Sensitivity with the 2KM some statistical issues

Maximilien Fechner

Goals

Answer several questions asked during the collaboration meeting

1. Where should the cuts on the chi^2 estimator be placed ?
2. How to deal with statistical fluctuations ?
3. How to deal with systematics ?

J. Dunmore has also been studying these issues (previous 2KM talks + T2K coll meeting talk)

In my presentation of march 9, 2006 (two meetings ago) I addressed issues #1 & #2 **in the absence of systematics**

Today I will briefly show :
- Consistency checks that the minimization works as expected
- What I plan to do with systematics : coverage checks.

Choice of estimator

See Naho's talk

Use a Poisson likelihood ratio estimator, including :

- SK 1 ring e-like sample (after all appearance cuts), recontructed Eν, 10 bins
- SK 2 ring e-like sample, invariant mass, 28 bins
- 2KM 1 ring e-like sample (after all appearance cuts), reconstructed Eν, 20 bins
- 2KM 2 ring e-like sample, invariant mass, 28 bins

$$\chi^2 = \chi^2_{1R,SK} + \chi^2_{1R,2km} + \chi^2_{2R,SK} + \chi^2_{2R,2km} + \sum_{k=1}^{N_s} \varepsilon_k^2/\sigma_k^2$$

$$= \sum_{i=1}^{N} 2 \left(E_i^{MC}(1 + \sum_{k=1}^{N_s} F_i^k \varepsilon_k) - O_i + O_i \log \left(\frac{O_i}{E_i^{MC}(1 + \sum_{k=1}^{N_s} F_i^k \varepsilon_k)} \right) \right) + \sum_{k=1}^{N_s} \left(\frac{\varepsilon_k}{\sigma_k} \right)^2$$

E : expected by MC
O : observed
F_{ik} : effect of kth nuisance parameter on bin i
σ_k : width of kth nuisance parameter

Equation must be solved iteratively
(Poisson stats -> non linear)

19 systematic parameters so far

Comments on systematics

Also possible to use a minimizer :

For each systematic term, reweight the event by (1+sigma*epsilon)

-> non linear in the free parameter epsilon

-> empirical "proof" that this method and the linearized one are equivalent for 2 systematic errors (N. Tanimoto's T2K Coll. Meeting talk)

Systematics implemented in the linearized method :

nue contamination 30%

9 cross section errors : model differences + absolute normalisation in main channels + NC/CC 30%

FV : 2.8% for each detector, uncorrelated

E scale : 2.1% for each detector, uncorrelated

PID for 1 ring & 2 ring events

Ring counting

differences between SK & 2KM for PID and ring counting

-> All relevant ATMPD errors have been implemented by N. Tanimoto (see previous talks)

