Progress on the T2K sensitivity studies

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Outline

- Description of the fitter
- Statistics issues
- Results

I. Description of the fitter

- General technique and data samples
- Treatment of systematics errors

Choice of estimator

Use a Poisson likelihood ratio estimator ; changed the number of bins to have always more than ~5 events per bin

- 1 ring e-like sample (after $\pi 0$ cuts), at SK & 2KM
- 2 ring-elike sample, (invariant mass), at SK & 2KM

$$\chi^2 = \chi^2_{1R,SK} + \chi^2_{1R,2km} + \chi^2_{2R,SK} + \chi^2_{2R,2km} + \sum_{k=1}^{N_s} \varepsilon_k^2/\sigma_k^2$$

$$= \sum_{i=1}^{N} 2 \left(E_i^{MC}(1 + \sum_{k=1}^{N_s} F_i^k \varepsilon_k) - O_i + O_i \log \left(\frac{O_i}{E_i^{MC}(1 + \sum_{k=1}^{N_s} F_i^k \varepsilon_k)} \right) \right) + \sum_{k=1}^{N_s} \left(\frac{\varepsilon_k}{\sigma_k} \right)^2$$

E_i^{MC} : expected by MC without any systematic effect

O_i : observed in bin i

F_{ik} : effect of kth nuisance parameter on bin i

σ_k : width of kth nuisance parameter

Equation must be solved iteratively (Poisson stats $\rightarrow$ non linear)

We now use a different Fij matrix at each point on a "grid" in oscillation parameter space

Estimator with systematics

Systematics implemented in the linearized method (N. Tanimoto's work)

- ν_e contamination : 30%
- 9 ν–interaction errors : M_A in QE and single-pi, CCQE models, CCQE normalization

single-pi production normalization, multi-pi production models and normalization, coherent pi production, NC/CC ratio, Nuclear effects in ^{16}O (pi reinteractions)
[Bug fixed by N.Tanimoto for this last error source]

- Fiducial Volume : 2.8% for each detector, uncorrelated (4% total)
- Energy scale : 2.1% for each detector, uncorrelated
- PID for 1 ring & 2 ring events
- Ring counting

These last two errors are "split" into a common error (identical at SK and 2KM) and an "SK-only error" to take advantage of cancellations with a 2KM detector

In summary : 76 bins (single ring e-like nue energy and 2 ring elike invariant mass)
19 operational sources of systematics in this analysis : main relevant
ATMPD errors for T2K (only 2 errors available in january).
possible cancellations between SK and 2KM are accounted for.

II. Statistics issues

- Definition of sensitivity

- Reminder : LOI analysis, results of latest SK analysis

- "Where do we place the cut on the $\Delta\chi^2$?"

Definition of "sensitivity"

- There are two main questions that one can ask about T2K :

1. **Sensitivity** : limit on θ_{13} in the absence of signal, i.e.

If θ_{13}=0, what limit will T2K set on θ_{13} at a given CL ?

Technique : make fake data at θ_{13}=0 and set cut on $\Delta\chi^2 = \chi^2 - \min \chi^2$

2. **Discovery potential** : true values of θ_{13} for which T2K will be able to rule out the no-oscillation hypothesis (θ_{13}=0) at a given CL

Technique : for each point X, make fake data at X, set cut on estimator

$D\chi^2 = \chi^2(\text{no-osc } \theta_{13}=0) - \min \chi^2$ to check if X is in/out.

- We want to obtain a "typical" contour, i.e. a contour that is "neutral" with respect to statistical fluctutations. Usually people make fake data without any fluctuations (ie observation=ouput of the Monte-Carlo).

For each method I propose to compute the median of the estimator over N experiments and set the cut on the median (does not depend on variable changes in estimator).

Sensitivity

- Applying method 2 (discovery potential) is time consuming (many fake experiments are required)
- Computing the median of the estimators is also time consuming, and is therefore not always done in practice

- LOI analysis : T2K = simple counting experiment. It is a sensitivity contour (method 1). No fluctuations were applied. In that simple case this is the same as the median contour.

- Long standing question : where should we place the cut on the estimator ?

- Need to study the coverage of the method. To ensure proper coverage generation of many fake experiments is necessary.

Two types of analyses

• LOI-like analysis : T2K is a simple counting experiment, with 10% systematics on background subtraction.
 i.e. $\chi^2 = (S+B-data)^2/(S+B+(\alpha \times B)^2)$; data is random with mean S+B

• Use full-fledged fitter, with spectral information, and with all systematics
Do contours in Δm^2-$\sin^2 2\theta_{13}$ and δ-$\sin^2 2\theta_{13}$ planes

• Check coverage by using Monte-Carlo in all cases i.e.
$\rightarrow$ Get the critical values of the estimator
In this talk I will always consider 90% CL critical values and contours.

Get the critical values

Use a 30x30 "logarithmic" grid in 2D parameter space

- Pick a point *A* on the map
- Make fake data from MC(*A*)
- Compute "true chi2" = chi2(*A*) and min(chi2) (which will be at another point)
- Get $\Delta\chi^2(A)$ = chi2(*A*) – min(chi2) distribution --> will depend on *A* (non linearities,etc.)

Only if chi2 is linear in the parameters AND the errors are gaussian will this be a 2dof χ^2 distribution !

