Sensitivity with the 2KM some statistical issues

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Goals

Answer several questions asked during the collaboration meeting

1. Where should the cuts on the χ^2 estimator be placed ?
2. How to deal with statistical fluctuations ?
3. How to deal with systematics ?

J. Dunmore has also been studying these issues (previous 2KM talks + T2K coll meeting talk)

Item #1 is related to the long standing issue "1-sided χ^2 vs 2-sided χ^2" aka "1.64 / 2.71" cuts on the LOI estimator, which was discussed again at KEK last january.
One argument is that we are not sensitive to Δm^2 when ν_μ disappearance is not studied, so that there is only 1 dof.

In this talk I will cover steps 1 & 2, when there are no systematics

Choice of estimator

See Naho's talk

Use a Poisson likelihood ratio estimator, including :

- SK 1 ring e-like sample (after all appearance cuts), Eν, 10 bins
- SK 2 ring e-like sample, invariant mass, 28 bins
- 2KM 1 ring e-like sample (after all appearance cuts), Eν, 20 bins
- 2KM 2 ring e-like sample, invariant mass, 28 bins

$$\chi^2 = \sum_{n=1}^{370} \left[2\left\{ N_{exp}^n \left(1 + \sum_{i=1}^{45} f_i^n \cdot \epsilon_i\right) - N_{obs}^n \right\} + 2N_{obs}^n \ln\left(\frac{N_{obs}^n}{N_{exp}^n \left(1 + \sum_{i=1}^{45} f_i^n \cdot \epsilon_i\right)}\right)\right] + \sum_{i=1}^{43} \left(\frac{\epsilon_i}{\sigma_i}\right)^2$$

N_{obs}^n	Number of observed events in n-th bin
N_{exp}^n	Number of expected events in n-th bin
ϵ_i	i-th systematic error term
f_i^n	Systematic error coefficient
σ_i	1 sigma value of systematic error

Equation must be solved iteratively (Poisson stats -> non linear)

19 systematic parameters so far

FIRST I WILL TURN OFF ALL SYSTEMATICS

Comments on systematics

Also possible to use a minimizer :

For each systematic term, reweight the event by (1+sigma*epsilon)

-> non linear in the free parameter epsilon

-> empirical “proof” that this method and the linearized one are equivalent for 2 systematic errors (N. Tanimoto's T2K Coll. Meeting talk)

Systematics implemented in the linearized method :

nue contamination 30%

9 cross section errors : model differences + absolute normalisation in main channels + NC/CC 30%

FV : 2.8% for each detector, uncorrelated

E scale : 2.1% for each detector, uncorrelated

PID for 1 ring & 2 ring events

Ring counting

differences between SK & 2KM for PID and ring counting

-> All relevant ATMPD errors haven't been implemented yet, N. Tanimoto will report at the next meeting

+ can treat Δm^2 as a nuisance parameters with 20% error for (δ,θ_{13}) plots

Get the critical values

Use a 30x30 "logarithmic" grid in (Δm^2,$\sin^2 2\theta_{13}$) plane

- Pick a point *A* on the map
- Make fake data from MC(*A*)
- Compute "true chi2" = chi2(*A*) and min(chi2) (which will be at another point)
- Get $\Delta\chi^2(A)$ = chi2(*A*) – min(chi2) distribution --> will depend on *A* (non linearities etc.)
- Determine α CL cut position on $\Delta\chi^2(A)$ distribution --> critical value $C_\alpha(A)$
- Use this cut on χ^2(data,*A*)-minχ^2(data), to decide if data accepts point *A* or not
- Repeat for all points on the map

Things to remember :

- The grid is a subset of the physical region $\Rightarrow$ <u>the minimum cannot escape the physical region $\Rightarrow$ I obtained Feldman&Cousins critical values</u>
- <u>How to deal with systematics ?</u> Not done yet but strategy is:

- make fake data by applying stat fluctuations BUT keep nuisance paramters fixed to 0

