The Numerical Description of Neutral Sheet Drift Effects

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Abstract

This technical contribution shows that three methods to handle neutral sheet drift in cosmic ray modulation, one exact and two approximate, are equivalent. All three have been used extensively, each having their merits, but as far as we are aware, they have not been compared with one another before.

1. Introduction

The magnetic state of the heliosphere is divided in a Northern and Southern sector by a neutral sheet. For one 11-year cycle, the field North (South) of this sheet points away from (towards) the sun. In the next cycle the field directions are reversed. In the former, $qA > 0$ drift cycle, positive particles drift from the poles towards the sheet, and radially outward along the sheet. In the opposite $qA < 0$ cycle, they drift inwards along the sheet, and from there towards the poles.

2. Neutral sheet drift

The theory of gradient and curvature drift in the inhomogeneous heliospheric magnetic field was developed mainly by Jokipii *et al.* (1977), Jokipii and Kopriva (1979), and Isenberg and Jokipii (1978, 1979), and is perfectly well understood: the average drift velocity for a near isotropic particle distribution function is $< v_d >= \nabla \times \kappa_T \mathbf{e_B}$, causing a particle flux $\mathbf{S} = \kappa_T \mathbf{B} \times \nabla f$, where $\kappa_T = \beta P/(3B)$. The drift velocity in the thin ($\ll$ gyroradius) neutral sheet is singular, but this has the physical meaning that particles within two gyroradii from the sheet can experience this drift, depending on their phase angle, and their average drift velocity is along the sheet, perpendicular to $\mathbf{B}$, with magnitude $v_{ns} = v/6$ (exactly), where v is the particle speed (*e.g.* Burger, 1987). This result is related to the requirement that the flux transverse to the sheet must be zero (see, *e.g.*, Jokipii and Kopriva, 1979). When the sheet is flat along the ecliptic plane, this leads to

$$S_\theta \mid_{\theta=\pi/2} = (1/r)\kappa_{\theta\theta}\partial f/\partial\theta + \kappa_T \sin\psi \partial f/\partial r = 0, \qquad (1)$$

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which can readily be programmed as a boundary condition of a numerical solution of the transport equation (TPE) when solved from from $\theta = 0$ (North pole), to $\theta = \pi/2$ (ecliptic). If the sheet is tilted and/or wavy, however, as expressed by

$$\cos\theta_s = \sin[\alpha(\sin\phi - \phi_0 + \Omega(r - r_{\mathrm{sun}})/V] \tag{2}$$

and shown for $\phi = \phi_0$ in panel G of the Figure, it lies neither along a grid line nor at the boundary, and one must use other methods to calculate the neutral sheet drift. Two such methods are now described: The first was used in Moraal (1990) and many subsequent papers. It calculates the drift velocity components

$$v_{dr} = v_{sh}\cos\beta\sin\psi, \quad v_{d\theta} = v_{sh}\sin\beta, \quad v_{d\phi} = v_{sh}\cos\beta\cos\psi, \tag{3}$$

$$\text{with } \cos\beta = 1/\sqrt{1 + [\alpha\Omega(r - r_{\mathrm{sun}})/V\cos(\phi - \phi_0 + \Omega(r - r_{\mathrm{sun}})/V)]^2}. \tag{4}$$

Here Ω is the angular velocity of the sun and ψ is the spiral angle of the magnetic filed, given by $\tan\psi = (\Omega(r - r_{sun})\sin\theta)/V$. The magnitude of these components, v_{sh}, is calculated from the fact that the average neutral sheet drift speed over two gyroradii ($R_L = P/Bc$) on both sides of the sheet is $v_{ns} = v/6$. The drift "line flux" is therefore $(v/6)4R_L = 2\kappa_T$. This flux is then redistributed with a triangular profile, centered along the sheet, with a baseline $2r\triangle\theta$, with $\triangle\theta$ typically two or three grid intervals in the latitudinal direction. Equating these two fluxes gives $v_{sh} = 2\kappa_T r\triangle\theta$, which is placed in the grid point nearest to the sheet, with linearly decreasing values at adjoining grid points. The art of the method is to calculate the grid point in θ nearest to the sheet for each r. For large tilt angles, these points oscillate violently with r, and it is not immediately clear whether this method is stable. Numerical solutions of the TPE show, however, that it is, because the diffusive terms limit large intensity fluctuations.