+ can treat Δm^2 as a nuisance parameters with 20% error for (δ,θ_{13}) plots

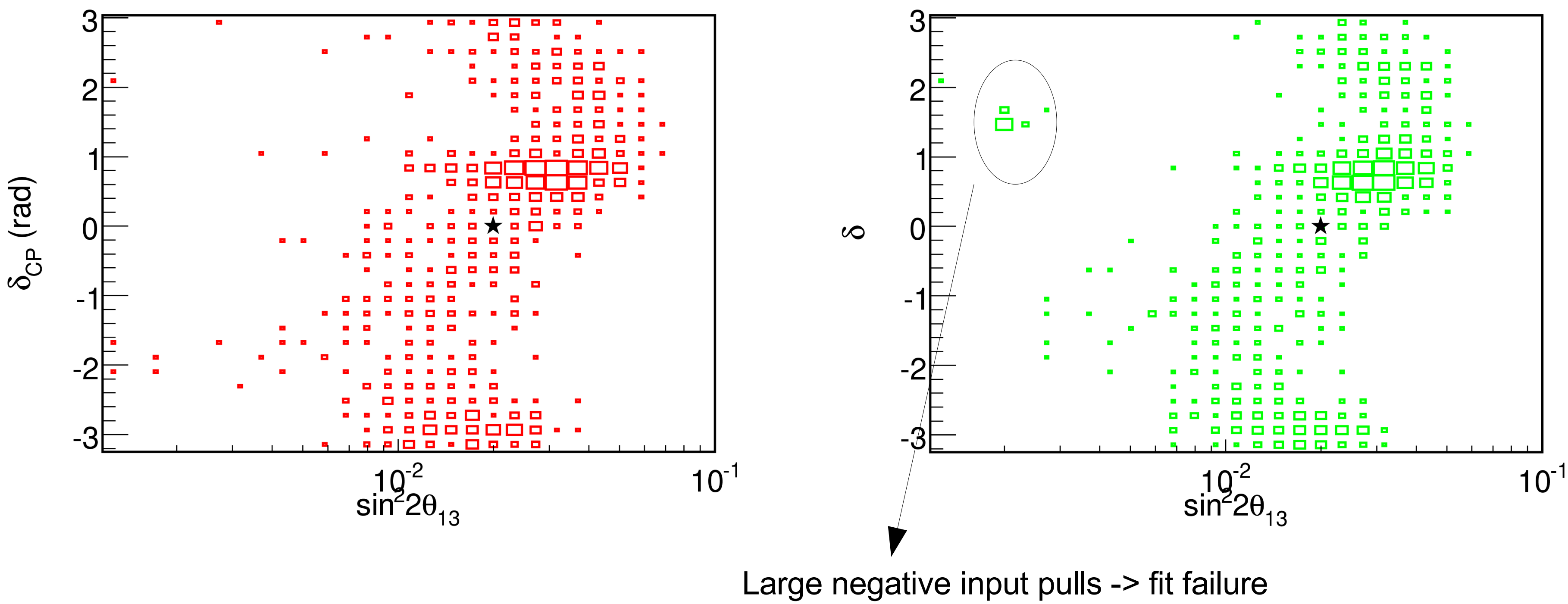
Performance of the fitter

Simple case with 2 nuisance parameters : beam ν_e contamination (σ=30%) and NC/CC (σ=10%)

Make fake data at Δm_{32}^2=2.5e-3 eV^2, $\theta_{23}=\pi/4$, Δm_{21}^2 = 8e-5 eV^2, θ_{12}=0.592, δcp=0, $\sin^2 2\theta_{13}$=2e-2