- minimize wrt to nuisance parameters before computing $\Delta\chi^2$

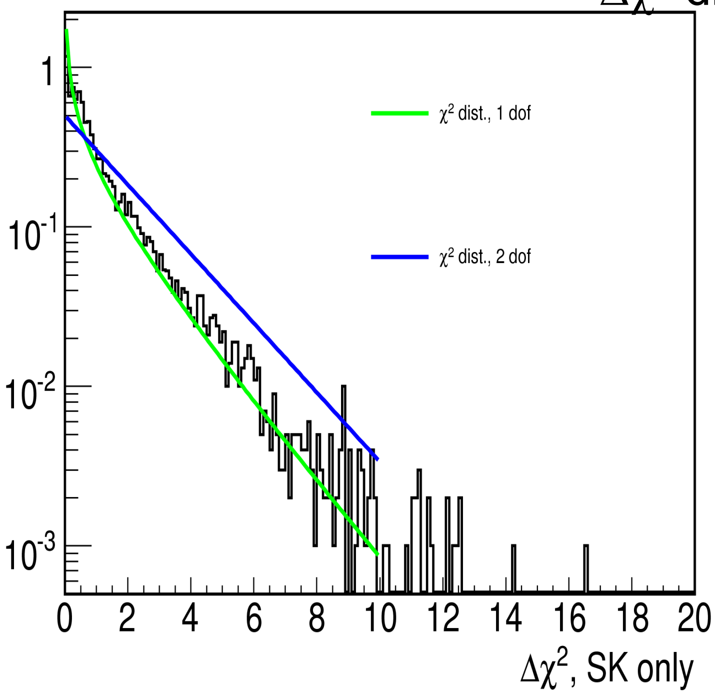
otherwise do the same

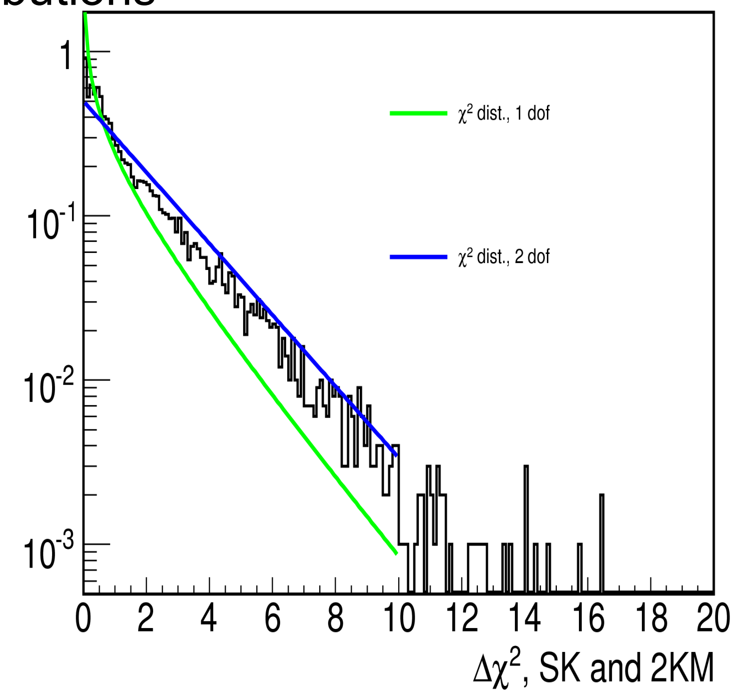
distributions at (2.24e-3 eV^2,$\sin^2 2\theta_{13}$=1.2e-3)

point on the grid closest to "no oscillation"

No systematics, solar oscillation turned off (-> no delta CP effect)

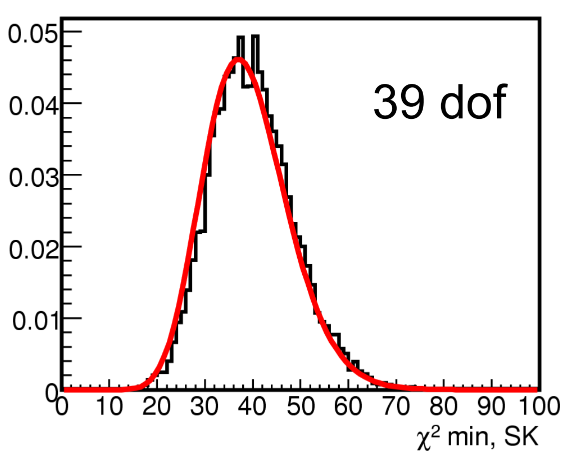
$\Delta\chi^2$ distributions

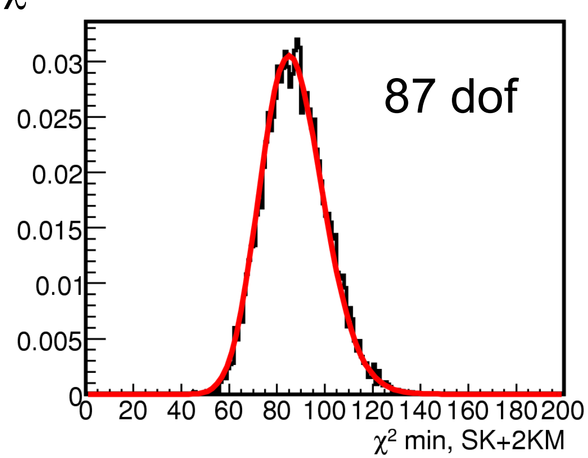




Almost χ^2
Closer to 1 dof than 2 dof
(F&C effect)

