THE CALCULATION OF ATMOSPHERIC NEUTRINO FLUX.

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The processes of the atmospheric neutrino generation is overviewed. Not all the processes are well known, but some of them are remained as the uncertainties of the calculation. From the view point of the calculation, the atmospheric neutrino experiment is reviewed, and efficient determination of neutrino characteristics is discussed. A work to reduce the uncertainty using the secondary cosmic rays is introduced shortly.

1. An overview as the introduction

The discovery of the neutrino oscillations and the neutrino masses using the the atmospheric neutrinos are illustrious achievements in the recent physics[1]. However, the uncertainty of the flux of atmospheric neutrino is a crucial for further study of the neutrino. and several experiments using accelerator neutrinos are being carried out. It is important to use the atmospheric neutrino flux properly with the knowledge of the uncertainties, and to reduce the uncertainty.

Most of the calculation[2] of the atmospheric neutrino flux is the simulation of the atmospheric neutrino generation processes. When high energy cosmic rays come in the atmosphere, the cosmic ray interact with the air nuclei and produce many mesons. The neutrinos are the decay product of mesons and

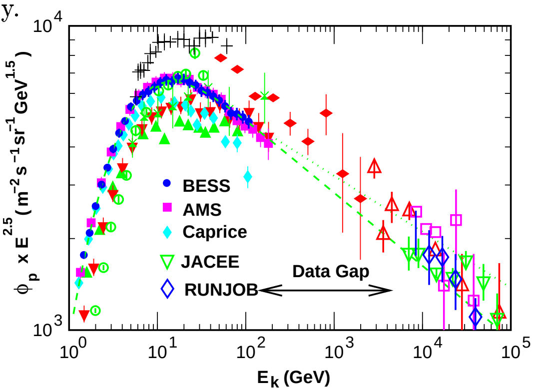

Figure 1. Observed primary proton flux.

the muons, which is also the decay product of the meson. The probability for mesons to decay or interact with air nuclei is determined by the meson energy and mass, decay life time, and the air density where these processes take place. The geomagnetic field sets a lower limit to the momentum of cosmic rays to come in the atmosphere. This is a source of directional variation of the atmospheric neutrino flux at lower energies.

Thus, the primary flux of cosmic rays and the interaction model are the two major components in the calculation atmospheric neutrino flux. However, they are also the main sources of the uncertainties. The primary cosmic spectra below 100 GeV are well determined by the AMS[3] and BESS[4]experiments. But, there are large uncertainties left above 100 GeV (Fig. 1). The situation for the hadronic interaction is similar or worse. The number of useful experiments are limited and concentrate in the lower energies ($\lesssim$ 30 GeV). As the decay of main neutrino source mesons, kaons and pions, is well studied. Also the density structure of atmosphere and the geomagnetic field are monitored frequently and are known well at least as the average of a long period.

The flux ratio of electron and muon neutrinos is almost free from the uncertainties of primary cosmic rays and interaction models, since it is determined by the $\pi - \mu$ decay process. This is the main source of atmospheric neutrinos below 100 GeV. The energy differences between 3 neutrinos in the $\pi - \mu$ decay and the steep spectra of pions give some effect on this ratio. However, even with large variation of interaction model, air density, etc, the calculation give a very close value to the naive value, 1/2. At energies above a few GeV, the muons tend to go into the ground before they decay. Therefore, the flux of electron neutrinos decreases more quickly than the flux of muon neutrino.

The zenith angle variation of neutrino flux is related to the production height of neutrinos. When the mesons are created at higher altitudes, the decay is more favored in the competition process of the decay and interaction than when they are created at lower altitudes, due to the difference of air density.

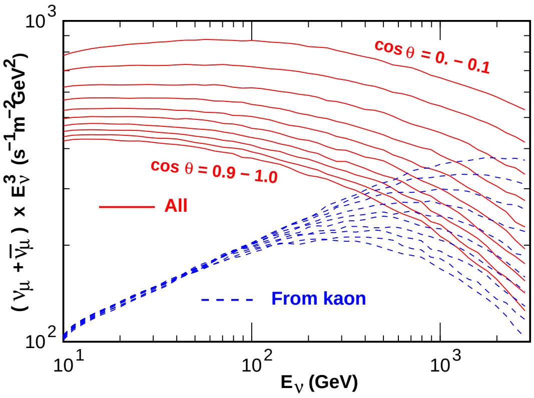

Figure 2. Muon neutrino spectra for different zenith angles.

