84) $\nu_\mu \to \nu_\mu$ $\quad 0.3\,\mathrm{eV}^2 \lesssim \Delta m^2_{CDHSW} \lesssim 90\,\mathrm{eV}^2$

disappearance (negative)

If $\Delta m^2_{LSND} \gtrsim 0.4\,\mathrm{eV}^2$ and $\left.|1-P(\nu_\mu\to\nu_\mu)|\right|_{\Delta m^2_{32}=\Delta m^2_{LSND}} > 0.3$

then the scheme doesn't work.

$\Rightarrow \Delta m^2_{LSND} = 0.3\,\mathrm{eV}^2$ as reference value here.

results of analysis of $\nu_{atm}$

$$\chi^2 = \chi^2(\text{SK contained}) + \chi^2(\text{SK upward through going } \mu)$$

best fit: $\Delta m^2_{atm} = 1.0\times 10^{-3}\,\mathrm{eV}^2$

$\delta_1 = 0$, $\theta_{24} = 40^\circ$, $\theta_{34} = 15^\circ$, $\theta_{23} = 20^\circ$

$\chi^2_{min} = 43$ (d.o.f. = 45)

NB the point $\theta_{24} = \frac{\pi}{4}$, $\theta_{34} = \frac{\pi}{2}$, $\theta_{23} = \frac{\pi}{6}$ is not pure $\nu_\mu \leftrightarrow \nu_s$:

$$P(\nu_\mu \to \nu_\tau) = \frac{3}{8}\sin^2\left(\frac{\Delta m^2_{LSND} L}{4E}\right) \simeq \frac{3}{16}$$

$$P(\nu_\mu \to \nu_s) = \frac{1}{16}\sin^2\left(\frac{\Delta m^2_{LSND} L}{4E}\right) + \frac{3}{4}\sin^2\left(\frac{\Delta m^2_{atm} L}{4E}\right)$$

$$\simeq \frac{1}{32} + \frac{3}{4}\sin^2\left(\frac{\Delta m^2_{atm} L}{4E}\right)$$

NB $\theta_{24} = 30^\circ$ ($\sin^2 2\theta_{24} = 0.75$) is allowed @ 90% CL only for $\theta_{23} \sim 20^\circ$.

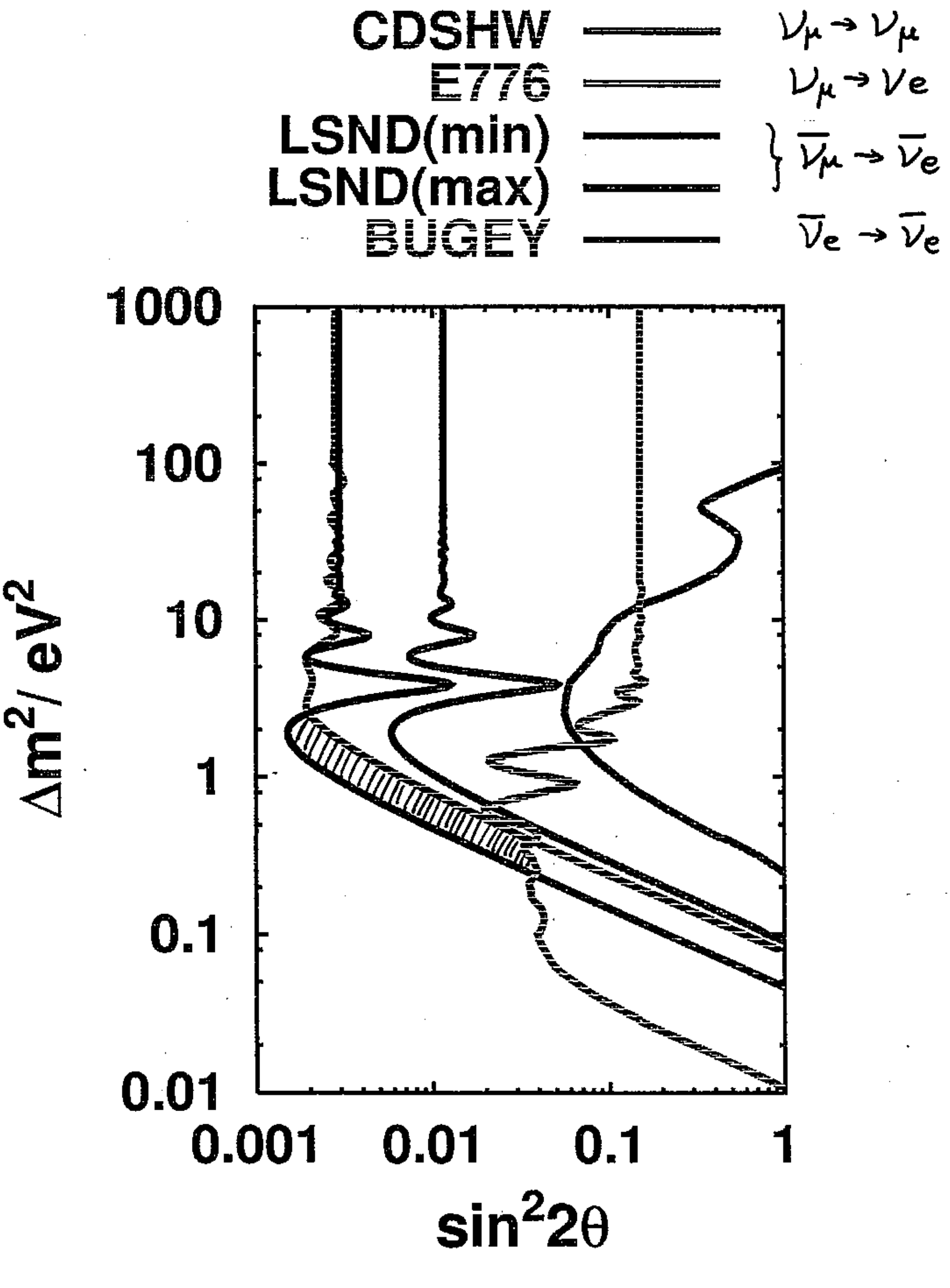
cf. De Rújula - Gavela - Hernandez hep-ph/0001124
"The atmospheric $\nu$ anomaly w/o maximal mixing?"