Minimum χ^2 distributions



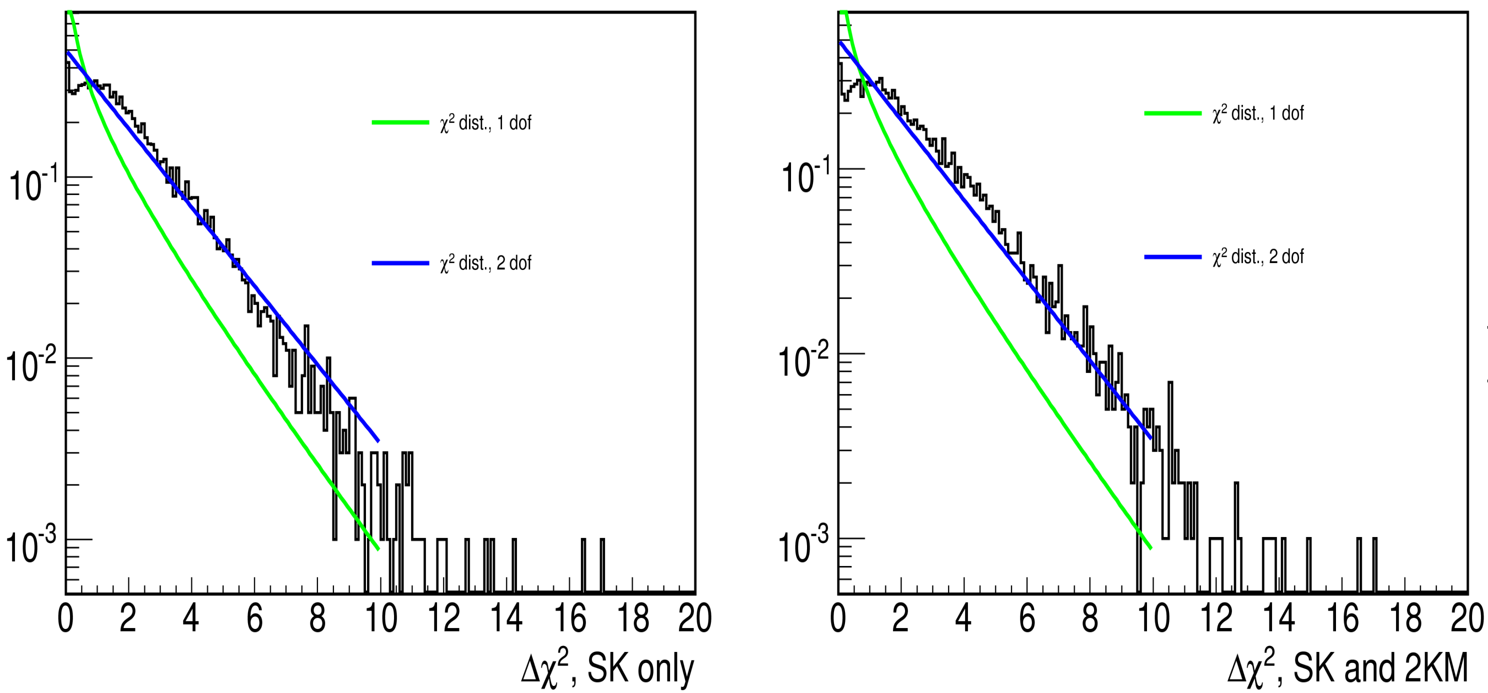


Unexpected : I expected 36 and 84 dof resp.
consequence of FC + Poisson likelihood ratio ?

Distributions at (2.24e-3 eV^2, $\sin^2 2\theta_{13}$=1.2e-2)

