# Neutrino energy reconstruction in quasi elastic $\nu A$ interactions

Anatoli Butkevich (INR, Moscow)

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## Outline

- Motivations
- Nucleons Fermi motion and neutrino energy
- Analysis and Results
- Conclusions

# Motivations

## QE neutrino scattering off free nucleon

$$\varepsilon_\nu = \frac{M\varepsilon_\mu - m_\mu^2/2}{M - \varepsilon_\mu + k_\mu \cos\theta},$$

where $M$ is mass of nucleon, $\varepsilon_\mu, k_\mu$, $\cos\theta$ are muon energy, momentum, and scattering angle, $m_\mu$ is mass of muon.

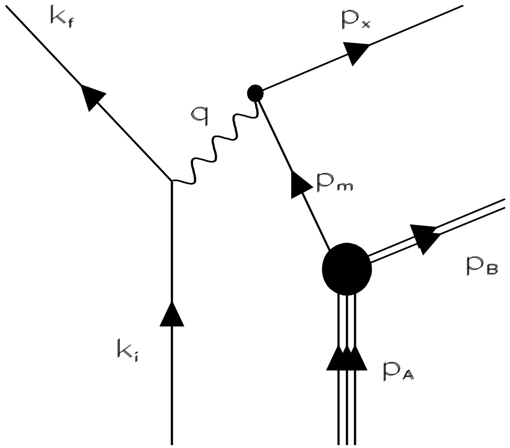
## QE neutrino scattering off nucleus

$$\varepsilon_\nu = \frac{(M - \epsilon_b)\varepsilon_\mu + (2M\epsilon_b - m_\mu^2 - \epsilon_b^2)/2}{(M - \epsilon_b) - \varepsilon_\mu + k_\mu \cos\theta},$$

where $\epsilon_b$ is binding energy of nucleon in nucleus.

**Nucleon Fermi motion is not taking into account.**

# Fermi motion and Nuclear binding




## Kinematics

$k_i = (\varepsilon_\nu, \mathbf{k}_\nu)$, $k_f = (\varepsilon_\mu, \mathbf{k}_\mu)$, $p_m = (\varepsilon_m, \mathbf{p}_m)$, $p_x = (\varepsilon_x, \mathbf{p}_x)$, $\mathbf{q} = \mathbf{p}_x - \mathbf{p}_m = \mathbf{k}_\nu - \mathbf{k}_\mu$

Initial nucleon is bound with energy $\varepsilon_m = \varepsilon - \epsilon_b$, $\varepsilon = (M^2 + \mathbf{p}_m^2)^{1/2}$
Knocked-out nucleon is free with energy $\varepsilon_x = (M^2 + \mathbf{p}_x^2)^{1/2}$

# Neutrino energy as a function bound nucleon momentum

From the conditions
$\mathbf{p}_x = \mathbf{p}_m + \mathbf{q}, \quad \mathbf{q} = \mathbf{k}_\nu - \mathbf{k}_\mu, \quad \varepsilon_\nu + \varepsilon_m = \varepsilon_\mu + \varepsilon_x$
the square equation for the neutrino energy can be obtained

$$A\varepsilon_\nu^2 + B\varepsilon_\nu + C = 0,$$

where

$$A = a^2 - p_m^2 \cos^2\theta_{pq}, \quad p_m = |\mathbf{p}_m|,$$
$$B = 2p_m^2 \cos^2\theta_{pq} \cdot (k_\mu \cos\theta) - ab,$$
$$C = b^2/4 - p_m^2 \cos^2\theta_{pq} k_\mu^2,$$
$$a = \varepsilon_{ef} - k_\mu \cos\theta_\mu,$$
$$b = \varepsilon_{ef}^2 - \varepsilon^2 - k_\mu^2,$$
$$\varepsilon_{ef} = \varepsilon_\mu - \epsilon - \epsilon_b,$$

$\theta_{pq}$ is an angle between momenta $\mathbf{q}$ and $\mathbf{p}_m$.

This equation has two solutions

$$\varepsilon_\nu^+ = B + \left(B^2 - 4AC\right)^{1/2} \quad \text{and} \quad \varepsilon_\nu^- = B - \left(B^2 - 4AC\right)^{1/2},$$

where $\varepsilon_\nu^+$ corresponds to $\cos\theta_{pq} > 0$ and $\varepsilon_\nu^-$ to $\cos\theta_{pq} < 0$

Asymptotic form of the solutions at $p_m \to 0$ is well known formula

$$\varepsilon_\nu = b/2a = \frac{(M-\epsilon_b)\varepsilon_\mu + (2M\epsilon_b - m_\mu^2 - \epsilon_b^2)/2}{(M-\epsilon_b) - \varepsilon_\mu + k_\mu\cos\theta}.$$

So, neutrino energy is a function $\varepsilon_\nu = f(k_\mu, \cos\theta, p_m, \cos\theta_{pq})$ of nucleon momentum $p_m$.