The second approximate method, described in Steenkamp (1995), is particularly well suited for an azimuthally symmetric solution of the TPE. For all tilt angles, it distributes the neutral sheet drift symmetrically along the ecliptic plane, such that the peak value is in the ecliptic, falling off linearly to zero in grid points up to the maximum excursion, α, of the sheet. (When $\alpha = 0$, this distribution in θ is set at one grid interval, typically 0.5 degrees, above and below the sheet.) From panel G of Figure 1, this requires the azimuthally averaged value of $\cos\beta$ in (4). This function can not be integrated, but the approximation

$$<\cos\beta>_\phi \;\approx\; \log_{10}[\alpha\Omega(r - r_{\mathrm{sun}})/4V + 1]/\sqrt{1 + [2\alpha\Omega(r - r_{\mathrm{sun}})/\pi V]^2} \tag{5}$$

is better than 10% for all values of α and r. In this case $v_{d\theta} = 0$ because $<\sin\beta>= 0$, while $< v_{d\phi} >$ does not play a role in an azimuthally symmetric heliosphere. In this approxmation the gradient/curvature drift within the excursions of the neutral sheet is scaled down by an amount $(2/\pi)\arcsin[(\pi/2 - \theta)/\alpha]$, to account for the fact that this drift alternates in direction as function of ϕ in the alternating field states between the excursions of the sheet.

3. Results and Discussion

The top two rows of Figure 1 show the numerical solution of the TPE in a heliosphere with a boundary at 90 AU where an interstellar proton spectrum, given by $j_T = 10\beta(T + 0.5E_0)^{-2.6}$, is specified, containing an unmodified Parker spiral magnetic field with a magnitude of 5 nT at Earth, solar wind speed $V =$ 400 km/s, $\kappa_\| = 1.8 \times 10^{22}\beta P(B_e/B)$ cm^2/s, $\kappa_\perp = 0.01\kappa_\|$, and with the curvature/gradient and neutral sheet drift calculated as described above. The no-drift solutions are shown in dotted lines for comparison. For the $qA > 0$ solution, the calculated intensities for the three methods differ by less than a line breadth. For the $qA < 0$ solution, the two approximate methods produce the same intensities, but they differ with up to 20% from the (exact) boundary condition method. This is insignificant in relation to the magnitude of the drift effect, and it proves that the two approximate neutral sheet drift descriptions are adequate. Notice the well-known behavior that in the $qA > 0$ case the radial gradients are negligible, while in the $qA < 0$ case both the radial and latitudinal gradients are much larger.

Having demonstrated the validity of the two approximations for the case of a flat neutral sheet, panels H and I of Figure 1 show the well-established intensity profiles (*e.g.* Kota and Jokipii, 1983) as function of tilt angle, α, calculated from the second approximate drift flux method. These intensities are shown in the ecliptic plane, for 1 AU and 60 AU respectively. The first method produces indistinguishable results for all tilt angles.

4. Conclusion

The two approximate methods to describe neutral sheet drift effects produce almost identical results to the exact boundary condition method for a flat neutral sheet. When the sheet is wavy, and the exact method can not be used, the two approximate methods produce indistinguishable results, which suggests that both are correct. Method two is designed for use in azimuthally symmetric solutions, but method one can also be used in three-dimensional solutions.