point on the grid near the 90% CL limit

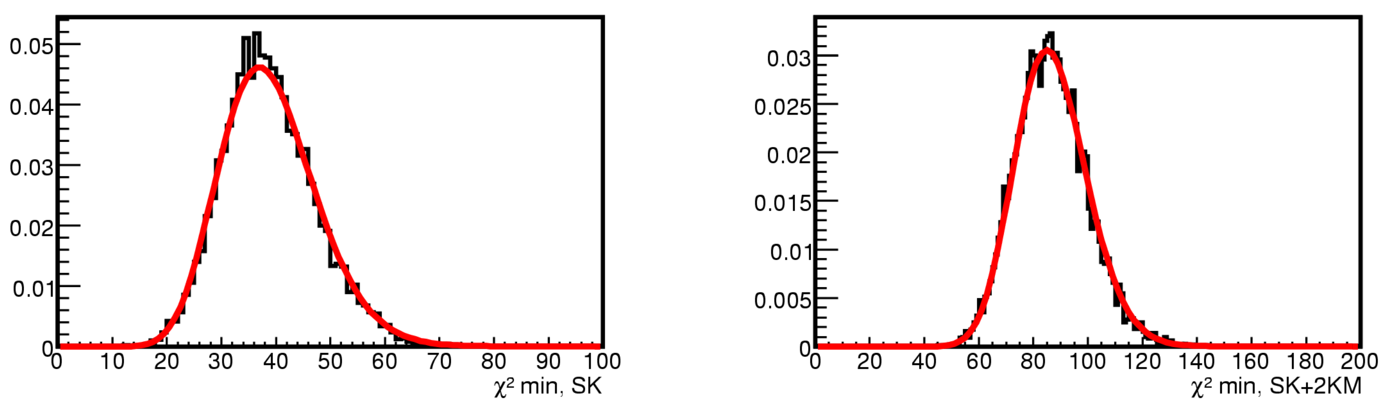
$\Delta\chi^2$ distributions



not exactly χ^2 distributed

BUT close to 2 dof
Critical value
can't be 2.71

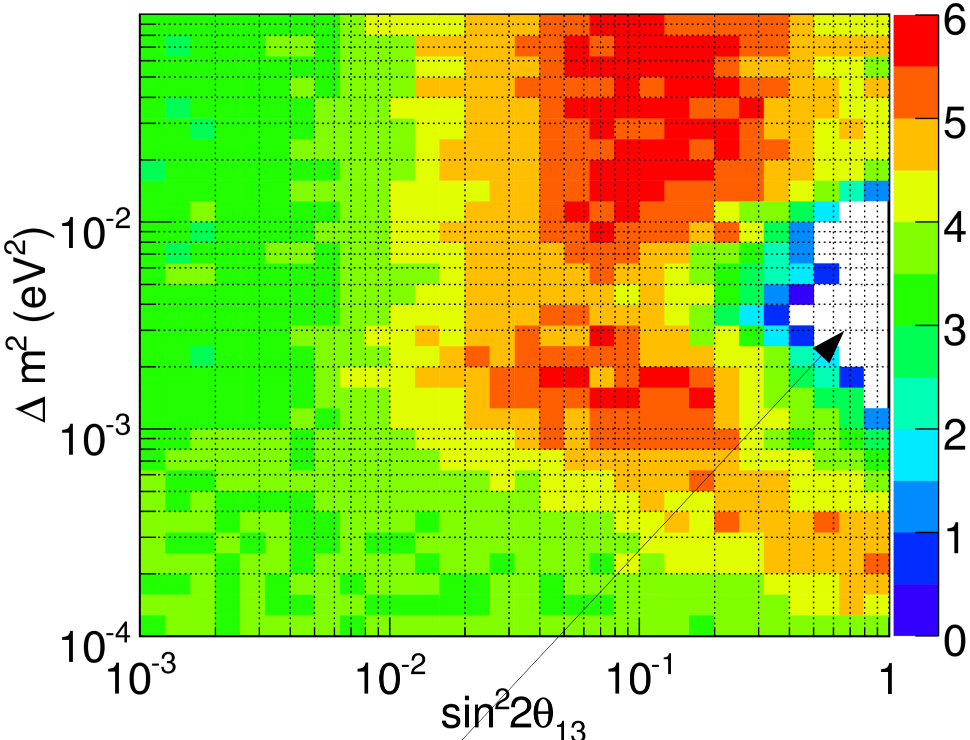
Minimum χ^2 distributions



Same remarks as previous page

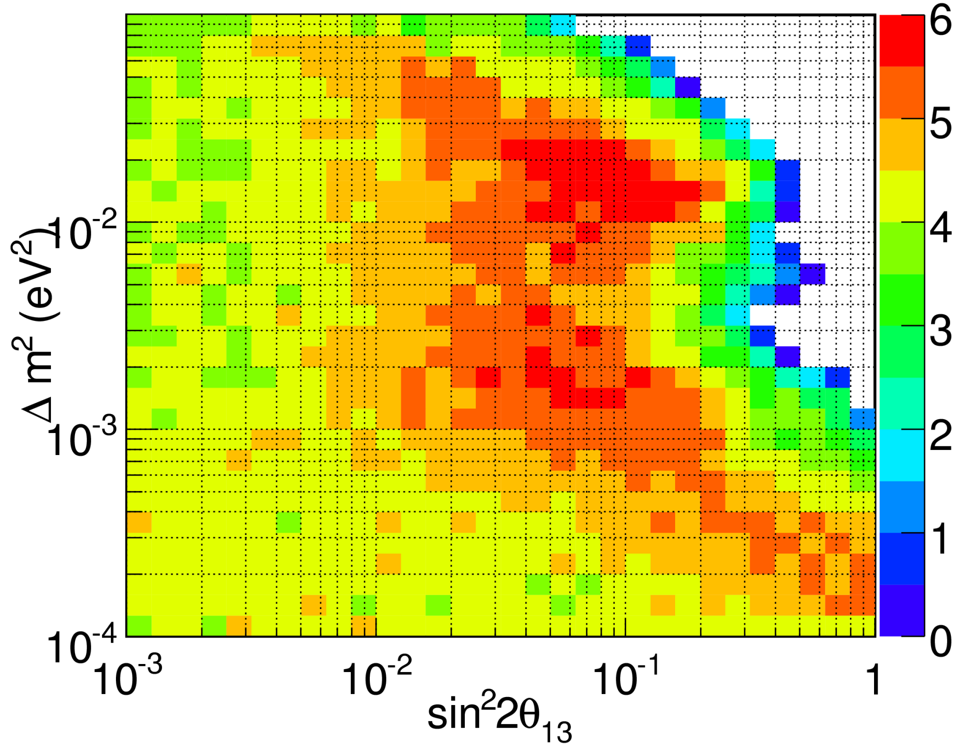
Map of critical values

Map of 90% CL Feldman-Cousins critical values, SK alone



artifact of the grid :
min chi2 found exactly
at "true" point so $\Delta\chi^2$=0

