Effect of Cross-helicity on the Ab Initio Formulation of Solar Modulation of Cosmic Rays

S. Parhi[1], J. W. Bieber[1], W. H. Matthaeus[1] and R. A. Burger[2]
(1) Bartol Research Institute, University of Delaware, Newark, DE 19716, USA
(2) School of Physics, Potchefstroom University, Potchefstroom, South Africa

Abstract

Cross-helicity affects magnetic variance and correlation length which are vital in the calculation of diffusion tensor needed for solar modulation of cosmic rays. A global solar wind turbulence model is discussed through the governing turbulence equations which consider magnetic variance, magnetic correlation length, plasma temperature, and cross-helicity. Developing such model we study the effect of the cross-helicity on the formulation of an ab initio theory for solar modulation of cosmic rays. For this the poorly understood perpendicular diffusion is considered to have been derived from the use of velocity-correlation functions using Green-Kubo-Taylor formalism.

1. Introduction

The theories of plasma turbulence and transport of solar wind fluctuations are very important to understand many phenomena in the heliosphere including charged particle scattering and cosmic ray modulation. The plasma turbulence depends on several factors which include magnetic field, plasma shear, pick-up ions, Elsässer variables, and cross-helicity. Pickup ions, cross-helicity, and Elsässer variables play important roles in the calculation of b^2, where $\mathbf{b}$ is the fluctuated magnetic field. The cross-helicity H_c (outward propagating energy - inward propagating energy) is defined as $H_c = \langle \mathbf{v} \cdot \mathbf{v_a} \rangle /2$, where the angle brackets denote the ensemble average. Here $\mathbf{v}$ and $\mathbf{v_a} = \mathbf{b}/(4\pi\rho)^{1/2}$ are fluctuations in solar wind and Alfvén speed respectively. Elsässer variables $\mathbf{z}^{\pm} = \mathbf{v} \pm \mathbf{v_a}$ represent two oppositely directed waves with an inward and outward travelling sense of velocity-magnetic field correlation. Elsässer variables are useful [6] in transport theory.

There are clear indications of non-zero cross-helicity in the high latitude [4], since Ulysses observations show dominance of outward-propagating waves, out to at least 4 AU [1, 5]. In this paper we study the effect of cross-helicity on the ab initio formulation of the solar modulation of cosmic rays by extending the 1D turbulence code [9] valid along one radial direction to a 2D version (both radial and latitudinal).

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2. Model description

For solar wind solutions, we assume only radial dependence of solar wind. The governing steady state equations [8] for an accurate description of magnetic energy, magnetic correlation scale (l_c), and temperature (T) at every point in the heliosphere are:

$$\frac{dz^2}{dr} = -\alpha \frac{(z^2)^{3/2}}{V_w l_c} - (\eta - C_{sh}) \frac{z^2}{r} + \frac{1}{V_w} C_{PI} \exp\left(-\frac{\lambda_I}{r}\right), \tag{1}$$

$$\frac{dl_c}{dr} = \beta \frac{(z^2)^{1/2}}{V_w} - c_p \frac{l_c}{r} - \beta \frac{l_c}{z^2 V_w} C_{PI} \exp\left(-\frac{\lambda_I}{r}\right), \tag{2}$$

$$\frac{dT}{dr} = -\gamma \frac{T}{r} + c_t \frac{\alpha}{V_w} \frac{(z^2)^{3/2}}{l_c}, \tag{3}$$

where z^2 represents the energy in the magnetic fluctuations, and V_w is the solar wind speed. Both slow and fast solar wind are considered. The factor C_{sh} is related to compression and shear driven turbulence in the solar wind, C_{PI} is another factor related to the pick-up ion driven turbulence in the solar wind through a constant f, and λ_I is the length scale of the ionization cavity. The term c_p represents the contributions from the couplings of the small-scale correlations to the large-scale gradient tensors. The factor α can be shown to correspond to cross-helicity. In Eq. 3, the polytropic index $\gamma = 4/3$ and $c_t = \frac{1}{3} m_p / k_B$, where m_p is proton mass in gram and k_B is the Boltzmann constant in erg/deg (K). For our simulation we consider [8] the following values: $C_{sh} = 1.7$, $f = 0.04$, $\eta = 0.9$, $\beta = 0.5$, $c_p = 0.65$, and $\lambda_I = 8\ AU$.

The initial values for all latitudes at inner boundary (0.4 AU) are taken to be $z^2 = 650$ km^2/s^2, $l_c = 0.03\ AU$, and $T = 60000\ K$; the resulting values at 1 AU are $z^2 = 400$ km^2/s^2, $l_c = 0.035\ AU$, and $T = 45000\ K$ which are comparable with typical observed values at 1 AU, namely, $z^2 = 200 - 1000$ km^2/s^2, $l_c = 0.02 - 0.05\ AU$, and $T = 3 \times 10^4 - 1.5 \times 10^5\ K$.