5. References

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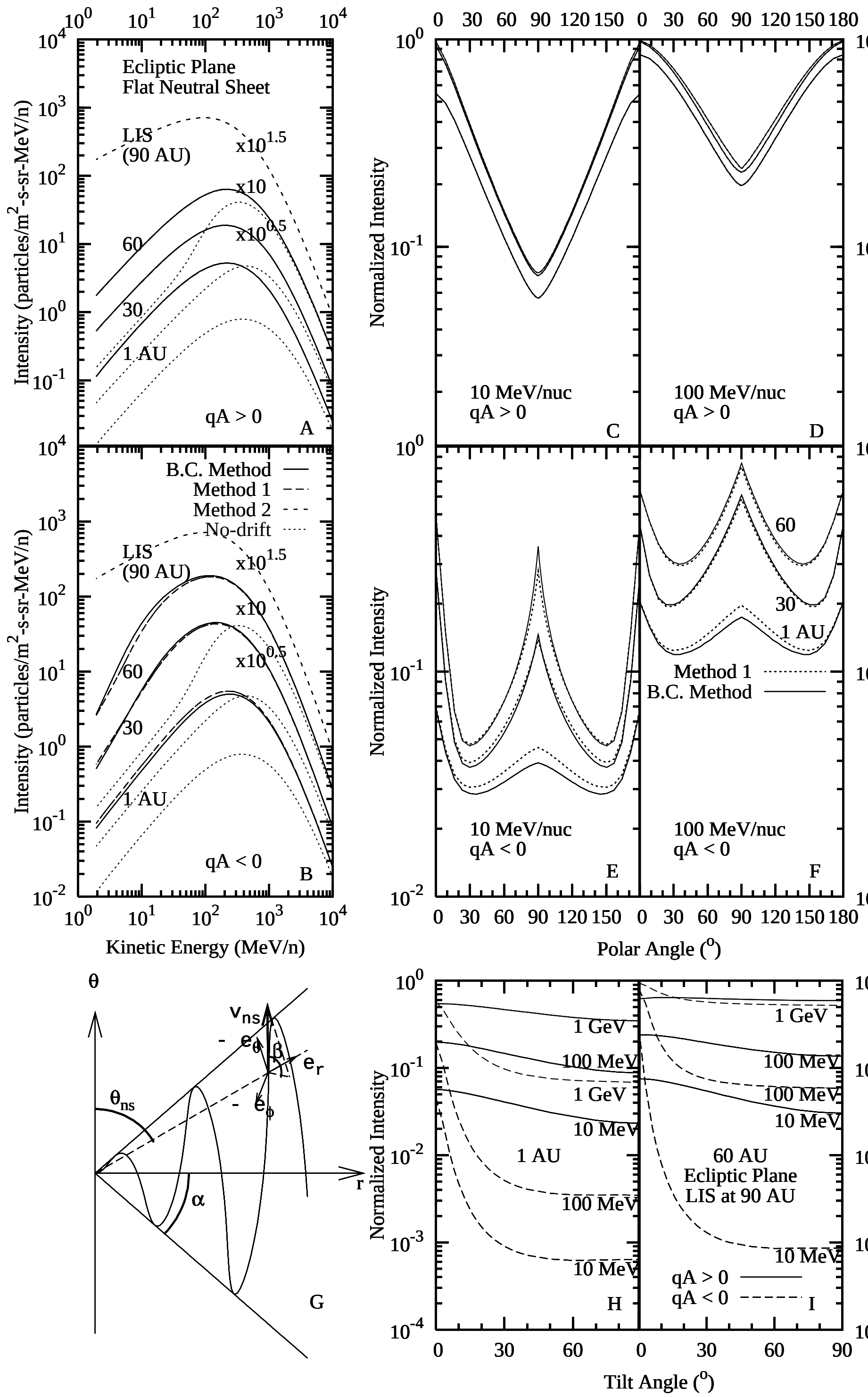

Fig. 1. Comparative analysis of the neutral sheet drift with the boundary condition (B.C.) method and two approximate methods described in the text.