Map of 90% CL Feldman-Cousins critical values, SK+2KM



The critical value is always > 3. Near the limit it is close to 4-5 not 2.7

No external information is used on Δm^2, only nue appearance is used : I estimate BOTH parameters → ~ 2 dof

Definition of sensitivity

- I use a different definition of sensitivity from what J. Dunmore presented at the collaboration meeting

2 questions :

- Sensitivity : limit on θ_{13} in the absence of signal, for a typical experiment

"Typical" = "neutral" with respect to statistical situations -> as many chances of fluctuating above and under --> Use the median of $\Delta\chi^2 = \chi^2(\text{true}) - \min \chi^2$

Procedure : shoot many fake expts at $\theta_{13}=0$, build the median $\Delta\chi^2$ at every point, and compare to critical value at every point

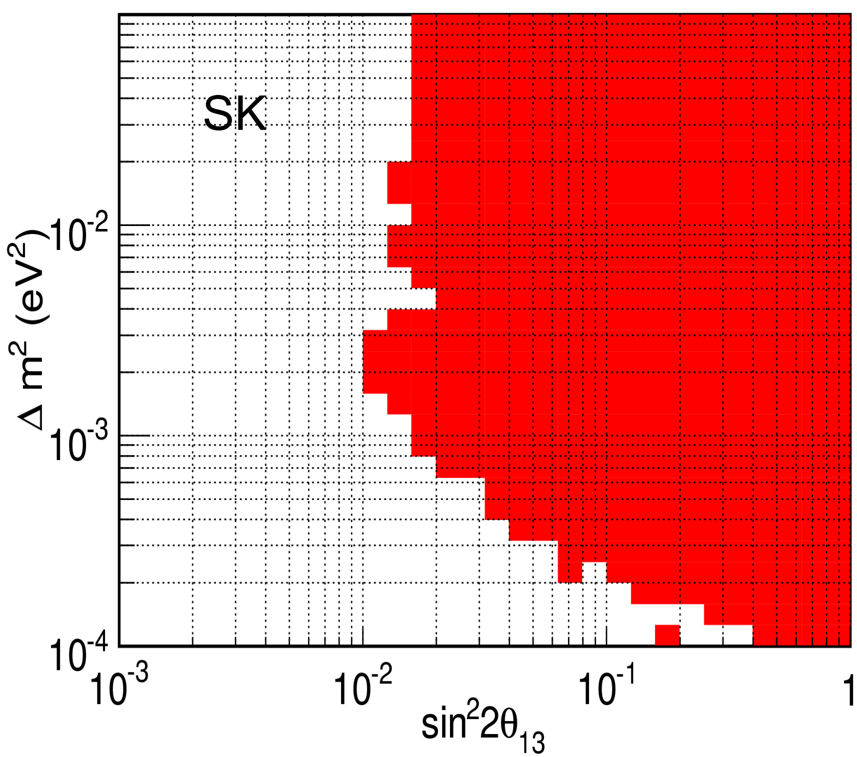
- "Discovery potential" : values of oscillation parameters for which we can rule out the no-oscillation hypothesis ($\theta_{13}=0$).

Similar technique : for every point in parameter space, shoot an experiment ***at this point***, compute $D\chi^2 = \chi^2(\text{no-osc}) - \min \chi^2$ (which is never χ^2 distributed), and compare to critical value ***at the no-osc point***.

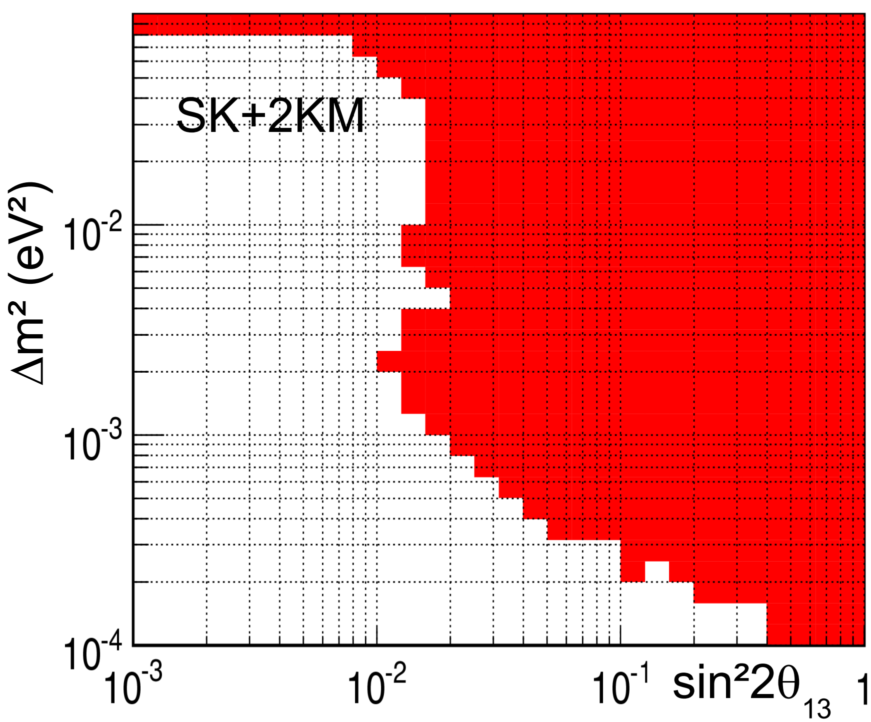
"Neutral" contour : using N experiments at each point build the median of $D\chi^2$ and compare to critical value.