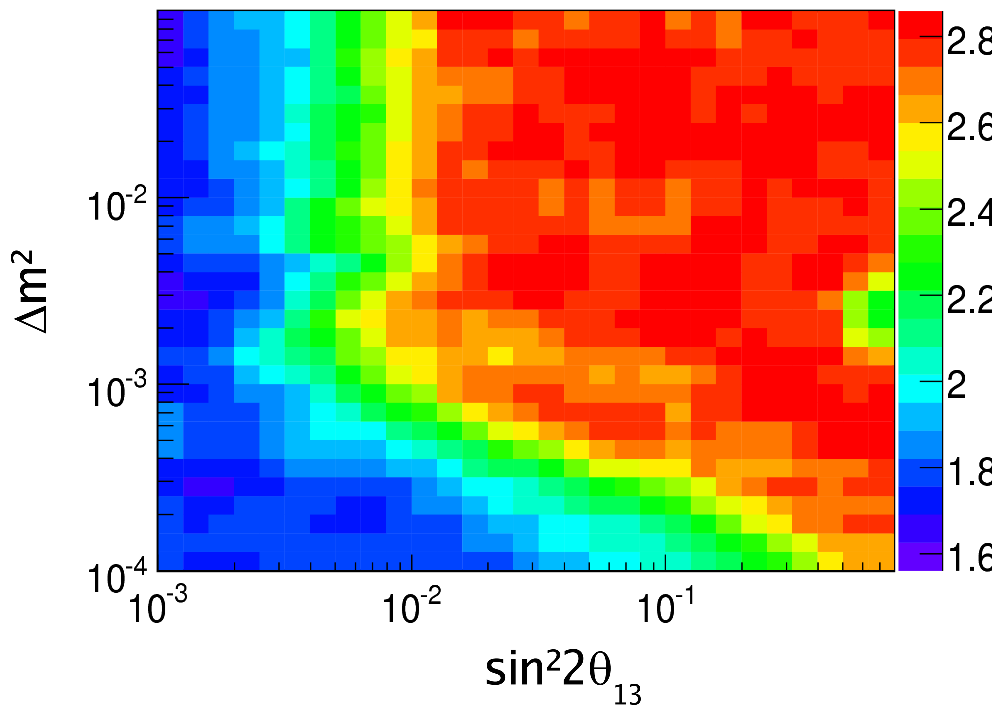
- Determine α CL cut position on $\Delta\chi^2(A)$ distribution --> critical value $C_\alpha(A)$
- Use this cut on $\chi^2(\text{data},A)-\min\chi^2(\text{data})$, to decide if data accepts point *A* or not
- Repeat for all points on the map

Things to remember :

- The grid is a subset of the physical region $\Rightarrow$ <u>the minimum cannot escape the physical region $\Rightarrow$ I obtained Feldman-Cousins critical values</u>
- <u>No systematics so far.</u>

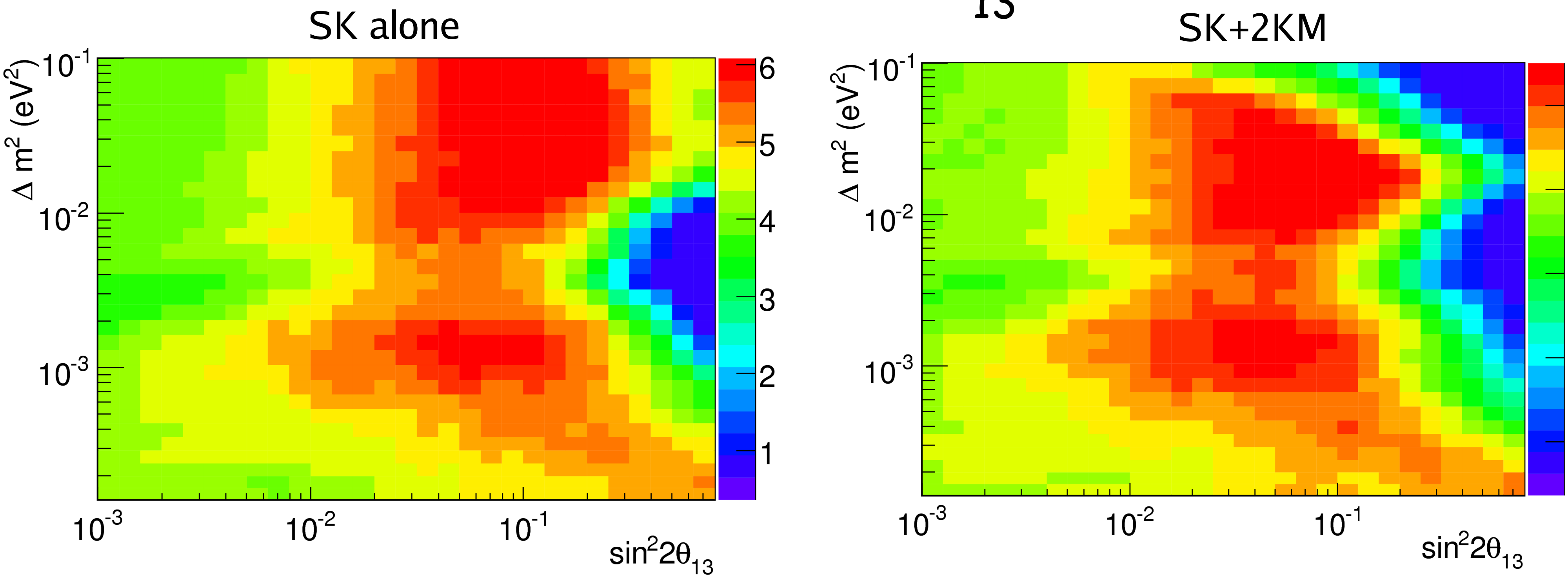
Critical values : LOI analysis

LOI analysis (simple counting experiment)



Cut value is higher than 1.64 contrary to what is used for the LOI.
Caculation of the $\Delta\chi^2$ map shows that it is around 2.7, as expected for a 1 dof χ^2 distribution

Critical values with the fitter: Δm^2-$\sin^2 2\theta_{13}$



Critical values in the Δm^2-$\sin^2 2\theta_{13}$ plane are ~4-5 for 90% CL

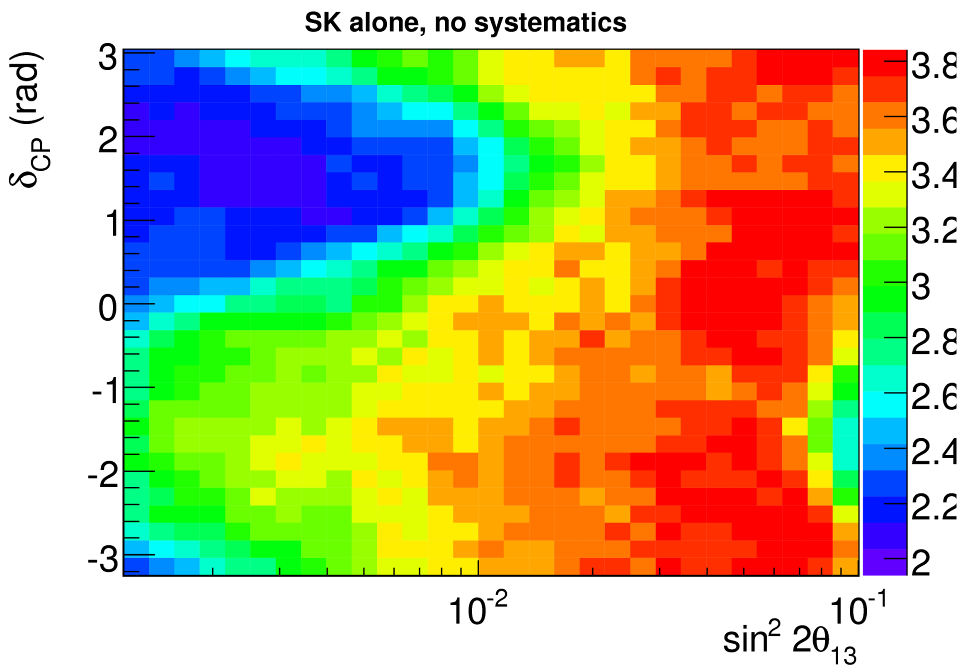
→ we do estimate 2 parameters ; a 1 dof cut (~2.7) will undercover badly