$$\theta \neq \frac{\pi}{4} \xrightarrow{\text{matter effects}} \theta_M \simeq \frac{\pi}{4}$$

in our case

$$\begin{pmatrix}\nu_\mu\\ \nu_\tau\\ \nu_s\end{pmatrix} = U \begin{pmatrix}\nu_2\\ \nu_3\\ \nu_4\end{pmatrix}$$

no strong constraint on U



CDSHW
E776
LSND(min)
LSND(max)
BUGEY
$\nu_\mu \to \nu_\mu$
$\nu_\mu \to \nu_e$
$\bar{\nu}_\mu \to \bar{\nu}_e$
$\bar{\nu}_e \to \bar{\nu}_e$
1000
100
10
1
0.1
0.01
$\Delta m^2 / eV^2$
0.001
0.01
0.1
1
$\sin^2 2\theta$

- Jannakos @ Moriond 99
KARMEN 2

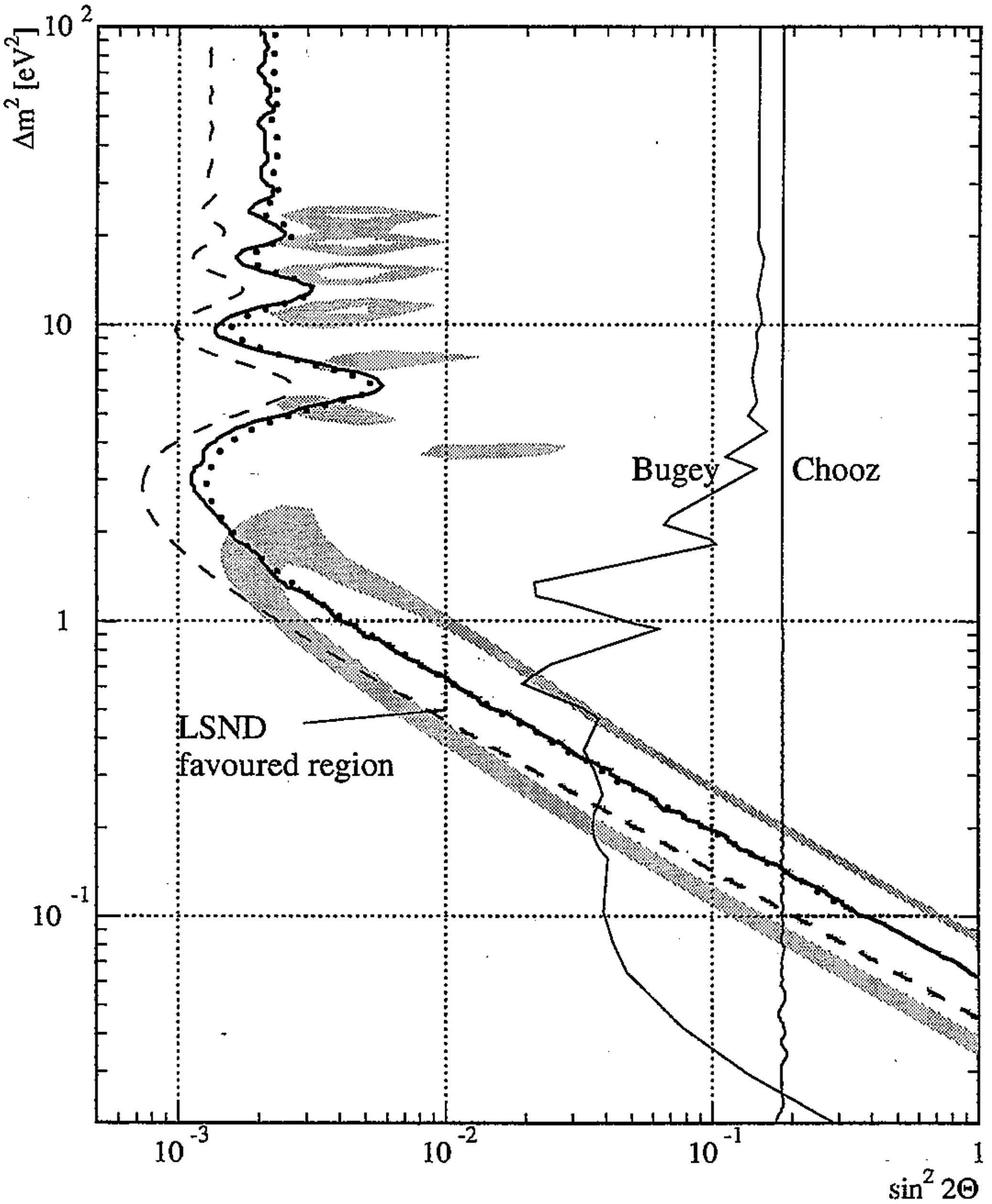
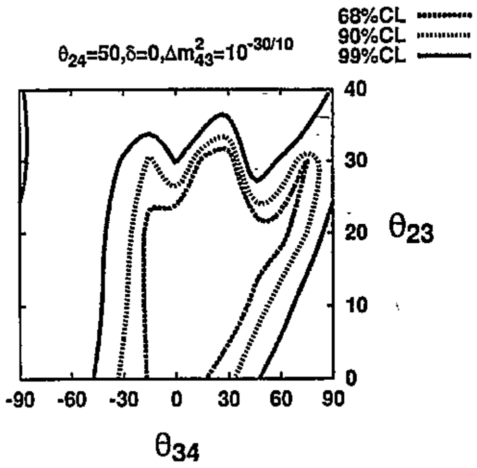
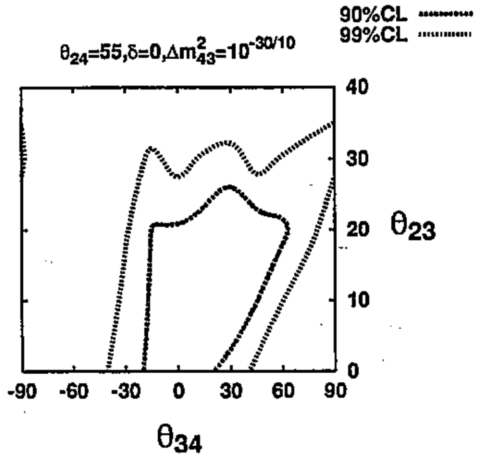
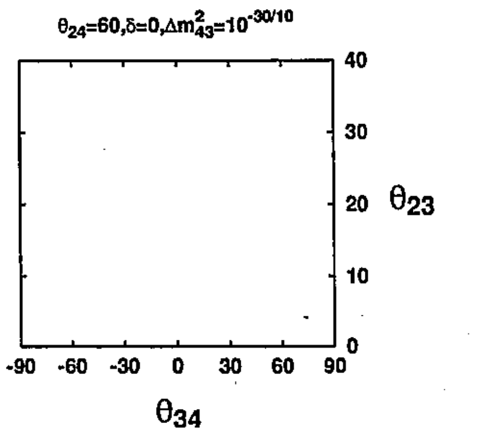
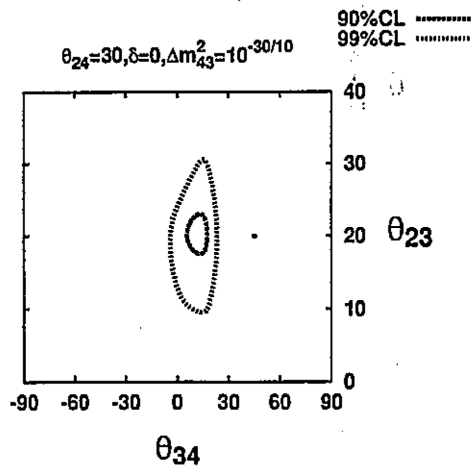