Mean value of neutrino energy $\langle \varepsilon_\nu \rangle$ and its variance $\sigma(\varepsilon_\nu)$, can be estimated using a momentum distribution of nucleons $S(\mathbf{p}_m)$ in nucleus as follow

$$\langle \varepsilon_\nu \rangle = \int \varepsilon_\nu(\mathbf{p}_m) S(\mathbf{p}_m) d\mathbf{p}_m,$$
$$\langle \varepsilon_\nu^2 \rangle = \int \varepsilon_\nu^2(\mathbf{p}_m) S(\mathbf{p}_m) d\mathbf{p}_m,$$
$$\sigma^2(\varepsilon_\nu) = \langle \varepsilon_\nu^2 \rangle - \langle \varepsilon_\nu \rangle^2.$$

In framework of the Fermi gas model the nucleon momentum distribution can be written as

$$S(\mathbf{p}_m) = \frac{3}{4\pi p_F^3}, \quad \text{and} \quad \int S(\mathbf{p}_m) d\mathbf{p}_m = 1,$$

where $p_F$ is the Fermi momentum. Using this momentum distribution we obtain

$$\langle \varepsilon_\nu \rangle = \frac{3}{2p_F^3} \int_0^{p_F} p_m^2 dp_m \int_0^1 (\varepsilon_\nu^+ + \varepsilon_\nu^-) d\cos\theta_{pq},$$

$$\langle \varepsilon_\nu^2 \rangle = \frac{3}{2p_F^3} \int_0^{p_F} p_m^2 dp_m \int_0^1 \left[(\varepsilon_\nu^+)^2 + (\varepsilon_\nu^-)^2\right] d\cos\theta_{pq}.$$

# Analysis and Result

**' Averaged energy' method**

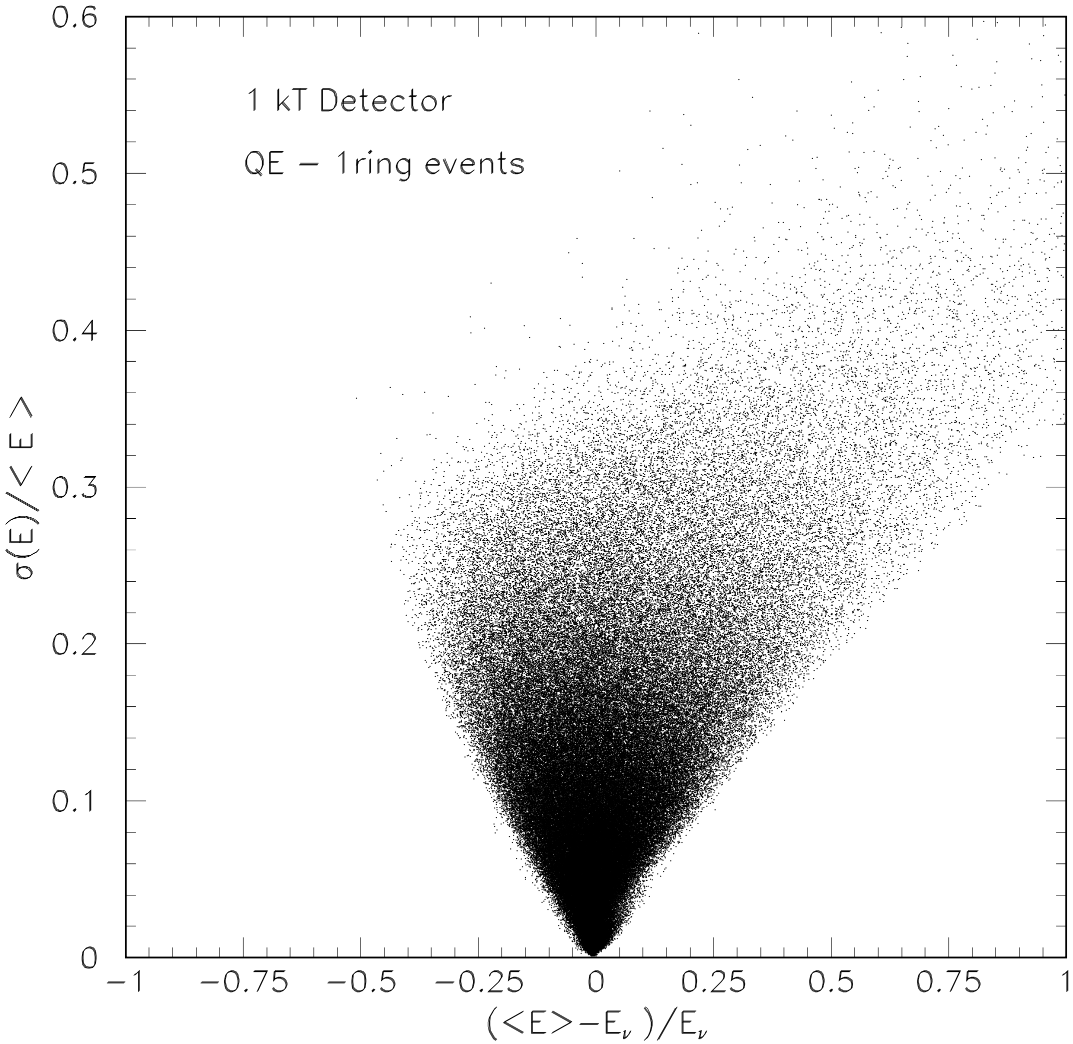
For each event the mean value of neutrino energy $\langle\varepsilon_\nu\rangle$ , its variance $\sigma(\varepsilon_\nu)$, and parameter $R = \sigma(\varepsilon_\nu)/\langle\varepsilon_\nu\rangle$ are estimated.