As the first interaction point of inclined cosmic rays are higher than the vertical ones, the energy of inclined neutrinos is generally larger than that of vertical neutrinos for the same energy and same kind primary cosmic rays. Therefore, the horizontal atmospheric neutrino flux is generally larger than that of vertical ones, with the steep energy spectra of primary cosmic rays.

If only the $\pi - \mu$ decay is the source of the atmospheric neutrinos, there are almost no uncertainties for the zenith angle dependences. However, as the kaon is an important source of atmospheric neutrino at higher energies, the uncertainty of K/π-ratio causes a uncertainty for the zenith angle variation. In Fig. 2, the energy spectra of muon neutrino are depicted for different zenith angles, and the contributions from kaons and pions separately. However, it is seen that the contribution of kaons are still small at around 70 GeV. The flux ratio (vertical/horizontal) changes only 5 % by the change of 20 % change of K/π-ratio. The median energy of neutrinos, which cause the upward-through-going-muon events, is around 70 GeV.

We often ignore the transverse momentum of the hadronic interaction, and geomagnetic filed in air, in the calculation of atmospheric neutrinos (1-dimensional calculation). This is because the full treatment (3-dimensional calculation) consumes a huge computation power, and it is difficult to get a useful results within a reasonable computation time until recently. The differences of atmospheric neutrino flux calculated in 1-dimensional of 3-dimensional calculation are found at low energies and at the near horizontal directions. For the flux ratio, there are almost no differences. The difference disappears except for near horizontal directions at the neutrino energy above 1 GeV. The difference remains to higher neutrino energies $\sim$ 10 GeV due to the curvature of muons in the air. In principle, the difference can be calculated accurately, but we consider it as the uncertainty from the practical reason. Note, even in the 1-dimensional calculation, the geomagnetic filed is taken into account outside the atmosphere. It is used to check if a cosmic ray can come in the atmosphere (rigidity cutoff test). It causes larger directional variation than the zenith angle variation at lower energies ($\lesssim$ 1 GeV).

Although there are some directional variations, there is an important symmetry for the atmospheric neutrino flux, if there are no neutrino oscillations. The neutrino flux arriving from the zenith angle θ_z $(0 \leq \theta \leq 90)$ is the same as the neutrino flux arriving from zenith angle of $180 - \theta_z$ (upward going) above a few GeV. This up-down symmetry is explained by the pure geometry. At the production position, the zenith angles of both neutrinos are the same, then the zenith angle variation works the same for both neutrinos. Also the neutrino flux is almost free from the rigidity cutoff above a few GeV.

2. Atmospheric neutrino experiments and uncertainties

The neutrino events in a detector are categorized by the topology as the fully-contained-events, vertex-contained events, upward-stopping-muon events, and upward-through-going-muon events. For the fully-contained events, the neutrino energy is relatively well determined but the energy is limited to $\lesssim$ 3 GeV. For other type of events, it is difficult to determine the neutrino energy, but the typical energies are estimated as $\sim$ 10 GeV for upward-stopping-muon events and vertex-contained events, and $\sim$ 70 GeV for upward through-going-muon events. We expect an up-down symmetry in the observation for fully and vertex contained events, but only upward going neutrinos are observed by upward-stopping and upward-through-going muon events observation.