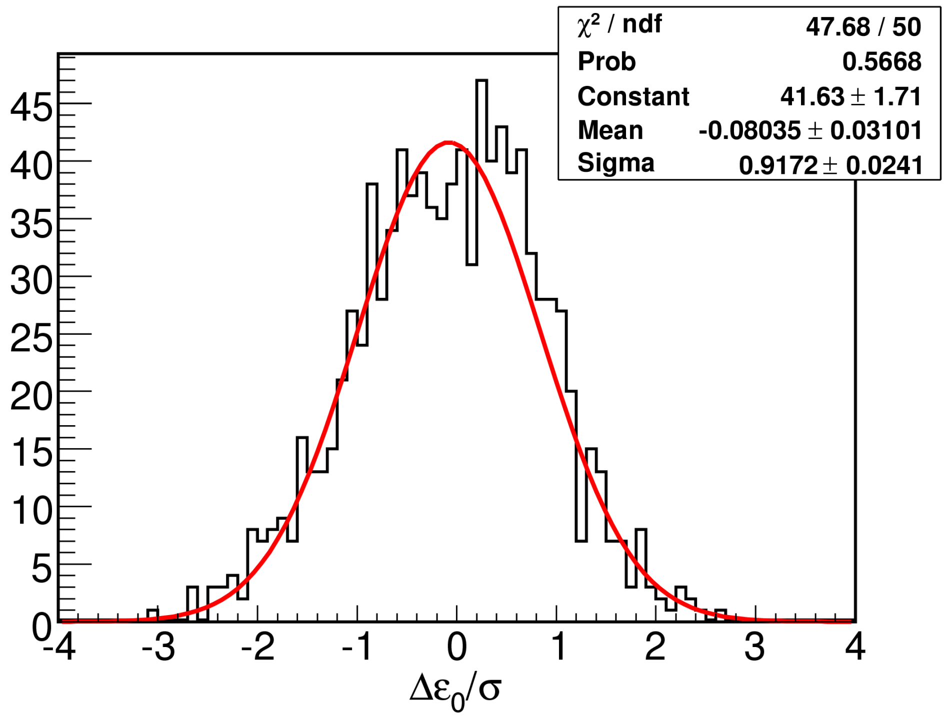
Poisson fluctuations in each bin

Gaussian fluctuations of nuisance parameters with their respective variances

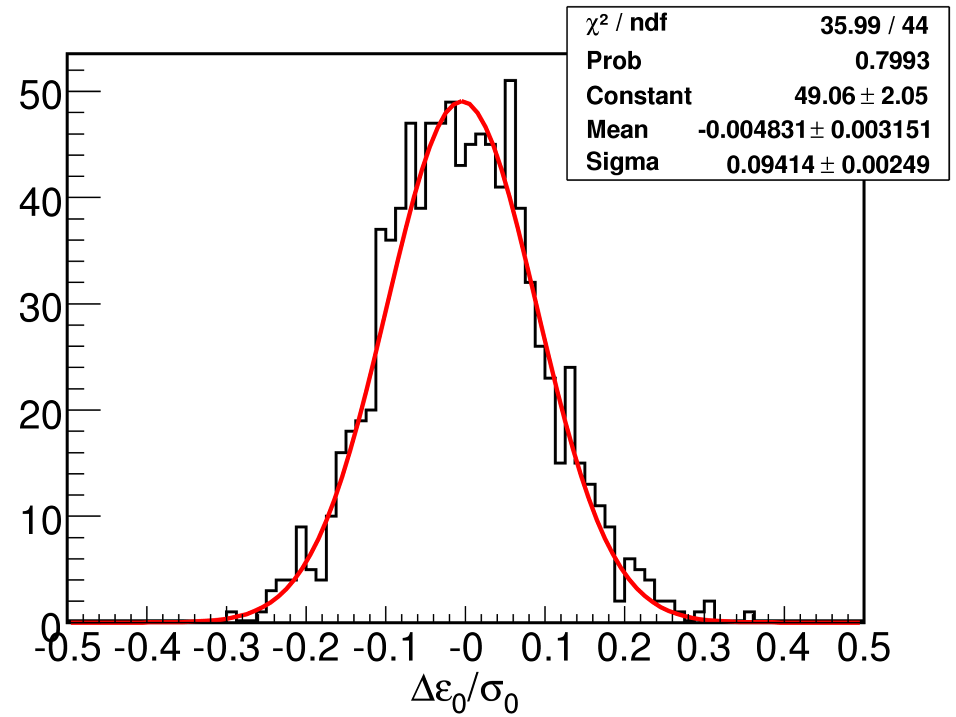


Performance of the fitter

(Fitted epsilon – Input epsilon)/sigma



SK alone

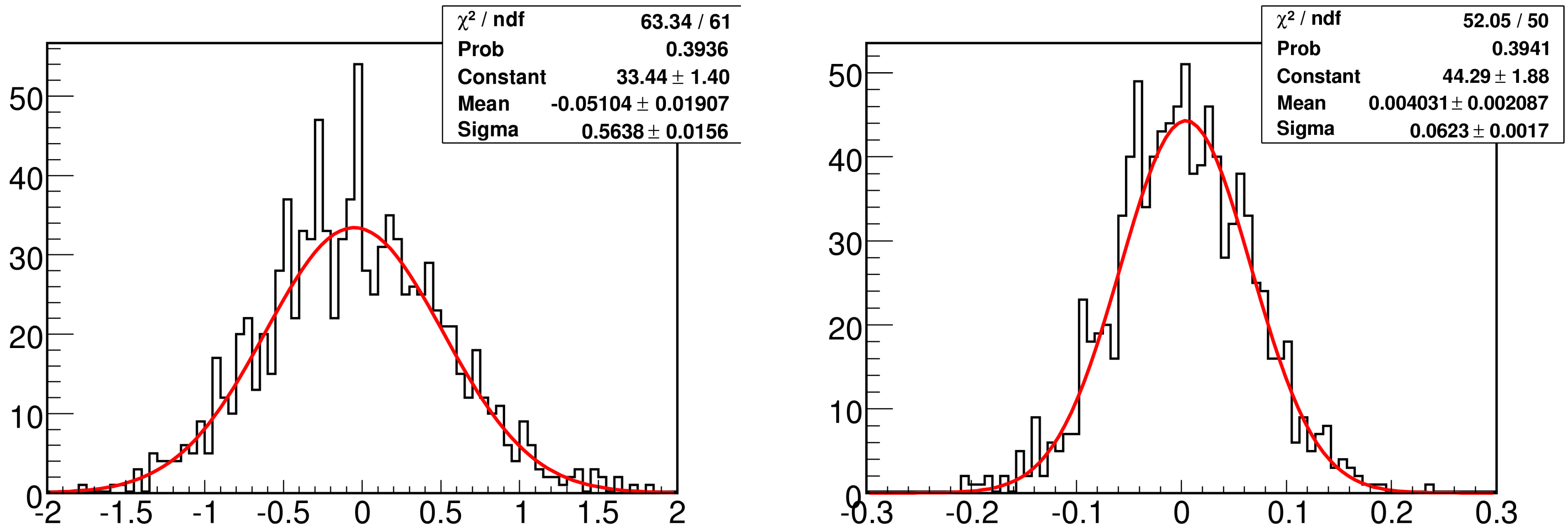


SK+2KM

beam nue contamination : SK alone cannot fit this error contribution -> output sigma roughly equal to input sigma
2KM improves our knowledge of this parameter (factor of ~10 on the width).