Sensitivity

Using the "median" contour definition + the critical values from the previous slides



90% CL sensitivity with SK alone
no systematics



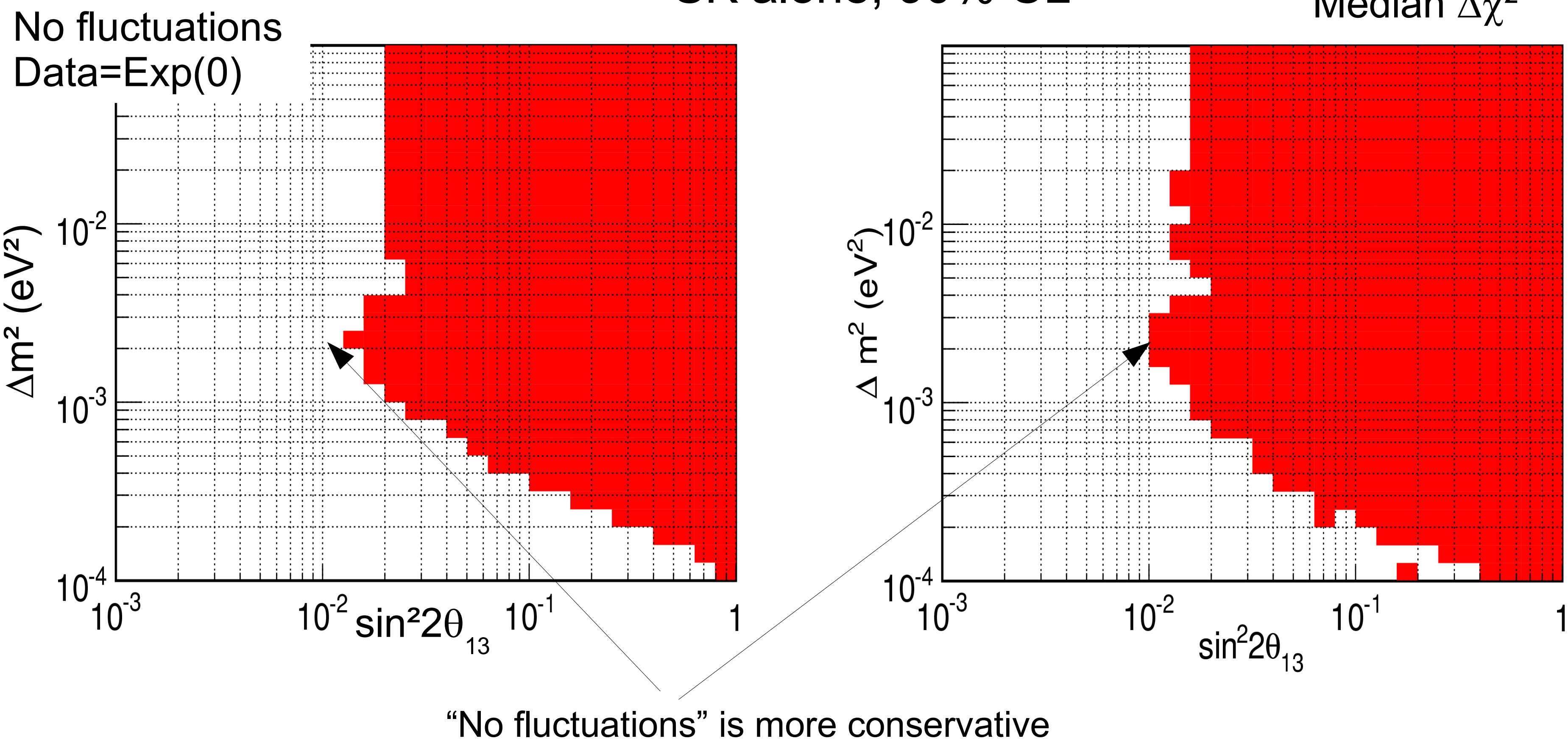
90% CL sensitivity with SK+2KM
no systematics

Comment on the "typical contour"

The "simple" approach is to use data=expectation(0) to make the sensitivity plot
Intuitively, expectation(0) is what the data should "on average" look like...