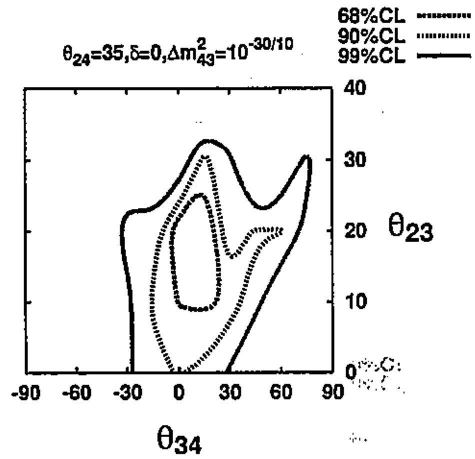

Figure 4: KARMEN 2 90 % confidence limits and sensitivity according to the Unified Approach compared to other experiments: The full line is the 90 % C.L. of the data presented here, the dotted line the corresponding sensitivity and the dashed line the 90 % C.L. derived from the Feb. 97-Apr. 98 data. Also shown are the 90 % C.L. of the two reactor experiments Chooz [12] and Bugey [13] and the favoured region for $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$oscillations as reported by LSND [11].

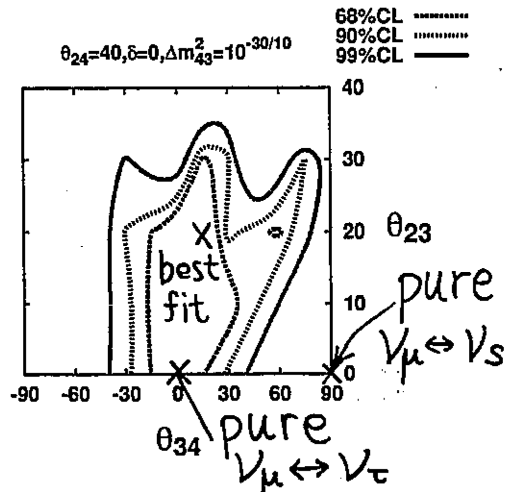

θ24=50,δ=0,Δm²43=10^-30/10
68%CL
90%CL
99%CL
θ23
θ34

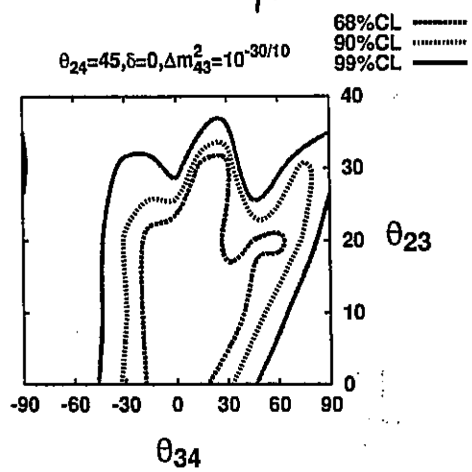

θ24=55,δ=0,Δm²43=10^-30/10
90%CL
99%CL
θ23