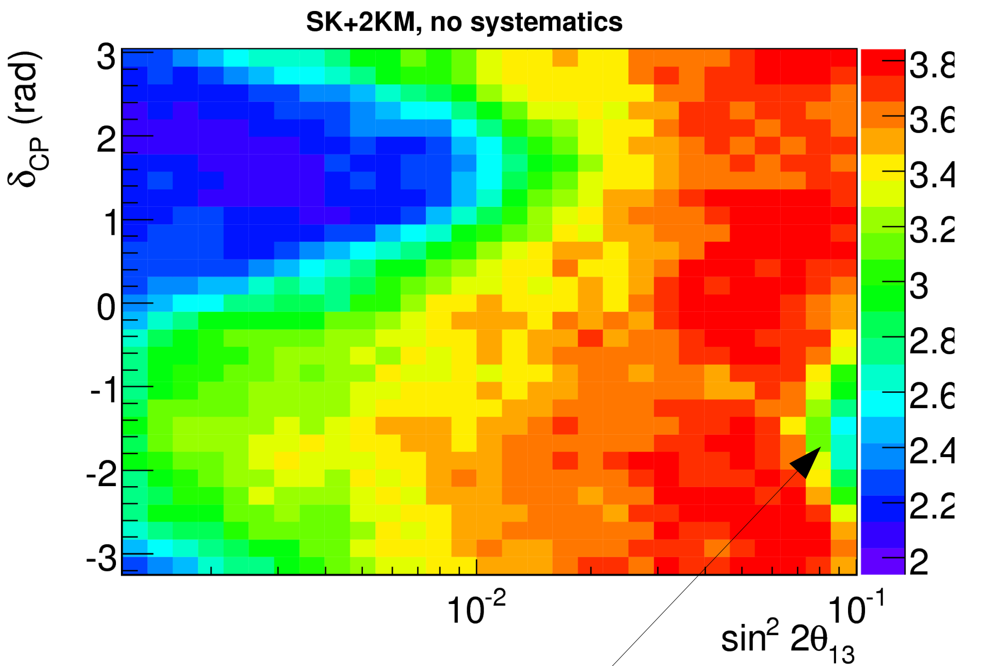
→ the 2KM detector is sensitive at high Δm^2 which causes the different shape

→ "Feldman-Cousins effect" : near the edges the values are lower.

Note: the solar parameters and δ are kept fixed in this fit (best fit value + δ=0)

Critical values : δ-$\sin^2 2\theta_{13}$





90% CL critical values :

Almost no perceptible difference between SK and SK+2KM **as expected.**

Critical values definitely lower than those of a 2-dof χ^2.

Warning : this is not exactly F-C, because $\sin^2 2\theta_{13}$ does not span the whole physical region.

So there is an edge effect on the right that shouldn't be there.

With systematics

Procedure : χ^2 now is a function of X(oscillation) and ε (nuisance parameters)

- Pick a point A on the map
- Make fake data from MC(A), setting all the nuisance parameter to 0
- Compute min χ^2(A,best fit ε) and min(χ^2) = χ^2(best fit X, best fit ε')
- Get $\Delta\chi^2$(A) =min χ^2(A,best fit ε) – min χ^2(best fit X, best fit ε') distribution

--> will depend on A (non linearities etc.)

- Determine α CL cut position on $\Delta\chi^2$(A) distribution --> critical value C_α(A)
- Use this cut on χ^2(data,A,best fit ε)- min(X,ε) χ^2(data),

  to decide if data accepts point A or not
- Repeat for all points on the map

Basically same procedure as before, but with a minimization of the nuisance parameters at each step.

This is an approximation, considered to be very good (Kendall& Stuart, Feldman) and certainly much faster than making a full Neyman construction over many (nuisance) parameters.

<u>**Question**</u> : is it correct to fix the nuisance parameters to their "central value" 0 ? Does it change the coverage when they are set to some other value ? Should they also be randomized ?

$\rightarrow$ **Preliminary tests suggest that nuisance parameters must be randomized !**

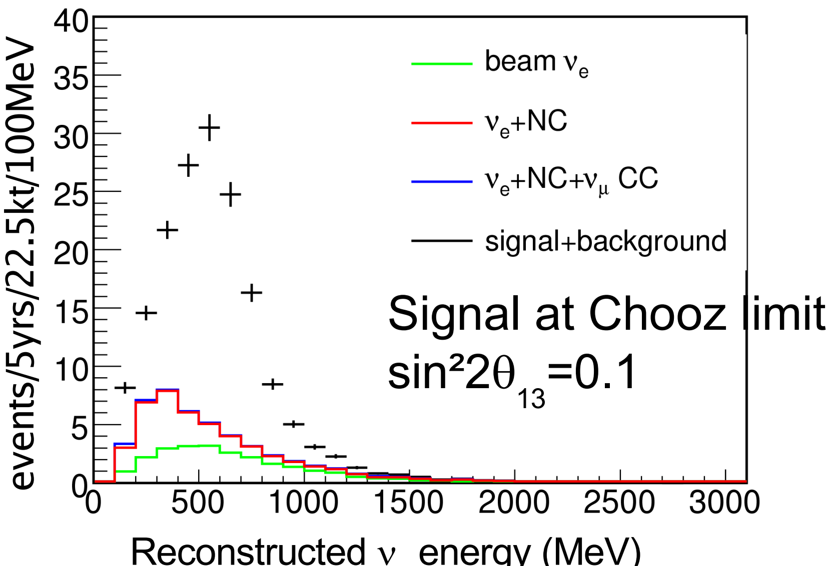
III. Results

- Reminder : status of SK ν_e appearance analysis
- Δm^2-$\sin^2 2\theta_{13}$ contours
- δ_{CP}-$\sin^2 2\theta_{13}$ contours