Performance of the fitter

(Fitted epsilon – Input epsilon)/sigma



SK alone

SK+2KM

NC/CC error already constrained by SK 2 ring sample (sigma ~ 0.56)
Very high statistics of 2KM 2 ring sample improves this again by a factor of ~9

These are simple consistency checks, using a small number of “easy” systematic terms

Get the critical values

Use a 30x30 "logarithmic" grid in $(\delta, \sin^2 2\theta_{13})$ plane

- Pick a point A on the map
- Make fake data from MC(A)
- Compute "true χ^2" = $\chi^2(A)$ and min(χ^2) (which will be at another point)
- Get $\Delta\chi^2(A) = \chi^2(A)$ – min(χ^2) distribution --> will depend on A (non linearities etc.)
- Determine α CL cut position on $\Delta\chi^2(A)$ distribution --> critical value $C_\alpha(A)$
- Use this cut on χ^2(data,A)-minχ^2(data), to decide if data accepts point A or not
- Repeat for all points on the map

This ensures that the interval indeed has the quoted coverage

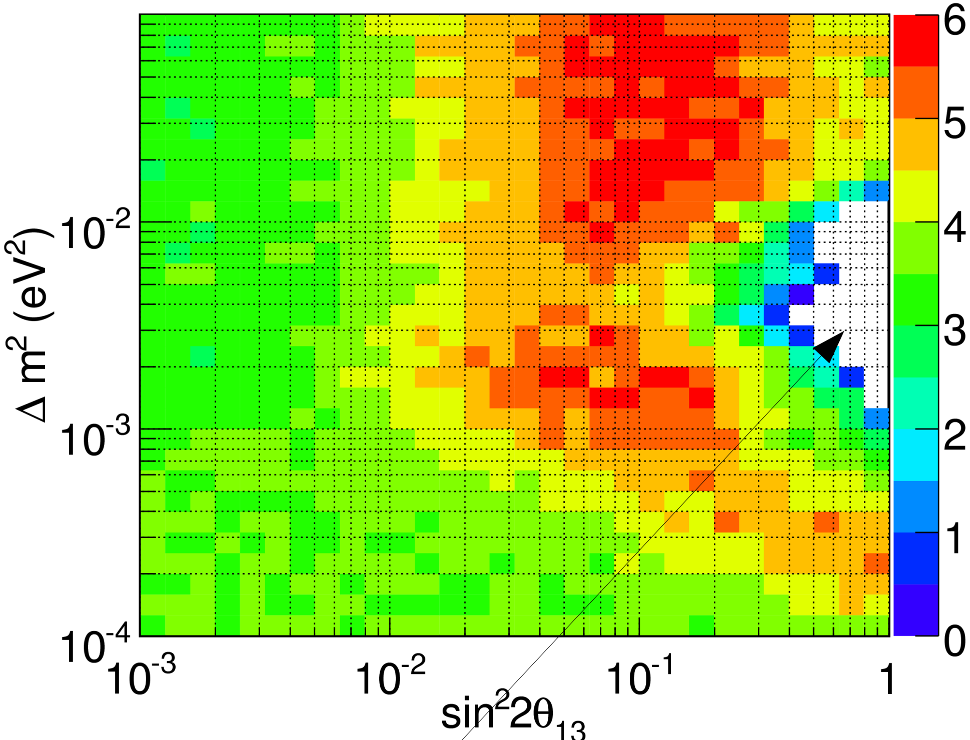
Things to remember :

- The grid is a subset of the physical region $\Rightarrow$ <u>the minimum cannot escape the physical region</u> $\Rightarrow$ <u>I applied the Feldman-Cousins prescription</u>

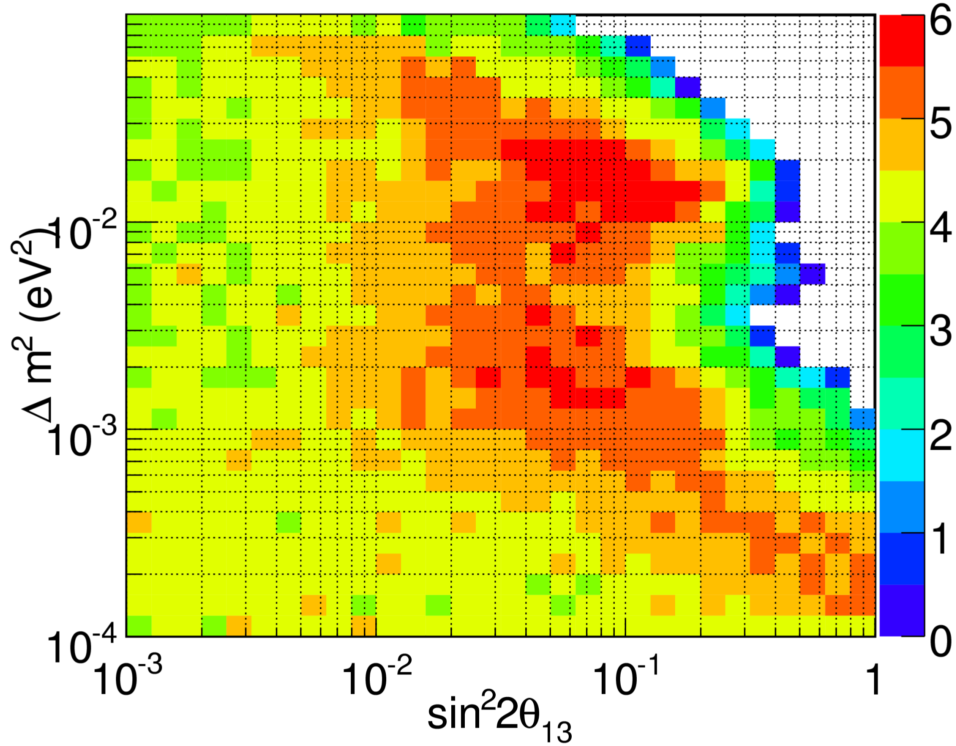
i.e. the ordering principle I used is χ^2(data|A) – χ^2(data|best fit **in plane**)