θ34


θ24=60,δ=0,Δm²43=10^-30/10
θ23
θ34


θ24=30,δ=0,Δm²43=10^-30/10
90%CL
99%CL
θ23
θ34


θ24=35,δ=0,Δm²43=10^-30/10
68%CL
90%CL
99%CL
θ23
θ34


θ24=40,δ=0,Δm²43=10^-30/10
68%CL
90%CL
99%CL
θ23
θ34
best fit
pure νμ↔νs
pure νμ↔ντ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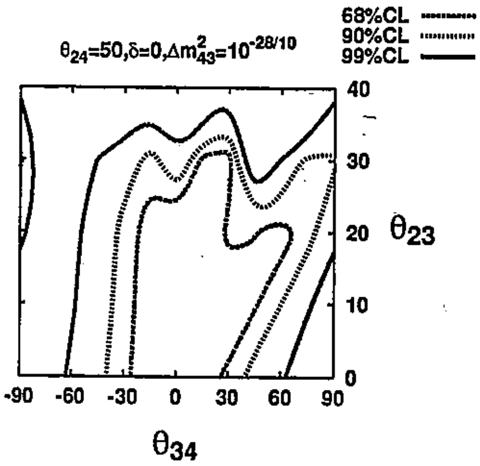
90%CL
99%CL
θ23
θ34


θ24=50,δ=0,Δm²43=10^-28/10
68%CL
90%CL
99%CL
θ23
θ34
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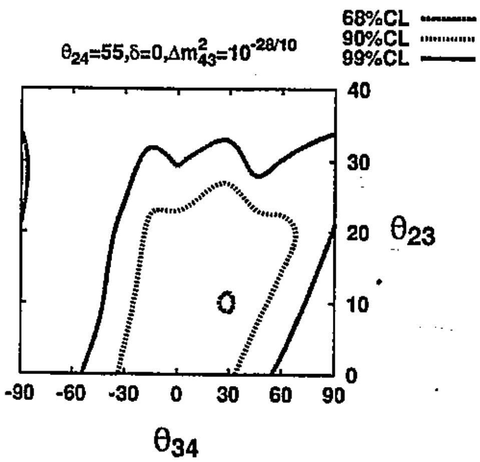
0 10 20 30 40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68%CL
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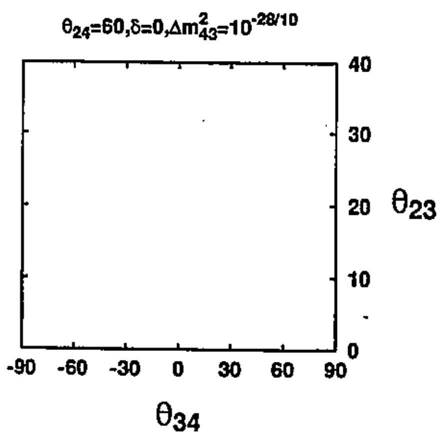
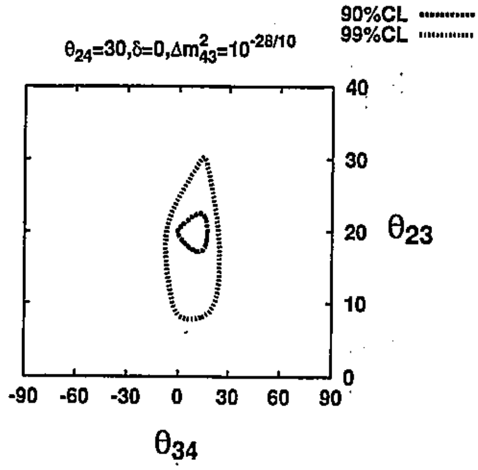
-90 -60 -30 0 30 60 90
0 10 20 30 40