Reminder: Selection efficiencies at SK

Monte-Carlo Super-K GEANT3, 22.5 kt, 5 years, Δm^2_{23} = 2.5e-3 eV²:

	$\nu\mu$ CC mis-ID	NC	beam νe CC	Signal (chooz)
FC,FV,Evis>100 (MeV)	2077.3	828.6	156.7	217.9
Single ring	978.7 (47.1%)	221 (26.7%)	82.2 (52.4%)	1843 (84.6%)
E-like	39.0 (1.9%)	173.5 (20.9%)	81.6 (52.1%)	182.2 (83.6%)
No decay e-	13.4 (0.65%)	154.2 (18.6%)	68.1 (43.5%)	166.4 (76.2%)
0.35<Eν<0.85 (Gev)	1.36 (0.07%)	52.7 (6.4%)	19.2 (12.3%)	127.2 (58.3%)
Cos$\theta_{\nu lepton}$<0.9	0.96 (0.05%)	38.4 (4.6%)	16.4 (10.5%)	111.4 (51.1%)
Polfit M$\gamma\gamma$ < 100 MeV/c²	0.46 (0.02%)	12.7 (1.5%)	13.5 (8.6%)	94.1 (43.2%)
ΔlogLikelihood < 80	0.36 (0.017%)	10.2 (1.2%)	13.2 (8.4%)	91.9 (42.2%)



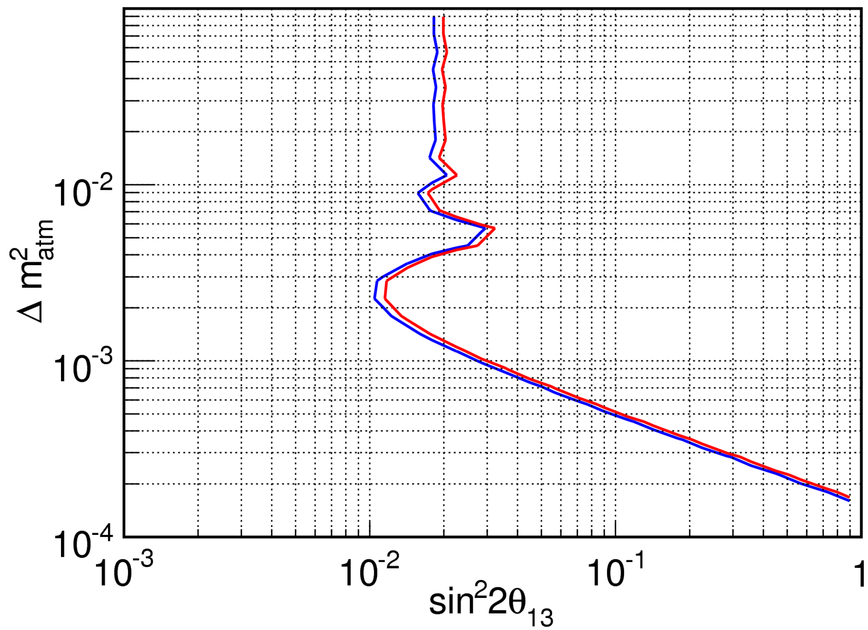
92 events < 103 events in official analysis

because of a bug fix in the event rates (by Hayato-san)

The official version of this table should be updated

LOI Analysis : based on total number of events

- First we use the LOI technique : $\chi^2 = S^2/(S+B+(\alpha \times B)^2)$
- Lower number of signal events
- Use cut at 2.7 based on previous critical value calculations



No systematics

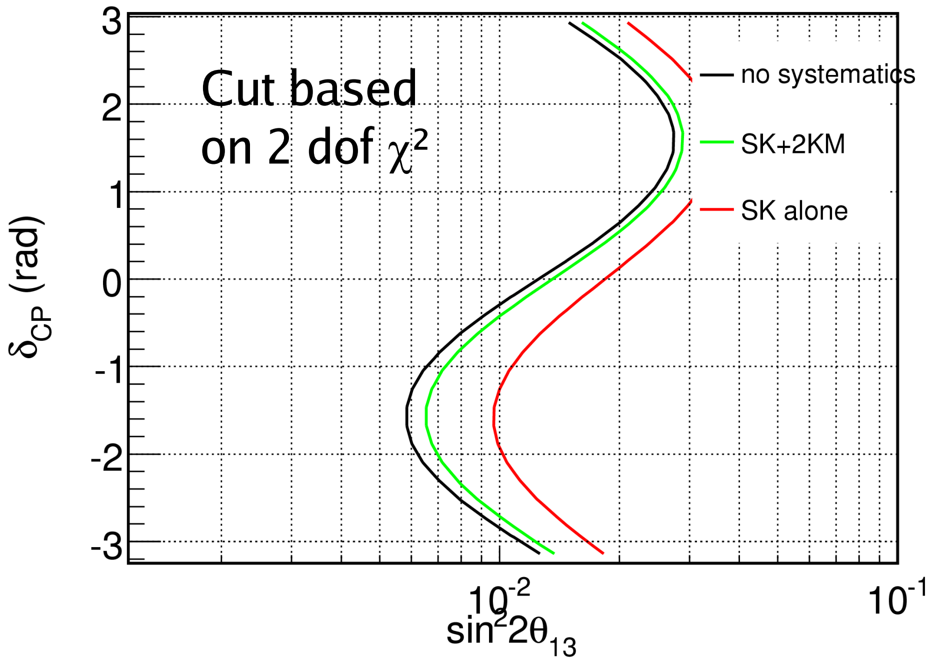
rescaled : 10% systematics on BG subtraction

Cut @2.71 (1 dof)

Solar oscillation turned off

- **Note : if we shorten the beam pipe, we heard that the event rate will be decreased by (↓5%). The limit should be worse again.**

Sensitivity contours (δ-$\sin^2 2\theta_{13}$)