Map of critical values

Map of 90% CL Feldman-Cousins critical values, SK alone



Map of 90% CL Feldman-Cousins critical values, SK+2KM

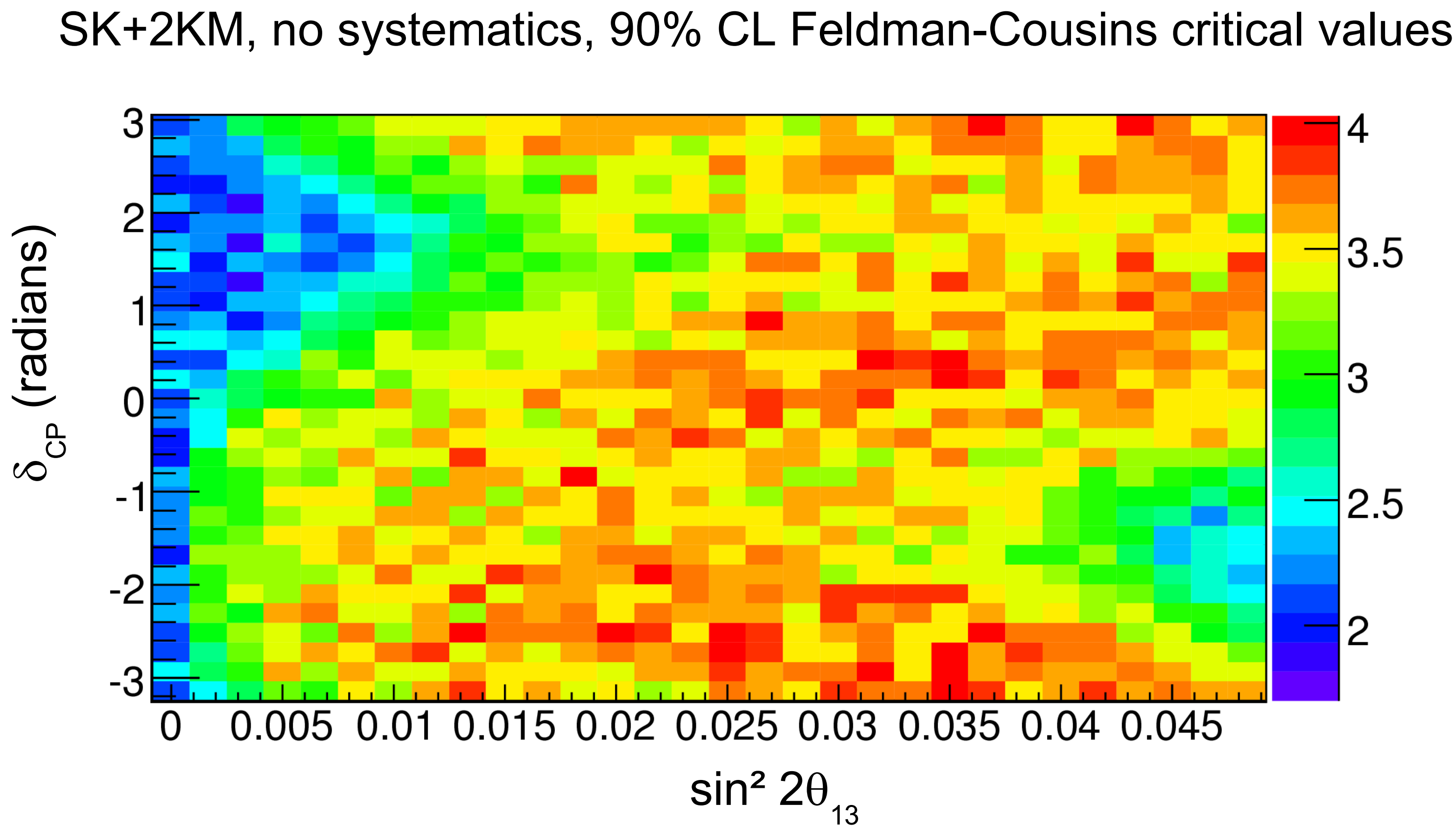


artifact of the grid : min chi2 found exactly at "true" point so $\Delta\chi^2$=0

Reduction of the χ^2 near the edge is the "F-C effect"

No external information is used on Δm^2, only nue appearance is used : I estimate BOTH parameters → ~ 2 dof

Map of critical values



Preliminary (only 1000 expts / point)

Cutting at 4.6 (usual linear 2-dof χ^2 prescription) seems to be too conservative
Edge effect ? Non linearity of the χ^2 ?

