Heliospheric Solar Wind Turbulence Model with Implications for Latitudinal Transport of Cosmic Rays

S. Parhi¹, J. W. Bieber¹, W. H. Matthaeus¹ and R. A. Burger²

(1) Bartol Research Institute, University of Delaware, Newark, DE 19716, USA
(2) School of Physics, Potchefstroom University, Potchefstroom, South Africa

Abstract

A global solar wind turbulence model is discussed considering the governing equations for magnetic correlation length, magnetic variance, and plasma temperature. The model is integrated with the present version of our ab initio modulation code. Thus the diffusion coefficients which determine the final make-up of the ab initio modulation model are calculated from the first principles. By formulating such a theory for solar wind turbulence we describe its implications for latitudinal transport of cosmic rays by appropriately varying the variance and correlation length latitudinally.

1. Introduction

The theories of plasma turbulence and transport of solar wind fluctuations are very important to understand many phenomena in the heliosphere including charged particle scattering and cosmic ray modulation. Thus it is necessary to understand how turbulence is driven by plasma shear and by excitation of fluctuations by scattering of interstellar pickup ions. Moreover, fluctuations in interplanetary turbulence may comprise of two or more components, each with a known symmetry [5]. Boundary data in the inner heliosphere can further complicate a complete understanding of the origins of solar wind turbulence. Another important aspect is the radial variation of the ordinary correlation length which is very poorly understood partly due to the uncertain impact of the pickup ion-driven turbulence in the outer heliosphere [9, 11], and partly due to the difficulties in measuring the correlation length parallel to the magnetic field especially in the outer heliosphere.

In this paper we extend the 1D turbulence code [7] valid along one radial direction to a 2D version (radial and latitudinal). This simulates the 2D turbulence in the heliosphere and we consider the latitudinal dependence of variance and correlation lengths. Other parameters which vary latitudinally will be added in future. The resulting code is integrated with our 2D modulation code to establish an ab initio theory for solar modulation of cosmic rays in the heliosphere, building on recent efforts along these lines [2, 4, 8, 11].
2. Model description

The governing steady state equations [6] for an accurate description of magnetic energy, magnetic correlation scale \((l_c)\), and temperature \((T)\) at every point in the heliosphere are:

\[
\frac{dz^2}{dr} = -\alpha \frac{(z^2)^{3/2}}{V_w \gamma l_c} - (\eta - C_{sh}) \frac{z^2}{r} + \frac{1}{V_w} C_{PI} \exp \left( -\frac{\lambda_I}{r} \right),
\]

\[
\frac{dl_c}{dr} = \beta \frac{(z^2)^{1/2}}{V_w} - \frac{c_p}{r} - \beta \frac{l_c}{z^2 V_w} C_{PI} \exp \left( -\frac{\lambda_I}{r} \right),
\]

\[
\frac{dT}{dr} = -\gamma \frac{T}{r} + c_t \frac{\alpha (z^2)^{3/2}}{V_w l_c}.
\]

where \(z^2\) represents the energy in the magnetic fluctuations, and \(V_w\) is the solar wind speed. Both slow and fast solar wind are considered. The factor \(C_{sh}\) is related to compression and shear driven turbulence in the solar wind, \(C_{PI}\) is related to the pick-up ion driven turbulence in the solar wind through a factor \(f\), and \(\lambda_I\) is the length scale of the ionization cavity. The term \(c_p\) represents the contributions from the couplings of the small-scale correlations to the large-scale gradient tensors. The factor \(\alpha\) can be shown to correspond to cross-helicity. In Eq. 3, the polytropic index \(\gamma = 4/3\) and \(c_t = \frac{1}{3} m_p / k_B\), where \(m_p\) is proton mass in gram and \(k_B\) is the Boltzmann constant in erg/deg (K). For our simulation we consider [6] the following values: \(C_{sh} = 1.7, f = 0.04, \alpha = 1, \eta = 0.9, \beta = 0.5, c_p = 0.65, \) and \(\lambda_I = 8\) AU.

The initial values at inner boundary (0.4 AU) for colatitudes \(0^\circ - 42^\circ\) are taken to be \(z^2 = 800 \text{ km}^2/\text{s}^2\), and \(l_c = 0.04\) AU, those at colatitudes \(42^\circ - 74^\circ\) are \(z^2 = 900 \text{ km}^2/\text{s}^2\), and \(l_c = 0.035\) AU, and those at colatitudes \(72^\circ - 90^\circ\) are \(z^2 = 1000 \text{ km}^2/\text{s}^2\), and \(l_c = 0.03\) AU. The temperature at all latitudes at inner boundary is \(T = 60000\) K. This is our first attempt to vary these quantities latitudinally in this manner. The resulting values at 1 AU in the equatorial plane are found to be comparable with typical observed values there.

3. Results and discussion

The turbulence in the heliosphere is calculated from the governing Eqs 1-2. The temperature Eq. 3 merely checks the effectiveness and accuracy of our procedure. The variance and correlation lengths at inner boundary (0.4 AU) are assumed to vary latitudinally as explained in the last section. The temperature in the inner boundary is kept fixed. Once these quantities are prescribed at the inner boundary the governing equations can find the corresponding quantities along each radial distance in the entire heliosphere using a fourth order Runge Kutta scheme. The model is integrated with the present version of the modulation
Fig. 1. Results from the integration of turbulence model with an ab initio modulation model. Panels display model predictions for (a) 1 AU spectrum, equatorial plane (c) radial profile for 200 MeV particles, equatorial plane, and (e) latitudinal gradient from equator to 80° at 2.1 AU. Right panels display key elements of the diffusion tensor. Specifically the radial mean free path $\lambda_{rr}$, latitudinal mean free path $\lambda_{\theta\theta}$, and the drift scale $\lambda_A$ are plotted vs. (b) rigidity, (d) radius at high latitude (80°) for 200 MeV protons, and (f) radius in the equatorial plane for 200 MeV protons. Dashed line / diamond (solid line / star) used in panels for both model and observational results for negative (positive) solar polarity. Radial profile data [10] from Voyager and IMP, and latitudinal gradient data [3] from Ulysses. Error bars on observations are comparable to size of data points and hence omitted.
code which incorporates the perpendicular diffusion derived from the use of the velocity-correlation functions using Green-Kubo-Taylor formalism [1].

The first result is presented in Figure 1. The spectrum and radial profile match very well with the observational data. In particular, the comparison is better than the best result in [8] where the turbulence is not calculated from the governing equations. The latitudinal gradient does not compare well with data. This could mean that the latitudinal variation of the key parameters (shear, pick-up ion effects, cross-helicity, and temperature) need closer scrutiny.

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4. References