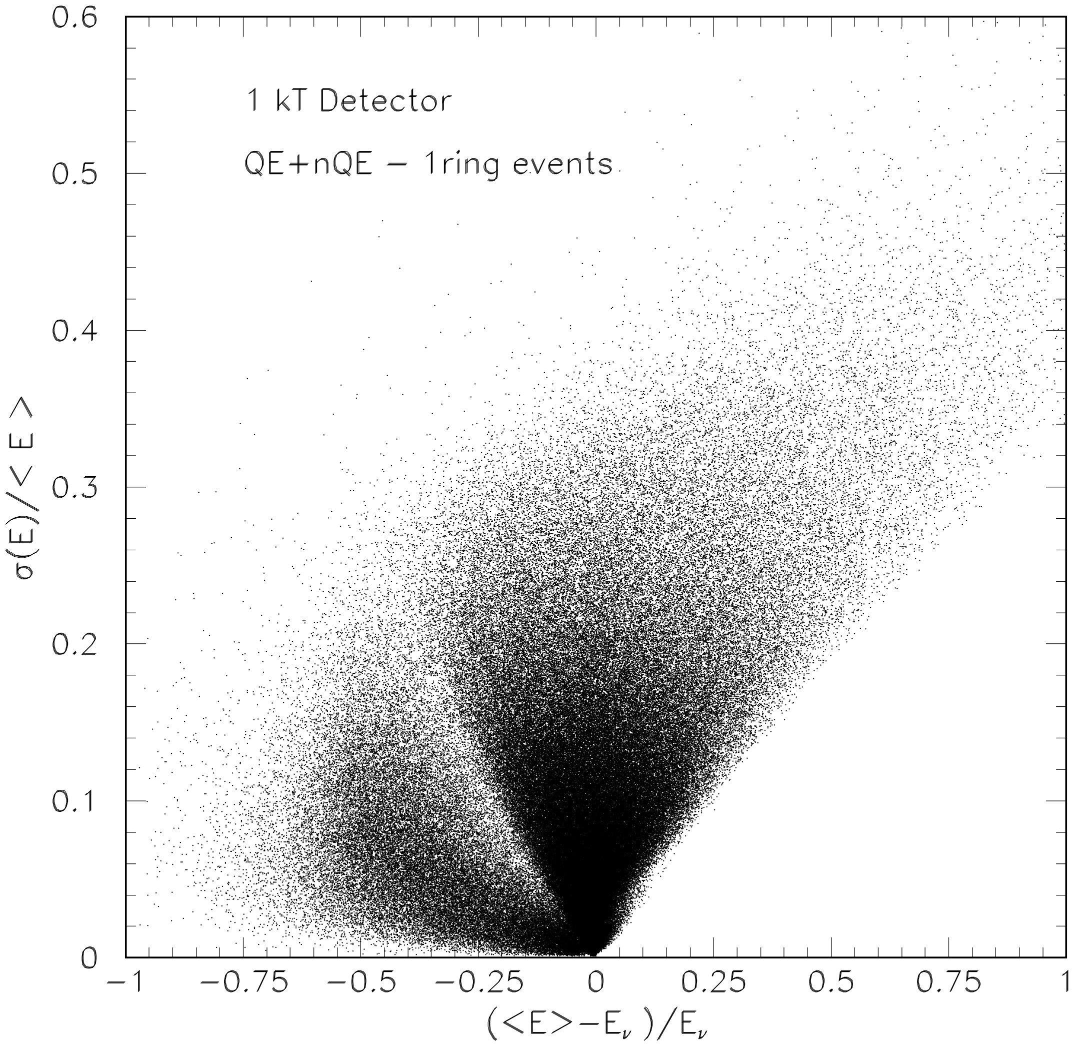
**' Motionless nucleon' method**