With systematics

Procedure : χ^2 now is a function of X(oscillation) and ε (nuisance parameters)

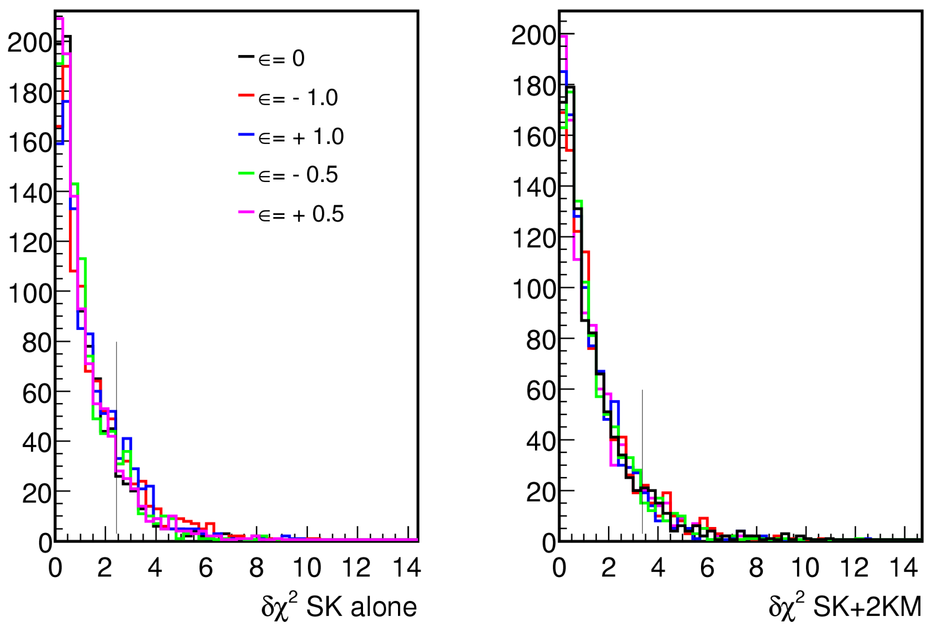
- Pick a point A on the map
- Make fake data from MC(A), setting all the nuisance parameter to 0
- Compute min χ^2(A,best fit ε) and min(χ^2) = χ^2(best fit X, best fit ε')
- Get $\Delta\chi^2$(A) =min χ^2(A,best fit ε) – min χ^2(best fit X, best fit ε') distribution

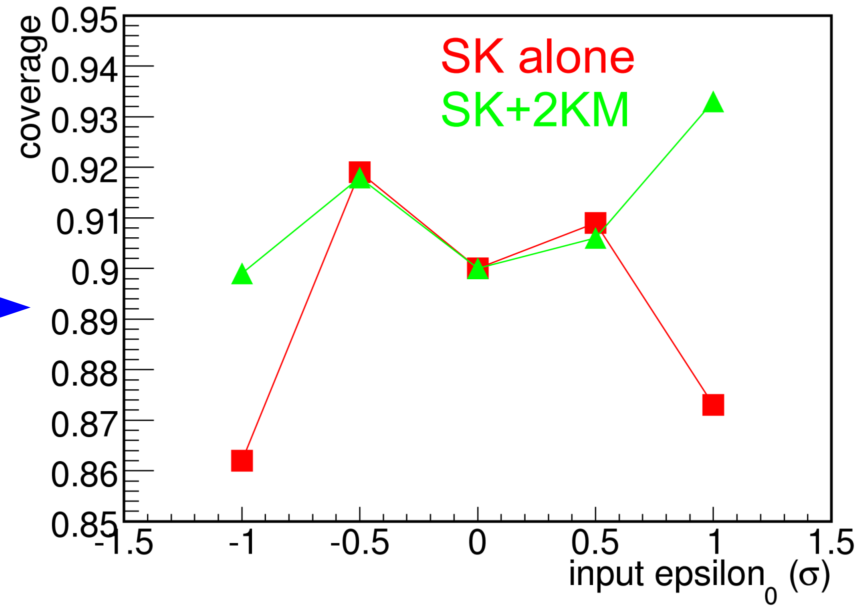
--> will depend on A (non linearities etc.)
- Determine α CL cut position on $\Delta\chi^2$(A) distribution --> critical value C_α(A)
- Use this cut on χ^2(data,A,best fit ε)- min(X,ε) χ^2(data),
 to decide if data accepts point A or not
- Repeat for all points on the map