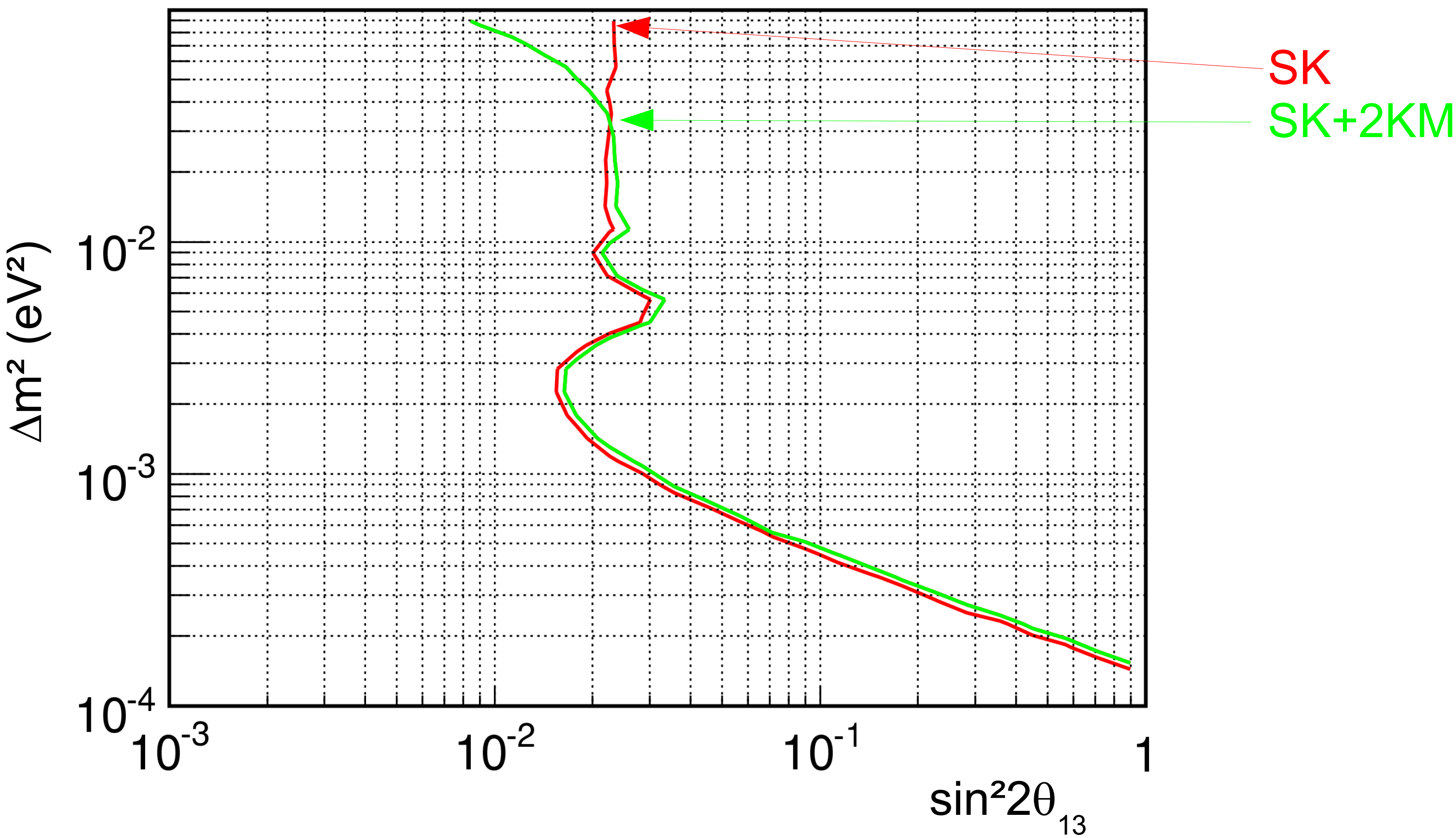
The contour built with this particular set is slightly more conservative (compared to the contour from the median $\Delta\chi^2$) → this choice biases the result

SK alone, 90% CL



Discovery potential

99% CL discovery potential
for SK alone (red) and SK+2KM (green)
<u>without systematics</u> --> lowest bound