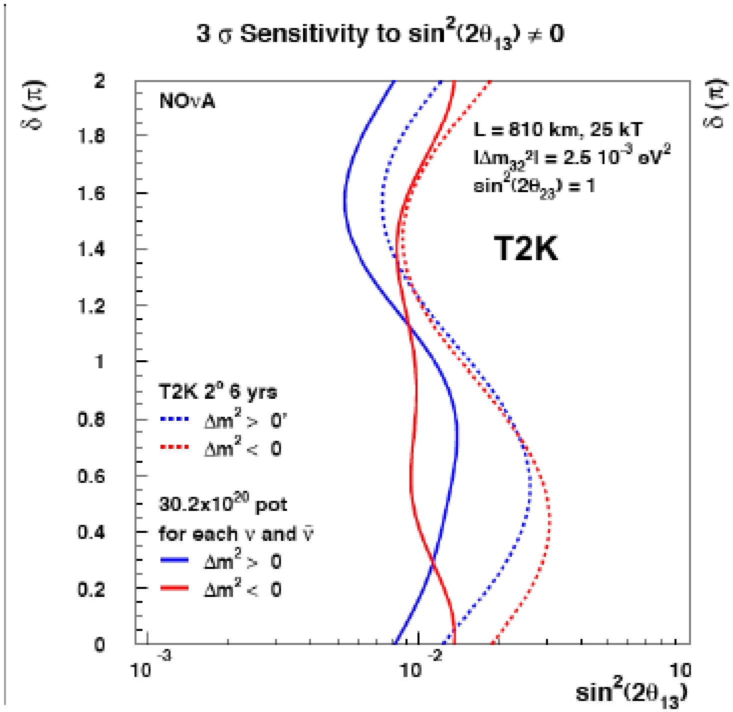


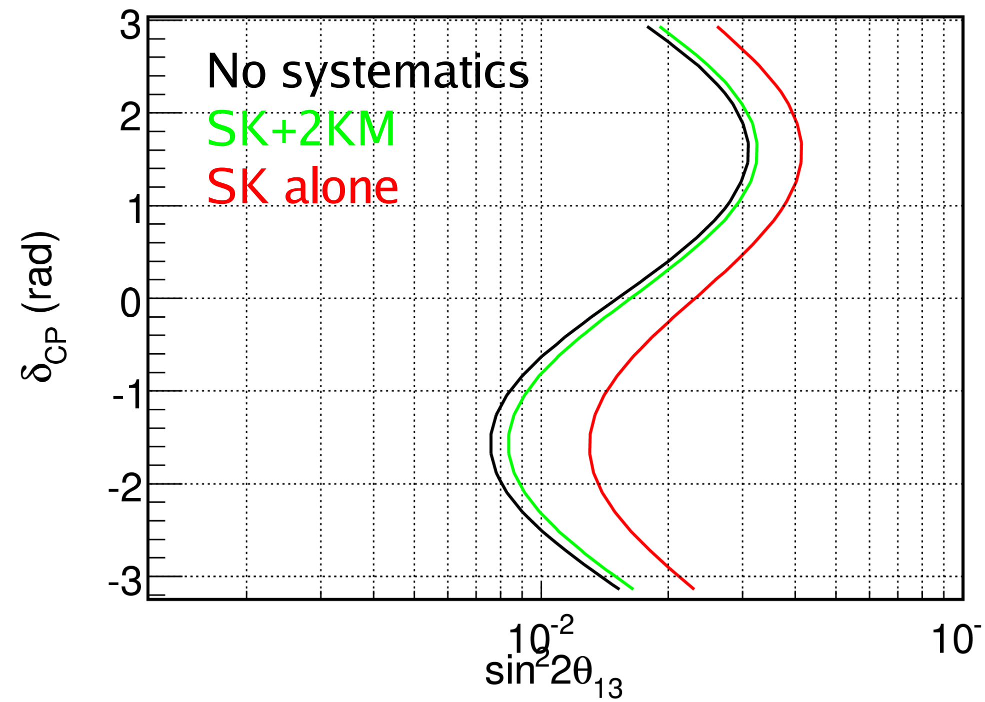
90% CL <u>sensitivity</u> contours using Δm^2=2.5e-3 eV^2 (fake data made at $\theta_{13}=0,\delta=0$

Fake data has no fluctuations

- Using the usual 2 dof cut is conservative
- Upper value of sensitivity : ~1.4e-2 at 90% with 2KM detector

Discovery potential : Plots à la NOνA





From Gary Feldman, P5 at Fermilab, 18/04/06

- These are **raster scans** : δ is fixed, and the $\Delta\chi^2$ is minimized along horizontal lines
- These are **discovery potentials** : they show which true value of θ_{13} is necessary to claim that θ_{13} is non zero at **3σ**, for a fixed value of δ (critical value is 9 i.e. 1 dof χ^2).
- NOνA is considered to be a counting experiment only, with 5% systematics on background subtraction – note : use **6.10^{21} pot** for both experiments.
- T2K lines are made with all 19 systematic errors, and full fitting, but no matter effects.

Conclusion

- We have developped a fitter that uses the SK-ATMPD pull technique, to fit SK and 2KM together.
- 19 relevant systematic nuisance parameters have been implemented so far (N.Tanimoto) + bug fixes in related SK code

- Reminder : the latest ν_e appearance analysis at SK, based on Hayato-san's corrected event rates, finds fewer signal events (92 instead of 103) for a quasi-unchanged background $\rightarrow$ Drop in sensitivity in $\sin^2 2\theta_{13}$ from compared to previous collaboration meetings

- Statistics issues : coverage studies through toy Monte-Carlo helps define the “position of the cut” on the estimators.
- For a counting experiment based on the LOI estimator we should use “2.7” as a cut value
- In the Δm^2-$\sin^2 2\theta_{13}$ plane the critical values are close to those of a 2 dof χ^2 in the region of interest
- In the δ-$\sin^2 2\theta_{13}$ plane, the critical values are lower than that of a 2 dof χ^2 in the region of interest
- Sensitivity $\sin^2 2\theta_{13}$ to at (δ=0) is ~1.1 10^{-2} using a Feldman-Cousins analysis

- TODO : implement the statistical refinements outlined on slide 6.