θ24=60,δ=0,Δm²43=10^-28/10
θ23
θ34
-90 -60 -30 0 30 60 90
0 10 20 30 40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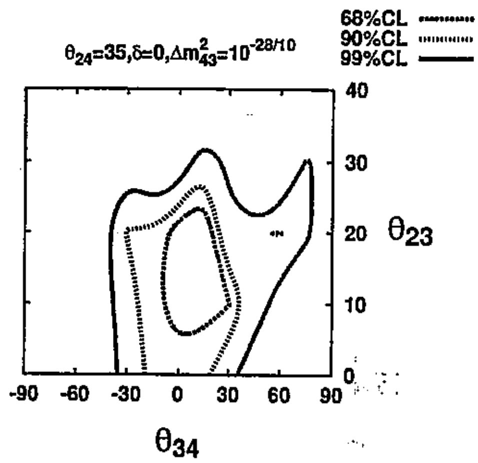
-90 -60 -30 0 30 60 90
0 10 20 30 40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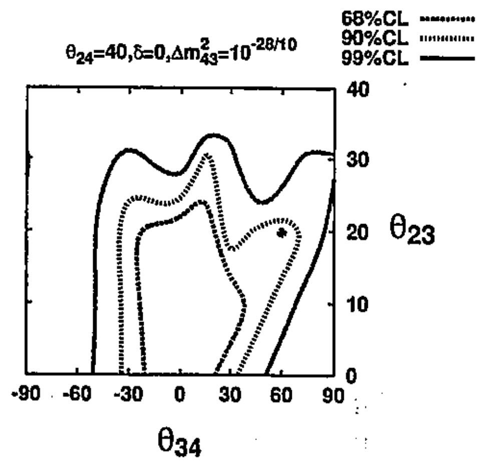
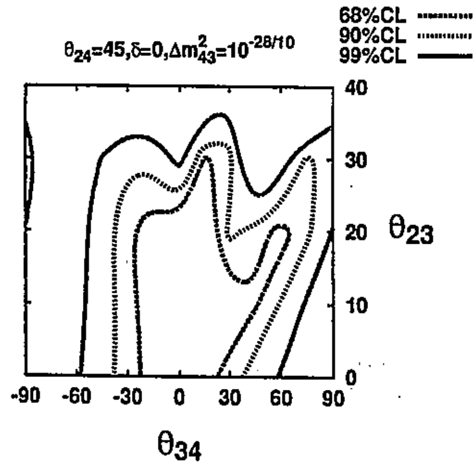
θ34
-90 -60 -30 0 30 60 90
0 10 20 30 40


θ24=40,δ=0,Δm²43=10^-28/10
68%CL
90%CL
99%CL
θ23
θ34
-90 -60 -30 0 30 60 90
0 10 20 30 40


θ24=45,δ=0,Δm²43=10^-28/10
68%CL
90%CL
99%CL
θ23
θ34
-90 -60 -30 0 30 60 90
0 10 20 30 40

# Fate of sterile $\nu$ scenarios

Okada - O.Y. '97

$\boxed{\text{"standard"}}$

$$U_{MNS} \sim \begin{pmatrix} c_\odot & s_\odot & 0 & 0 \\ 0 & 0 & c_{atm} & s_{atm} \\ 0 & 0 & -s_{atm} & c_{atm} \\ -s_\odot & c_\odot & 0 & 0 \end{pmatrix}$$

$\theta_\odot$ : SMA MSW

consistent with
- $\nu_\odot$ $\quad \nu_e \leftrightarrow \nu_s$ SMA
- $\nu_{atm}$ $\quad \nu_\mu \leftrightarrow \nu_\tau$
- $\nu_{LSND}$
- BBN $N_\nu < 4$

This will die if SNO & SK prove that $\nu_\odot$ is $\nu_e \leftrightarrow \nu_{active}$

$\boxed{\text{more general}}$

$$U_{MNS} \sim \begin{pmatrix} c_\odot & s_\odot & 0 & 0 \\ s_\odot c_{atm} s_{23} & -c_\odot c_{atm} s_{23} e^{i\delta_1} & c_{atm} c_{23} & s_{atm} \\ s_\odot w & -c_\odot w & U_{\tau 3} & c_{atm} c_{34} \\ -s_\odot z & c_\odot z & U_{s3} & c_{atm} s_{34} \end{pmatrix}$$

$w \equiv c_{23} s_{34} - c_{34} s_{23} s_{atm} e^{i\delta_1}$, $z \equiv c_{23} c_{34} - s_{34} s_{23} s_{atm} e^{i\delta_1}$

$U_{\tau 3} \equiv -c_{23} c_{34} s_{atm} - s_{23} s_{34} e^{-i\delta_1}$, $U_{s3} \equiv -c_{23} s_{34} s_{atm} - s_{23} c_{34} e^{-i\delta_1}$

consistent with
- $\nu_\odot$ $\quad$ mixtures of $\nu_e \leftrightarrow \nu_{active}$, $\nu_e \leftrightarrow \nu_s$
- $\nu_{atm}$ $\quad$ mixtures of $\nu_\mu \leftrightarrow \nu_\tau$, $\nu_\mu \leftrightarrow \nu_s$
- $\nu_{LSND}$

<u>not consistent with BBN $N_\nu < 4$</u>

If we forget about BBN constraint, this scenario won't die until it is shown that $\nu_\odot$ is almost of $\nu_e \leftrightarrow \nu_{active}$ and high statistics of upward going $\mu$ requires $\theta_{23} \simeq 0$, or that $\nu_{LSND}$ is wrong.

$\boxed{\text{May be they'll survive for a couple more years!}}$