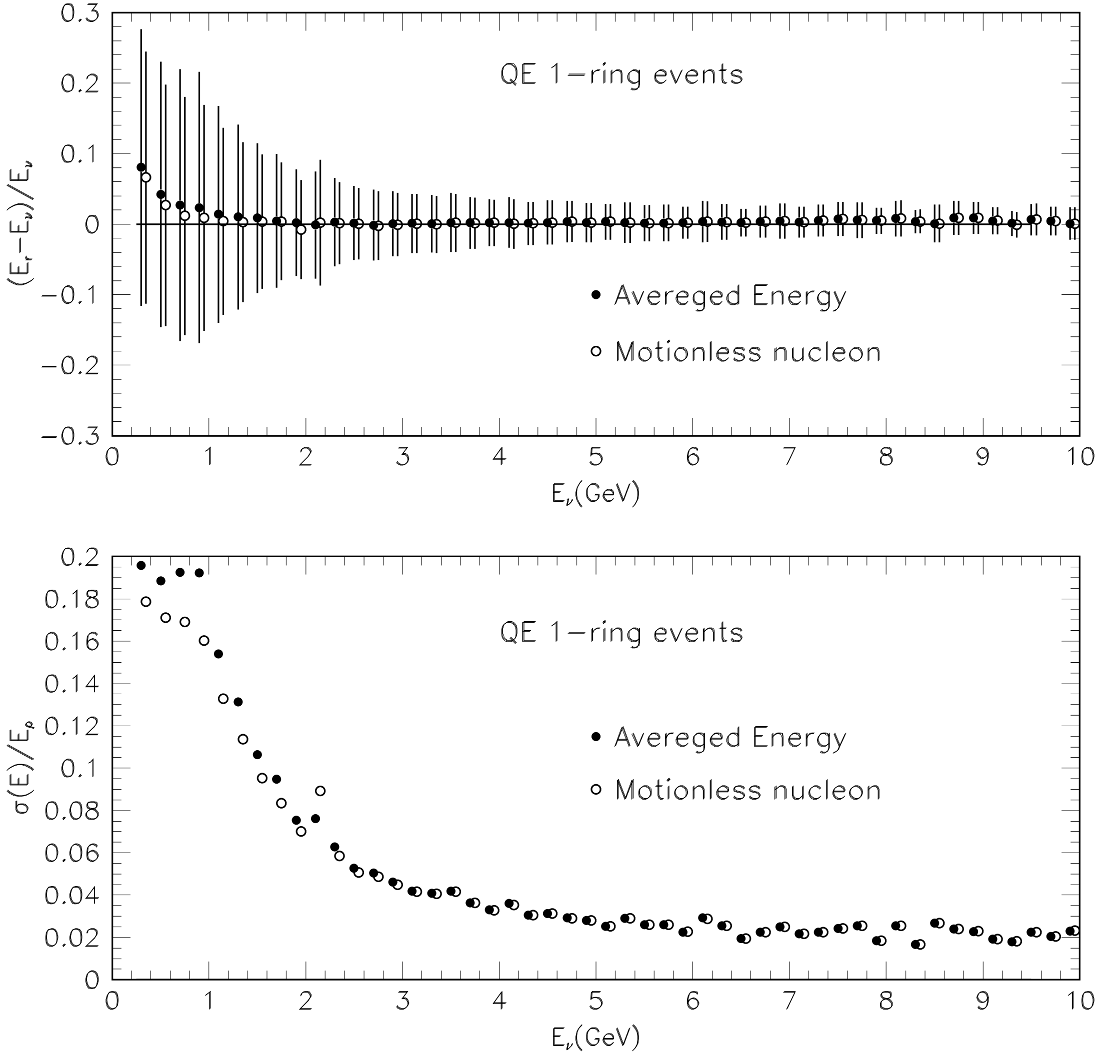
Neutrino energy is estimeted using known formula.