Here we assume the 2 component oscillations between μ and τ neutrinos. Starting from a ν_μ, the probability to find the ν_μ at the distance of x is expressed by the formula,

$$\begin{aligned} P(\nu_\mu, x) &= 1 - \sin^2 2\theta \sin^2 \Delta x \\ P(\nu_\tau, x) &= \qquad \sin^2 2\theta \sin^2 \Delta x \end{aligned} \tag{1}$$

Where, θ is the mixing angle of two neutrinos, and

$$\Delta x = 1.27 (\frac{\Delta m^2}{1\mathrm{eV}^2})(\frac{1\mathrm{GeV}}{E_\nu})(\frac{x}{1\mathrm{km}})$$

With the rough estimation for $\Delta m^2 \sim 3 \times 10^{-3}$ eV, the oscillation length $L = 1/1.27 \cdot (1\mathrm{eV}^2/\Delta m^2) \cdot (E_\nu/1\mathrm{GeV})$ km corresponding to the typical energy of neutrino experiment categories are 40 $\sim$ 400 km for fully contained events, 2600km for upward-stopping-muon and vertex-contained events, and 18000 km for upward through going muon events.

From eq. 1 and estimation of the oscillation length, the vertical down going neutrinos in the fully and vertex contained events are almost not affected by the neutrino oscillations. The oscillation length is far shorter than the diameter of the Earth for the fully and vertex contained events, and upward-stopping-muon events. We expect the phase averaged flux in neutrino oscillation, calculated by substituting $\sin^2 \Delta x = 1/2$ in eq. 1. For the upward-through-going muon events, however, the oscillation length is comparable to but longer than the diameter of the Earth.

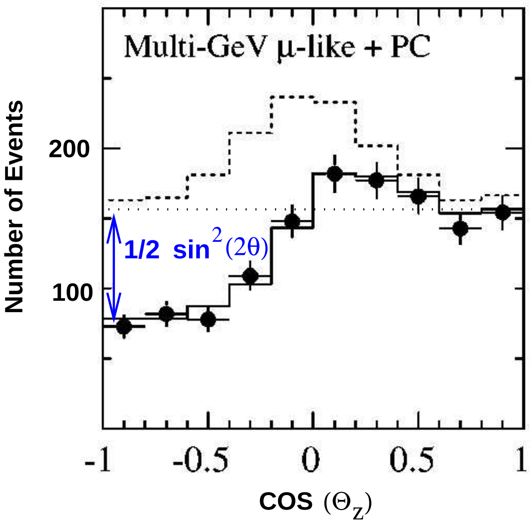

Figure 3. The angle dependence of event numbers.

The mixing angle can be determined from the difference of expectation and observed flux for vertical upward neutrinos in fully and vertex contained events, selecting a events with energies above a few GeV. The event number variation over zenith angle is shown in Fig. 3 from SK data. The direction difference between the neutrinos and induced muons, and the effect of rigidity cutoff is small for these events. The difference between expected and experimental fluxes for upward going neutrino, then the difference between upward and downward going vertical neutrino flux are directly related to the mixing angle. Here, the uncertainties of the atmospheric neutrino does not affect much to the analysis of the mixing angle, because of the up-down symmetry of the expected atmospheric neutrino flux.

The Δm^2 is determined by the analysis of oscillation length. Note, the distance between the neutrino production position and the detector is estimated by the formula:

$$d = \sqrt{(h^2 + 2R_e h) + (R_e \cos\theta_z)^2} - R_e \cos\theta_z \quad , \tag{2}$$

where θ_z is the arrival zenith angle of neutrino, $R_e \sim 6400$ km is the radius of the Earth, and h is the production height of neutrinos ($10 \sim 15$ km).

We find the distance varies non-linearly and very quickly for $\cos\theta_z = 0.1 \sim -0.1$ corresponding to the distances from 100 to 1500km. Using the experiment category whose oscillation length is in this range, it is difficult to determine the oscillation length accurately. The neutrino direction is not determined well for neutrino energies below 1 GeV due to the large scattering angle in the detector. Also for the near horizontal directions and energies $\gtrsim$ 1 GeV, the difference between 3-dimensional and 1-dimensional calculations is seen. It would be better not to use the flux at near horizontal directions, until we can calculate the atmospheric neutrino flux accurately in the 3-dimensional framework.

Therefore, the measurement of Δm^2 is the main task of experiment categorized as vertex-contained events, upward-stopping and upward-through-going muon events. Among them, the event rate is largest for upward-through going muon events. Therefore, we discuss this category observation here.