Basically same procedure as before, but with a minimization of the nuisance parameters at each step.
This is an approximation, considered to be very good (Kendall& Stuart ?) and certainly much faster than making a full Neyman construction over many (nuisance) parameters.
<u>Question</u> : is it correct to fix the nuisance parameters to their “central value” 0 ? Does it change the coverage when they are set to some other value ? Should they also be randomized ?
 $\rightarrow$ Check this in a simple scheme with 2 systematics

Coverage checks

- Use 2 systematic errors : nue contamination (30%) and NC/CC (10%)
- Pick one point on the map (δ=0,sin²2θ_{13}=2e-2)
- Fix the 2[nd] to 0, let the first one vary from -1sigma to +1sigma
- Measure the actual coverage given by the 90% CL critical value obtained for epsilon=0





Very similar distributions : changing this input epsilon has little effect on the coverage

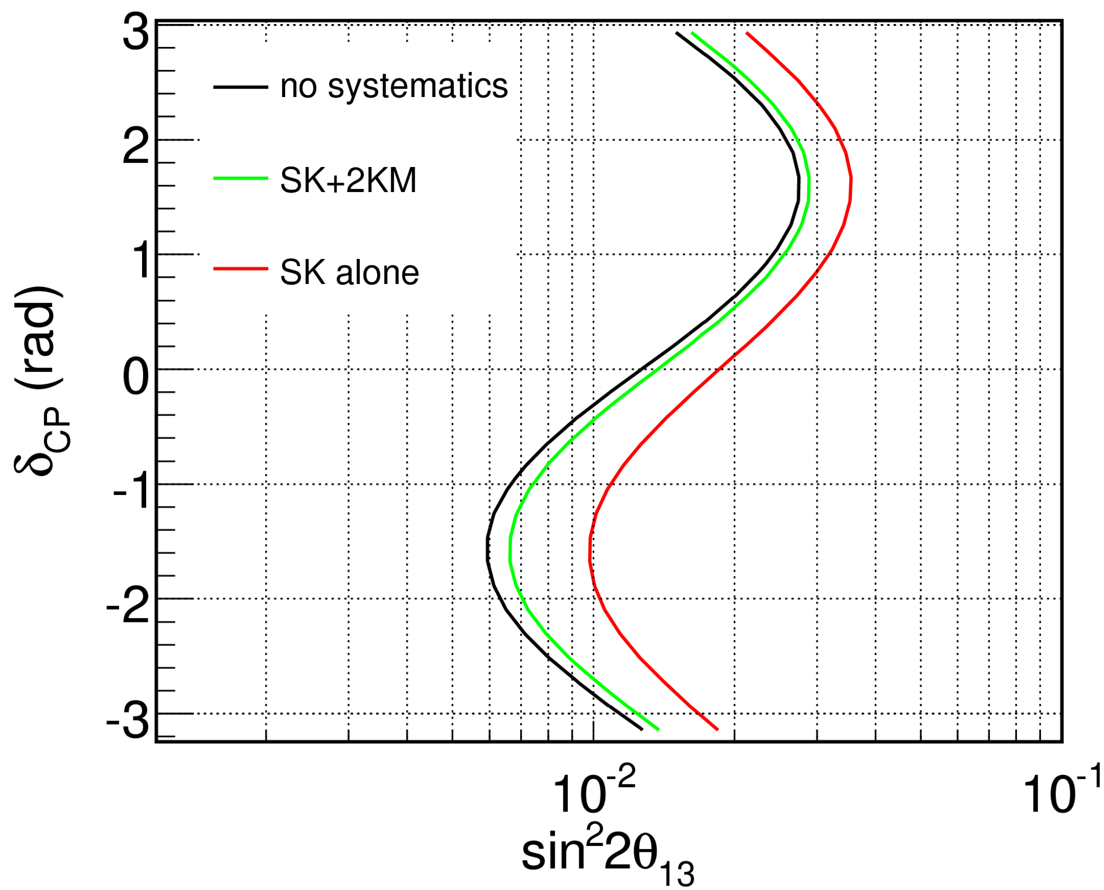
Variations <4%
(need to estimate stat uncertainty on coverage)