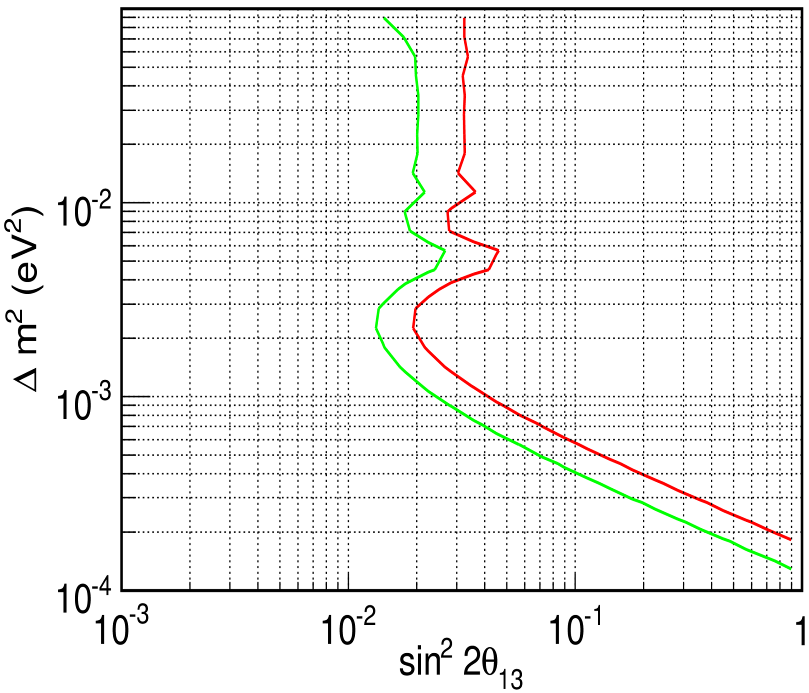


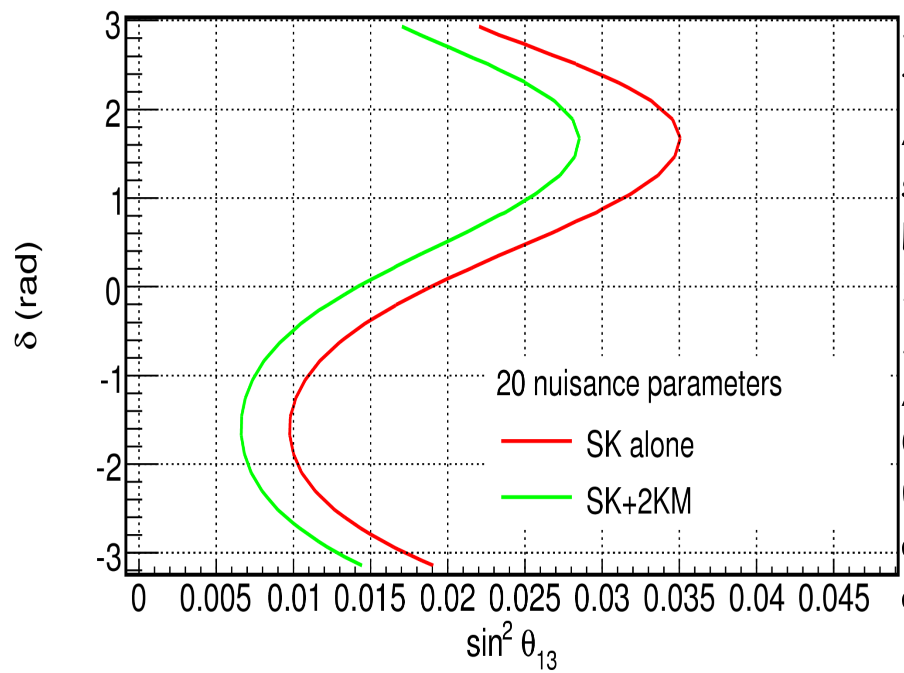
With no systematics, SK & SK+2KM should be equivalent :
differences due to rounding + too small stats ?

Preliminary results with systematics

At the moment, none of the statistics refinements are available with systematics
-> no critical values, no set of random experiments to take the median...

If I don't apply statistical fluctuations ie data=mean expectation(θ_{13}=0;δ=0),
AND if I use cut(90%)=4.61 (based on the previous results this is acceptable) :





Solar oscillation turned on at Δm^2=7.92e-5 eV^2 $\sin^2\theta$=0.314 Lisi's best fit hep-ph/0506083
Atm oscillation Δm^2=2.5e-3 eV^2 $\theta=\pi/4$
uncertainty on atm. Δm^2 treated as 20th parameter

T2K sensitivity with 2KM : $\sin^2 2\theta_{13} \sim 1.4\ 10^{-2}$

Conclusion

- Our framework is operational for statistics studies
- Technical issues : use a grid fit for oscillation parameters, linearized method for systematics -> see N. Tanimoto's talk
- Computed the critical values on the map using toy MC with systematics turned off; since minimum is restricted to physical region, this is F&C's method
- Critical value is never < 3, actually close to 4.6 for 90% CL → the use of 2.71 in our talk at the collaboration meeting was wrong

The cut value for the 1 bin LOI estimator should be tested in a similar fashion.

- Sensitivity : make fake data with no oscillations, and set limit on θ13
- Discovery potential : make fake data everywhere, and check where the null hypothesis is rejected
- Use the median $\Delta\chi^2$ in both cases ; using data=expectation(0) is conservative

Future plan : turn systematics back on

- Systematics : not done yet ; need to be careful with Fij calculations

define $\Delta\chi^2(X_0) = min_{\varepsilon}\chi^2(X_0) - min_{\varepsilon,X}\chi^2$

Make faka data with stat. fluctuations but pulls fixed at 0,

and minimize pulls at all points on the map.