The certainty of the zenith angle variation of the atmospheric neutrino flux is very important in this analysis. Remember, the oscillation length is longer than the diameter of the Earth, for the upward-through-going muon events, and the variation from the no oscillation flux is small. Also there is a large uncertainty of absolute normalization expected for the atmospheric neutrino flux of this energy region. Note, the up-down symmetry of the atmospheric neutrino flux is not useful to determine the absolute normalization. This is very crucial situation to determine Δm^2, since we have to assume a large uncertainty for the absolute normalization. As an example, we show a χ^2-test in Fig 4. Assuming the normalization is free, the χ^2 values are almost the same for the 3 curves. This is the main reason why the atmospheric neutrino experiment can not pin down the Δm^2 accurately.

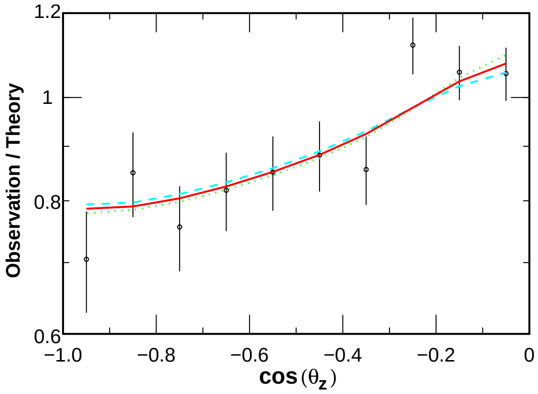

Figure 4. Up going muon data and fit curve with free normalization. They give almost the same χ^2-values.

3. Reduction of the uncertainties

For the the uncertainty of the primary flux above 100 GeV, we have nothing to do but wait for new results from the observation[5]. However, for the uncertainty of the interaction model, a calibration with the secondary cosmic rays may be useful. Here introduced is the calibration study using the muon flux observed at the balloon altitude. The 2001 BESS flight kept relatively lower altitude than the normal flight, and collected a large number of muon events. The data are very useful for this purpose.

In Fig.5, we show the comparison of calculation and observation for two interaction models, which are used in the calculation of atmospheric neutrino flux. It is seen that the dpmjet3 interaction model is favored by the experiment. It would be possible to select the interaction model applying the comparisons to all the available interaction models. More details will be published elsewhere[6].

4. Summary

We have given an overview for the calculation the atmospheric neutrino flux and summarize the uncertainties. The experiments for atmospheric neutrino experiment is reviewed from the view point of the calculation and uncertainties. The mixing angle may be determined accurately by selecting a higher energy events from the fully and vertex contained events. However, the uncertainty of the absolute normalization of the atmospheric neutrino flux is crucial to determine Δm^2. The reduction of the uncertainties in the absolute normalization is desired.

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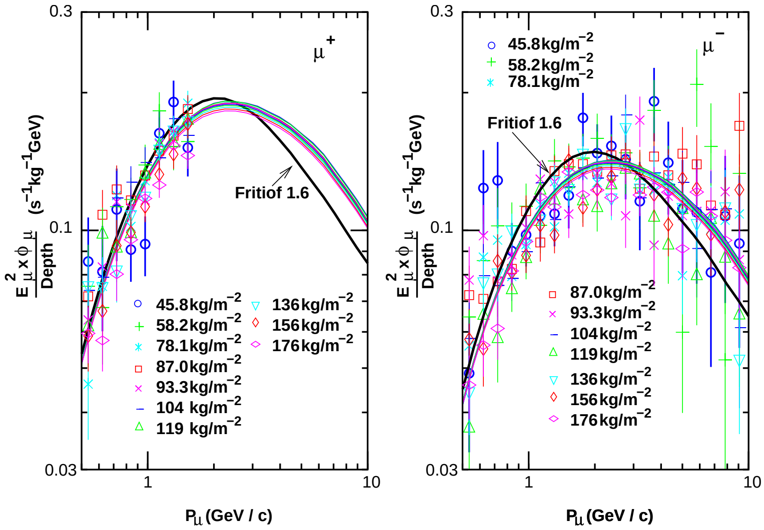

Figure 5. Muon flux normalized with depth at balloon altitude. Lines are calculated muon fluxes with dpmjet3, and Fritiof 1.6.

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