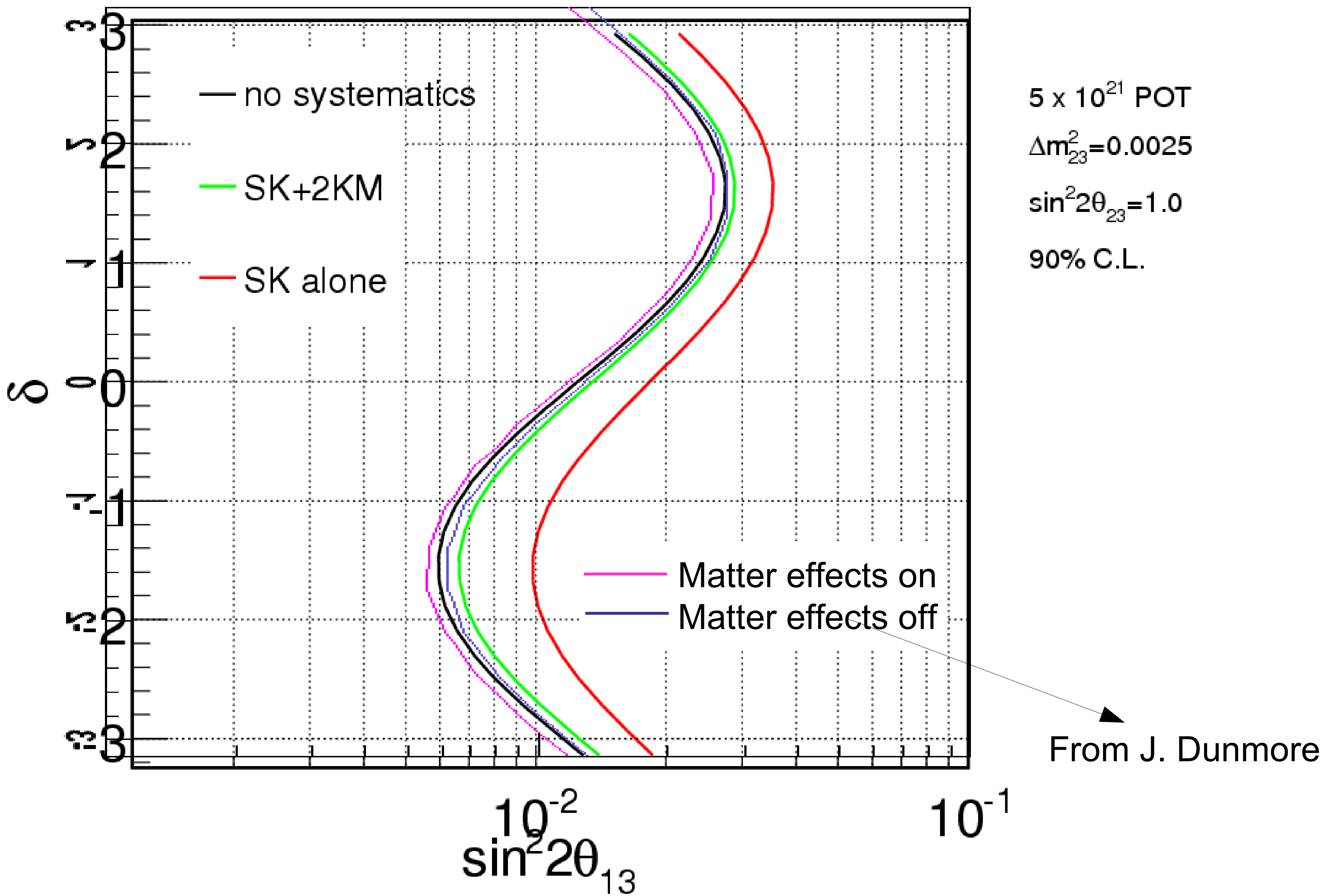
Current sensitivity plot

Make fake data at Δm_{32}^2=2.5e-3 eV^2, $\theta_{23}=\pi/4$, Δm_{21}^2 = 8e-5 eV^2, θ_{12}=0.592, δcp=0, $\sin^2 2\theta_{13}$=0 (sensitivity plot), and epsilons = 0

No statistical fluctuations in the fake data

Use a 90% CL cut at 4.61 ie assuming linear χ^2 which is not correct





With matter effects turned off, and no systematics, there is good agreement between the 2 independant analysis codes !

Many thanks to Jessica Dunmore for this test !

Conclusion

- Obtained fitted nuisance parameter distributions for a simple situation with 2 systematics and confirmed that the code works correctly
- Obtained critical value map in (δ,$\sin^2 2\theta_{13}$) plane WITHOUT systematics
- Described a standard method to get critical values with systematics, avoid doing a full Neyman construction
main issue : what do we do with the nuisance parameters (fix or randomize) ?
- Preliminary tests that this method with fixed input parameters will provide acceptable coverage
- Comparisons by J. Dunmore show that both independant methods agree WITHOUT systematics ; checking systematics etc.

- TODO : profile the code and run it on many CPUs to get the critical value maps WITH systematics