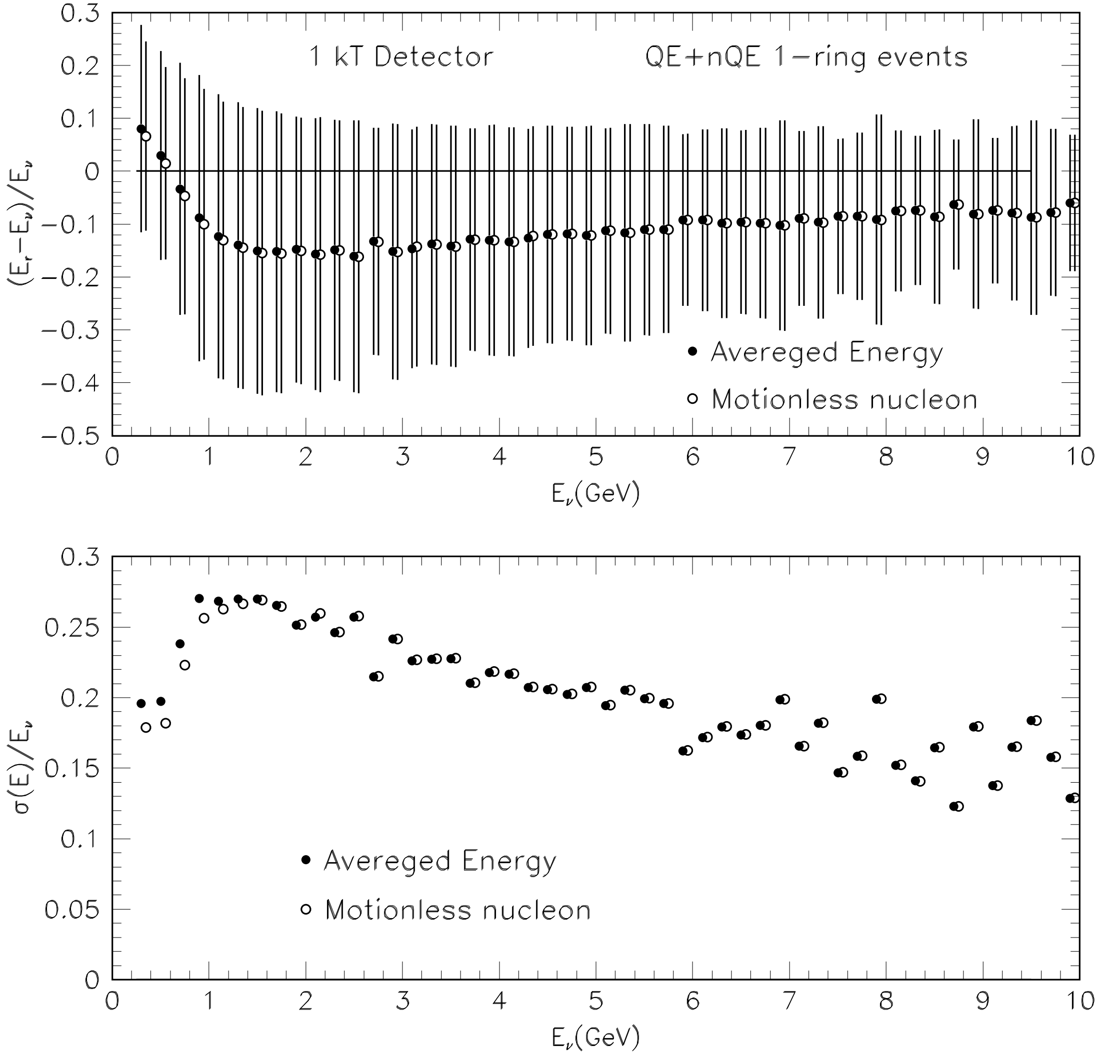
In this analysis the accuracies of two methods were compered. Simulated and reconstracted (1-ring) neutrino events in 1 kT detector were used.