Removed slides

General technique

- Build a χ^2 – like estimator that includes systematics
- We use Poisson statistics + iteration to solve equation
  - Same as SK Atmospheric Neutrino and Proton Decay (ATMPD) fitting technique
- Use 3 flavor oscillation probabilities
- MC is reweighted for the systematic terms on an event by event basis (See next page)
- We will use 19 systematic terms
- Value of the sigmas of the systematic errors are taken from
  1. SK ATMPD
  2. 2KM systematics analysis showed in january

How to reweight MC events

- Use the fully reconstructed MC at 2km and at SK produced in Dec, 2005
- Detector response is not parameterized
- Read in events one-by-one, then:
- Multiply each event by a weighting factor when it is placed in a histogram, taking into account oscillations and systematics e.g. $(1+\sum_{i\in systematics} F_i \varepsilon_i) P_{osc}$ where the ε_i are free parameters (linearization of the systematic effects)
- In january we showed that this linearized method was equivalent to the non-linear method with a minimizer (in a simplified case)

Technique used in SK analyses : PRD 71, 112005 (2005), PRD 66, 053010 (2002)

3-flavor nu oscillation analysis in SK (not published yet)...

Choice of estimator

Use a Poisson likelihood ratio estimator ; changed the number of bins to have always more than ~5 events per bin

- SK 1 ring e-like sample (after all appearance cuts), recontructed Eν, 9 bins
- SK 2 ring e-like sample, invariant mass, 17 bins
- 2KM 1 ring e-like sample (after all appearance cuts), reconstructed Eν, 22 bins
- 2KM 2 ring e-like sample, invariant mass, 28 bins

$$\chi^2 = \chi^2_{1R,SK} + \chi^2_{1R,2km} + \chi^2_{2R,SK} + \chi^2_{2R,2km} + \sum_{k=1}^{N_s} \varepsilon_k^2/\sigma_k^2$$

$$= \sum_{i=1}^{N} 2\left(E_i^{MC}(1+\sum_{k=1}^{N_s} F_i^k \varepsilon_k) - O_i + O_i \log\left(\frac{O_i}{E_i^{MC}(1+\sum_{k=1}^{N_s} F_i^k \varepsilon_k)}\right)\right) + \sum_{k=1}^{N_s}\left(\frac{\varepsilon_k}{\sigma_k}\right)^2$$

E_i^{MC} : expected by MC without any systematic effect

O_i : observed in bin i

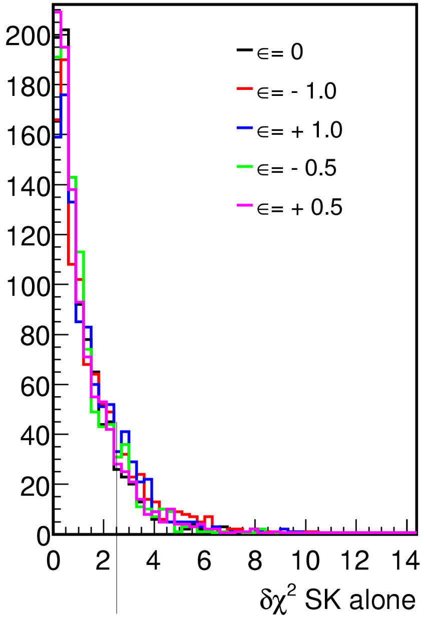
F_{ik} : effect of kth nuisance parameter on bin i

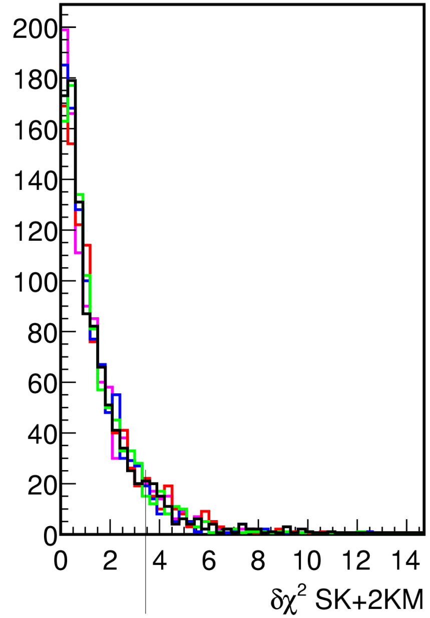
σ_k : width of kth nuisance parameter

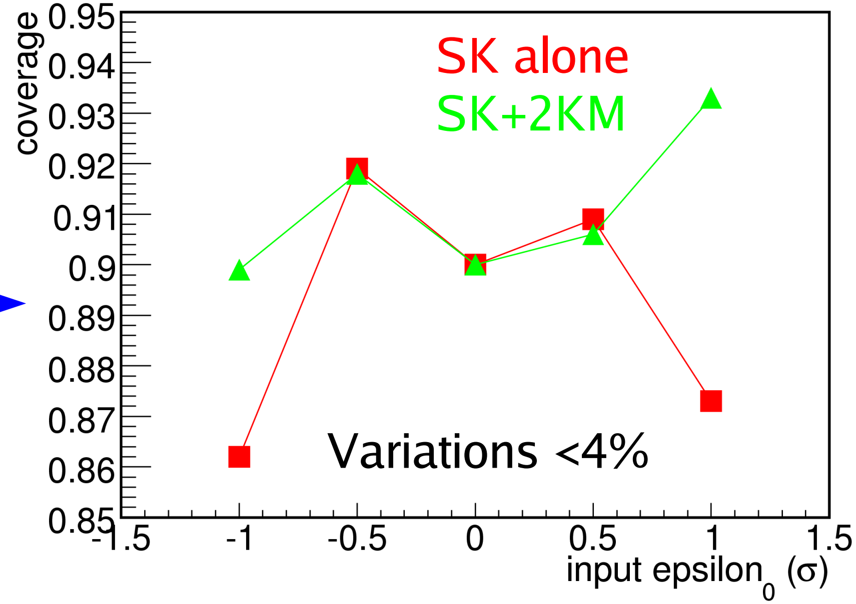
Equation must be solved iteratively (Poisson stats → non linear)

Coverage checks

- Use 2 simple systematic errors : nue contamination (30%) and NC/CC (10%)
- Pick one set of oscillation parameters (δ=0,$\sin^2 2\theta_{13}$=2e-2)
- Fix the 2nd nuisance parameter to 0, let the first one vary from -1sigma to +1sigma
- Measure the actual coverage given by the 90% CL critical value obtained for epsilon=0