3. Results and discussion

The turbulence in the heliosphere is calculated from the governing Eqs 1-2. The temperature Eq. (3) merely checks the effectiveness and accuracy of our procedure. The variance, correlation length, and temperature at inner boundary (0.4 AU) are assumed to be constant at all latitudes. Once these quantities are prescribed at the inner boundary the governing equations can then find the corresponding quantities along each radial direction in the entire heliosphere using a fourth order Runge Kutta scheme. The model is integrated with the modulation code which solves Parker's transport equation, building on recent efforts along these lines [3, 10, 12]. The poorly understood perpendicular diffusion in transport

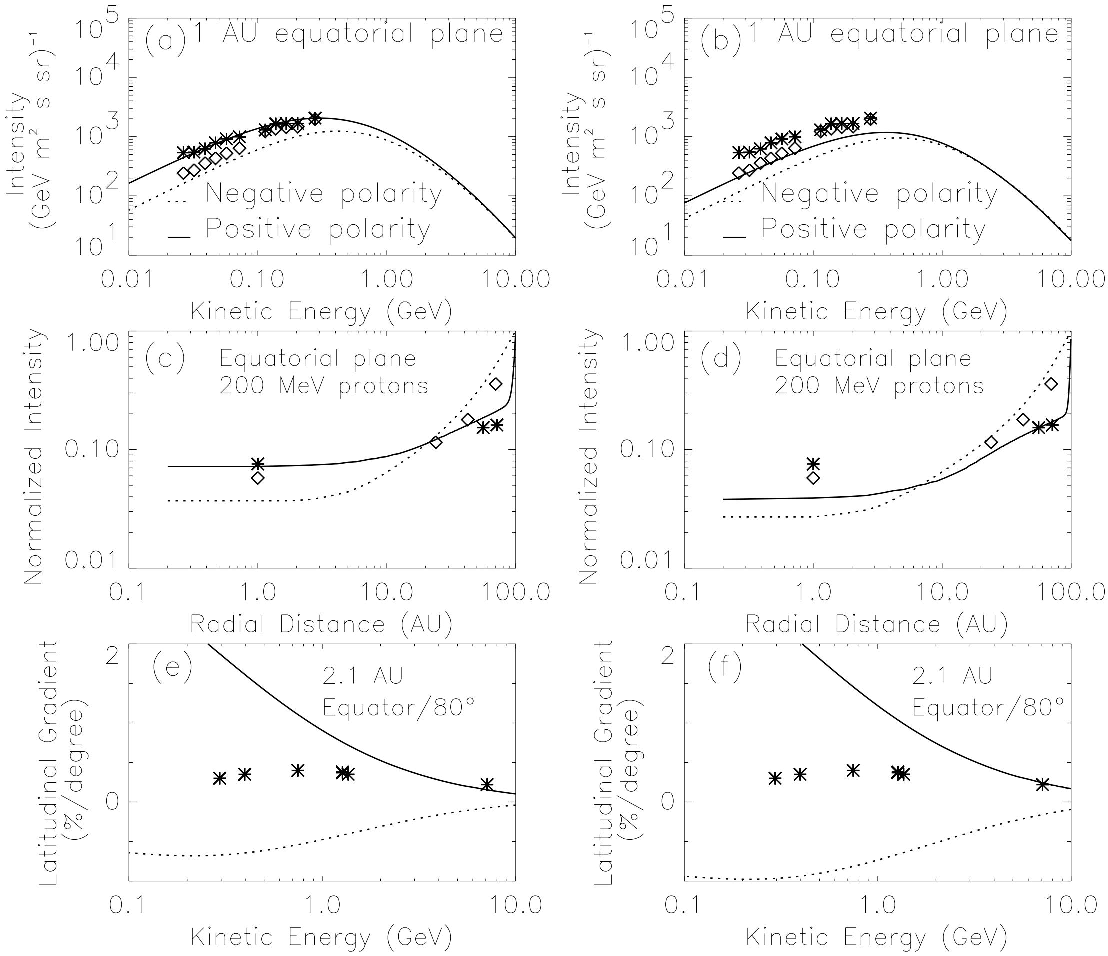

Fig. 1. Results from the integration of turbulence model with an ab initio modulation model. Left panels are for constant cross-helicity $\alpha = 0.5$ at all latitudes whereas the right panels are for varying cross-helicity along latitudes. Left panels display model predictions for (a) 1 AU spectrum, equatorial plane (c) radial profile for 200 MeV particles, equatorial plane, and (f) latitudinal gradient from equator to 80° at 2.1 AU. Right panels denote the same. Dashed line / diamond (solid line / star) used in panels for both model and observational results for negative (positive) solar polarity. Radial profile data [11] from Voyager and IMP. Error bars on observations are comparable to size of data points and hence omitted.

equation is considered to have been derived from the use of velocity-correlation functions using Green-Kubo-Taylor formalism [2].

The initial result is presented in Figure 1. For the left panels the cross-helicity $\alpha = 0.5$ at all latitudes. For the right panels $\alpha = 0.1$ at high latitudes, $\alpha = 0.8$ at mid-latitudes, and $\alpha = 1.0$ at low latitudes. The intensity and radial profile clearly show that a non-uniform cross-helicity can mismatch the simulated data with observation. It appears some kind of average cross-helicity works well for this model. The latitudinal gradient for negative polarity is less when α is varying along latitudes compared with the same when α is constant. The radial profiles suggest that the cross-over of both polarities [10] occur at around 25 AU when $\alpha = 0.5$ in compared with the same at around 7 AU when α varies along latitudes. One has to keep in mind that several parameters (variance, correlation length, and temperature) have to be changed latitudinally so as to get a proper evaluation of the effect of the cross-helicity on the solar modulation of cosmic rays.

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4. References

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