Statistics: QE 1-ring events - 183249; QE+nQE 1-ring events - 236205


1 kT Detector
QE – 1ring events
σ(E)/< E >
(<E>–E_ν )/E_ν
0.6
0.5
0.4
0.3
0.2
0.1
0
–1
–0.75
–0.5
–0.25
0
0.25
0.5
0.75
1


1 kT Detector
QE+nQE – 1ring events
0.6
0.5
0.4
0.3
0.2
0.1
0
σ(E)/< E >
–1
–0.75
–0.5
–0.25
0
0.25
0.5
0.75
1
(<E>–E_ν )/E_ν


QE 1−ring events
• Avereged Energy
○ Motionless nucleon
$(E_r - E_\nu)/E_\nu$
$E_\nu$(GeV)
QE 1−ring events
• Avereged Energy
○ Motionless nucleon
$\sigma(E)/E_\nu$
$E_\nu$(GeV)


1 kT Detector
QE+nQE 1−ring events
(E_r−E_ν)/E_ν
E_ν(GeV)
Avereged Energy
Motionless nucleon
σ(E)/E_ν
E_ν(GeV)
Avereged Energy
Motionless nucleon

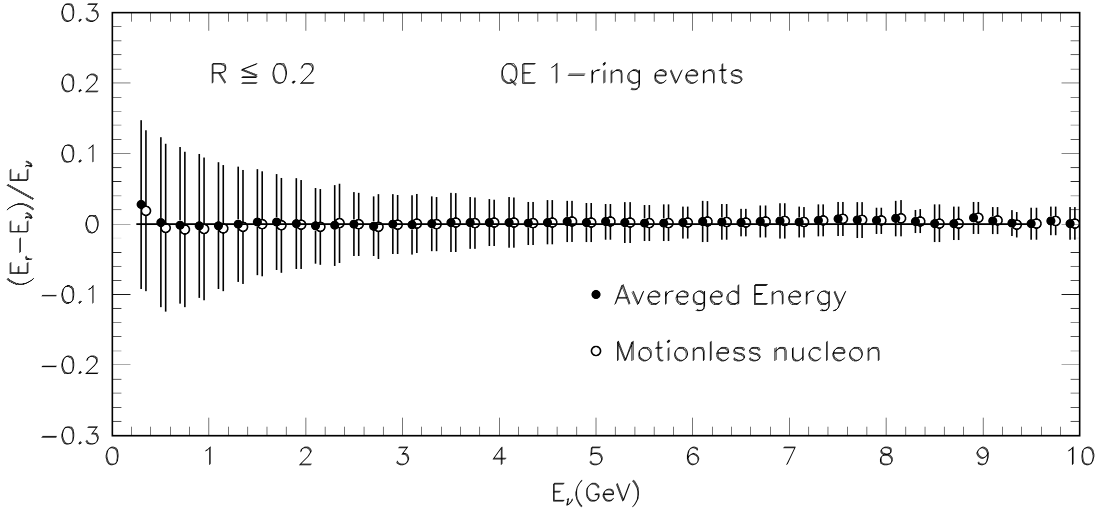
# R Cut

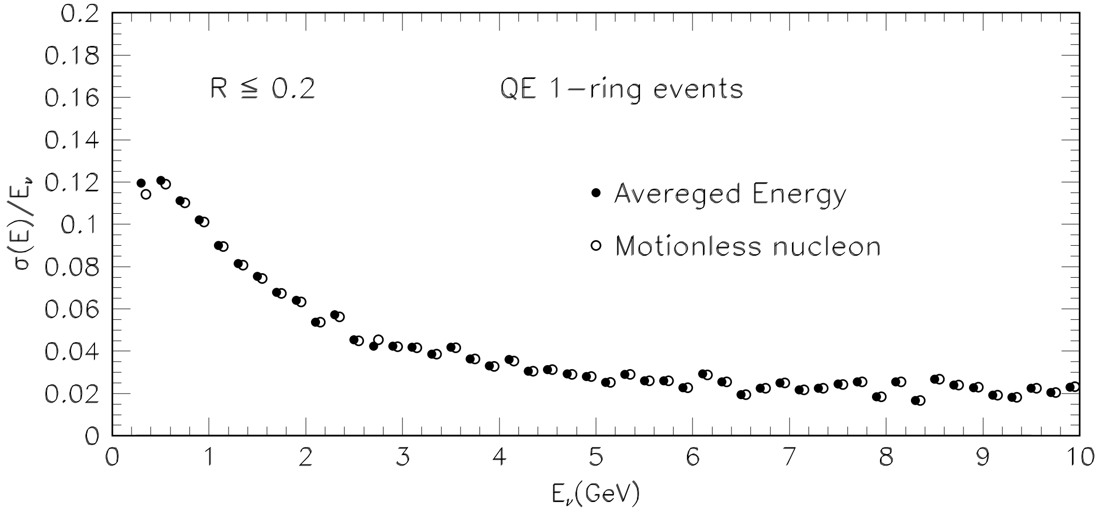