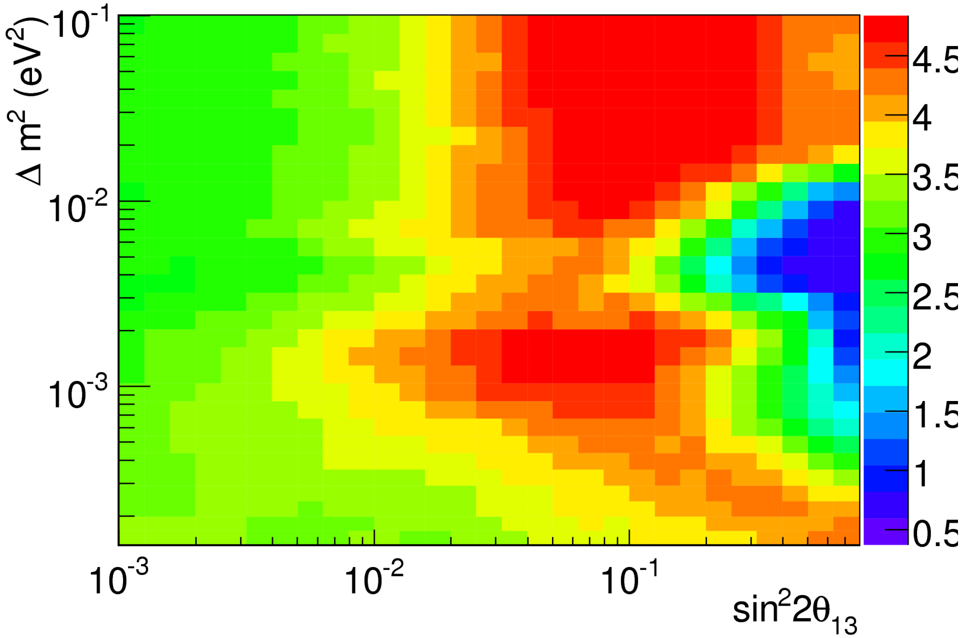


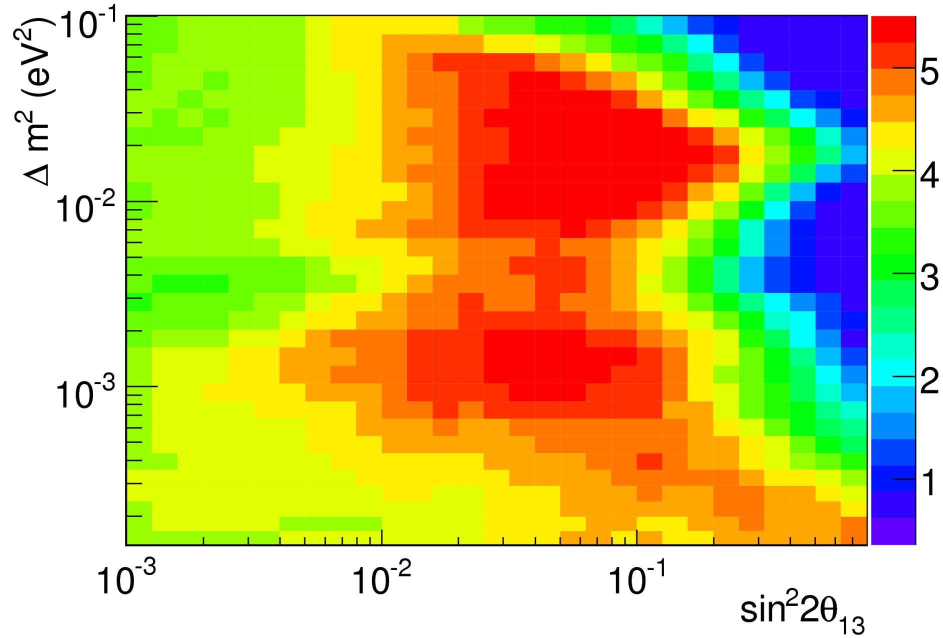
Very similar distributions : changing this
input nuisance parameters has little effect on the coverage

Fixing the nuisance parameters to 0 is acceptable for this study !

Critical values with systematics

Nuisance parameters fixed at 0 when making fake data, always fitted during the computations as explained on slide 2.





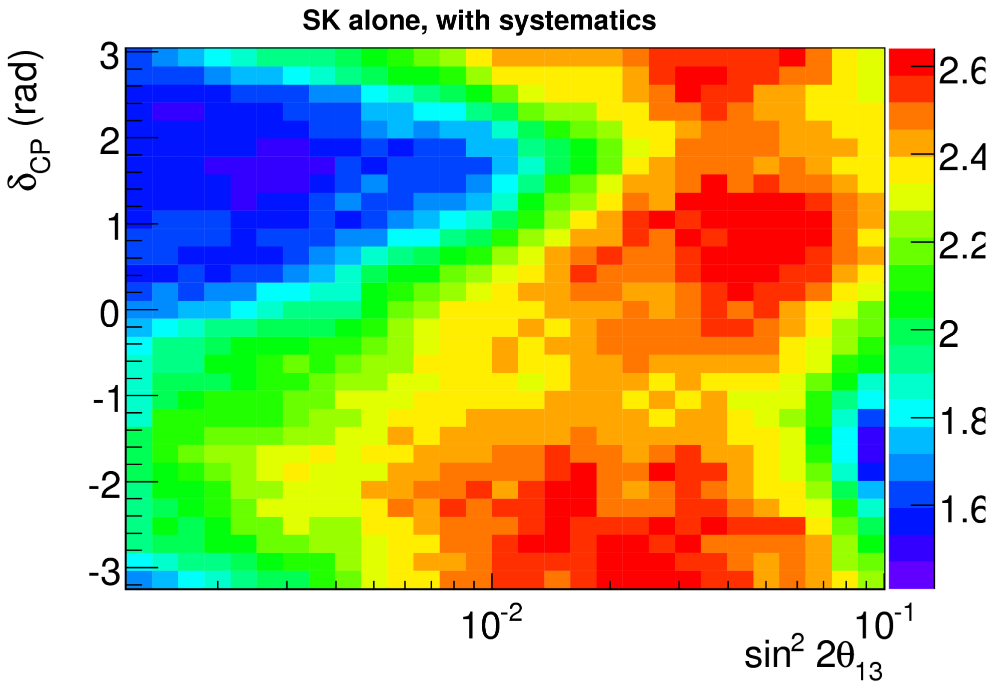
At 90% CL.