Cut: $R \leq$0.2

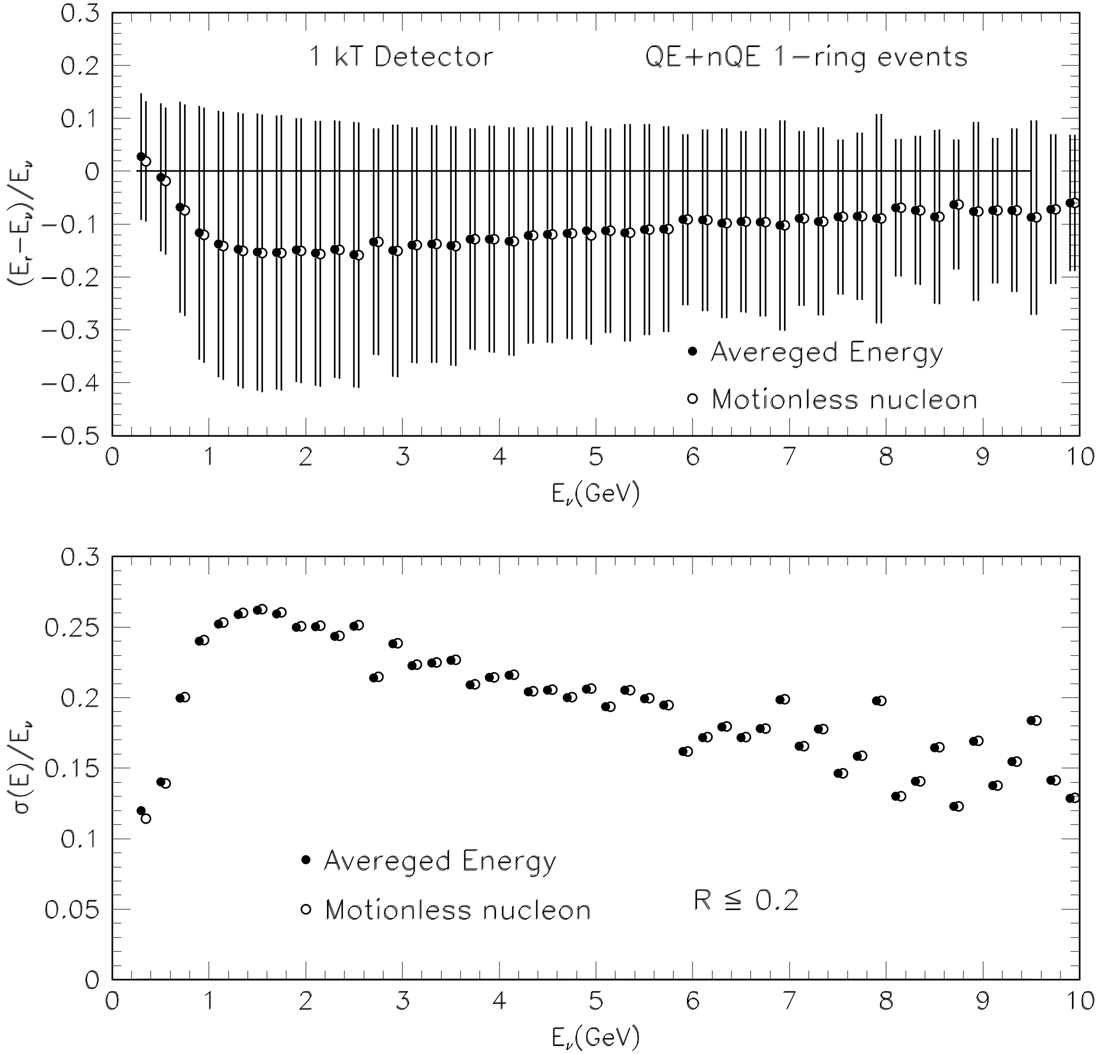
QE 1-ring events. Efficiency=0.825

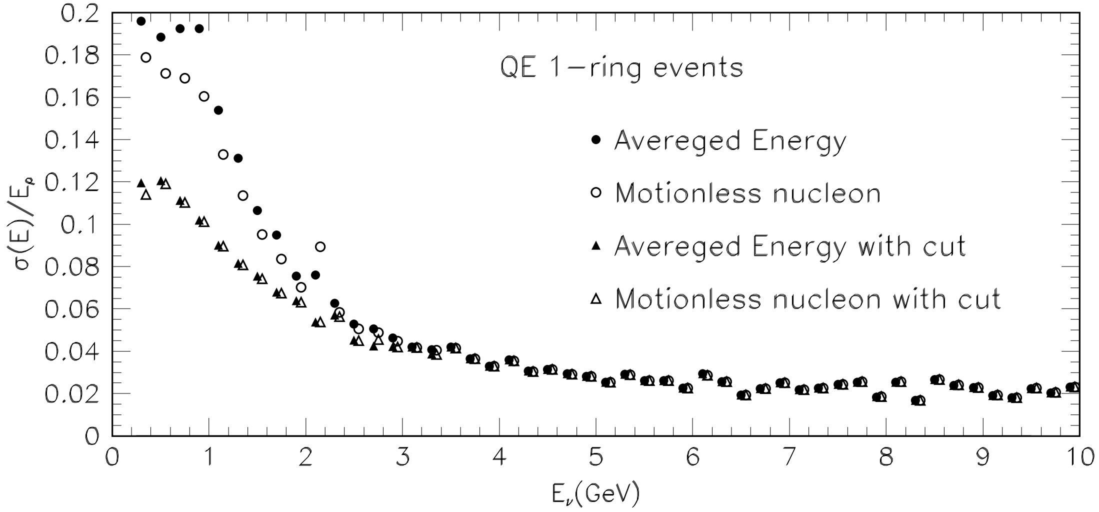
nQE 1-ring events. Efficiency=0.856

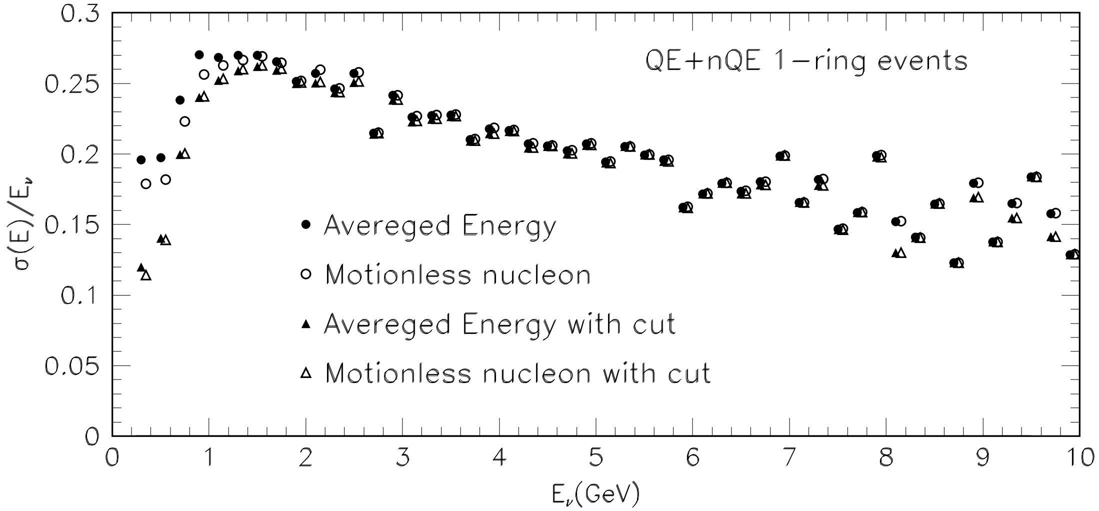
In the case of QE 1-ring events energy resolution was increased up to 8-12% in the energy range $E_\nu \leq$ 2 GeV


R ≦ 0.2
QE 1−ring events
• Avereged Energy
○ Motionless nucleon
$(E_r - E_\nu)/E_\nu$
$E_\nu$(GeV)


R ≦ 0.2
QE 1−ring events
• Avereged Energy
○ Motionless nucleon
$\sigma(E)/E_\nu$
$E_\nu$(GeV)


1 kT Detector
QE+nQE 1−ring events
(E_r−E_ν)/E_ν
• Avereged Energy
○ Motionless nucleon
E_ν(GeV)
σ(E)/E_ν
• Avereged Energy
○ Motionless nucleon
R ≦ 0.2
E_ν(GeV)


QE 1−ring events
Avereged Energy
Motionless nucleon
Avereged Energy with cut
Motionless nucleon with cut
σ(E)/E_ν
E_ν(GeV)


QE+nQE 1−ring events
Avereged Energy
Motionless nucleon
Avereged Energy with cut
Motionless nucleon with cut
σ(E)/E_ν
E_ν(GeV)

# Summary

- The energy reconstraction method which takes into account the Fermi motion of nucleon was presented.

- For QE interaction this method allow to estimat event by event an accuracy of reconstracted neutrino energy.

- Energy resolution of QE 1-ring events increases up to 8-12% in the energy range $E_\nu \leq 2$ GeV when R-cut is used.

- This method can be usefull in analisys of single muon track events which are detected by fully active detectors (LAr, SiBr, ...).