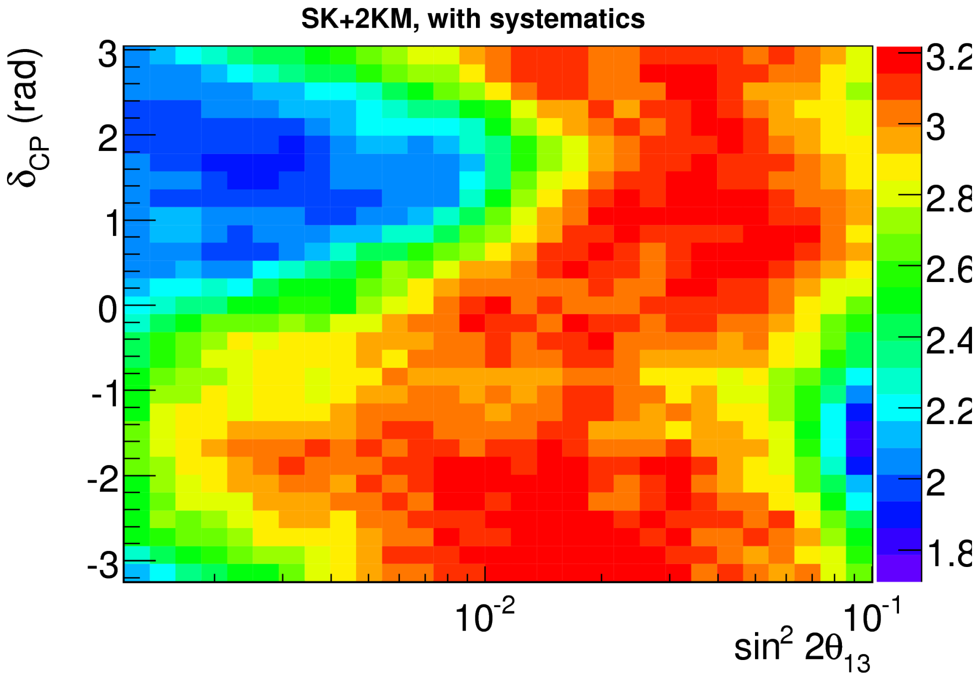
Two main comments :

- Values lower than in the absence of systematics (nuisance parameters give extra freedom to lower the $\Delta\chi^2$).
- Values lower at SK than at SK+2KM : same reason [fewer constraints when SK is alone]

Critical values with systematics

Nuisance parameters fixed at 0 when making fake data, always fitted during the computations as explained on slide 2.

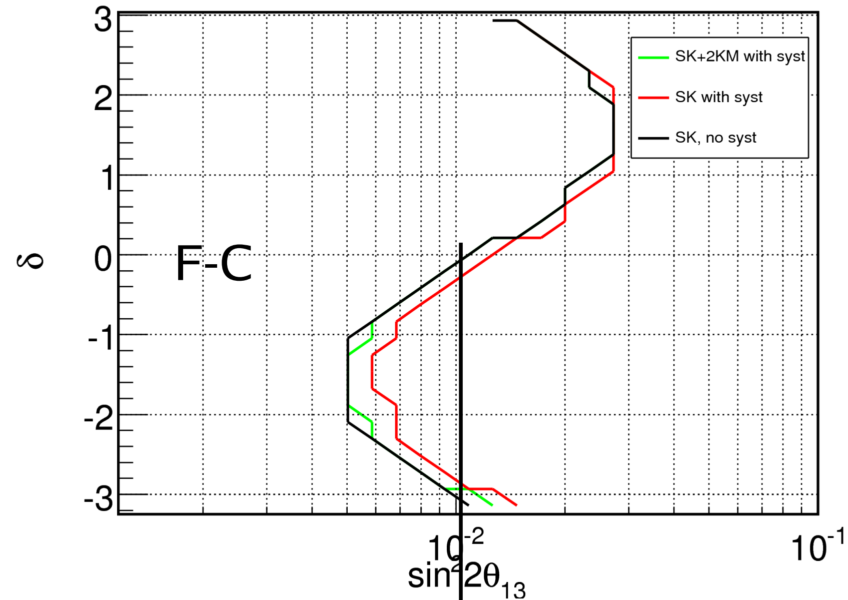


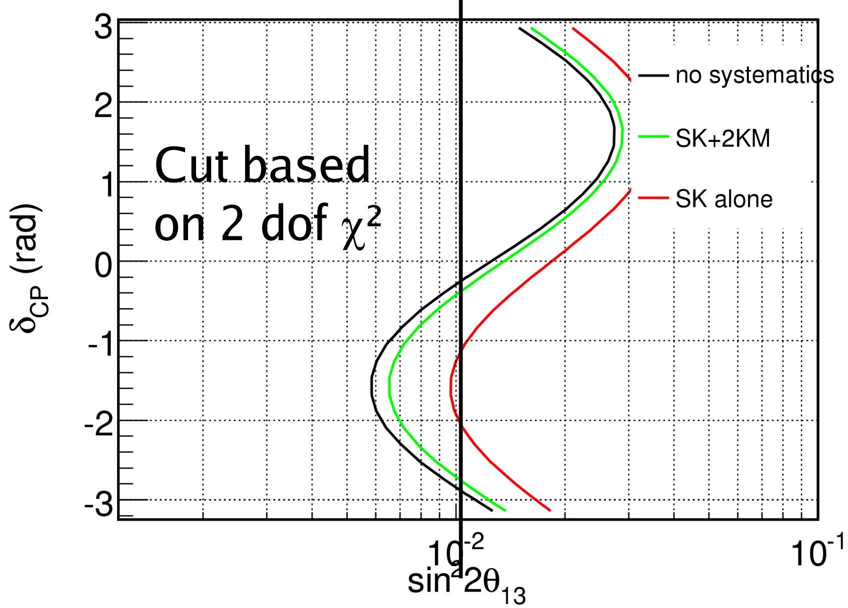


At 90% CL.

Same comments as previous slide.

Sensitivity contours (δ-$\sin^2 2\theta_{13}$)





90% CL <u>sensitivity</u> contours using Δm^2=2.5e-3 eV^2 (fake data made at θ_{13}=0,δ=0

Fake data has no fluctuations

- Using the usual 2 dof cut is conservative